



# Heavy-Quark Physics

## *Lecture 3: QCD Factorization and “ $\alpha$ ”*

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41. Intl. Universitätswochen für Theoretische Physik

Schladming, Styria, Austria

February 22–28, 2003

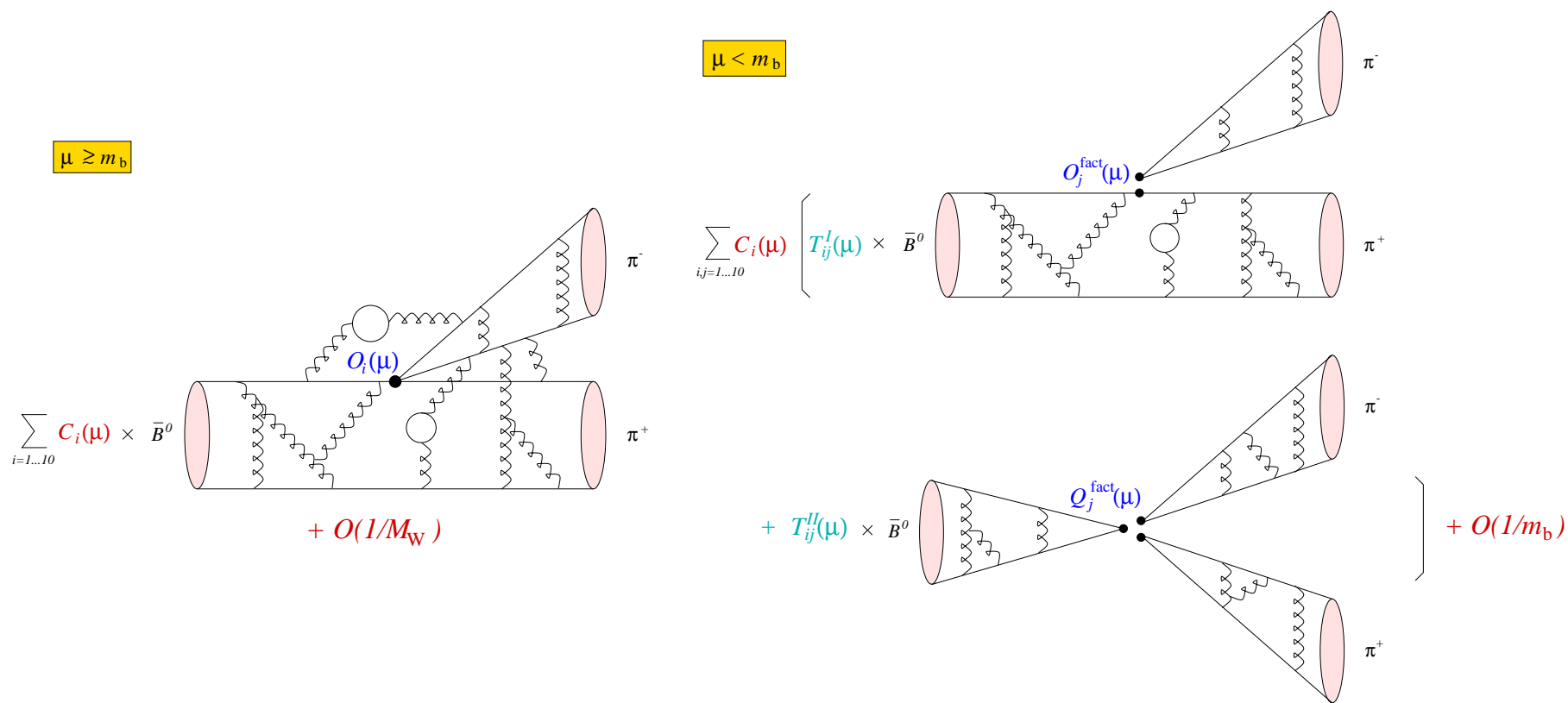
# QCD Factorization

Hadronic weak decays simplify greatly in the heavy-quark limit  $m_b \gg \Lambda_{\text{QCD}}$

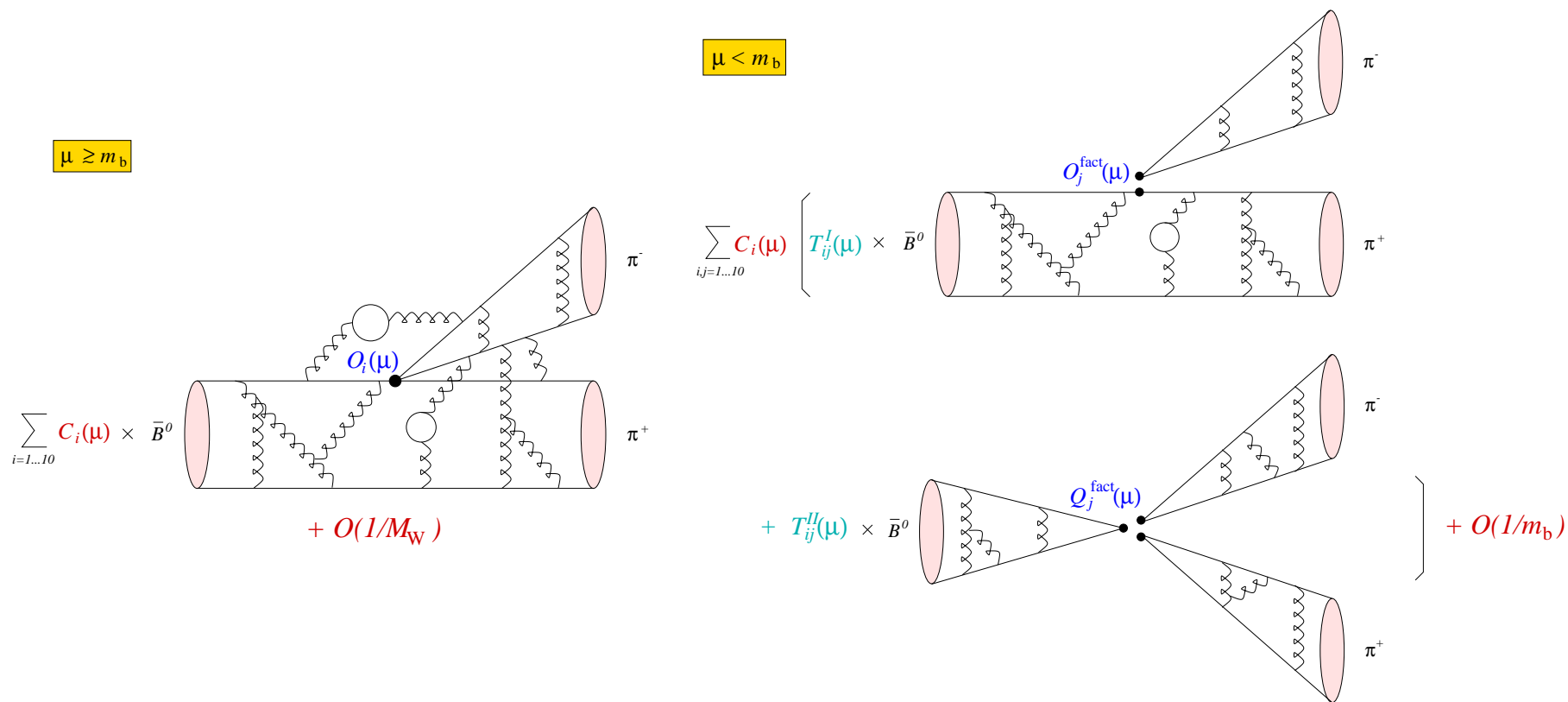
- fast-moving light meson produced by a point-like source decouples from soft QCD interactions (**color transparency**)
- systematic QCD treatment  $\Rightarrow$  QCD factorization
- rigorous results in the heavy-quark limit, valid to all orders of perturbation theory
- dominant power corrections are included in phenomenological analyses



Hard gluon effects with  $\mu \gg m_b$  can be calculated and factorized into the Wilson coefficients  $C_i(\mu)$  of an effective weak Hamiltonian



Hard gluon effects with  $\mu \sim m_b$  can be calculated and factorized into the hard-scattering kernels  $T_{ij}(\mu)$  of the factorization formula



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- QCD factorization makes testable predictions
- data will teach us about importance of power-suppressed effects

# Factorization in $\bar{B}^0 \rightarrow D^{(*)} + L^-$

Simplest application of the factorization formula leads to the quasi-universal prediction:

$$|a_1^{D^{(*)}L}| = 1.05 \pm 0.02 + O(\Lambda_{\text{QCD}}/m_b)$$

Experimental determination (Bjorken ratio):

$$\frac{\Gamma(\bar{B}^0 \rightarrow D^{*+} L^-)}{d\Gamma(\bar{B}^0 \rightarrow D^{*+} l^- \nu)/dq^2 \Big|_{m_L^2}} = 6\pi^2 |V_{ud}|^2 f_L^2 |a_1^{D^*L}|^2$$

Results (CLEO):

$$|a_1^{D^*\pi}| = 1.08 \pm 0.07, \quad |a_1^{D^*\rho}| = 1.09 \pm 0.10, \quad |a_1^{D^*a_1}| = 1.08 \pm 0.11$$

- good agreement within errors, indicating that power corrections are under control

# Factorization in $B \rightarrow \pi K, \pi\pi$

- factorization in decays  $B \rightarrow$  two light mesons can be probed using  $B^\pm \rightarrow \pi^\pm \pi^0$  ( $\simeq$  pure tree) and  $B^\pm \rightarrow \pi^\pm K^0$  ( $\simeq$  pure penguin), which have negligible amplitude interference
- crucial properties:
  - magnitude of tree amplitude
  - magnitude of  $T/P$  ratio
  - strong phase of  $T/P$  ratio
- once these tests are conclusive, factorization can be used to constrain the unitarity triangle

# Magnitude of the Tree Amplitude

Absolute prediction for  $B^\pm \rightarrow \pi^\pm \pi^0$  branching ratio:

$$\text{Br}(B^\pm \rightarrow \pi^\pm \pi^0) = \left[ 5.6^{+0.8}_{-0.4} \text{ (pars.)} \pm 0.3 \text{ (power)} \right] \cdot 10^{-6} \times \left[ \frac{|V_{ub}|}{0.0037} \frac{F_0^{B \rightarrow \pi}(0)}{0.28} \right]^2$$

$$\text{Exp.: } (5.54 \pm 0.89) \times 10^{-6}$$

Sensitivity to semileptonic form factor and  $|V_{ub}|$  can be eliminated in the ratio:

$$\frac{\Gamma(B^\pm \rightarrow \pi^\pm \pi^0)}{d\Gamma(\bar{B}^0 \rightarrow \pi^+ l^- \bar{\nu})/dq^2|_{q^2=0}} = 3\pi^2 f_\pi^2 \underbrace{|a_1^{(\pi\pi)} + a_2^{(\pi\pi)}|^2}_{1.33^{+0.20}_{-0.11} \text{ (pars.)} \pm 0.07 \text{ (power)}}$$

# Magnitude and Phase of the $T/P$ Ratio

Tree-to-penguin ratio can be measured via:

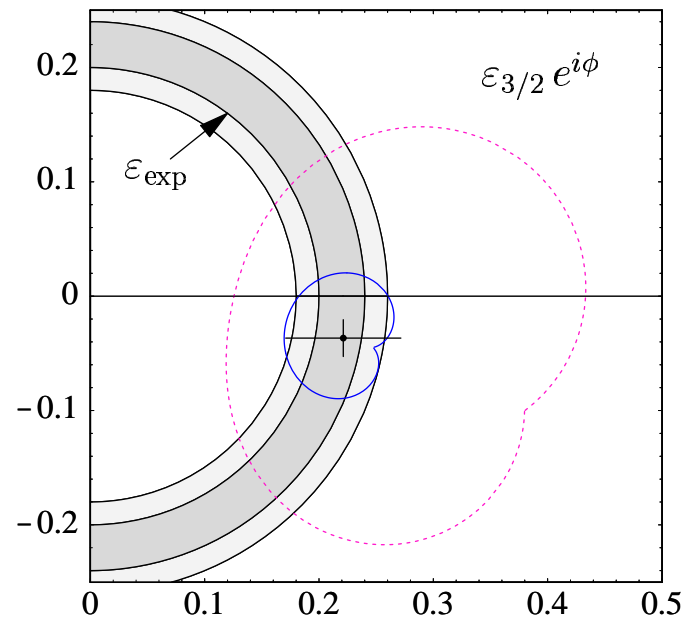
$$\varepsilon_{\text{exp}} = \left| \frac{T}{P} \right| = \tan\theta_C \frac{f_K}{f_\pi} \left[ \frac{2\text{Br}(B^\pm \rightarrow \pi^\pm \pi^0)}{\text{Br}(B^\pm \rightarrow \pi^\pm K^0)} \right]^{\frac{1}{2}} = 0.215 \pm 0.020$$

- independent of form factors, but  $\propto |V_{ub}/V_{cb}|$
- agrees well with theoretical prediction:

$$\varepsilon_{\text{th}} = 0.22 \pm 0.04 \text{ (pars.)} \pm 0.04 \text{ (power)} \pm 0.05 (V_{ub})$$

- strong indication that weak annihilation contributions are reasonably estimated!

## Direct comparison with data:



- excellent agreement (solid:  $\rho_A = 1$ , dashed:  $\rho_A = 2$ )
- larger values of  $\rho_A$  would require extreme fine-tuning!

QCD factorization predicts that (most) strong phases are **parametrically suppressed**:

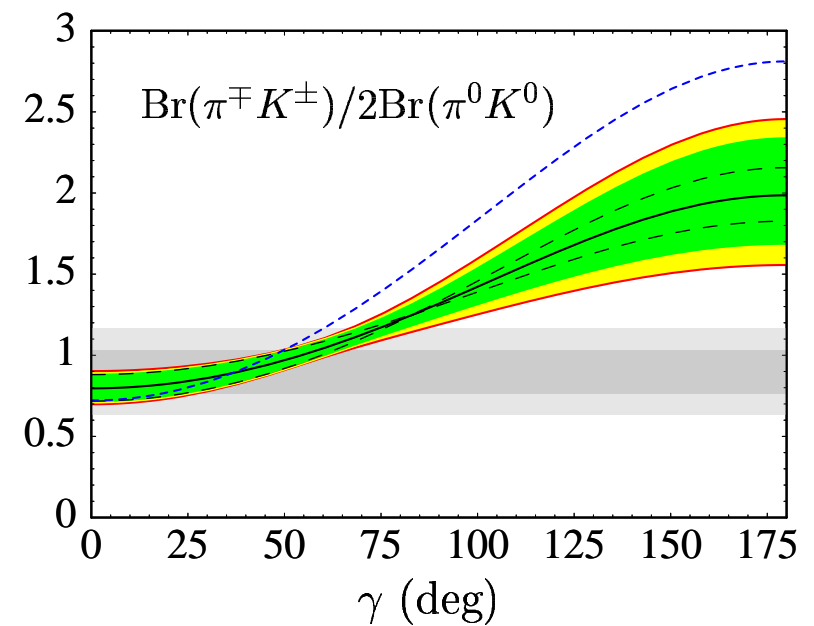
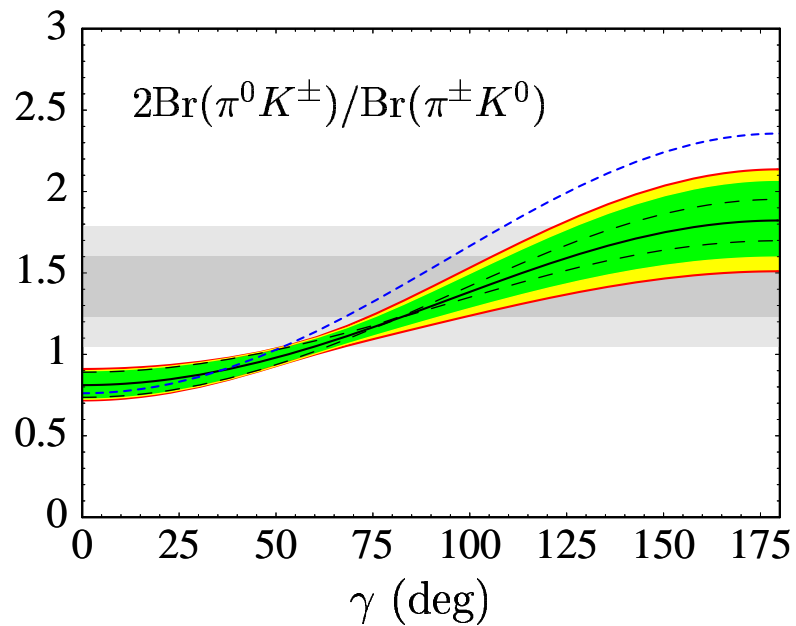
$$\sin \phi_{st} = O \left[ \alpha_s(m_b); \frac{\Lambda_{\text{QCD}}}{m_b} \right] \Rightarrow \text{small direct CP asymmetries}$$

	Experiment (CLEO, BaBar, Belle)	Theory		
		BBNS	Keum, Li, Sanda	Ciuchini et al.
$A_{\text{CP}}(\pi^+ K^-)$	$-8.8 \pm 4.4$	$5 \pm 9$	$-18$	$\pm(17 \pm 6)$
$A_{\text{CP}}(\pi^0 K^-)$	$-10.1 \pm 7.7$	$7 \pm 9$	$-15$	$\pm(18 \pm 6)$
$A_{\text{CP}}(\pi^- \bar{K}^0)$	$4.4 \pm 8.0$	$1 \pm 1$	$-2$	$\pm(3 \pm 3)$

● supported by data

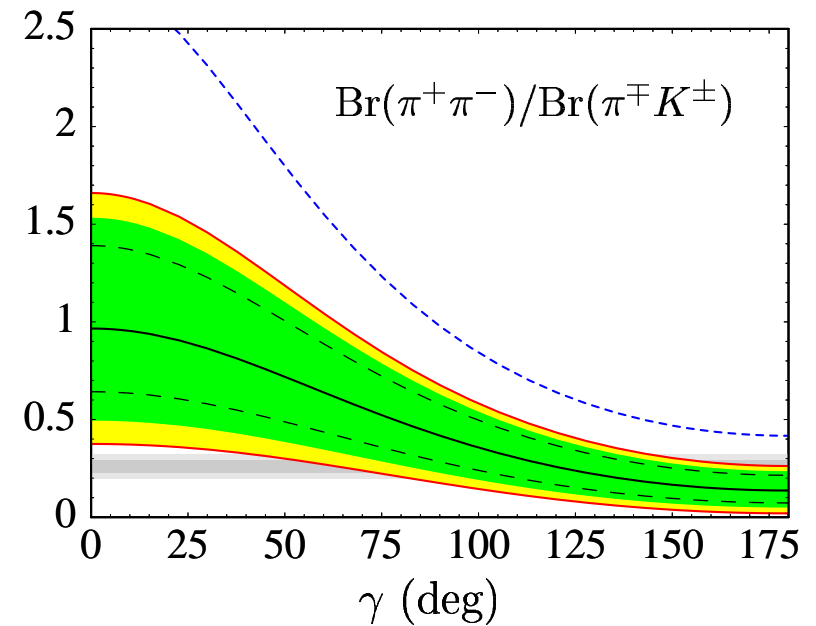
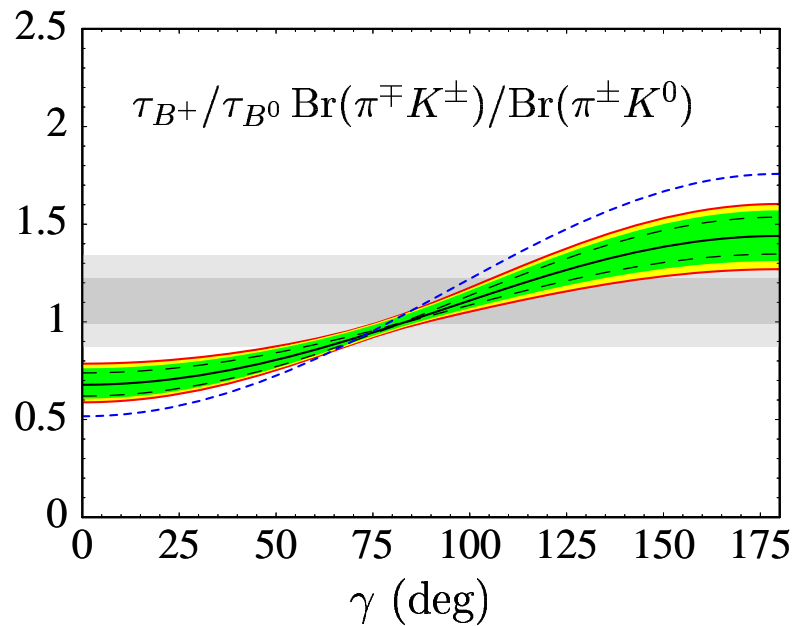
# Ratios of $B \rightarrow \pi K, \pi\pi$ Rates

Various ratios of CP-averaged  $B \rightarrow \pi K, \pi\pi$  branching fractions have a strong dependence on  $\gamma$  and  $|V_{ub}|$ , e.g.:



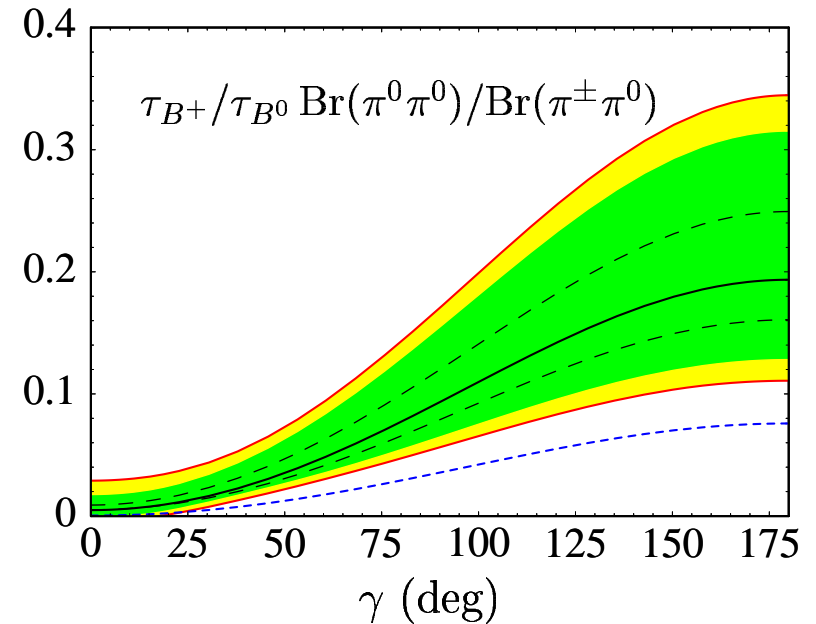
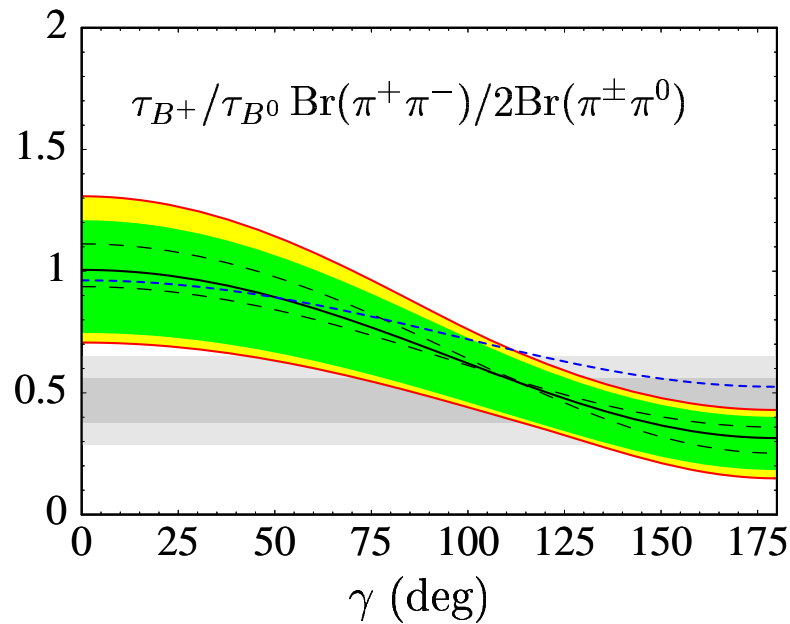
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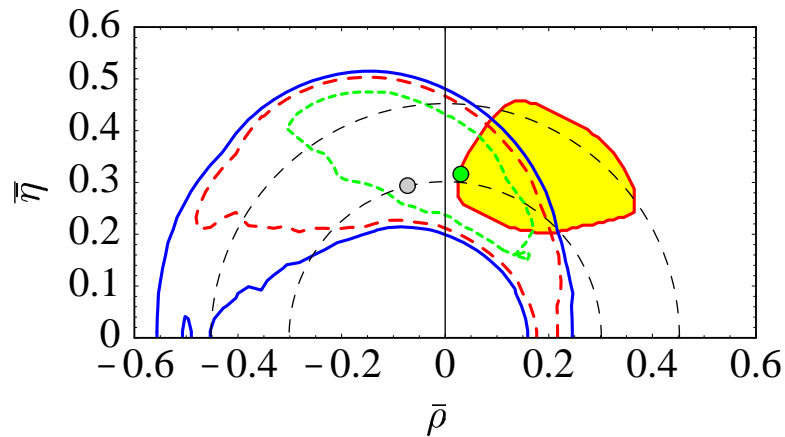
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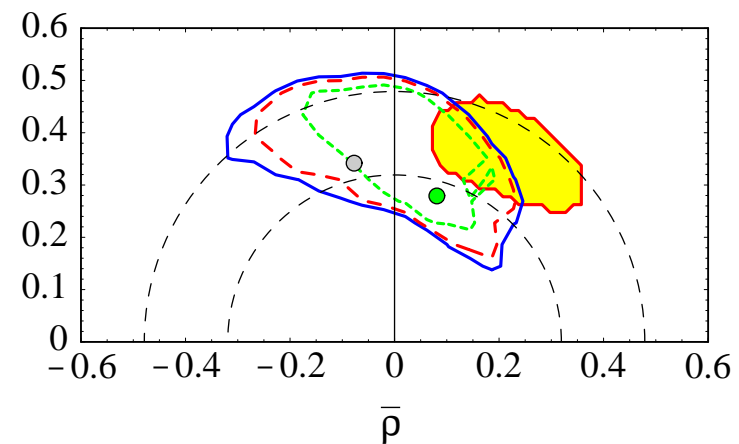
● tends to favor large  $\gamma$

# Global Fit

Derive constraints on  $\bar{\rho}$  and  $\bar{\eta}$  from a global analysis of the data in the context of QCD factorization: [BBNS]



spring 2001



summer 2002

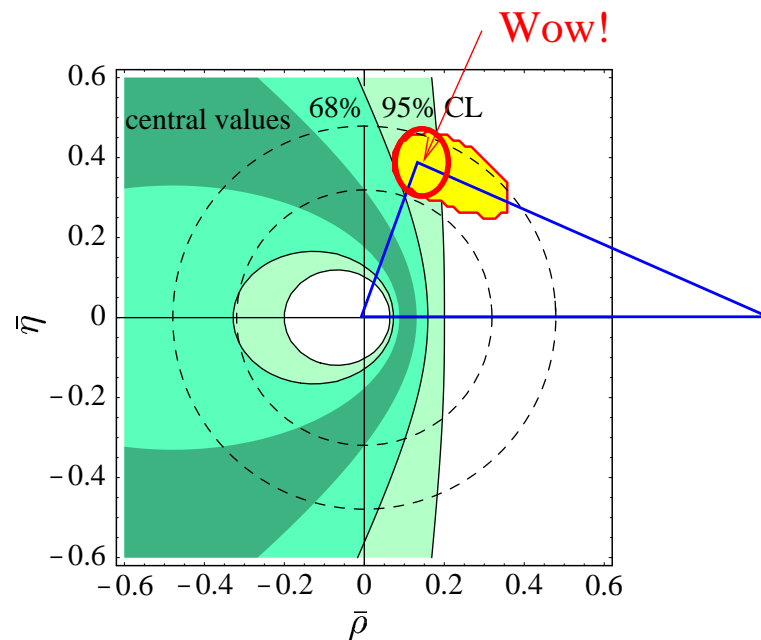
- fit is excellent, with  $\chi^2/\text{dof} = 4.4/3$
- allowed region has shrunk as data got more precise

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- allowed regions obtained from the fit to charmless hadronic decays are compatible with the standard fit, but once again tend to favor larger  $\gamma$  values

# Lesson

- QCD factorization has successfully passed several crucial tests, including the magnitudes of the tree and penguin amplitudes, their relative strong-interaction phase, and the size of weak annihilation effects
- results of the global fit are consistent with the constraint obtained from the analysis of the  $R_*$  ratio



# Lesson

- QCD factorization has successfully passed several crucial tests, including the magnitudes of the tree and penguin amplitudes, their relative strong-interaction phase, and the size of weak annihilation effects
- results of the global fit are consistent with the constraint obtained from the analysis of the  $R_*$  ratio
- will now use QCD factorization to estimate (with generous errors) the penguin-to-tree ratio in the decay  $B \rightarrow \pi^+ \pi^-$  to get the last constraint in the construction of the CP-b triangle

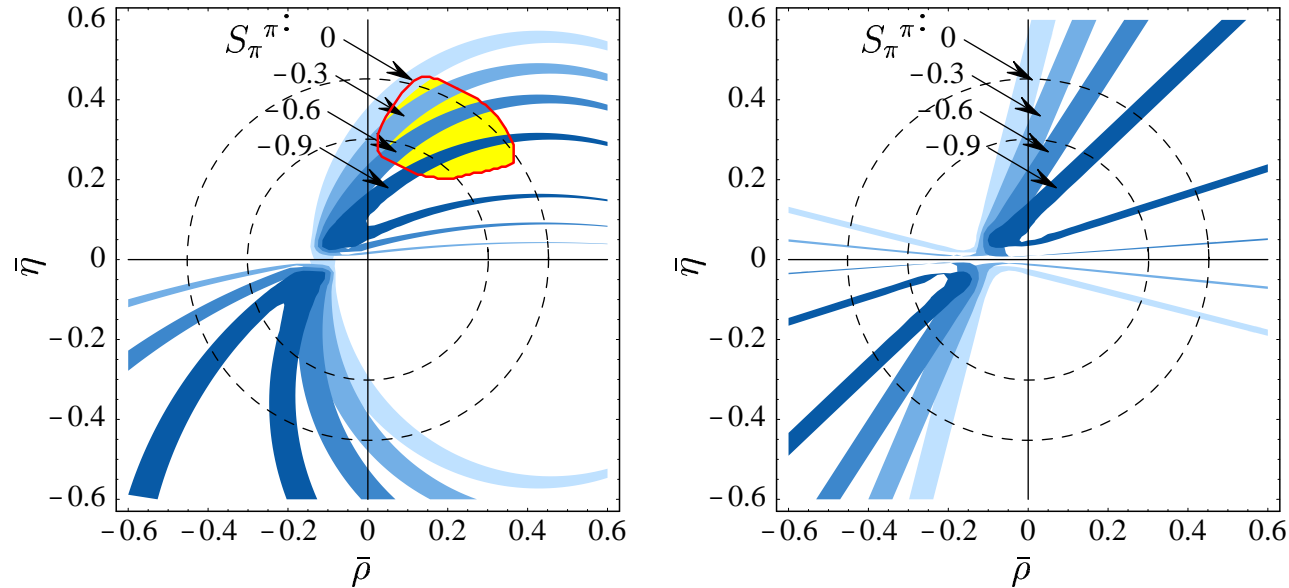
# Mixing-Independent Analysis of $S_{\pi\pi}$

If we define  $A_{\text{CP}}^{B \rightarrow \pi\pi}(t) = S_{\pi\pi} \sin(\Delta m_d t) + C_{\pi\pi} \cos(\Delta m_d t)$ , then the coefficient  $S_{\pi\pi}$  is given by the general formula ( $\phi_d = 2\beta$  in SM):

$$S_{\pi\pi} = \frac{2 \text{Im} \lambda_{\pi\pi}}{1 + |\lambda_{\pi\pi}|^2} \quad \text{with} \quad \lambda_{\pi\pi} = e^{-i\phi_d} \frac{e^{-i\gamma} + (P/T)_{\pi\pi}}{e^{+i\gamma} + (P/T)_{\pi\pi}}$$

- trick to get insensitive to New Physics in mixing is to use  $e^{-i\phi_d} = \pm(1 - s_{\text{exp}}^2)^{1/2} - i s_{\text{exp}}$  with measured value  $s_{\text{exp}} = (\sin 2\beta)_{\text{exp}} = 0.73 \pm 0.05$
- this turns circles in  $(\bar{\rho}, \bar{\eta})$  plane into straight lines, which intersect  $|V_{ub}|$  circles at (almost)  $90^\circ$  angles

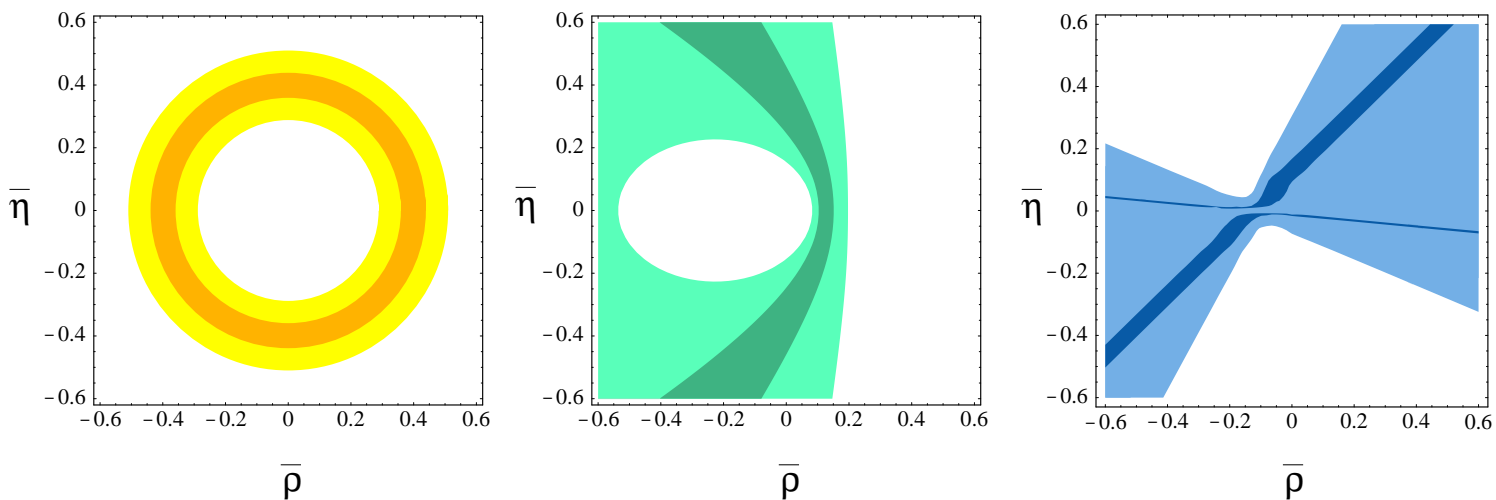
Hadronic uncertainties (QCD factorization) are large in  $\alpha$ , but small when displayed as bands in the  $(\bar{\rho}, \bar{\eta})$  plane:



exp. results:  $S_{\pi\pi} = 0.02 \pm 0.34 \pm 0.05$  (BaBar)

versus  $S_{\pi\pi} = -1.23 \pm 0.41^{+0.08}_{-0.07}$  (Belle)

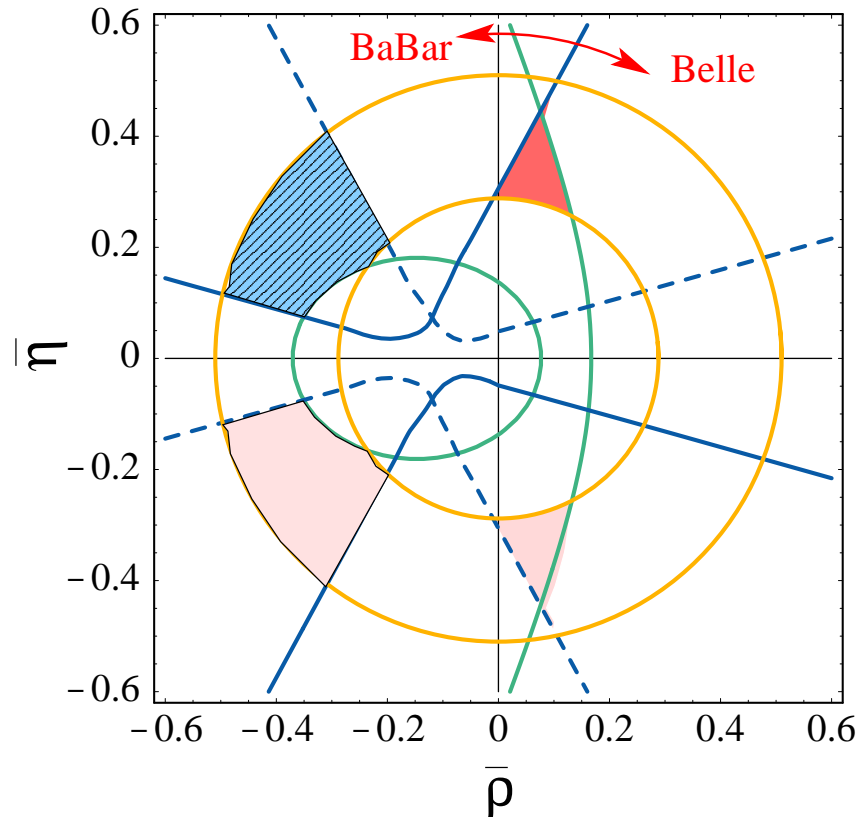
This provides the third constraint in the construction:



- based on above discussion we have good reasons to trust the error analysis within QCD factorization
- but even doubling the theoretical error on  $(P/T)_{\pi\pi}$  would at present not change the analysis at all!

# Resulting CP-b Triangle

Combine three constraints and construct the resulting allowed regions for the apex of the unitarity triangle:



- combination of the three constraints establishes CP violation in the bottom sector!
- if we use that  $\epsilon_K$  requires positive value of  $\bar{\eta}$ , only two solutions in the upper half-plane remain
- one of these lies close to the standard fit, though once again somewhat larger  $\gamma$  values are preferred
- a second allowed region, consistent with the constraints from  $\epsilon_K$  and charmless hadronic decays, is incompatible with the constraints from  $\sin 2\beta$  and  $\Delta m_s / \Delta m_d$

# Origins of a Possible Discrepancy?

errors in lattice calculations of matrix elements for  $B_d-\bar{B}_d$  and  $B_s-\bar{B}_s$  mixing may have been underestimated

[Kronfeld, Ryan]

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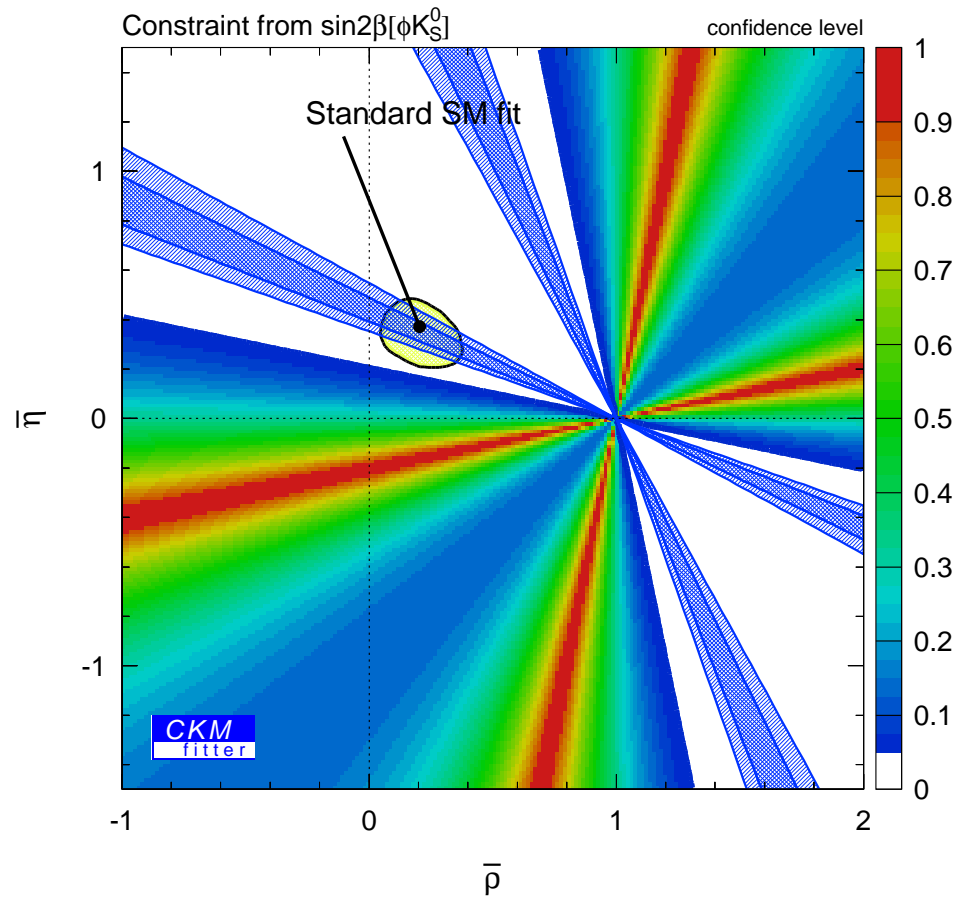
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- New Physics in  $B_d-\bar{B}_d$  mixing
- New Physics in  $b \rightarrow s$  or  $b \rightarrow d$  FCNC transitions (e.g. from penguin and box graphs with exchange of new heavy particles)  
⇒ clean signal would be a difference in the time-dependent CP asymmetries in  $B \rightarrow \phi K_S$  and  $B \rightarrow J/\psi K_S$  decays  
**(tentative evidence seen in recent data!)**

# The $\phi K_S$ Story



⇒ future will tell ...

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- after all, **New ( $B$ ) Physics** may be just around the corner ...