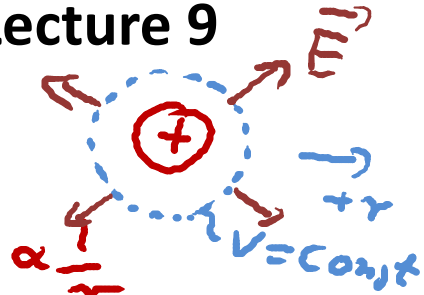
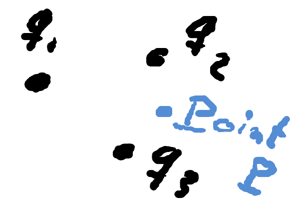
Recap



- Electric point charge:

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \propto \frac{1}{r^2} \iff V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \propto \frac{1}{r}$$

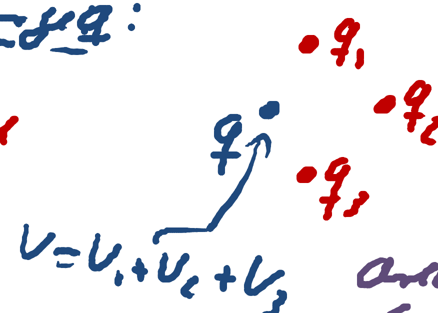
- Potential due to a group of point charges:



Principle of superposition: $V(P) = \sum_{i=1}^{N} V_i(P)$

- Potential energy of a point charge:

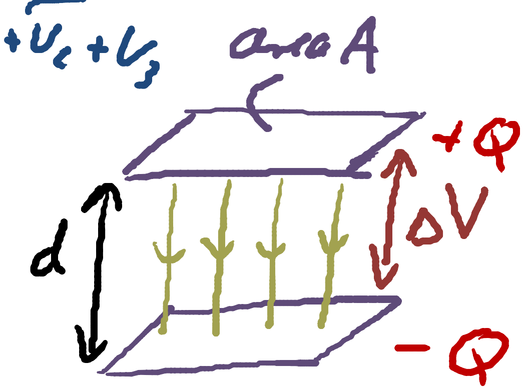
$$U = q\, V_{\text{at place of point charge by other charges}}$$



- Capacitors:

– capacitance $= C = \dfrac{Q \text{ per plate}}{\Delta V_{\text{between plates}}}$

$$[C] = \frac{C}{V} = \text{farad} = 1\,F$$



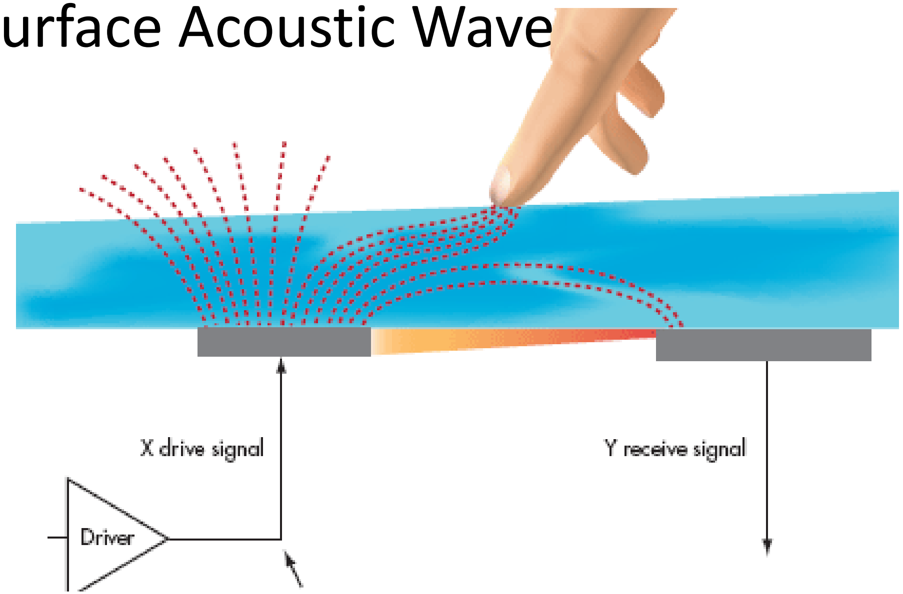
– for parallel plate capacitor:

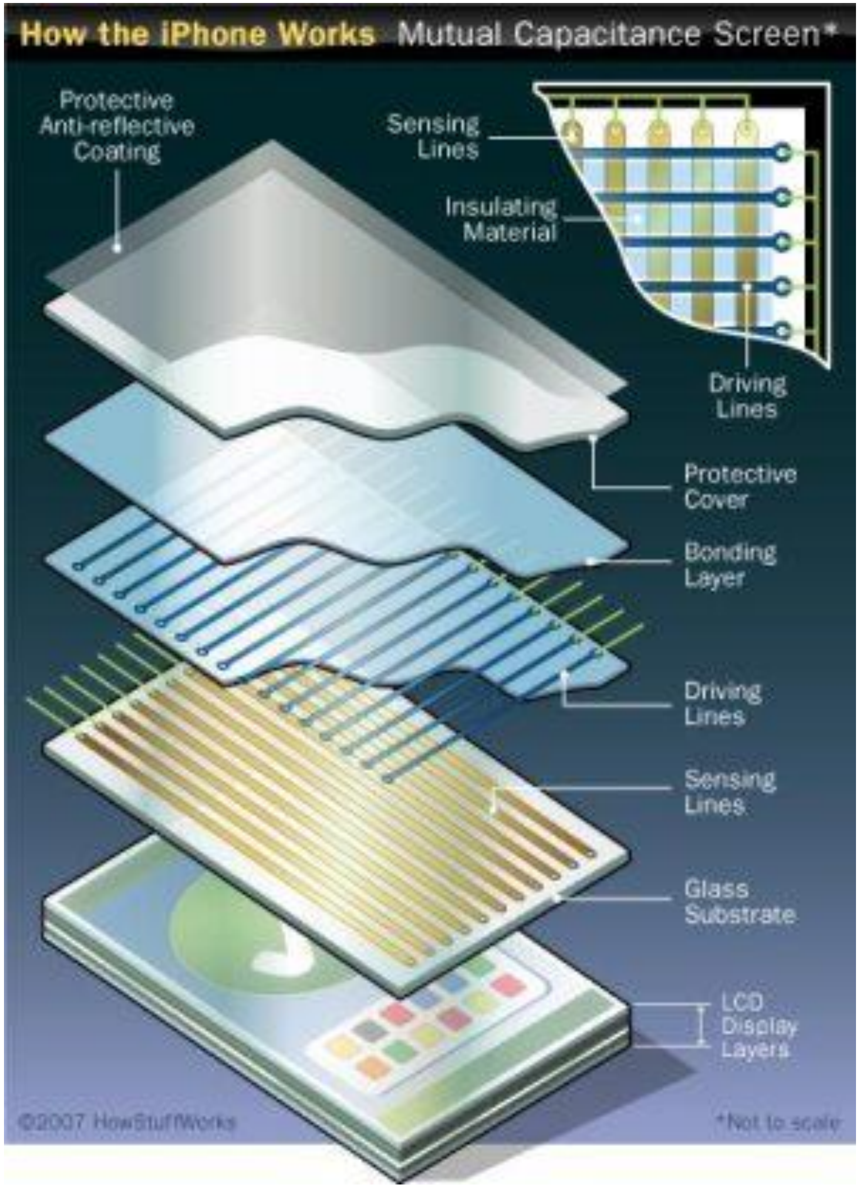
$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \qquad \Delta V = Ed \qquad C = \varepsilon_0\frac{A}{d}$$

$\sigma = \dfrac{Q}{A}$ = surface density of charge

Touch Screens

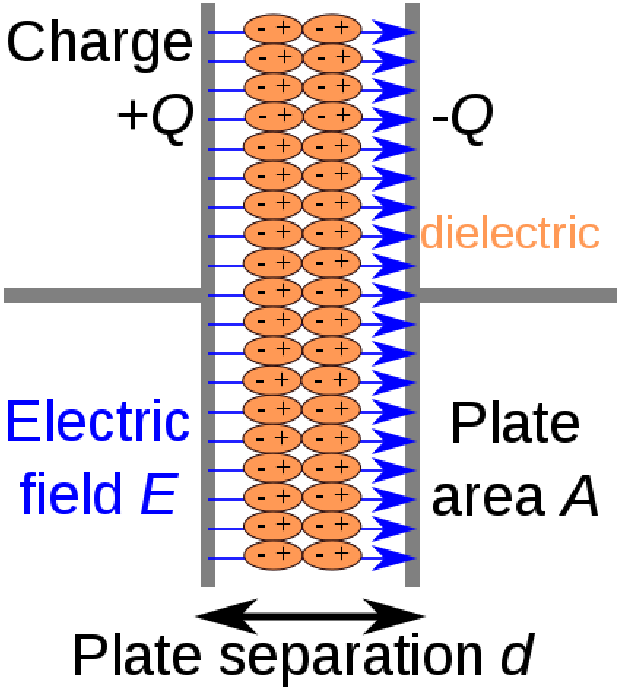
- **Technologies:**
  - Infrared of optical Touch
  - Capacitive Touch
    - touching the screen surface results in a distortion of the screen's electrostatic field, measurable as a change in capacitance
  - Resistive Touch Technology
  - Surface Acoustic Wave

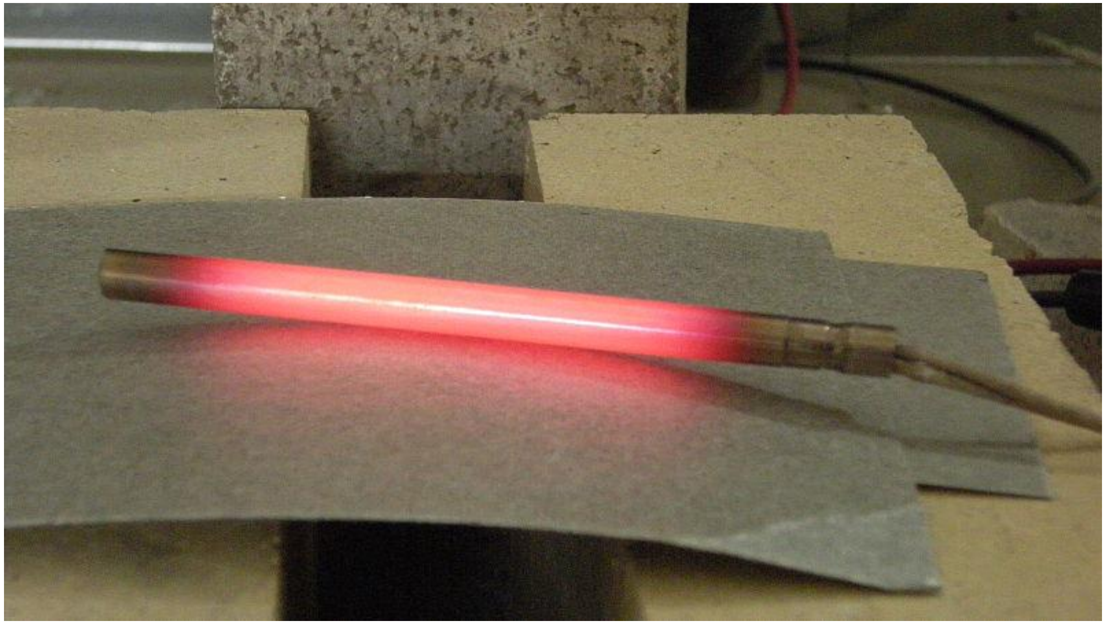




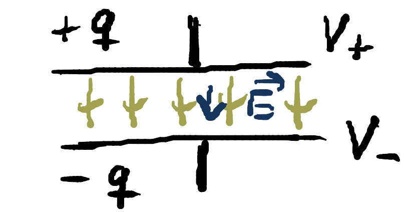
Today:

- Energy density of the electric field
- Dielectrics
- Electric current
- Electrical resistance





Energy stored in a Capacitor / Electric Field



- potential difference between plate:

$$\Delta V = V_+ - V_- = q/C$$

- move small positive charge dq from the − to the + plate ⇒ increase in potential energy stored in capacitor

$$dU = dq \cdot \Delta V = dq\,\frac{q}{C} = W_{\text{we need to do to move charge}}$$

⇒ total potential energy when charging capacitor from $q_i = 0$ to $q_f = Q$:

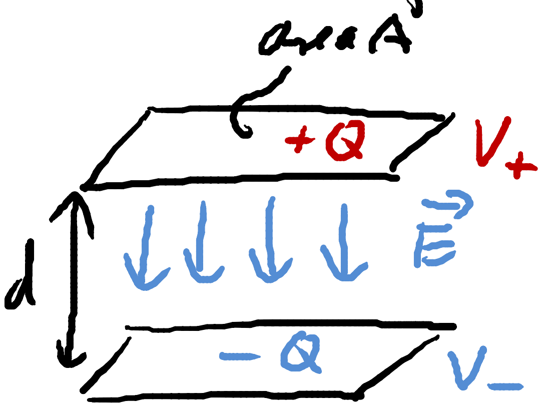
$$\left(\begin{array}{c}\text{Energy stored in}\\ \text{capacitor}\end{array}\right) = \underline{\underline{U}} = \int_0^Q dU = \int_0^Q \Delta V\,dq = \int_0^Q \frac{q}{C}\,dq = \underline{\underline{\frac{1}{2C}\,Q^2}}$$

= area under $\Delta V(q)$ graph

$$U_{capacitor} = \begin{pmatrix}\text{energy stored in}\\ \text{capacitor}\end{pmatrix} = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C \Delta V^2$$

$C = Q/\Delta V$

- can think of this energy as being stored in the electric field $\vec{E}$ between the two plates:



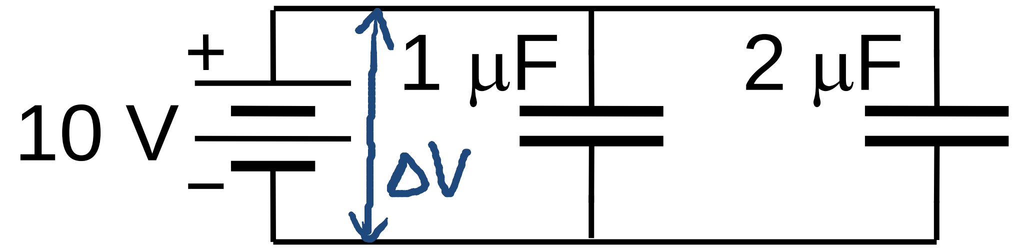
$$\Rightarrow \begin{pmatrix}\text{Energy of electric}\\ \text{field of capacitor}\end{pmatrix} = \frac{1}{2} C \Delta V^2$$

$\Rightarrow$ energy density u_{el} associated with an electric field:

$$u_{el} = \frac{\text{Energy in field}}{\text{Volume}} = \frac{\frac{1}{2}\varepsilon_0 \frac{A}{d} \Delta V^2}{A d} = \frac{1}{2}\varepsilon_0 \left(\frac{\Delta V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2$$

true for any electric field!

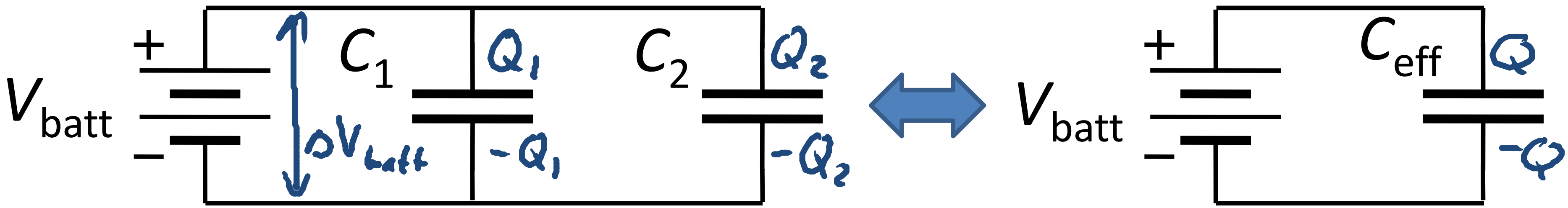
Which capacitor stores more charge?



A. 1 μF
B. 2 μF
C. Both store the same charge

$$Q = C \cdot \underbrace{\Delta V}_{\text{same for both}} \propto C$$

Capacitors in Parallel



What should be the value of C_{eff} in terms of C_1 & C_2 so that the battery delivers the same charge in both circuits?

same: $\Delta V_{batt} = \Delta V_1 = \Delta V_2$

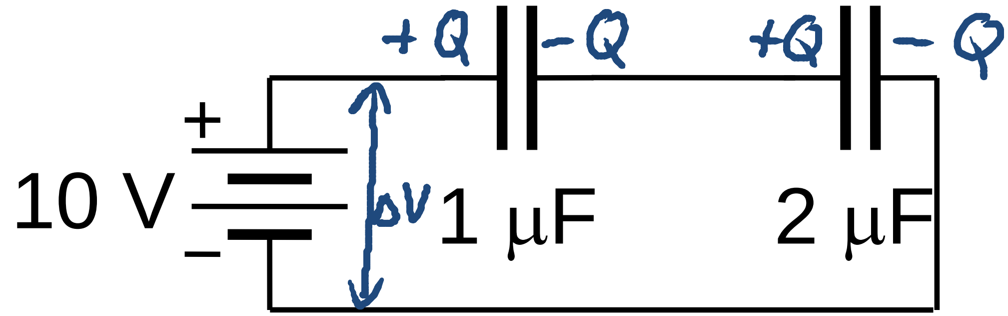
add: charge $Q = Q_1 + Q_2$

$$\Rightarrow Q = Q_1 + Q_2 = C_1 \Delta V_1 + C_2 \Delta V_2 = (C_1 + C_2)\Delta V_{batt}$$

$$= C_{eff} \Delta V_{batt}$$

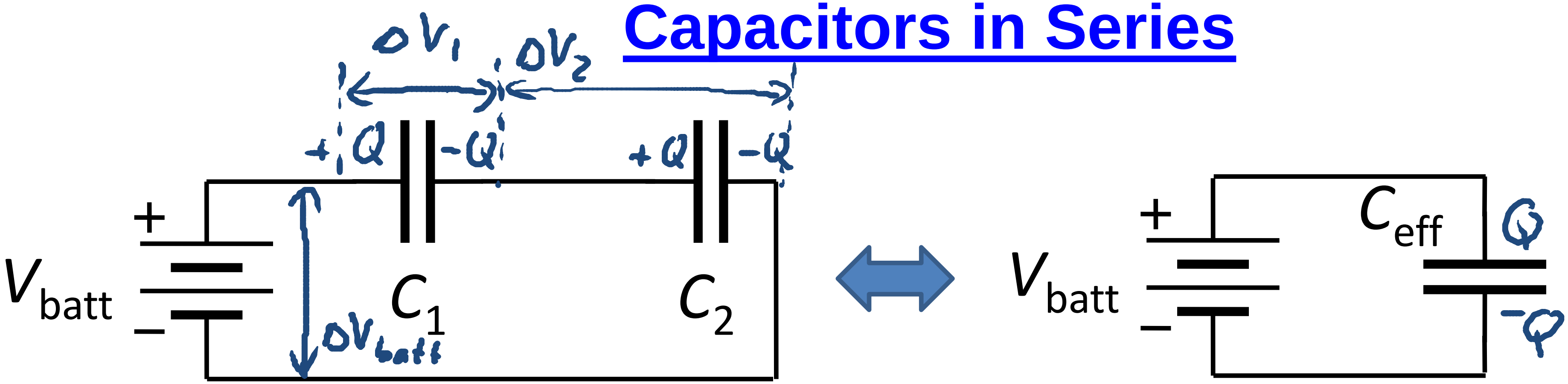
with $\boxed{C_{eff} = \sum_{i=1}^{N} C_i}$ for capacitors in parallel

Which capacitor stores more charge?



A. 1 μF
B. 2 μF
C. Both store the same charge

Capacitors in Series



What should be the value of C_{eff} in terms of C_1 & C_2 so that the battery delivers the same charge in both circuits?

same: charge Q : $Q = Q_1 = Q_2$

add : $\Delta V_{batt} = \Delta V_1 + \Delta V_2$

$$\Delta V_{batt} = \Delta V_1 + \Delta V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) Q = \frac{1}{C_{eff}} Q$$

with $\boxed{\frac{1}{C_{eff}} = \sum_{i=1}^{N} \frac{1}{C_i}}$ for capacitors in series

Dielectrics and Electric Fields

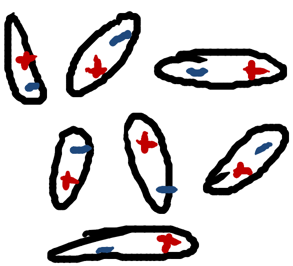
Dielectric: Insulator that can be polarized by an applied electric field

Two Types:

Polar Dielectric	**Nonpolar Dielectric**
molecules have permanent electric dipole moments	no permanent dipole moment, but external electric field induces dipole moments

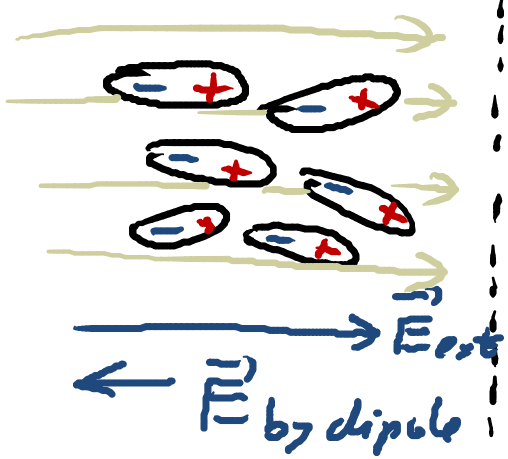
Polar Dielectric

no external field



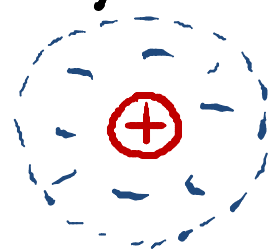
$\vec{E}_{avg} = 0$

with external field



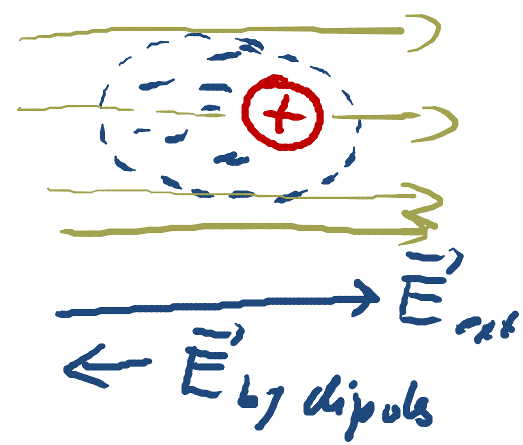
Nonpolar Dielectric

no external field

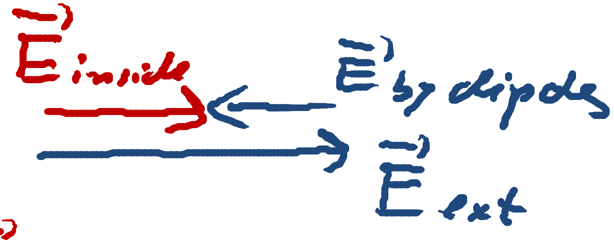


$\vec{E}_{avg} = 0$

with ext. field



In both cases:



$$\vec{E}_{\substack{inside\\dielectric}} = \vec{E}_{external} + \vec{E}_{by\ dipoles} \Rightarrow |E_{inside}| < |E_{ext}|$$

$\Rightarrow$ The effect of the dielectric is to weaken the original external field:

$$|E_{inside}| = \frac{|E_{external}|}{\kappa}$$

with dielectric constant: $\kappa \geq 1$ (dimensionless)

- κ is a property of the dielectric material
- examples: $\kappa_{vacuum} = 1$; $\kappa_{paper} \simeq 3.5$; $\kappa_{water} \simeq 80$

$\Rightarrow$ Effect of dielectric:

All electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\,\varepsilon_0$.

Examples:

since: $E_{\substack{\text{with}\\ \text{dielectric}}} = \dfrac{E_{\text{without}}}{\kappa}$

- **Point charge:**

$$E_{\text{without}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \longrightarrow E_{\text{with}} = \frac{1}{4\pi\kappa\varepsilon_0}\frac{Q}{r^2}$$

- **Capacitor:**

$$E_{\text{without}} = \sigma/\varepsilon_0 \longrightarrow E_{\text{with}} = \frac{\sigma}{\kappa\varepsilon_0}$$

$$\Rightarrow C_{\text{without}} = \frac{Q}{\Delta V} = \frac{Q}{-\int \vec{E}_{\text{without}}\cdot d\vec{s}} = \varepsilon_0 \underbrace{\frac{A}{d}}_{\text{for ll plots only}} \rightarrow \underline{C_{\text{with}}} = \frac{Q}{\Delta V_{\text{with}}} = \frac{Q}{-\int \vec{E}_{\text{with}}\cdot d\vec{s}} = \kappa C_{\text{without}}$$

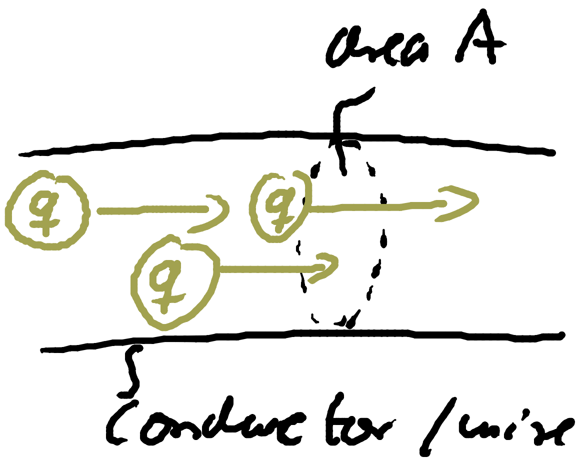
$$= \underline{\kappa\varepsilon_0 A/d}$$

- **Gauss' Law:**

$$Q_{\text{inside}} = \varepsilon_0 \underbrace{\oint \vec{E}_{\text{without}}\cdot d\vec{A}}_{\underline{\Phi}} \longrightarrow Q_{\text{inside}} = \varepsilon_0 \underline{\underline{\kappa}} \oint \vec{E}_{\text{with}}\cdot d\vec{A}$$

Moving Charges: Electric Current i

Electric Current: net flow of moving charges through an area per time



$$\boxed{i = \frac{dq}{dt} = \left(\begin{array}{c}\text{charge passing}\\ \text{through area per time}\end{array}\right)}$$

for constant / steady current:

$$\boxed{i = \frac{\Delta q}{\Delta t}}$$

Units: $[i] = \frac{C}{s} \equiv$ ampere $= 1A$