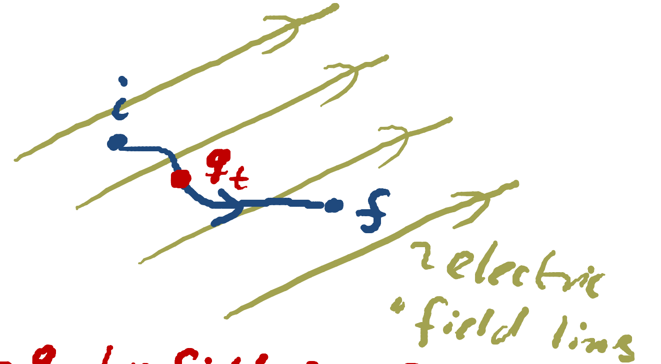
Recap

Lecture 7

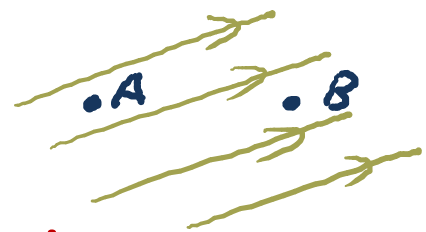
- **Electric Potential Energy**
  - change of potential energy of charge q_t in electric field:



$$\Delta U_{el} = U_{el,f} - U_{el,i} = -W_{el.\ on\ q_t\ by\ field,\ i \to f}$$

$\Rightarrow$ for $U_{el} = 0$ at infinity: $U_f = -W_{by\ electric\ force,\ \infty \to f}$

- **Electric Potential ("Voltage")**
  - property of an electric field
  - difference in potential between two points:



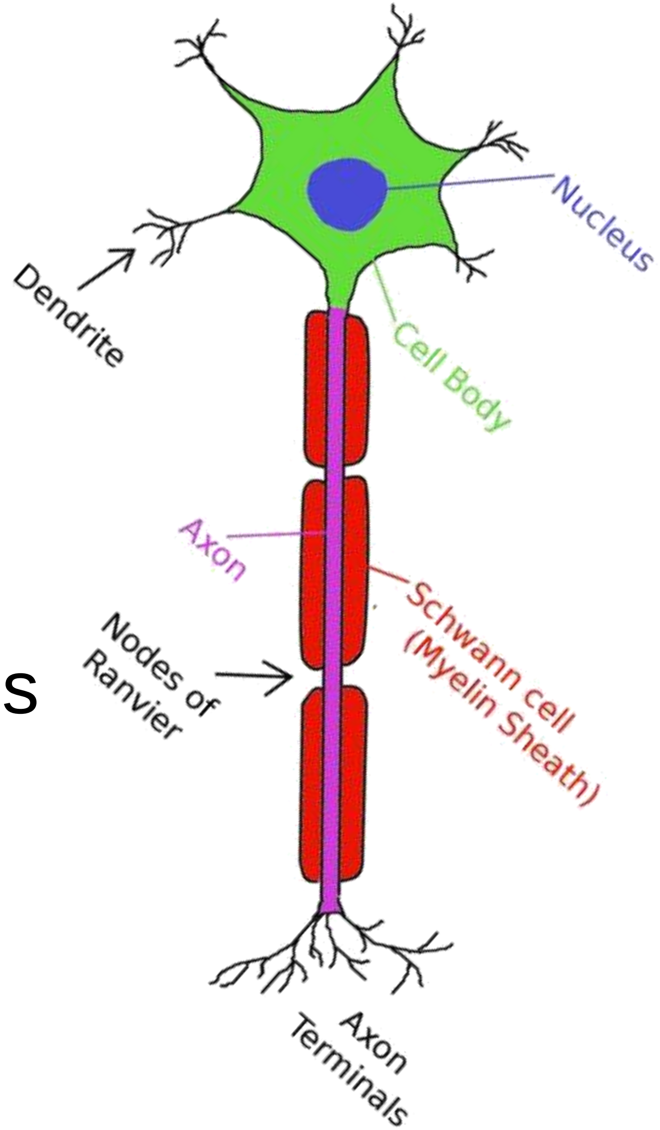
$$\Delta V = V_B - V_A = \frac{\Delta U_{el\ on\ q_t,\ A \to B}}{q_t} = \left(\text{change in potential energy per unit charge}\right)$$

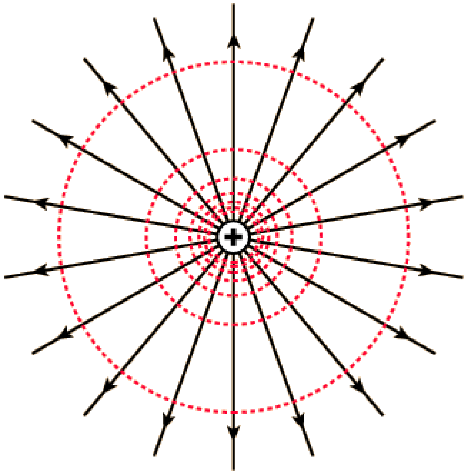
$\Rightarrow$ for $V = 0$ at infinity: $V = \dfrac{U_{el}}{q_t}$

  - $[V] = J/C = V = \text{Volts}$
  - $\vec{E}$ always points in direction of maximum electric potential decrease!

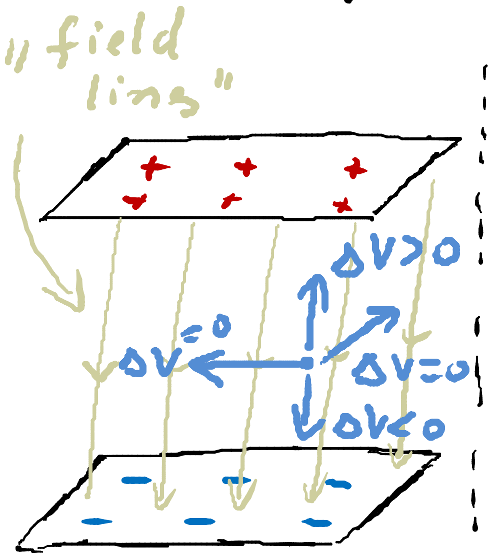
Today:

- More on the electric potential
  - Equipotential surfaces
  - How to find the potential from the electric field
  - How to find the electric field from the potential
  - Potential of a point charge
  - Transmission of nerve impulses

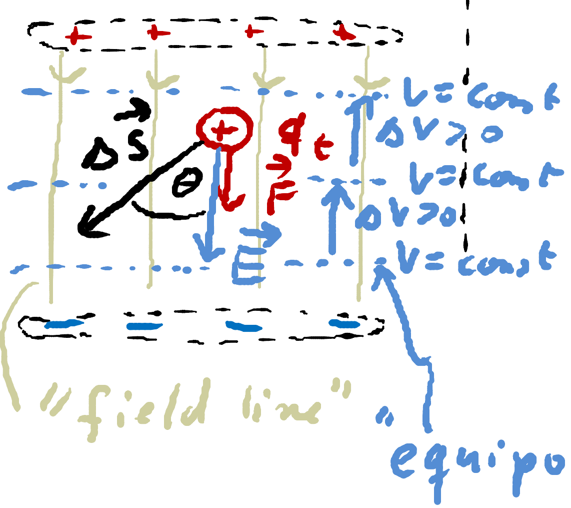




Example: Uniform Electric Field

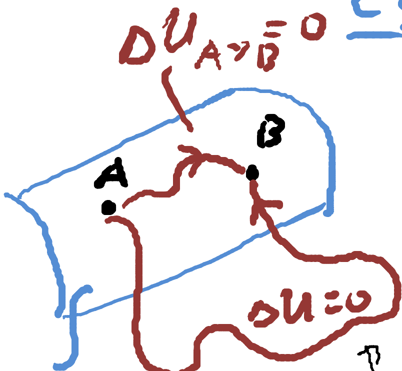


side view:



Properties of test charge q_t		Properties of point in electric field
Electric force $\vec F_{el} = q_t \vec E$	per unit charge →	**Electric field** $\vec E = \dfrac{\vec F_{el\ on\ q_t}}{q_t}$
↓ ΔU is (-1)· integral over $\vec F_{el}$ along path ↑ $\vec F$ is (-1)· gradient of U_{el}: $F_{el\ along\ \vec{\Delta s}} = -\dfrac{\Delta U_{el}}{\Delta s}$		↓ ΔV is (-1) · integral over $\vec E$ ↑ $\vec E$ is (-1)· gradient of V: $E_{along\ \vec{\Delta s}} = -\dfrac{\Delta V}{\Delta s}$
Electric Potential energy change $\Delta U_{el} = -W_{el}$ $= -\vec F \cdot \vec{\Delta s}$ $= -q_t \underbrace{\vec E \cdot \vec{\Delta s}}_{E\Delta s\cos\theta}$	per unit charge →	**Electric Potential** $\Delta V = \dfrac{\Delta U_{el}}{q_t} = -\dfrac{W_{el}}{q_t}$ $= -\vec E \cdot \vec{\Delta s}$ $= -E\Delta s\cos\theta$

Equipotential Surfaces:



equipotential surface
$V=\text{const}$

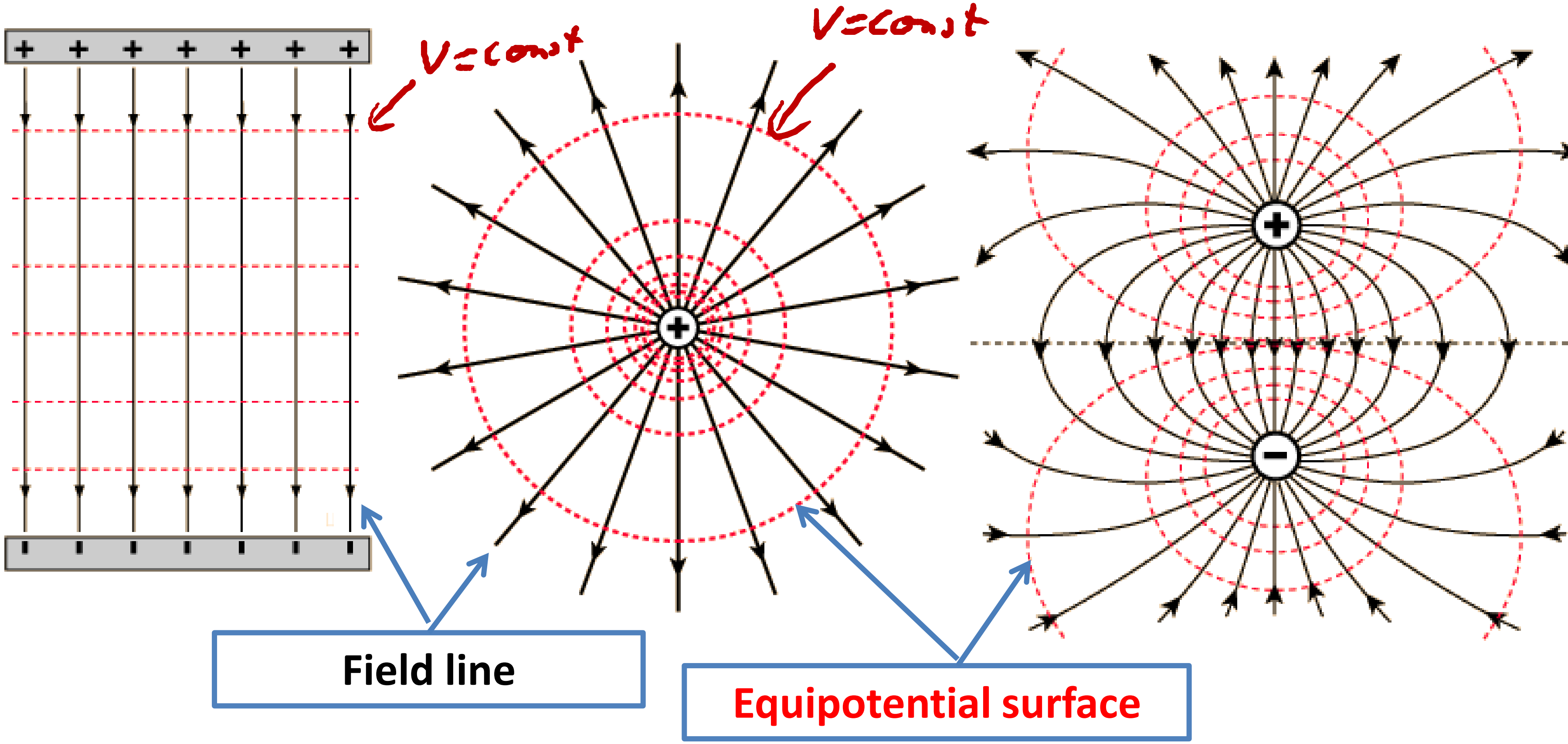
$\Delta V_{A\to B}=0$
$\Rightarrow \Delta U_{A\to B}=q_t\,\Delta V=0$
$\Rightarrow W_{el,A\to B}=0$

- has the same electric potential at every point on the surface ($V=\text{const} \Rightarrow \Delta V_{A\to B}=0$)

  $\Rightarrow$ change in electric potential energy $\Delta U_{A\to B}=0$ when a charge moves from one point on an equipot. surface to another point on the <u>same</u> surface
- is perpendicular ($\perp$) to the electric field $\vec{E}$ at all points
  (if $E_\parallel \neq 0$, $W_{A\to B}\neq 0 \Rightarrow \Delta V_{A\to B}\neq 0$)
- The electric field $\vec{E}$ always points in direction of <u>maximum decrease</u> of the potential energy V!

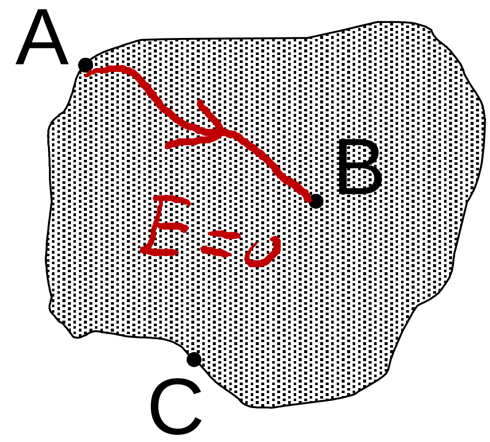
Equipotential surfaces: Examples

Uniform electric field | **Point charge** | **Electric dipole**



Note: In reality, all of these are 3D!

The isolated piece of metal (conductor) shown (in cross section) has a net charge and is in electrostatic equilibrium.



Which of the following is true concerning the potential difference between points A and B? (Point B is inside the metal.)

A. $V_B - V_A > 0$

B. $V_B - V_A < 0$

C. $V_B - V_A = 0$

D. Can't tell

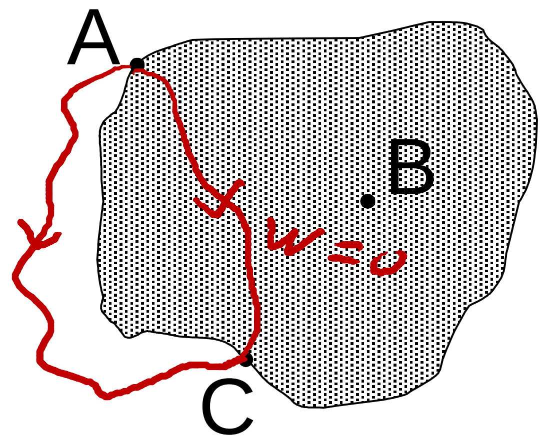
$\vec{E} = 0$ inside conductor

$\Rightarrow W_{el, A\to B} = 0 \Rightarrow \Delta U_{el} = 0 \Rightarrow \Delta V_{A \to B} = 0$

$\Rightarrow$ In electrostatic equilibrium, every point in a conductor must be at the same potential

$\Rightarrow V = \text{const}$ inside

The isolated piece of metal (conductor) shown (in cross section) has a net charge and is in electrostatic equilibrium.



Which of the following is true concerning the potential difference between points A and C?

also surface of conductor
is at V= const

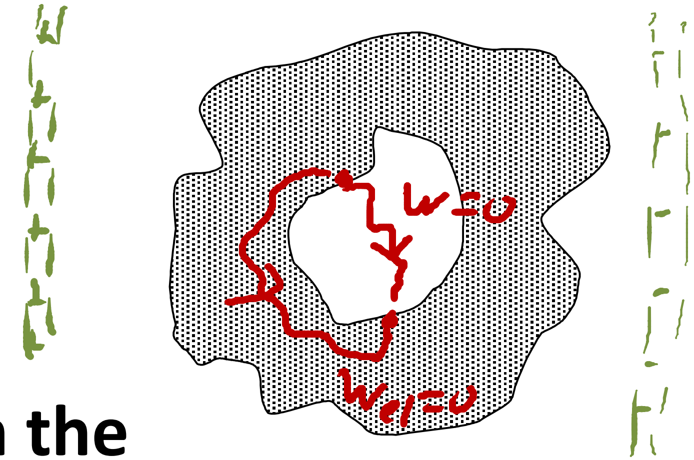
A. $V_C - V_A > 0$

B. $V_C - V_A < 0$

C. $V_C - V_A = 0$

D. Can't tell

The isolated hollow piece of metal (conductor) shown above (in cross section) has a net charge and is in electrostatic equilibrium.
Is there an electric field anywhere in the hollow inside the metal?



A. Yes

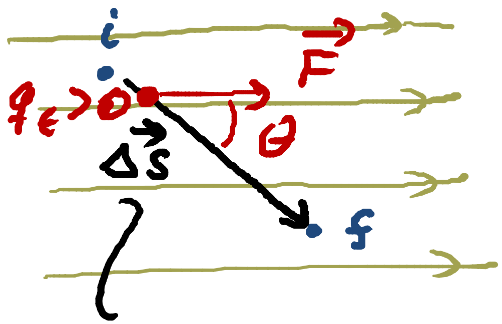
B. No

C. Can’t tell

→ even true if conductor is placed in some outside electric field!

Calculating the Potential V from the Field $\vec{E}$

(a) Uniform field



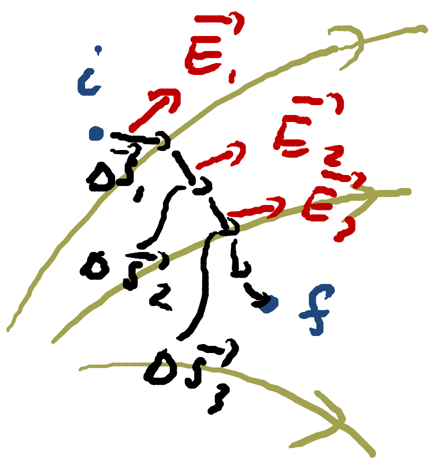
$\Delta\vec{s}$: displacement vector

$$\Delta V = V_f - V_i = \frac{\Delta U_{el,\,i\to f}}{q_t} = -\frac{W_{el\ on\ q_t,\,i\to f}}{q_t}$$

$$= -\frac{1}{q_t}\vec{F}\cdot\Delta\vec{s} = -\frac{1}{q_t}(q_t\vec{E})\cdot\Delta\vec{s}$$

$$= -\vec{E}\cdot\Delta\vec{s} = -E\,\Delta s\cos\theta$$

(b) General case:



⇒ Divide "path" into series of small displacements $\Delta\vec{s}_i$ with $\vec{E}_i \approx$ const

$$\Rightarrow \Delta V = V_f - V_i = \frac{\Delta U_{el,\,i\to f}}{q_t} = -\sum_{i=1}^{N}\left(\vec{E}_i\cdot\Delta\vec{s}_i\right)$$

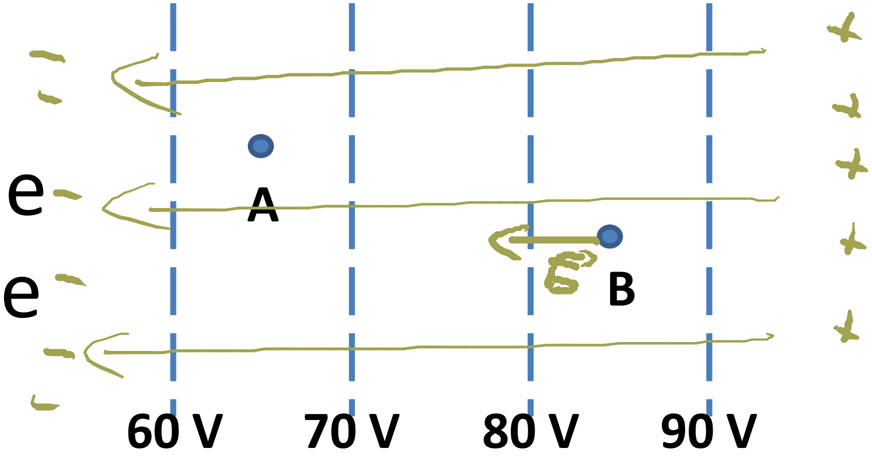
⇒ get integral along path:

$$\boxed{\Delta V = V_f - V_i = -\int_i^f\left(\vec{E}\cdot d\vec{s}\right)}$$

⇒ if we set $V_i = 0$ (usually for point at ∞)

$$V = -\int_i^f \vec{E}\cdot d\vec{s}$$

An electron (q<0) moves from point A to point B. The work on the electron by the electric field is…



A. $W_{el} > 0$

B. $W_{el} < 0$

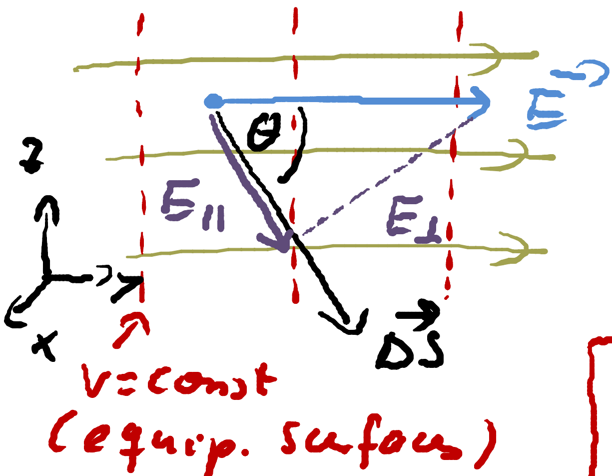
~~C.~~ $W_{el} = 0$

$$\Delta V = \frac{\Delta U_{el}}{q} = -\frac{W_{el}}{q}$$

$$\Rightarrow W_{el,\, on\, e^-} = -\underbrace{q}_{<0}\,\underbrace{\Delta V}_{>0} > 0$$

Calculating the Field $\vec{E}$ from the potential V

(a) Uniform field



from above:

$$\Delta V = -\vec{E}\cdot\vec{\Delta s} = -E\,\Delta s\cos\theta$$

$$\Rightarrow \frac{\Delta V}{\Delta s} = -E\cos\theta = -E_{\text{component along } \vec{\Delta s} \text{ direction}}$$

$\Rightarrow$ if we take $\vec{\Delta s}$ along x, y, or z-axis

$$\boxed{E_{\substack{\text{component along}\\ x\text{-axis}}} = E_x = -\frac{\Delta V}{\Delta x} \qquad E_y = -\frac{\Delta V}{\Delta y} \qquad E_z = -\frac{\Delta V}{\Delta z}}$$

(b) General case:

$\vec{\Delta s} \Rightarrow \vec{ds}$; $\Delta V \Rightarrow dV$, $\Delta x \Rightarrow dx$ ....

$$E_{\text{comp. along } x} = -\left.\frac{dV}{dx}\right|_{y,z=\text{const}} = -\left(\begin{array}{l}\text{rate of change of } V\\ \text{with distance along } x\text{-}\\ \text{direction}\end{array}\right)$$

$$E_y = -\left.\frac{dV}{dy}\right|_{x,z=\text{const}} \qquad E_z = -\left.\frac{dV}{dz}\right|_{x,y=\text{const}}$$

The component of $\vec{E}$ in any direction is the <u>negative</u> of the rate at which the potential changes with distance in that direction!

The graph shows the electric potential V as function of x.

$$E_x = -\frac{dV}{dx}$$

In which region has the x-component of the electric field the **largest positive value**?

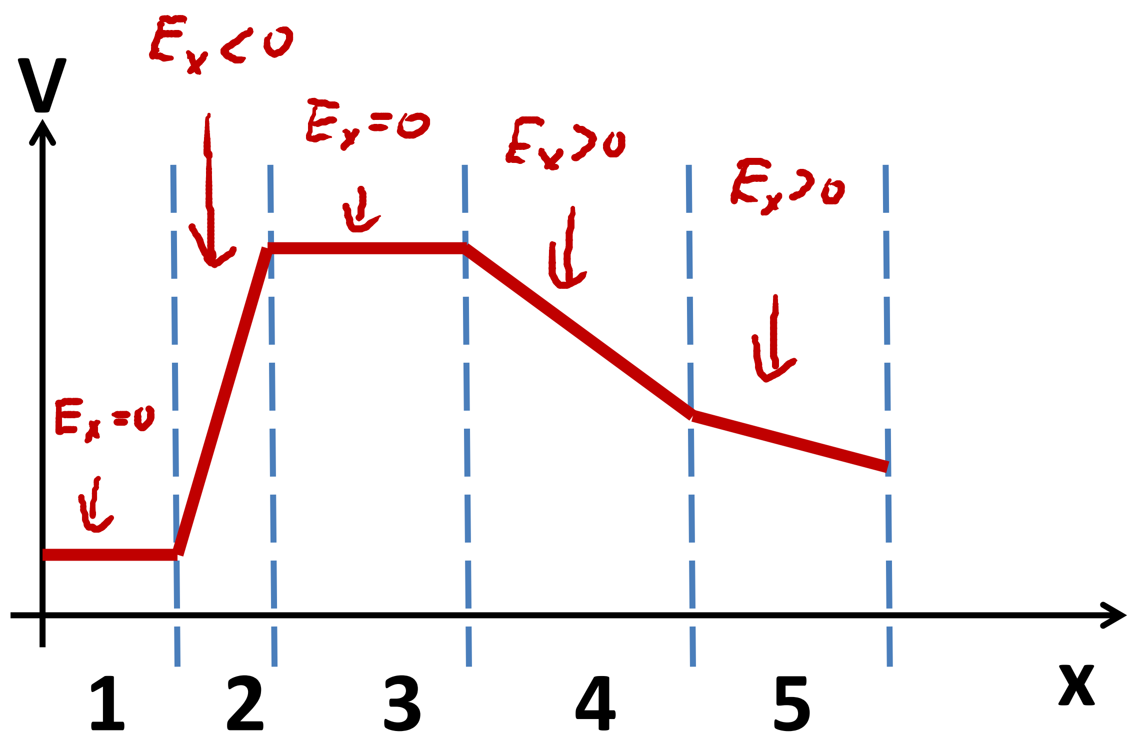
A. Region 1

B. Region 2

C. Region 3

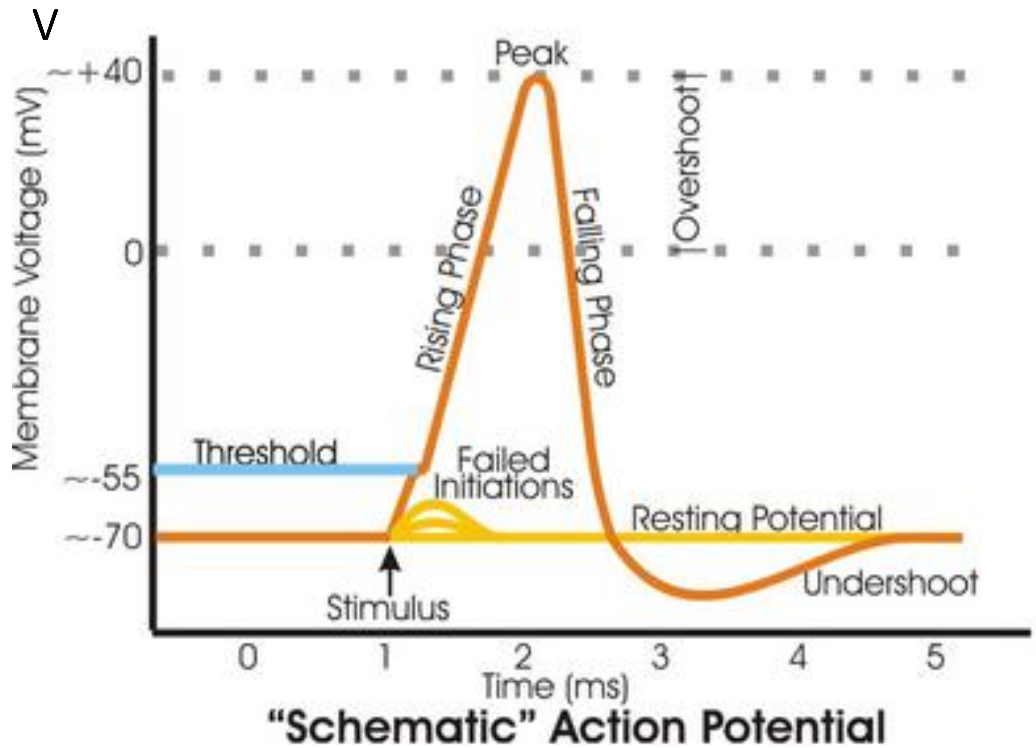
D. Region 4

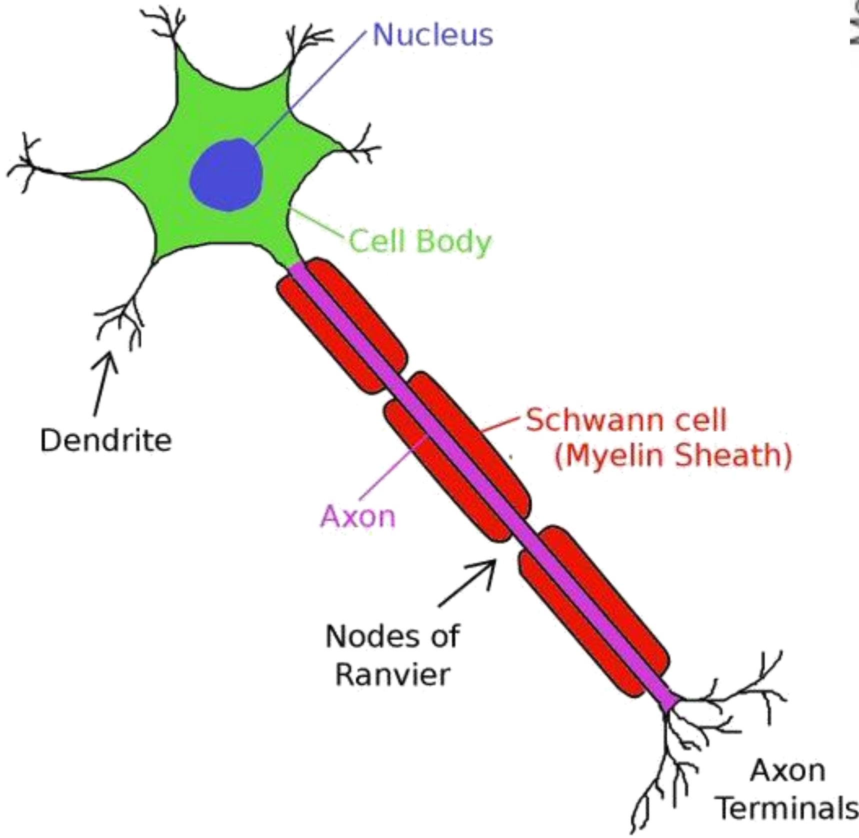
E. Region 5



Transmission of Nerve Impulses

- **Axon: transmits nerve impulses**
- **In resting state: -70 mV potential of fluid inside relative to fluid outside (negative ions on inner surface of membrane and positive ions on outside)**





- **Nerve impulse changes the potential difference across the membrane (by sodium ion flow though membrane) to ~+40 mV**
- **Action potential propagates with 30 m/s down the axon**
- **~20% of resting energy of human body goes into active pumping of sodium ions!**