Recap I

Lecture 41

- Heisenberg's Uncertainty Principle:

$$\Delta x \cdot \Delta p_x \geq \frac{h}{4\pi} \quad \text{always}$$

uncertainty in position x

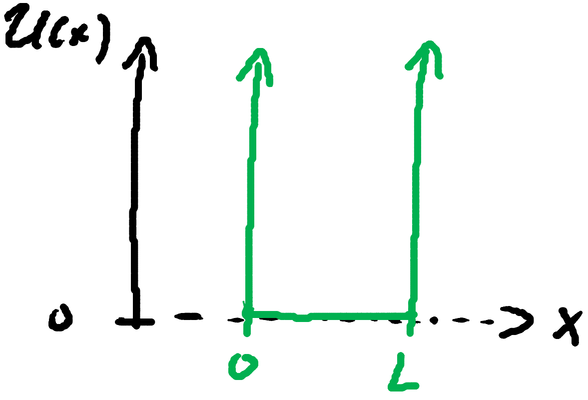
uncertainty in x-component of momentum

small uncertainty in position

⇓

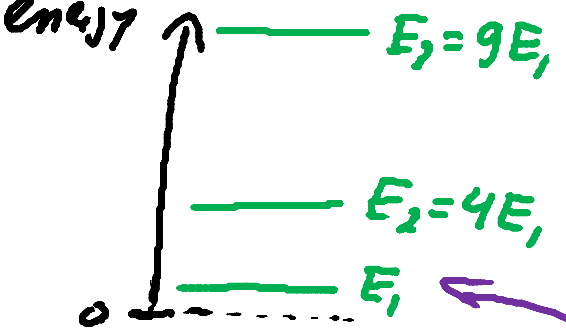
large uncertainty in momentum

- Particle confined in infinitely deep square potential energy well:



$U(x) = 0 \quad 0 < x < L$

$U(x) = \infty$ elsewhere



confinement of particle leads to quantization of energy!

$$\boxed{E_n = \frac{h^2}{8mL^2} n^2 = E_1 n^2}$$

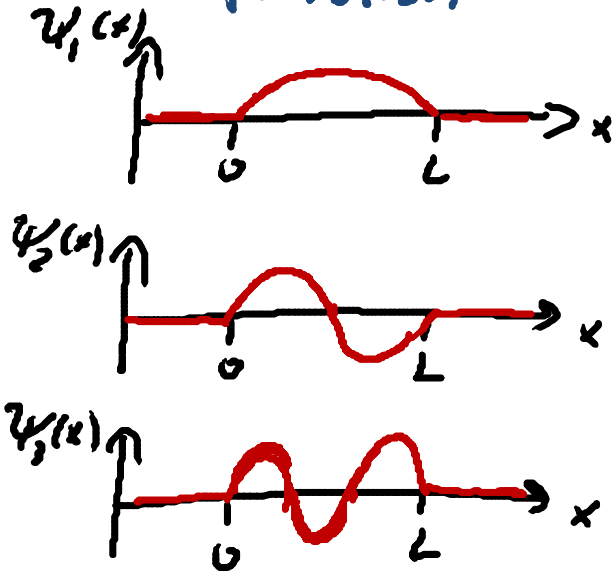
$n = 1, 2, 3 \ldots$

note: $E_1 > 0$!

"zero-point energy"

Recap II

→ corresponding wave function:



$$\boxed{\begin{array}{ll} \psi_n(x) = \sqrt{\frac{2}{L}}\,\sin\left(\frac{n\pi}{L}x\right) & \text{inside well} \\ \psi_n(x) = 0 \quad \text{outside of well} & n = 1, 2, \ldots \end{array}}$$

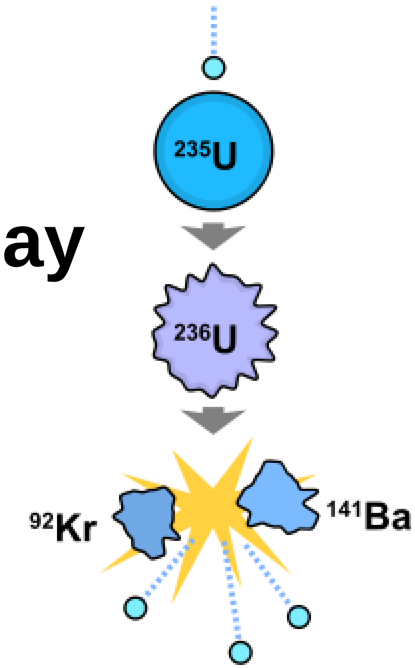
⇒ like standing waves on a string of length L with wavelength:

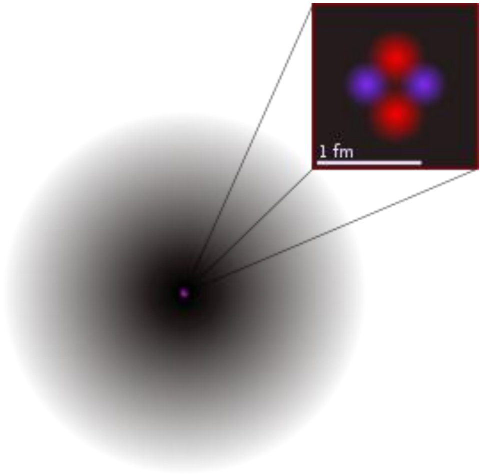
$$\boxed{\lambda_n = \frac{2\pi}{k_n} = \frac{2L}{n}}$$

- Spin angular momentum $\vec{S}$:
  - Intrinsic property of a particle: $|\vec{S}| = \sqrt{s(s+1)}\; h/2\pi$
  - For electrons: spin quantum number: $s = 1/2$
  - Component of $\vec{S}$ along any axis is quantized: $S_z = m_s\, h/2\pi$
  - for electron: spin magnetic quantum number $m_s = +1/2$ ("spin up") or $m_s = -1/2$ ("spin down")
- Pauli Exclusion Principle: No two electrons confined in the the same trap can have the same set of values for their quantum numbers!

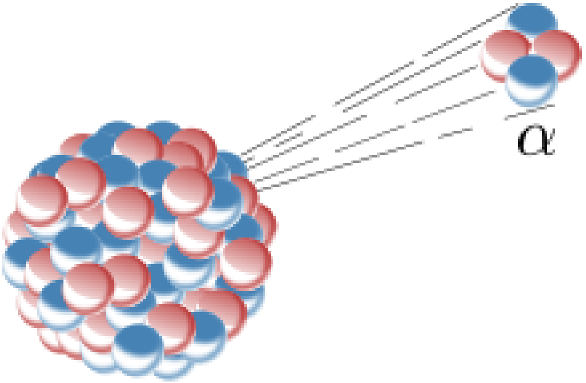
Today:

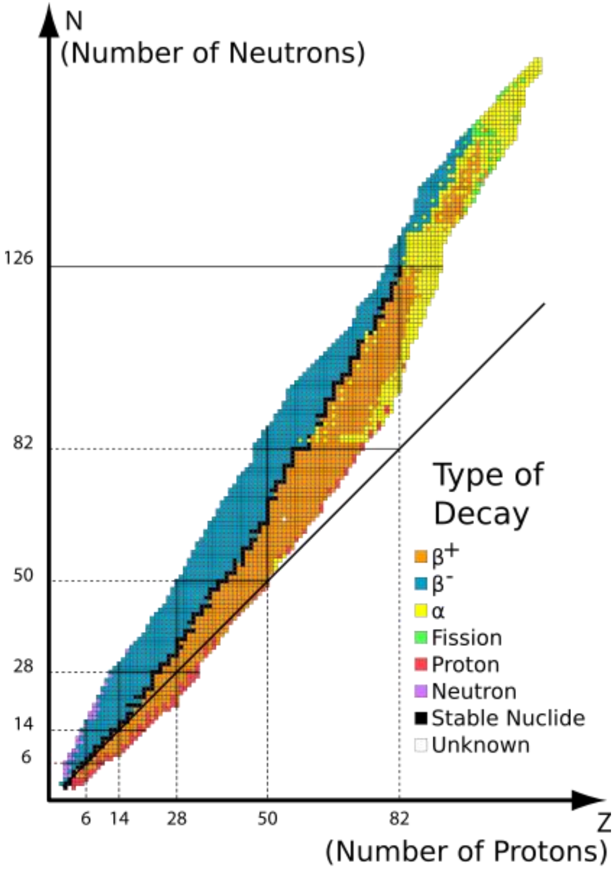
- **Nuclear Physics**
  - The nucleus
  - Radioactive decay
  - Fission
  - Fusion
- **Particle Physics:**
  - What is the Higgs?





1 Å = 100,000 fm



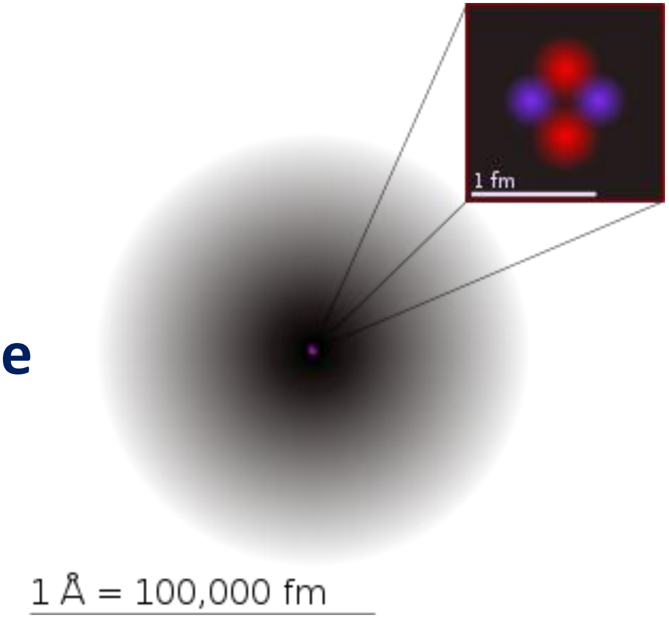


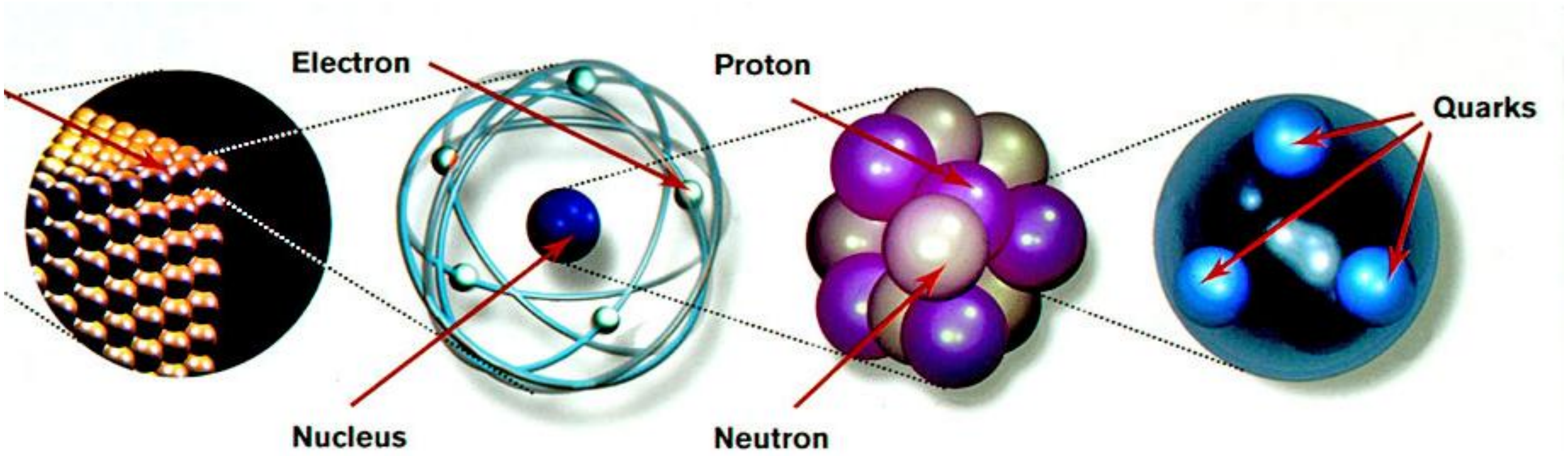
Nuclear Physics: The Nucleus

- **Positive charge and most of the atom's mass are concentrated in a tiny dense core of ~ 10^{-15} m to 10^{-14} m in diameter.**

  -> Atomic nucleus

- **Nuclei are composed of protons (charge = +e per proton) and neutrons (no electric charge)**
  - **Z = # of protons**
  - **N = # of neutrons**
  - **Atomic mass number: $A = Z + N$**





- **Most nuclei are spherical (some are ellipsoidal):**

-> Effective radius: $R = R_0 A^{1/3}$ **where $R_0 \approx 1.2 \cdot 10^{-15}$ m = 1.2 fm**

(1 femtometer = 1 fermi = 1 fm = $1 \cdot 10^{-15}$ m)

- **Element type and chemical properties are determined by *Z*.**
- **Different species (as determined by *Z* and *A*) are called nuclides.**

- **Notation to describe nuclides:**

$$ {}^{A}_{Z}X \quad \text{or} \quad {}^{A}X $$

chemical symbol of element

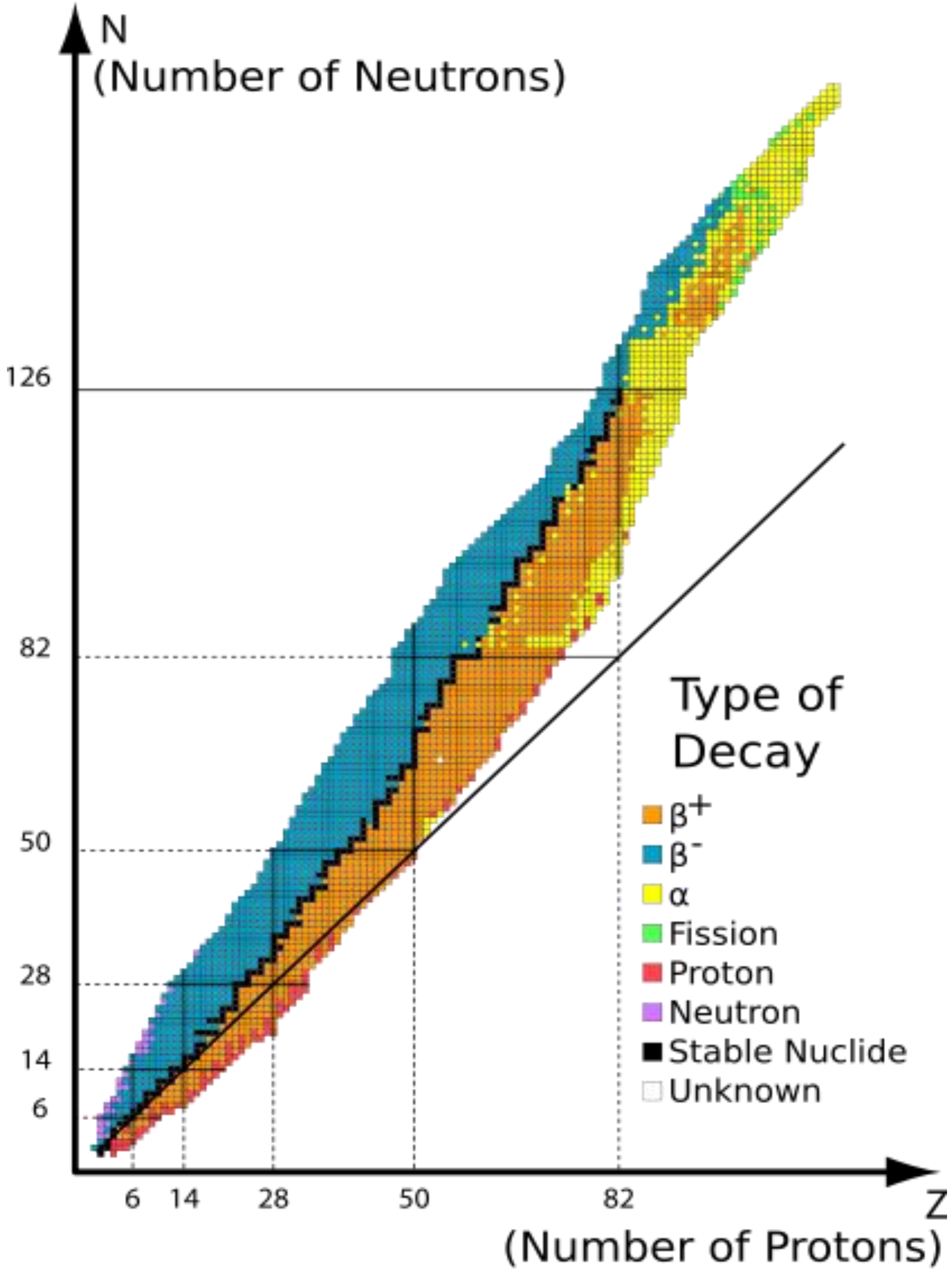
  - **Examples:** ${}^{4}_{2}He$ ${}^{16}_{8}O$ ${}^{56}_{26}Fe$ ${}^{197}_{79}Au$

- **Nuclides with same *Z* but different *A* (i.e. different *N*) are isotopes of each other.**

  - **Examples:**

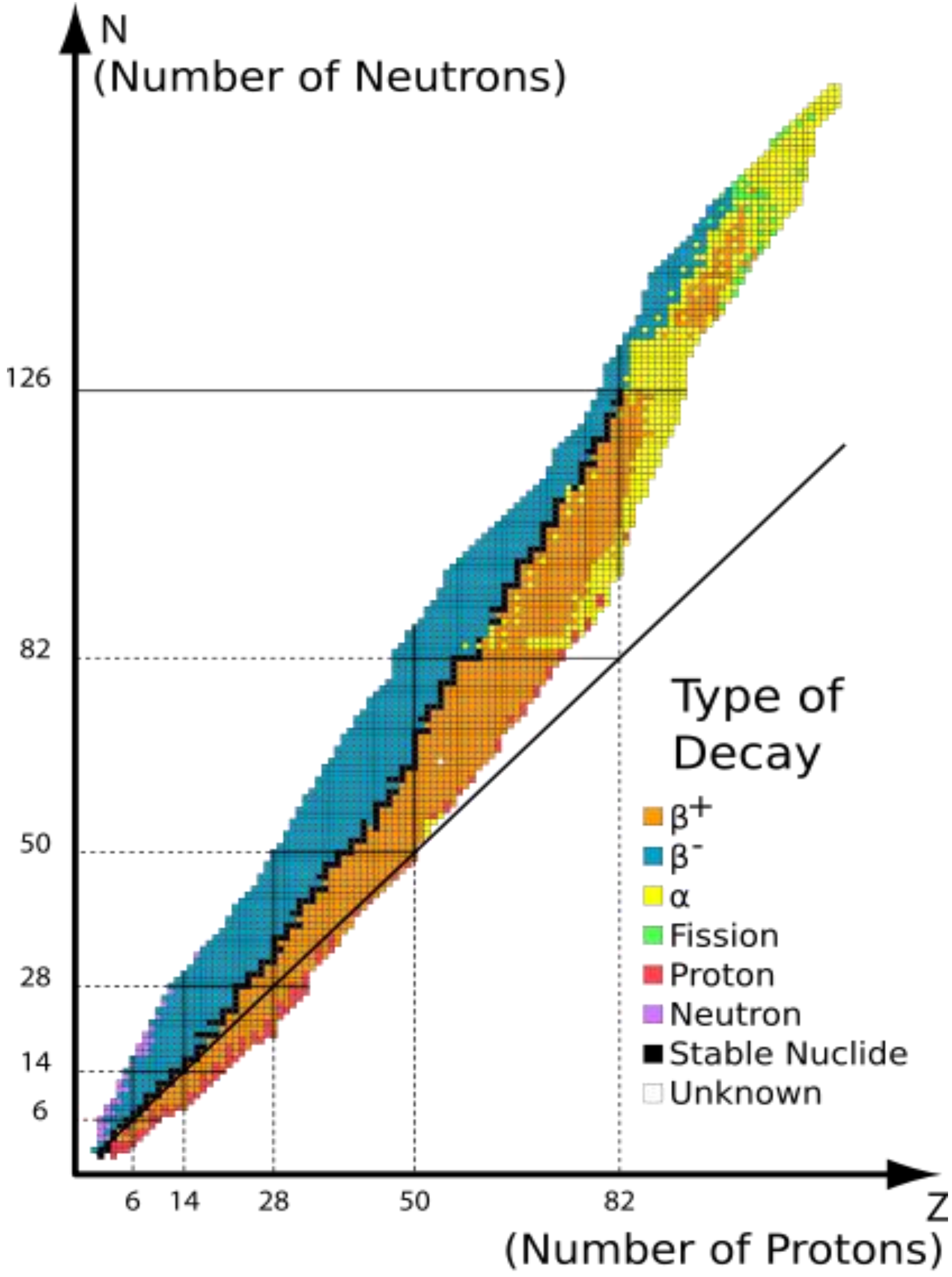
$$ {}^{3}_{2}He \qquad {}^{4}_{2}He $$

Plot of known Nuclides:



- **The black shading indicates the band of stable nuclides.**
- **Low-mass, stable nuclides have essentially equal numbers of neutrons and protons.**
- **More massive nuclides have an increasing excess of neutrons.**

Plot of known Nuclides:

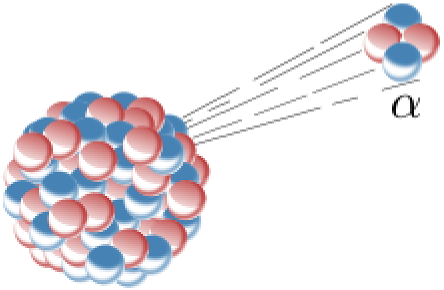


- **Most nuclides are <u>not</u> stable and undergo radioactive decay by emitting radiation and transferring into other nuclides.**
- **There are no stable nuclides with Z>83 (bismuth).**

Radioactive Radiation Types:

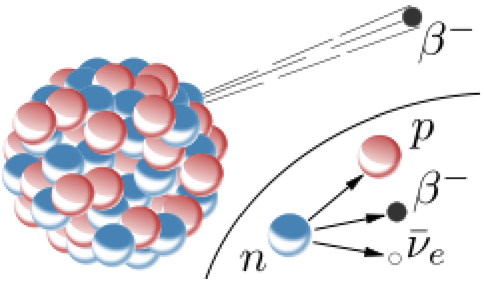
Alpha (α) particles

- **He^{+2} nuclei** (positively charged).
- Can be stopped by a thick piece of paper or several cm of air.



Beta (β) particles

- **Electrons** (negatively charged) or positrons (anti-electrons; positively charged).
- Can penetrate several sheets of paper, thin metal foils, ~ 1 m of air.



Gamma (γ)

- Very high energy **photons** (uncharged).
- Could pass through a human hand.
- Stopped by several cm of lead.

Radioactive Decay:

- Most nuclides are not stable & undergo radioactive decay by emitting radiation & transforming into other nuclides.
- **Radioactive decay is a statistical process**
- **Decay constant λ:**
  - ***λ = probability that a particular nuclide will decay in a unit time interval; [λ] = 1/s***
  - λ = fraction of nuclei in a large sample that are expected to decay on average per unit time interval
  - λ has a characteristic value for every radionuclide.
  - λ is independent of any external influence, including the decay of another nucleus.

Radioactive Decay:

- **If a sample has *N* radioactive nuclei of a given type then the average number decaying per unit time is λN.**

⇒ Define decay rate *R* as:

$$R = -\frac{dN}{dt} = \lambda N = \left[\text{average number of decays per time}\right]$$

⇒ Integrate to number of radioactive nuclei vs. time:

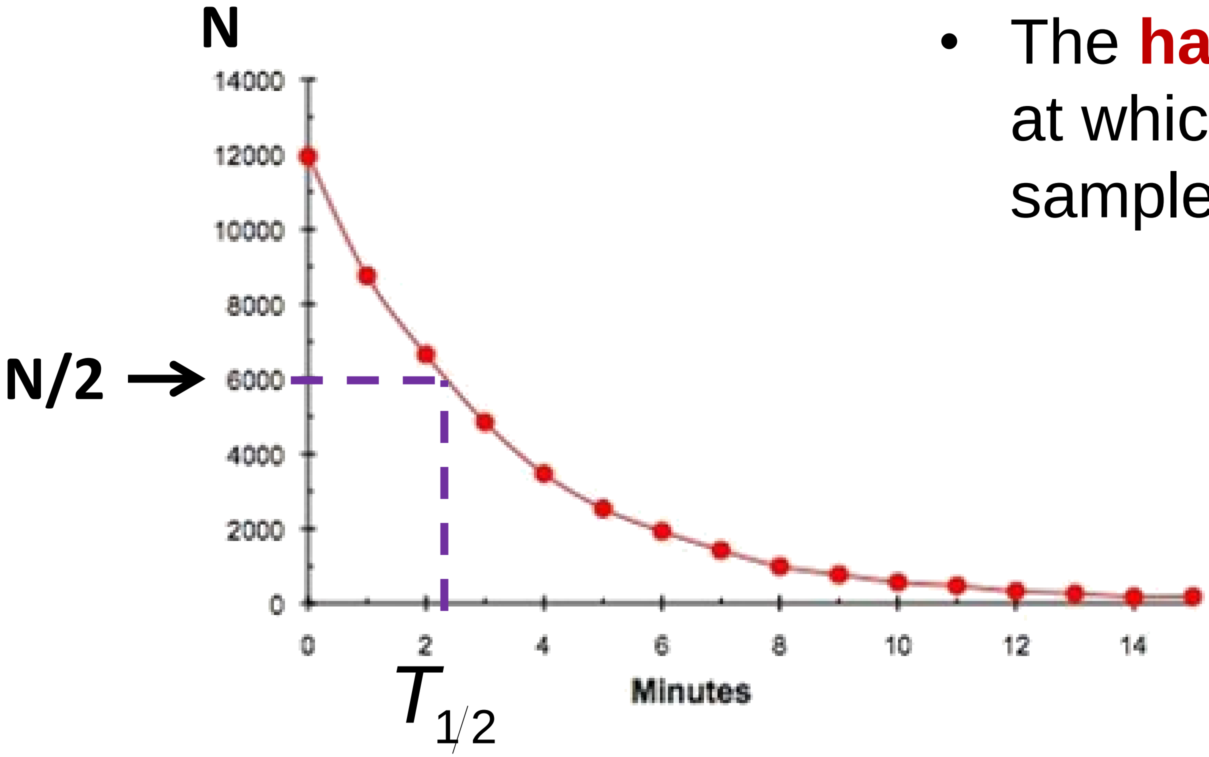
$$N(t) = N(0)e^{-\lambda t} = N(0)e^{-t/\tau}$$

with N(0) = number of radioactive nuclei at t = 0

⇒ Number of radioactive nuclei decreases exponentially with time constant:

$$\tau = \frac{1}{\lambda}$$

Radioactive Decay: Half Time



- The **half-life** $T_{1/2}$ is the time at which half of the original sample remains:

$$\frac{N(T_{1/2})}{N(0)} = \frac{1}{2} = e^{-T_{1/2}/\tau}.$$

$$T_{1/2} = \tau \ln(2)$$

⇒ Can rewrite the number of radioactive nuclei vs. time equation:

$$N(t) = N(0)e^{-t\ln(2)/T_{1/2}} = N(0)(e^{\ln(2)})^{(-t/T_{1/2})} = N(0)2^{-t/T_{1/2}} = N(0)(\tfrac{1}{2})^{t/T_{1/2}}$$

Application: carbon-14 dating:

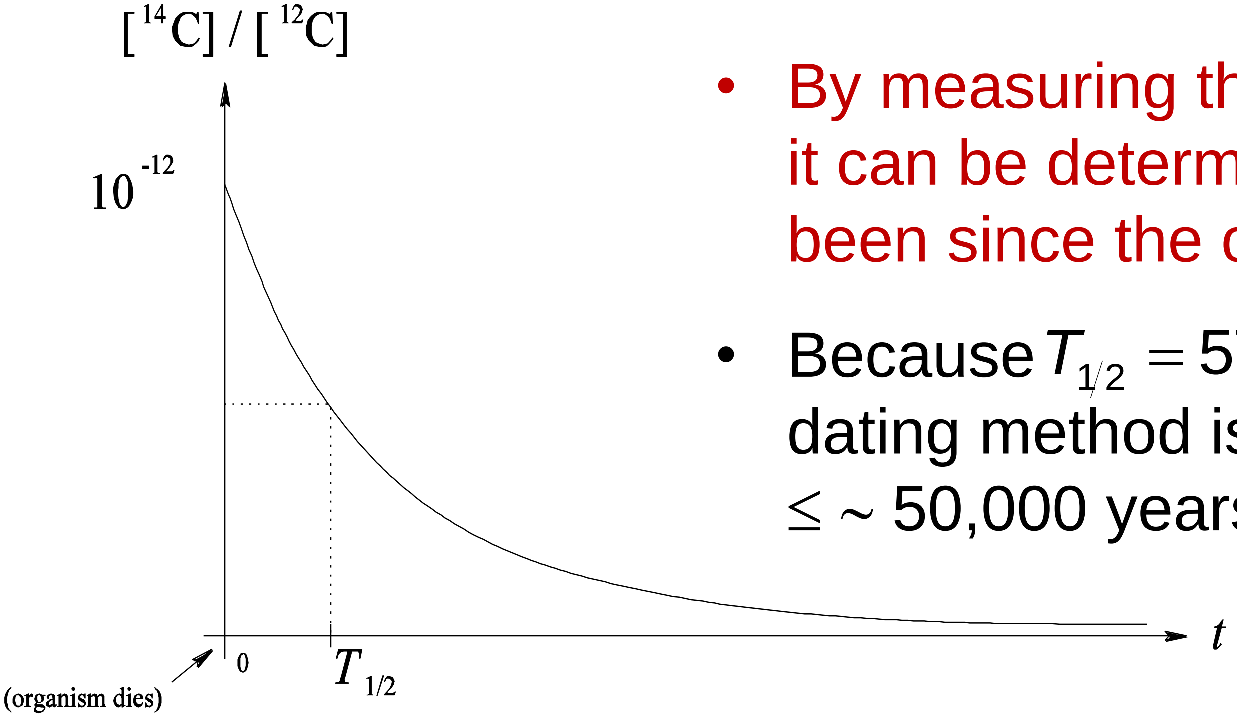
- Carbon-14 ($^{14}_{6}C$) is an **unstable isotope of carbon** ($T_{1/2} = 5730$ years) which is produced in the upper atmosphere by cosmic ray neutrons colliding with $^{14}_{7}N$:

$$^{14}_{7}N \quad + \quad n \quad \rightarrow \quad ^{14}_{6}C \quad + \quad p$$

- This ^{14}C is rapidly oxidized to $^{14}CO_2$ and thus can enter living organisms through photosynthesis & the food chain.
- In the atmosphere, $\dfrac{[^{14}CO_2]}{[^{12}CO_2]} \approx 10^{-12}$.
- A living organism that derives its carbon from the atmosphere will have the same $[^{14}C]/[^{12}C]$ in its tissues.
- But once the organism dies it stops taking in carbon & the amount of ^{14}C in its tissues decreases due to radioactive decay:

$$^{14}_{6}C \quad \rightarrow \quad ^{14}_{7}N \quad + \quad \beta^{-}$$

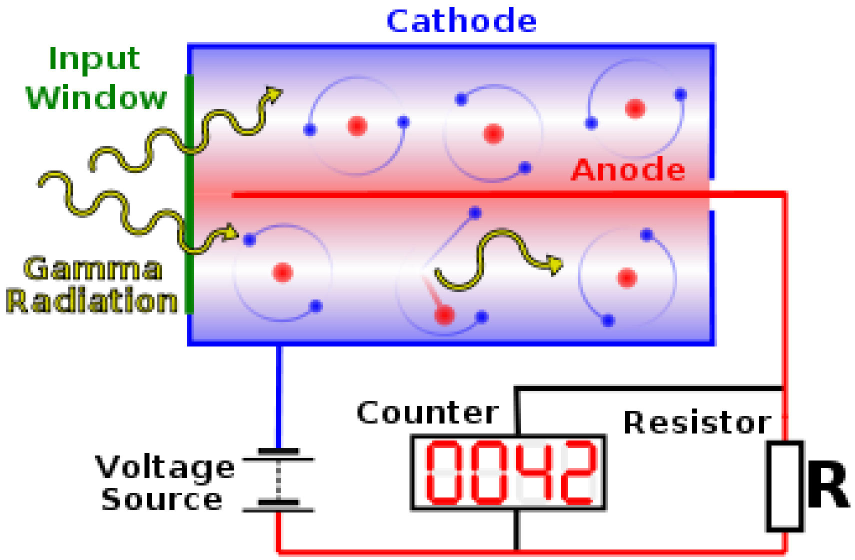
Application: carbon-14 dating:



- By measuring the ratio of [^{14}C]/[^{12}C], it can be determined how long it has been since the organism died.
- Because $T_{1/2} = 5730$ years, the ^{14}C dating method is good for ages $\leq \sim$ 50,000 years.
- ^{14}C dating may be complicated because the proportion of ^{14}C in the atmosphere has not been constant. So, other dating techniques are often used as 'calibrations' for ^{14}C dating.
- Modern human activity has altered the [^{14}C]/[^{12}C] in the atmosphere through nuclear weapons tests & the burning of fossil fuels.

Measuring Radiation:

Geiger tube radiation detector” (*Geiger counter)*



- Radiation (alpha particles, beta particles, or gamma ray photons) will cause electrons to be ejected from the gas or the metal in the tube.
- That electron will then cause more ejections -> number of electrons is multiplied by a factor of about 10^6 to 10^8 before reaching the thin wire.
- Electrons create a current in the thin wire at the center of the counter.

Radiation Dosage:

Absorbed dose = radiation energy absorbed by an object per unit mass

Units: 1 grey = 1 Gy = 1 J/kg = 100 rad

Example: Radiation dose from natural sources per year:
~ 2 mGy = 0.2 rad

The Nuclear Force:

Protons repel each other because of their charge (electric force).

⇒ **A totally different attractive force must bind protons & neutrons together in the nucleus. This nuclear force is thought to be a secondary effect of the strong force that binds quarks together to form neutrons & protons.**

⇒ The nuclear force must be a very short range force because its influence does not extend far beyond the nuclear “surface”.

- The atomic mass unit:

The atomic mass unit, u, is chosen so that the **atomic** (not nuclear) **mass of ^{12}C is exactly 12 u**.

$$1\text{u} = 1.661 \times 10^{-27}\ \text{kg.}$$

$$1\text{u} = 931.494013\ \text{MeV}/c^2.$$

Atomic mass is often reported in these ***atomic mass units***.

Nuclear binding energy:

$M =$ the mass of a nucleus.

$\sum_i m_i =$ the total mass of its individual protons & neutrons.

$$M < \sum_i m_i, \text{ or, } Mc^2 < \sum_i (m_i c^2).$$

- The **binding energy**, ΔE_{BE}, of the nucleus is:

$$\Delta E_{\text{BE}} \equiv \sum_i (m_i c^2) - Mc^2 .$$

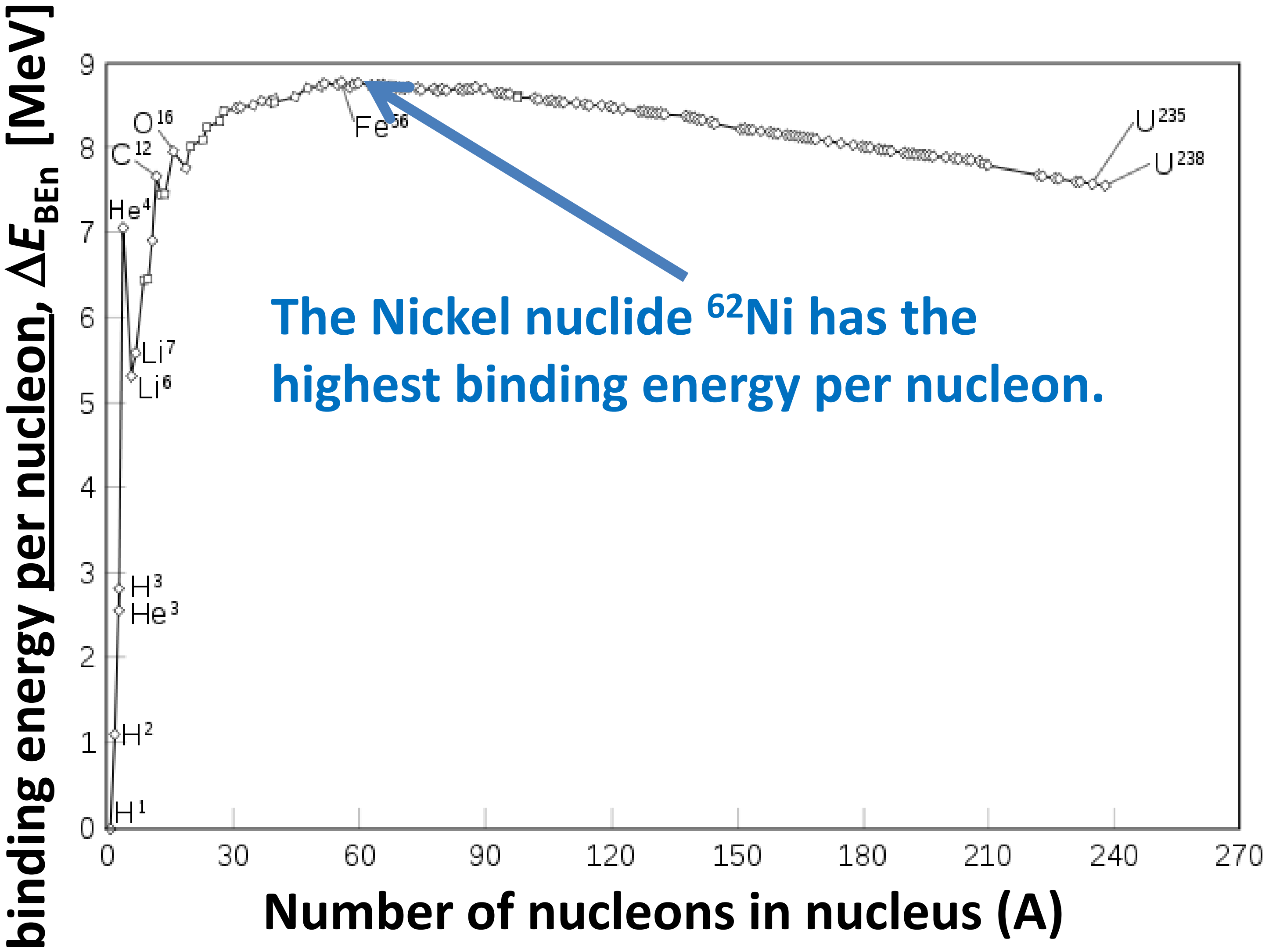
  - **It is the energy that would be required to separate a nucleus into its component nucleons.**
  - The binding energy per nucleon, ΔE_{BEn}:

$$\Delta E_{\text{BEn}} \equiv \frac{\Delta E_{\text{BE}}}{A}$$

The "curve of binding energy":

A graph of binding energy per nucleon of common isotopes.

More tightly bound.



The Nickel nuclide ^{62}Ni has the highest binding energy per nucleon.

Nuclear reactions:

- Conserved quantities are electric charge & total number of nucleons.
- The **energy Q released in a reaction** is:

$$Q = m_i c^2 - m_f c^2 = -\Delta m c^2,$$

where m_i is the total mass of the reactants and m_f is the total mass of the products.

- **Recall: $E = mc^2$, so can convert mass to energy!**
- $Q > 0$ when some mass is converted to energy.

Nuclear reactions: Example

Alpha decay of ^{238}U:

$$^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\alpha$$

$$\text{Atomic mass of } {}^{238}\mathrm{U} = 238.05079\ \mathrm{u}$$

$$\text{Atomic mass of } {}^{234}_{90}\mathrm{Th} = 234.04363\ \mathrm{u}$$

$$\text{Atomic mass of } {}^{4}_{2}\mathrm{He} = 4.00260\ \mathrm{u}$$

$$\Delta \mathrm{m} = \mathrm{m_f} - \mathrm{m_i} = (234.04363\ \mathrm{u} + 4.00260\ \mathrm{u}) - 238.05079\ \mathrm{u} = -0.00456\ \mathrm{u}$$

$$\Rightarrow \quad Q = -\Delta m c^2 = (0.00456\ \mathrm{u})c^2\left(\frac{931.5\ \mathrm{MeV}/c^2}{1\ \mathrm{u}}\right) = 4.25\ \mathrm{MeV}$$

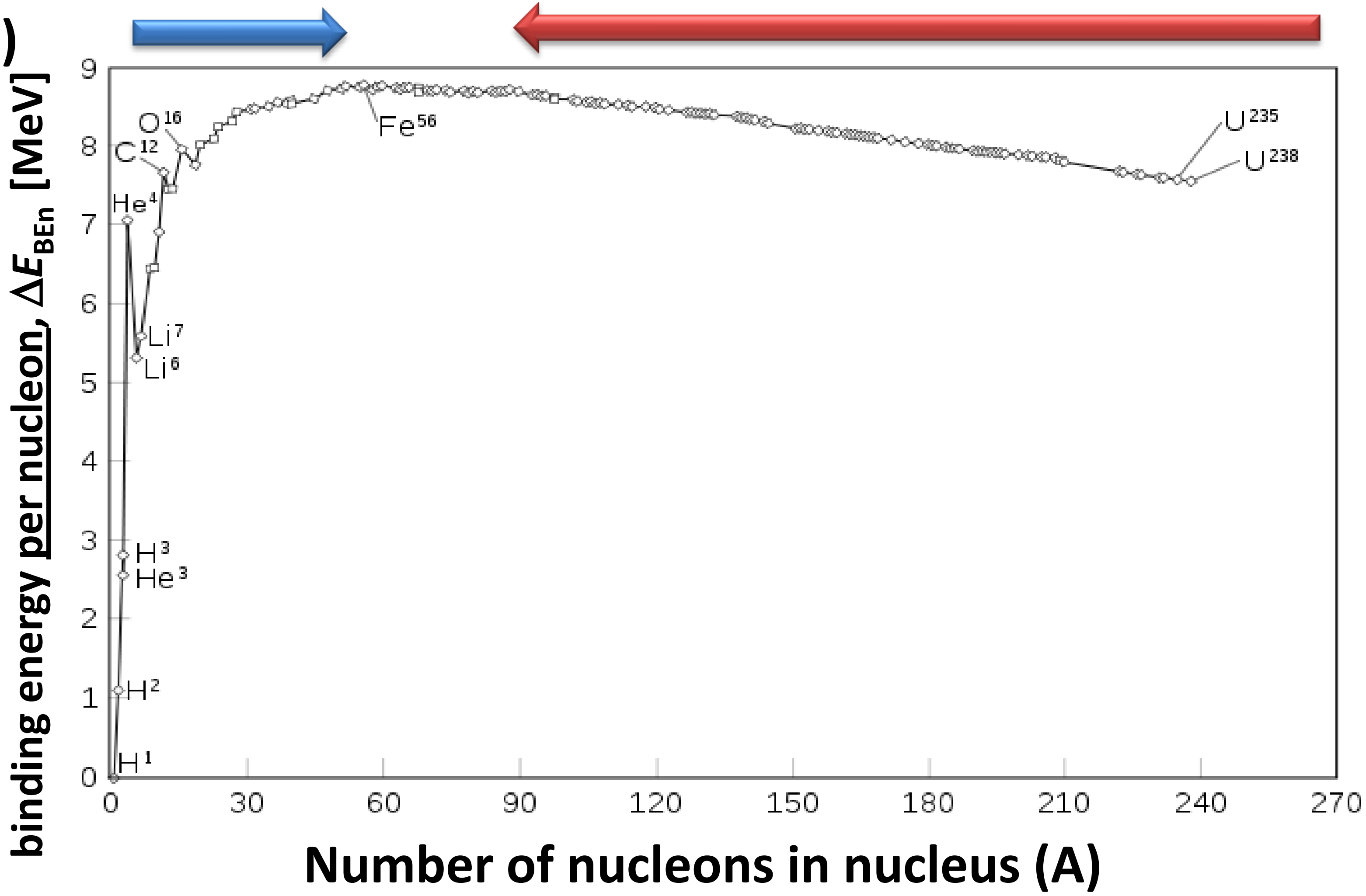
- **Q>0, so energy is released**
- **Almost all of this energy released is kinetic energy of the α particle.** (Why?)

(U: Uranium; Th: Thorium; α: helium nucleus)

The "curve of binding energy":

Nuclear fusion: light nuclei combine to form a larger nucleus (e.g. in stars)

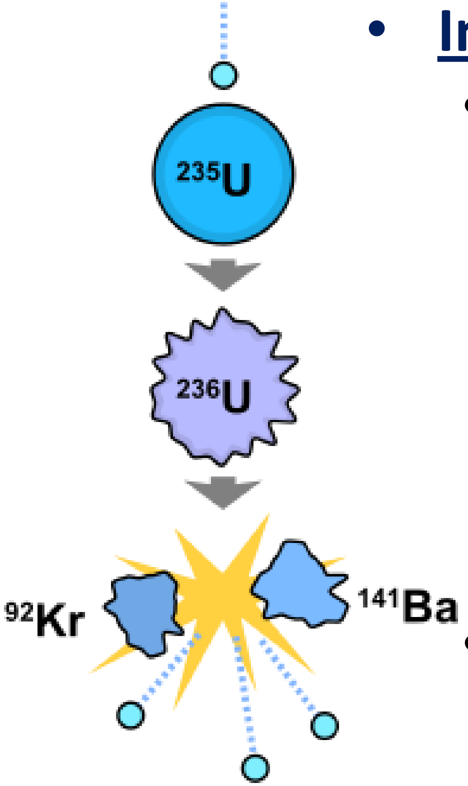
Nuclear fission: large nucleus is converted to smaller nuclei plus energy (e.g. in nuclear reactor)



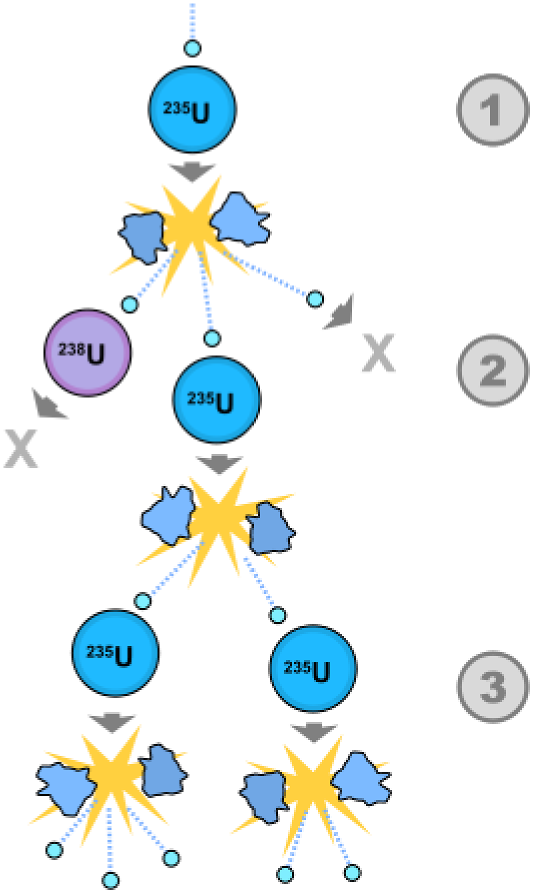
Nuclear Fission:

- **Large nucleus → smaller nuclei, neutrons, & energy.**
- **Example:** $^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{236}_{92}\mathrm{U}^{*}$ (excited state, unstable)

$$^{236}_{92}\mathrm{U}^{*} \xrightarrow{\text{one possibility}} {}^{141}\mathrm{Ba} + {}^{92}\mathrm{Kr} + 3^{1}_{0}\mathrm{n}$$

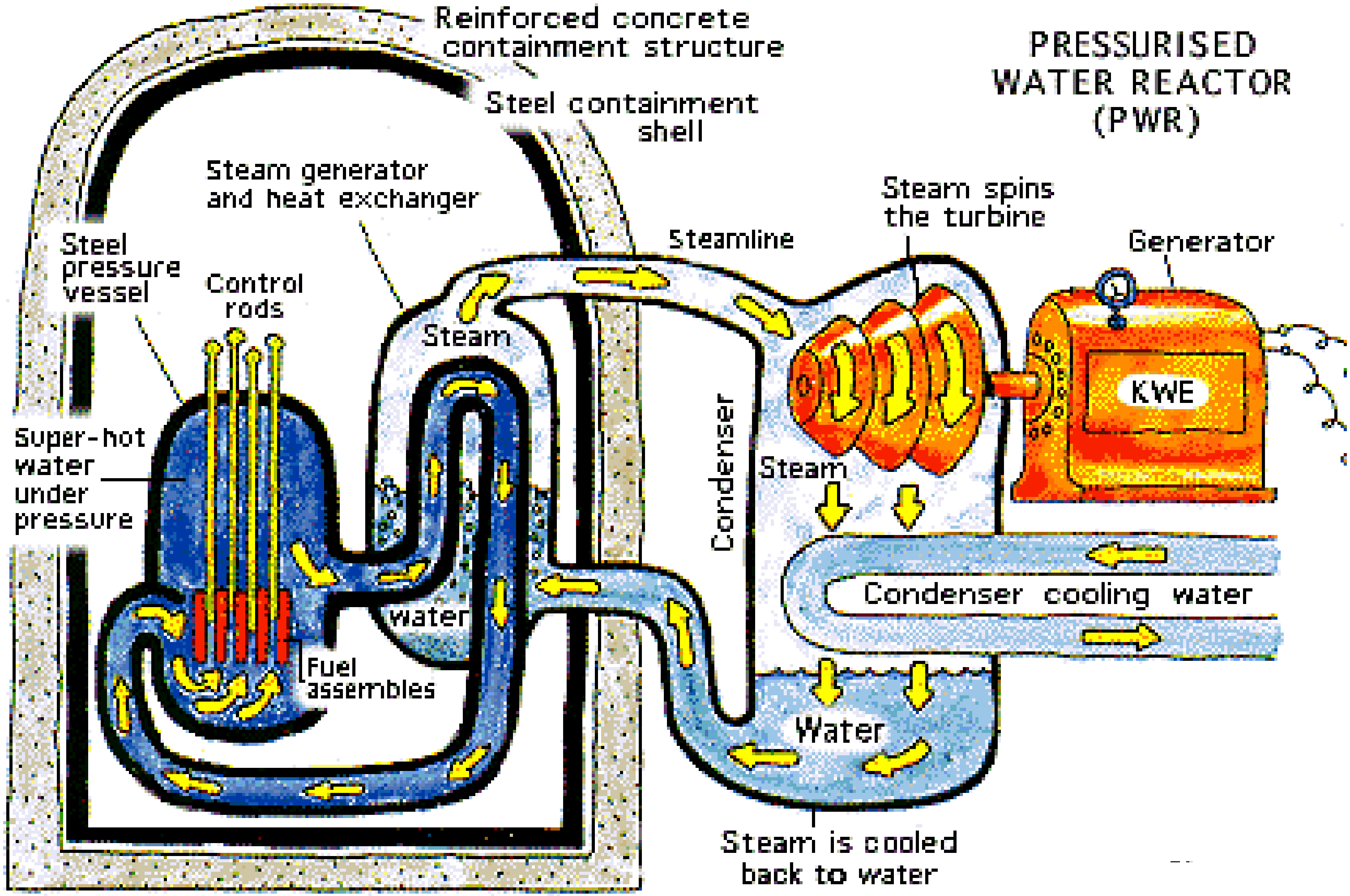


- **Induced nuclear fission event:**
  - **A neutron is absorbed by the nucleus of a uranium-235 atom, which in turn splits into fast-moving lighter elements (fission products) and free neutrons.**
  - **More neutrons produced than consumed -> chain reaction.**

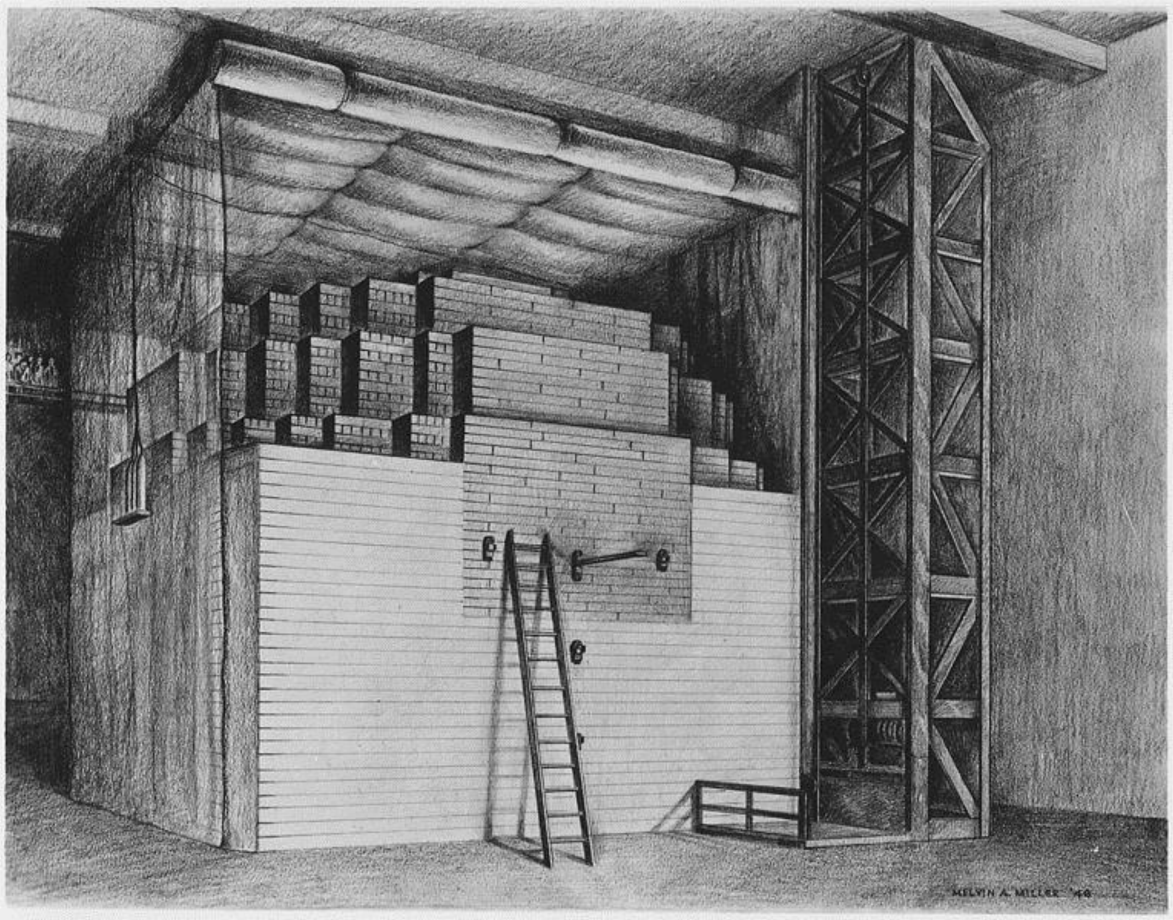


(U: Uranium; Ba: Barium; Kr: Krypton)

Nuclear Reactors



Drawing of the first artificial reactor, Chicago Pile-1

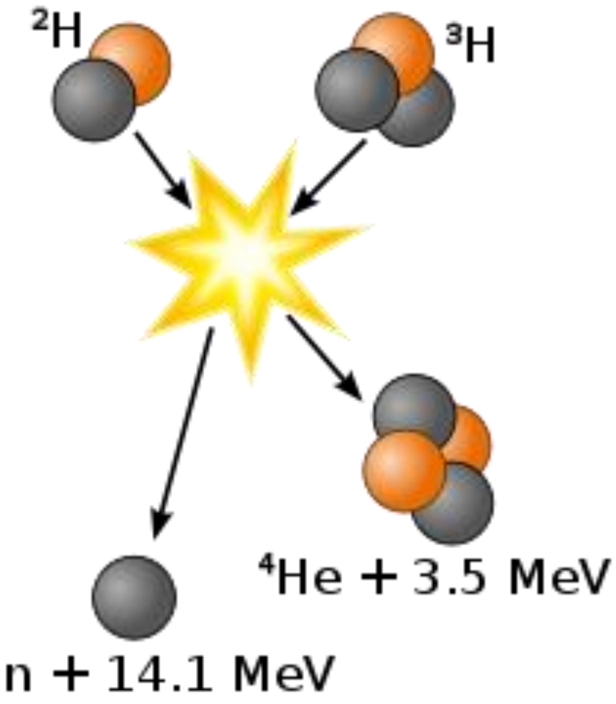


CP-1 was built on a rackets court, under the abandoned west stands of the original Alonzo Stagg Field stadium, at the University of Chicago. The first self-sustaining nuclear chain reaction was initiated in CP-1 on December 2, 1942.

Nuclear Fusion:

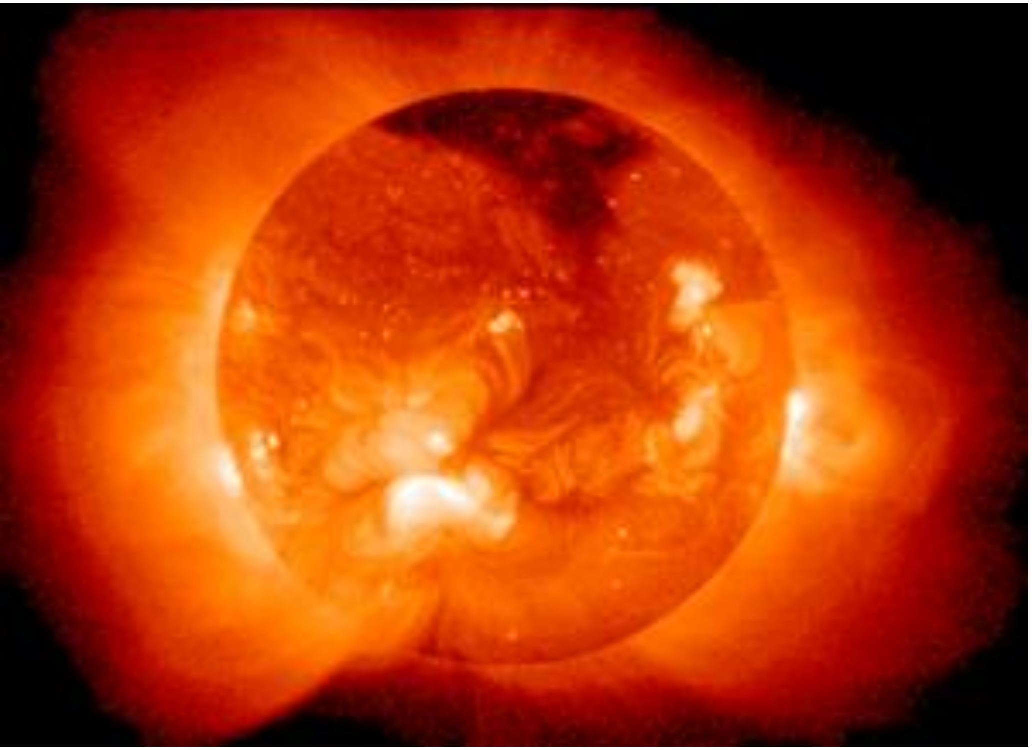
- Two or more small nuclei → single heavier nucleus, other particles, & energy.
- Example:

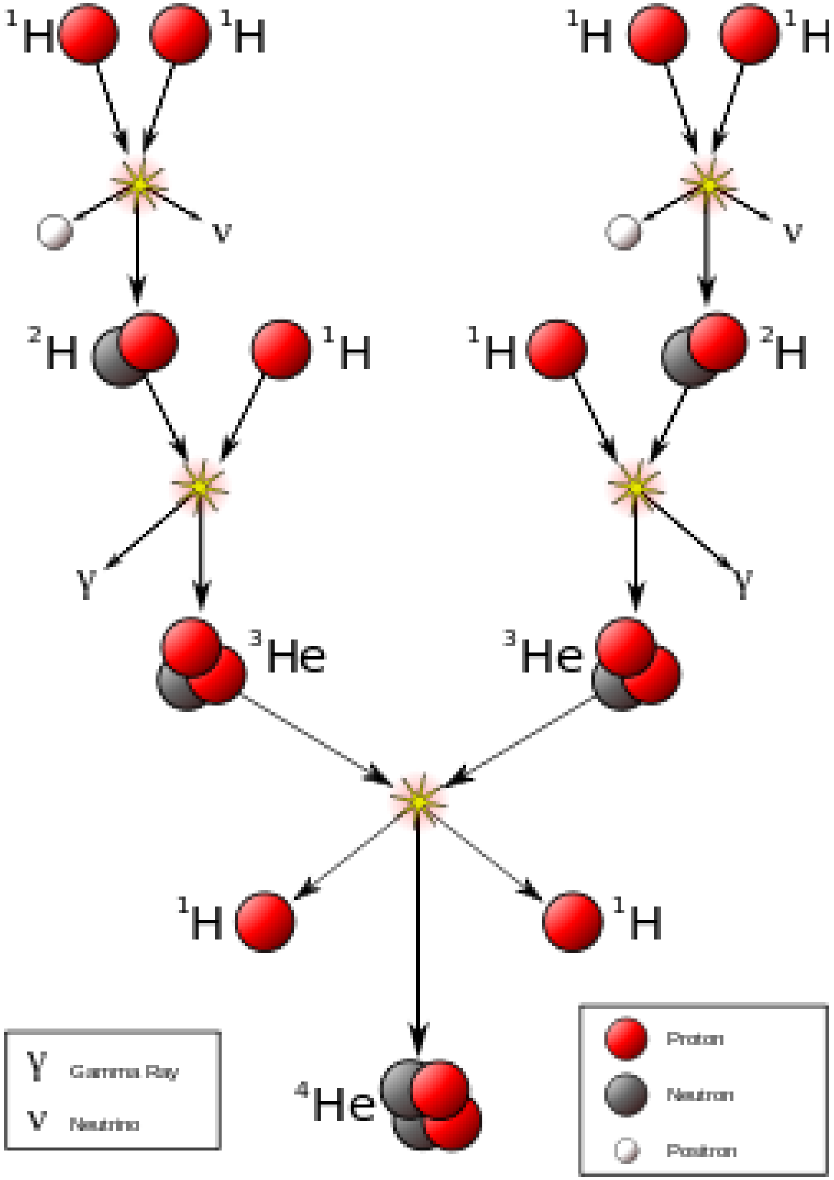
$$ {}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + \mathrm{n} + \text{energy} $$



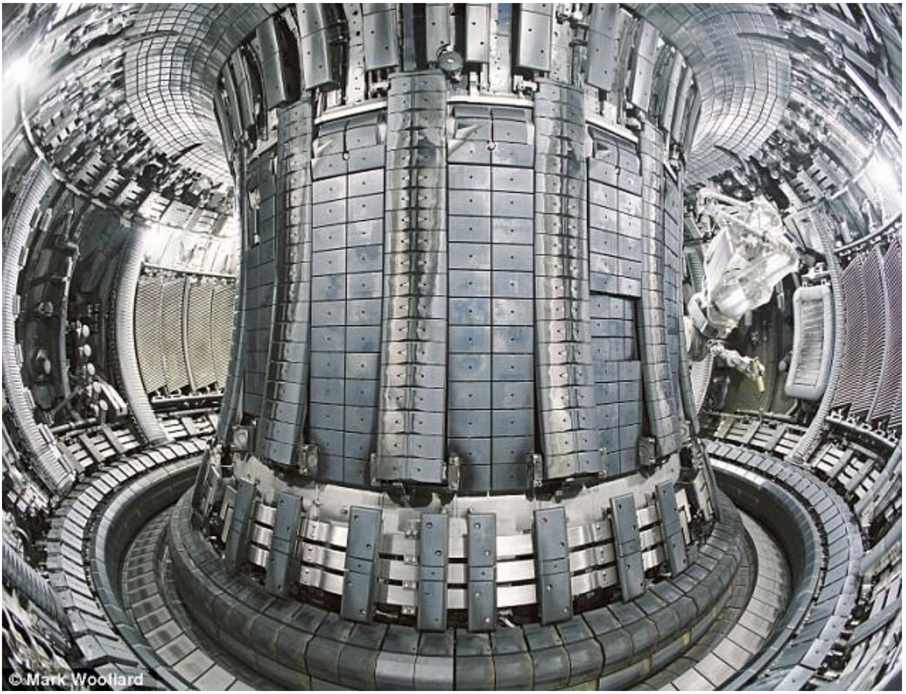
Fusion of deuterium with tritium creating helium-4, freeing a neutron, and releasing 17.59 MeV of energy

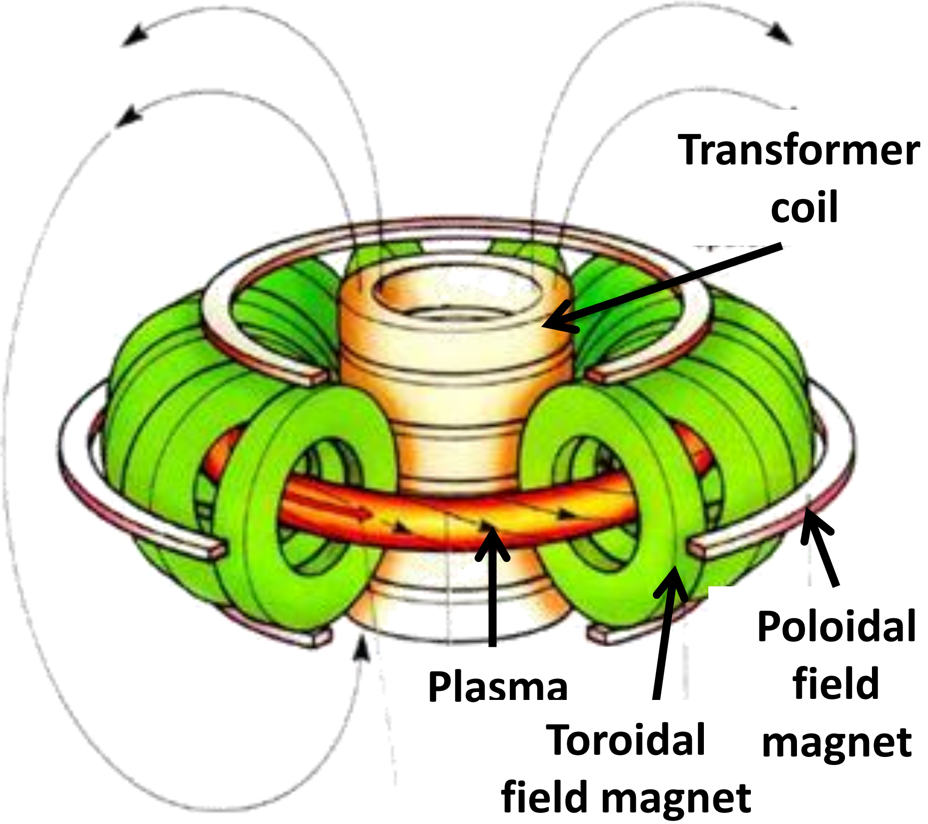
Natural Fusion Reactor: The Sun





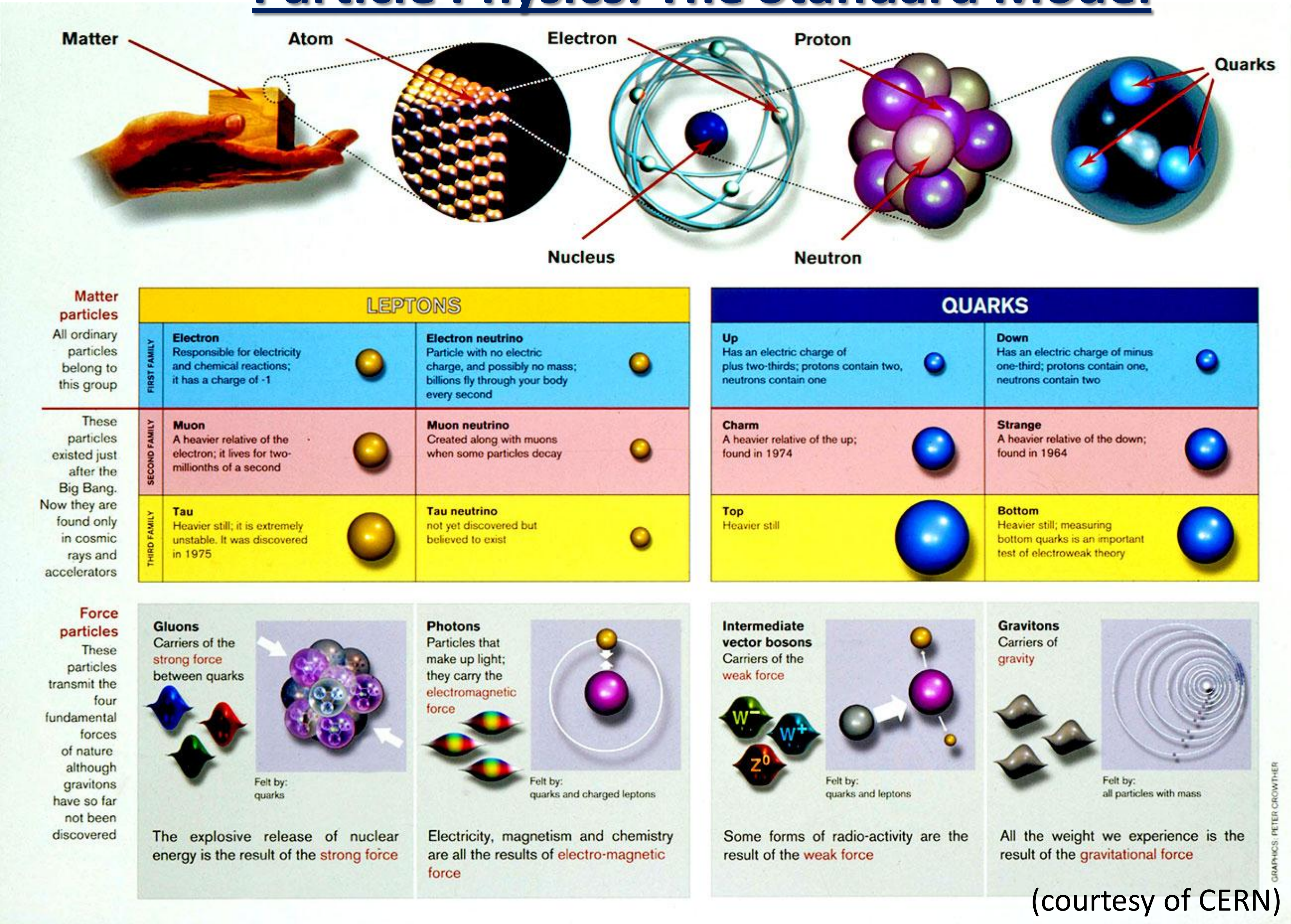
Artificial Fusion Reactors





In 1997, the Joint European Torus produced a peak of 16.1 megawatts (21,600 hp) of fusion power (65% of input power), with fusion power of over 10 MW (13,000 hp) sustained for over 0.5 sec.

Particle Physics: The Standard Model



Matter particles

All ordinary particles belong to this group

These particles existed just after the Big Bang. Now they are found only in cosmic rays and accelerators

	LEPTONS		QUARKS	
FIRST FAMILY	**Electron** Responsible for electricity and chemical reactions; it has a charge of -1	**Electron neutrino** Particle with no electric charge, and possibly no mass; billions fly through your body every second	**Up** Has an electric charge of plus two-thirds; protons contain two, neutrons contain one	**Down** Has an electric charge of minus one-third; protons contain one, neutrons contain two
SECOND FAMILY	**Muon** A heavier relative of the electron; it lives for two-millionths of a second	**Muon neutrino** Created along with muons when some particles decay	**Charm** A heavier relative of the up; found in 1974	**Strange** A heavier relative of the down; found in 1964
THIRD FAMILY	**Tau** Heavier still; it is extremely unstable. It was discovered in 1975	**Tau neutrino** not yet discovered but believed to exist	**Top** Heavier still	**Bottom** Heavier still; measuring bottom quarks is an important test of electroweak theory

Force particles

These particles transmit the four fundamental forces of nature although gravitons have so far not been discovered

Gluons
Carriers of the strong force between quarks

Felt by:
quarks

The explosive release of nuclear energy is the result of the strong force

Photons
Particles that make up light; they carry the electromagnetic force

Felt by:
quarks and charged leptons

Electricity, magnetism and chemistry are all the results of electro-magnetic force

Intermediate vector bosons
Carriers of the weak force

Felt by:
quarks and leptons

Some forms of radio-activity are the result of the weak force

Gravitons
Carriers of gravity

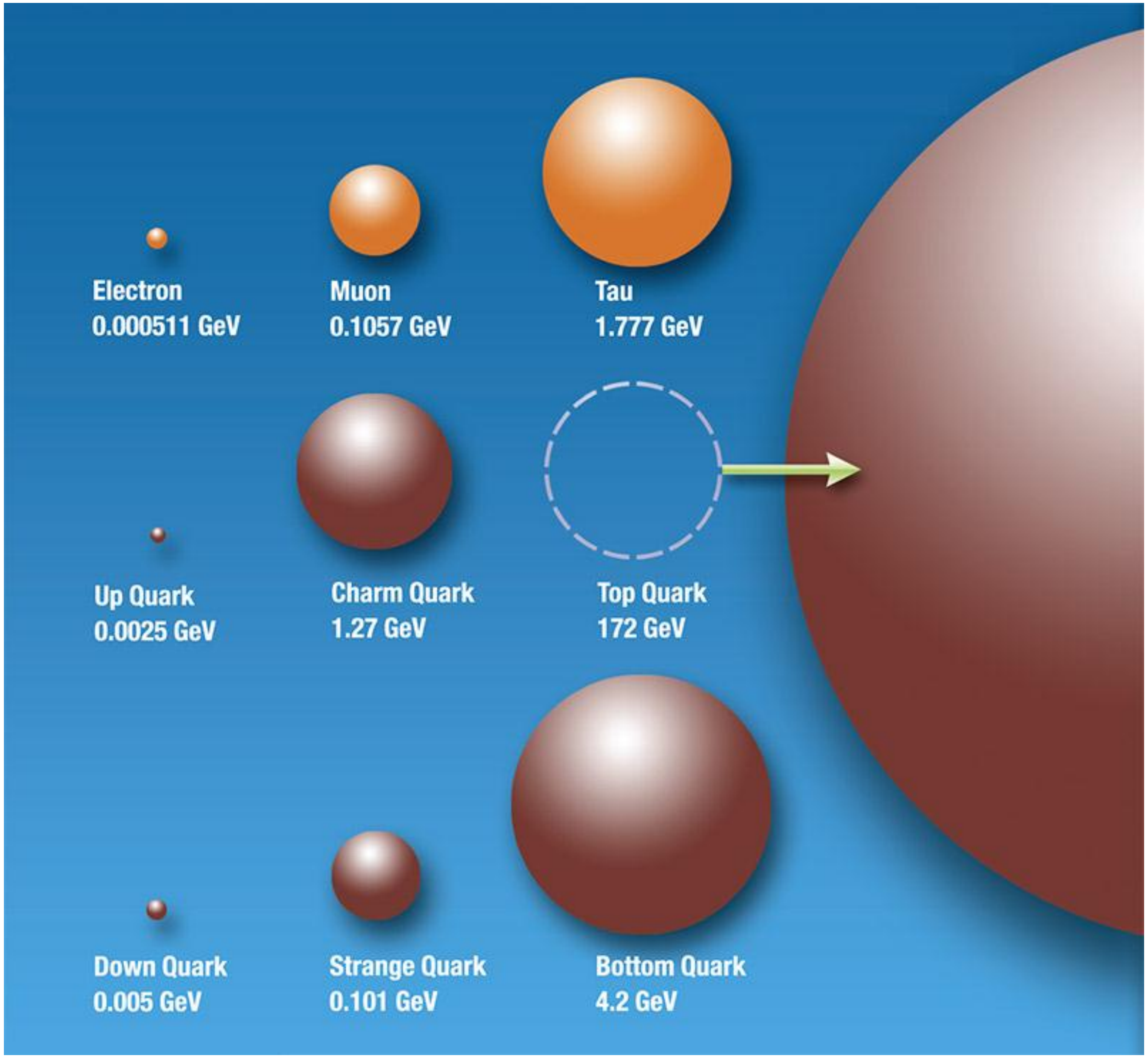
Felt by:
all particles with mass

All the weight we experience is the result of the gravitational force

GRAPHICS: PETER CROWTHER

(courtesy of CERN)

What is the Origin of Mass?

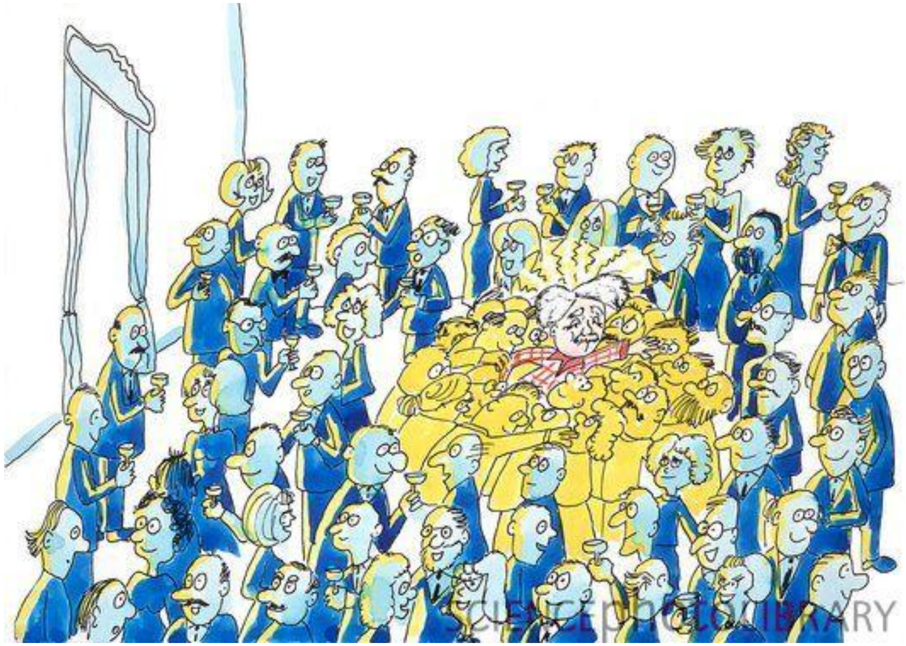


- Fundamental particles do not have any size. Here the different sizes represent the different masses. The masses of neutrinos are so small they would not be visible at this scale.
- Why do fundamental particles have such different masses?
- How do particles gain mass?
- To explain these mysteries, theories predict a new particle, the Higgs particle.

(image courtesy of CERN)

The Higgs Field Theory

To understand the Higgs mechanism, imagine that a room full of physicists chattering quietly is like space filled with the Higgs field …



A well-known scientist walks in, creating a disturbance as he moves across the room and attracting a cluster of admirers with each step … this increases his resistance to movement, in other words, he acquires mass, just like a particle moving through the Higgs field.

(cartoons courtesy of CERN)

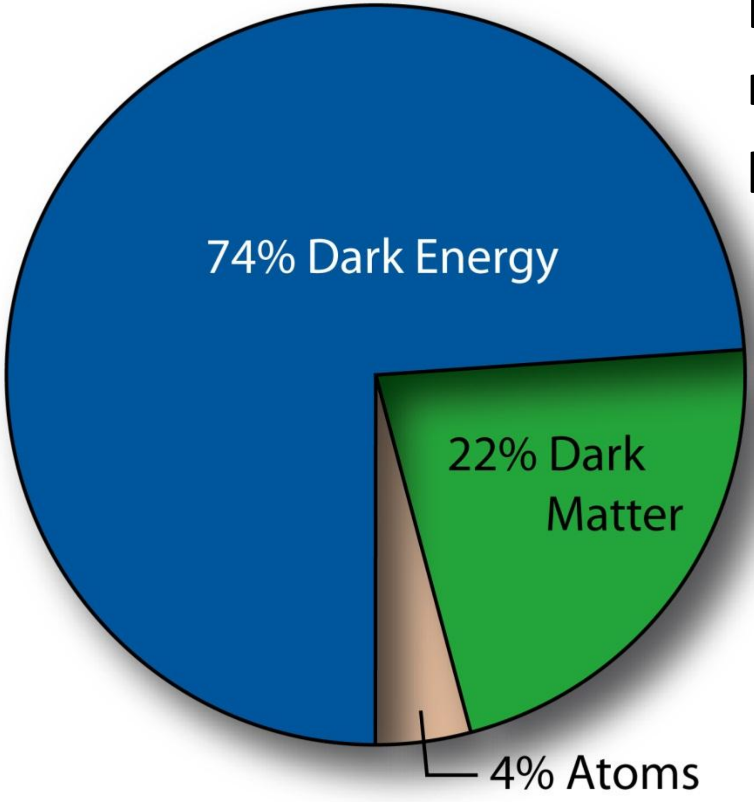
... if a rumor crosses the room,

... it creates the same kind of clustering, but this time among the scientists themselves. In this analogy, these clusters are the Higgs particles.

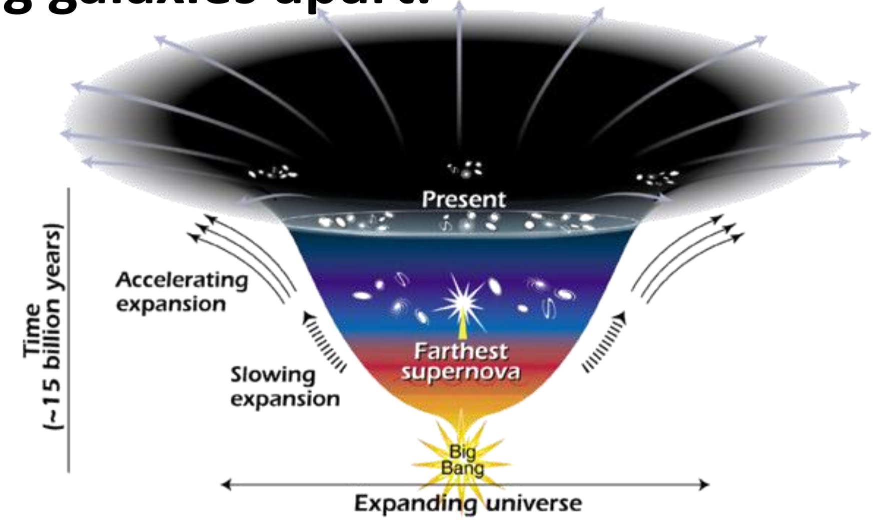
See also:
http://youtu.be/RIg1Vh7uPyw

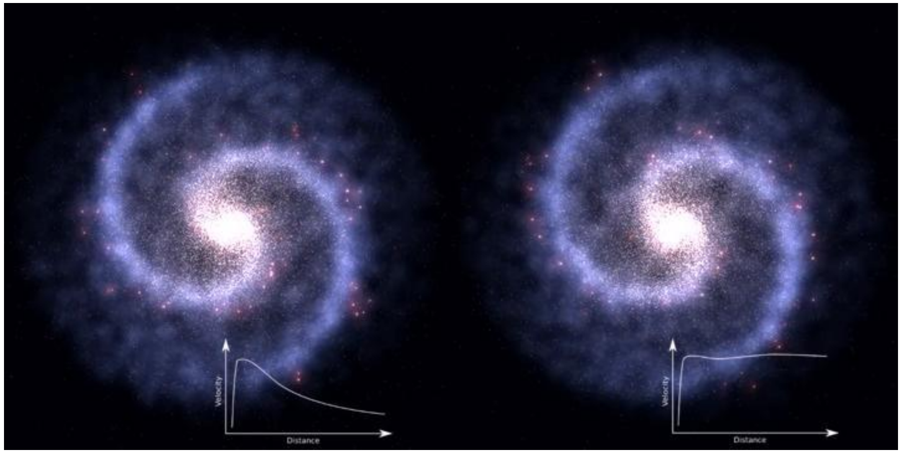
(cartoons courtesy of CERN)

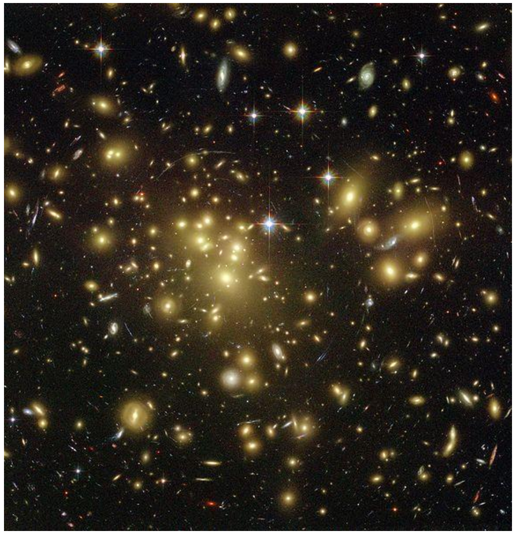
What is the Universe made of?



Faster expansion rate is attributed to a mysterious, dark energy / force that is pulling galaxies apart.







Gravitational lensing and galaxy rotation speeds indicates the presence of dark matter

(image courtesy of NASA)