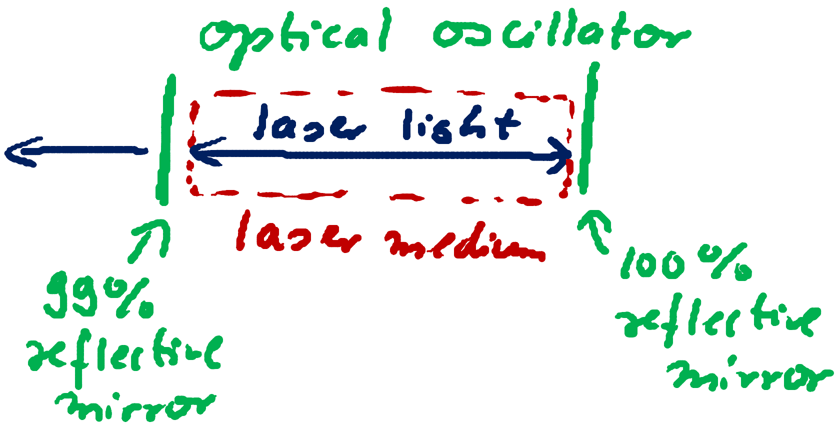
Recap I

Lecture 39

- **LASER**

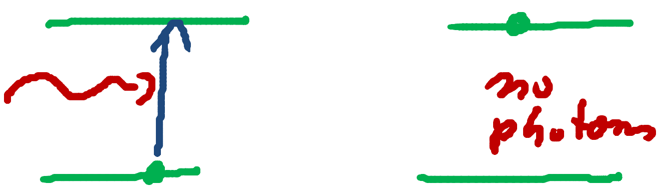


Laser Light is:

- highly monochromatic
- highly coherent
- highly directional
- can be sharply focused

⇒ Two competing processes:

Photon absorption



Stimulated Emission

⇒ for amplification of laser light in medium, need more atoms in the higher energy state than in the lower state

⇒ need population inversion, created artificially by "pumping" (energy input)

Recap II

- Particle Waves:
  - All particles have wave like and particle like properties!
  - A particle with momentum p has a "particle wave" associated with its motion with wavelength

$$\boxed{\lambda = h/p}$$ (de Broglie wavelength)

$\Rightarrow$ for particles with mass $m > 0$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{hc}{\sqrt{2E_0K}}$$

Kinetic energy

"rest mass energy" $E_0 = mc^2$

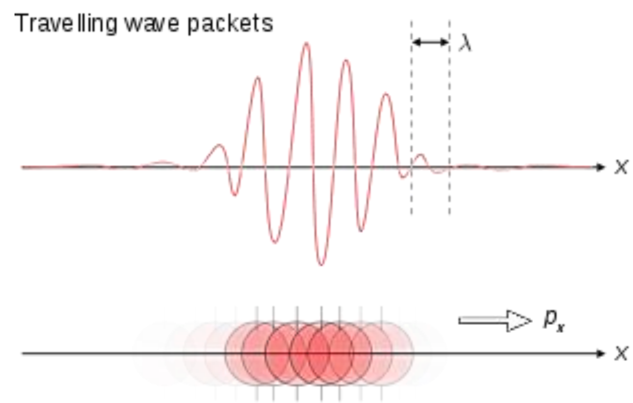
$\Rightarrow$ for photons:

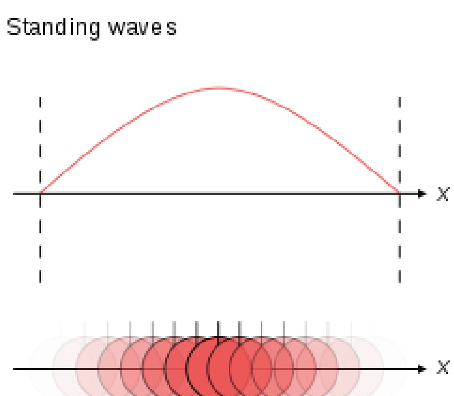
$$\lambda = \frac{h}{P_{Photon}} = \frac{hc}{E_{Photon}}$$

$$P_{Photon} = \frac{E_{photon}}{c}$$

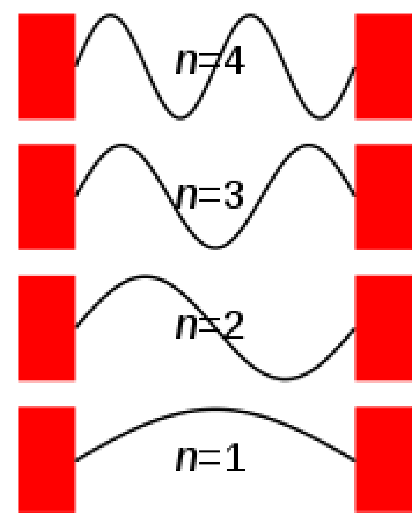
Today:

- **<u>More quantum mechanics</u>**
  - **Particle wave functions**
  - **Probabilities and uncertainty**
  - **Schrödinger's equation**
  - **Free particle**
  - **1-D infinite square well**





$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 \, \mathrm{d}x = 1$$



Quantum Mechanics

Wave function Ψ and Schrödinger's equation

- The (particle/matter) wave function Ψ [psi]

$$\Psi(x, y, z, t)$$

- function of position and time
- complex function in the mathematical sense:

$$\Psi = R(x,y,z,t) + i\,J(x,y,z,t)$$

  where R and J are both real functions, and $i^2 = -1$
- computational device, Ψ itself has no physical existance
- but: contains all the information about the particle! (position, momentum, energy...)

How? Example: Position (1-D case)

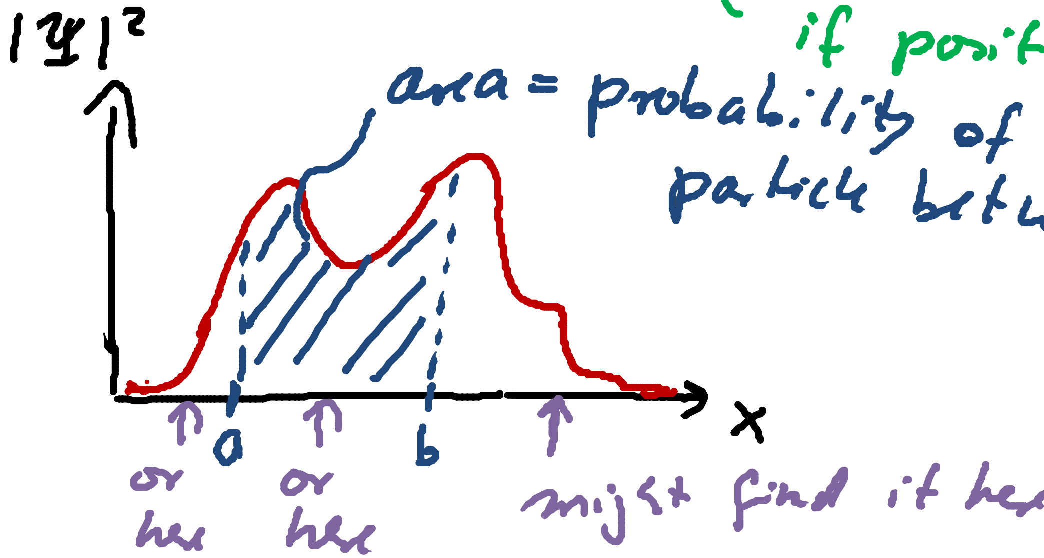
Born's statistical interpretation of the wave function:

If, at time t, a measurement is made to locate the position of a particle associated with the wave function $\Psi(x,t)$, then the probability $P(x,t)\,dx$ that the particle will be found at a coordinate between x and $x+dx$ is equal to:

$$\boxed{\underbrace{P(x,t)}_{\text{"probability density function"}}dx = |\underbrace{\Psi}_{}|^2\,dx \geq 0}$$

$|\Psi|^2 = [\text{absolute value of } \Psi]^2$

$$\Rightarrow \int_a^b |\Psi(x,t)|^2\,dx = \left\{ \begin{array}{l} \text{probability of finding} \\ \text{the particle between} \\ a \text{ and } b \text{ at time } t \\ \text{if position is measured} \end{array} \right\}$$



area = probability of finding particle between a and b

$\Rightarrow$ Probability of finding the particle somewhere must be 1

so require : $\boxed{\int_{-\infty}^{+\infty} |\Psi|^2\,dx = 1}$ Normalization condition

How to find the wave function of a given particle?

particle wave equation (differential equation) tells how Ψ changes with position and time $\longrightarrow$ solution of wave equation: wave function Ψ ("particle wave")

$\Rightarrow$ For non-relativistic particles:

particle wave equation = Schrödinger's equation

For a particle moving along x-direction with potential energy $U(x)$:

total mechanical energy of particle = kinetic + potential energy

$$\boxed{-\frac{h^2}{8\pi^2 m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\,\psi(x)}$$

time-independent Schrödinger equation

mass of particle

with $\Psi(x,t) = \psi(x)\,e^{-i\omega t}$

angular frequency of particle wave

Example 1: Free particle ($U(x)=0$) with momentum p

$U=0 \Rightarrow E = K + U = K = \frac{1}{2} m v^2 = \frac{p^2}{2m}$

$\Rightarrow$ Schrödinger's equation becomes: $-\frac{h^2}{8\pi^2 m}\frac{d^2\psi(x)}{dx} = \frac{p^2}{2m}\psi(x)$

$$\Rightarrow \frac{d^2\psi}{dx^2} = -\left(2\pi\frac{p}{h}\right)^2\psi$$

$$\Rightarrow \boxed{\frac{d^2\psi}{dx^2} = -k^2\psi}$$

Schrödinger's equation for a free particle

since $\lambda = h/p$

and $k = \frac{2\pi}{\lambda} = 2\pi\frac{p}{h}$

wave number of particle wave

$\Rightarrow$ General solution of this differential equation:

$$\psi(x) = A e^{ikx} + B e^{-ikx}$$

constants

$\Rightarrow$ get for the time dependent wave function $\Psi(x,t)$ of a free particle with definite momentum p:

$$\Psi_{\text{free particle}}(x,t) = \psi(x)\, e^{-i\omega t}$$

$$= \underbrace{A\, e^{i(kx-\omega t)}}_{\text{wave/particle traveling in } +x \text{ direction}} + \underbrace{B\, e^{-i(kx+\omega t)}}_{\text{wave/particle traveling in } -x \text{ direction}}$$

!

!

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math:

$$e^{i\theta} = \cos\theta + i \sin\theta$$

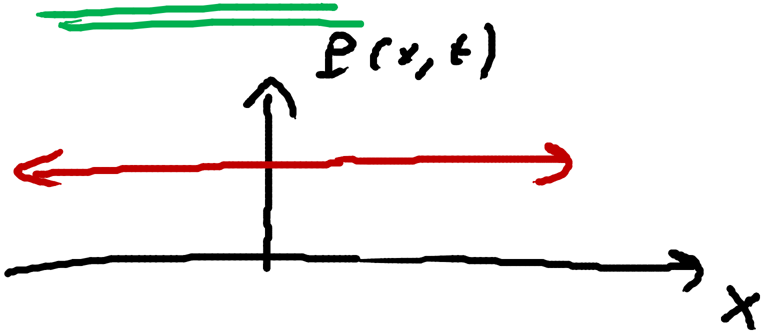
$$|e^{i\theta}| = 1 \quad \Rightarrow \quad |e^{i\theta}|^2 = 1 \quad \underline{\text{always}}$$

⇒ wave function of a free particle with **momentum** **p** moving in $+x$ direction:

$$\Psi(x,t) = A\,e^{i(kx-\omega t)}$$

⇒ Probability density function of this particle

$$\rho(x,t) = |\Psi(x,t)|^2 = |A|^2 \underbrace{\left|e^{i(kx-\omega t)}\right|^2}_{=1} = |A|^2 = \text{const!}$$



⇒ same for **all** values of x !?

⇒ equal probability of finding the particle anywhere along the x-axis

Note:

free particle with exact (definite) momentum p (no uncertainty in momentum p) ⟶ particle can be found anywhere with equal probability (infinitely great uncertainty in position of particle)

Heisenberg's Uncertainty Principle:

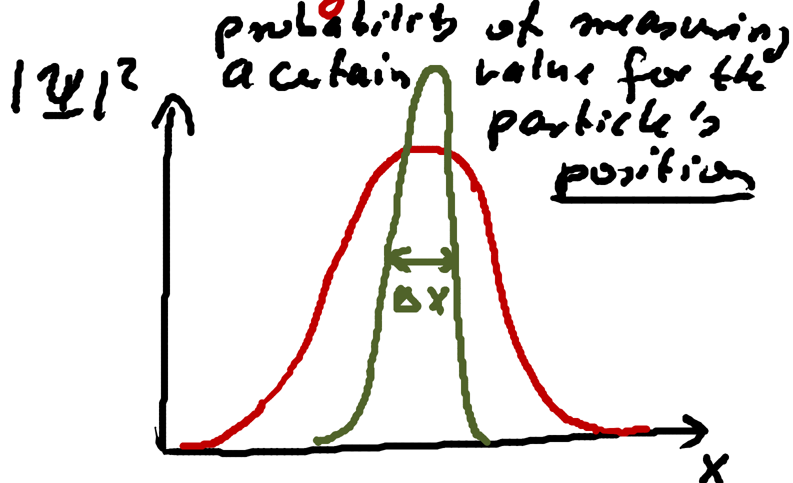
How then can a particle ever be localized?

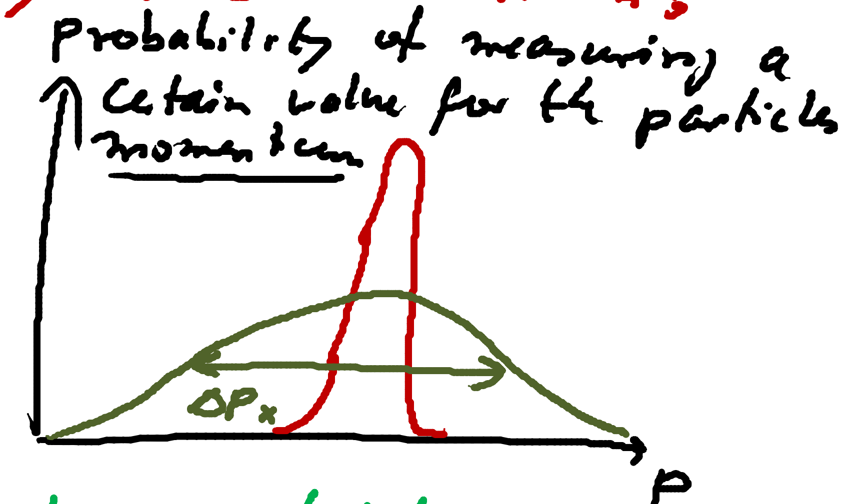
$\Rightarrow$ Need to add up wavefunctions with particle (de Broglie) wavelengths λ

$\Rightarrow$ get uncertainty in wavelength λ of particle

$\Rightarrow$ get uncertainty in momentum $p = h/\lambda$!

$\Rightarrow$ The more determined the position of a particle the larger the uncertainty is in its momentum!





$\Rightarrow$ small uncertainty Δx in position of particle

$\Rightarrow$ large uncertainty Δp_x in x-component of momentum!