

Recap I

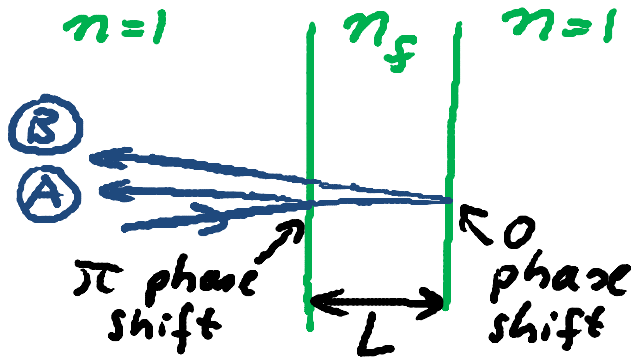
Lecture 33

- Sources of phase shifts between two waves: $\Delta\phi_{A,B} = \phi_B - \phi_A$
 - (1) waves start out with different phases
 - (2) path length difference: $\Delta\phi = \frac{2\pi}{\lambda} (\Delta \text{path length})$
 - (3) waves travel through mediums with different index of refraction: $\lambda = \frac{\lambda_{\text{vacuum}}}{n}$
 - (4) phase shift upon reflection

Reflection Type	Phase Shift
slow \rightarrow fast $n_{\text{incident}} > n_{\text{transm.}}$	0
fast \rightarrow slow $n_{\text{incident}} < n_{\text{transm.}}$	π

Recap II

• Thin-Film Interference:



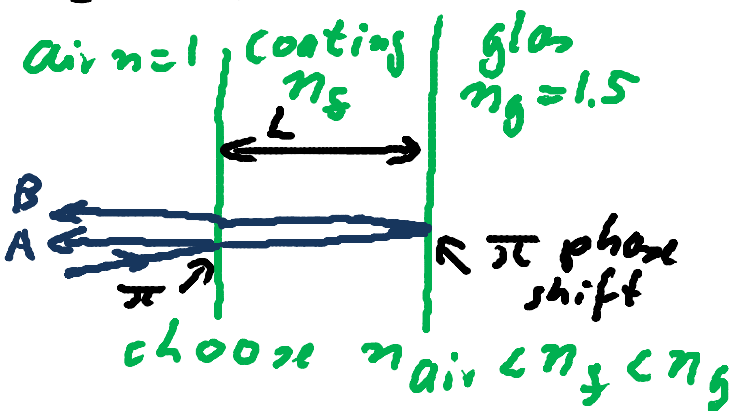
Total phase shift between reflected waves (A) and (B) results from

- Path length difference = $2L$
 - Different wavelength in film: $\lambda_f = \frac{\lambda_{vacuum}}{n_f}$
 - Phase shifts upon reflection
- \Rightarrow here:

$$2L \text{ constructive, } m = \left(m + \frac{1}{2}\right) \frac{\lambda_{vacuum}}{n_{film}}$$

$$2L \text{ destructive, } m = m \frac{\lambda_{vacuum}}{n_{film}}$$

$$m = 0, 1, 2, \dots$$



Anti reflective Coating on Glas:

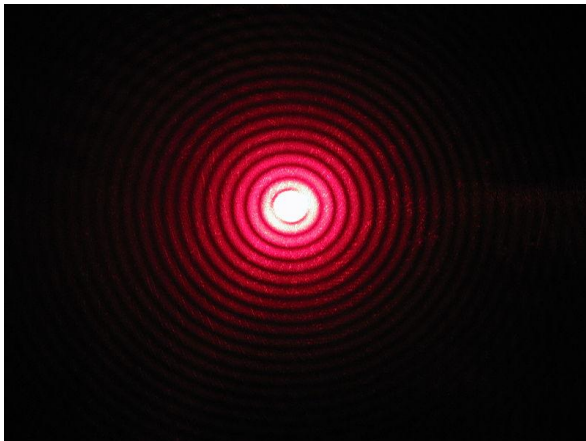
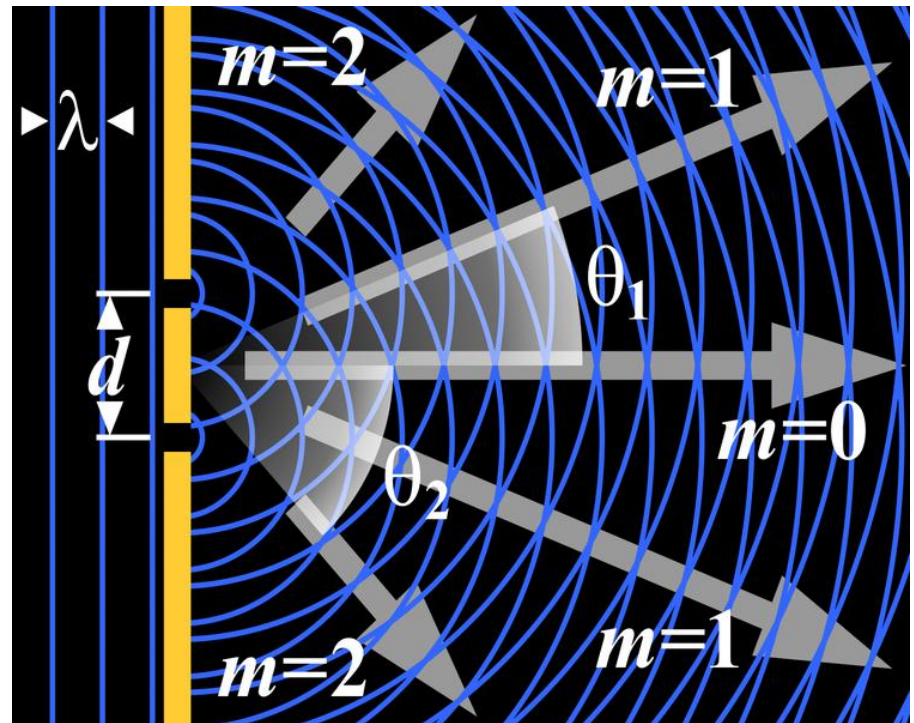
choose L to get destructive interference between waves (A) and (B) for visible light:

$$2L \text{ destructive, } m = \left(m + \frac{1}{2}\right) \frac{\lambda_{vacuum}}{n_{film}} \quad m = 0, 1, 2, \dots$$

Today:

- **Diffraction**

- Single slit
- Circular aperture
- Double slit (again)



What is the smallest object (finest detail) the human eye can resolve?

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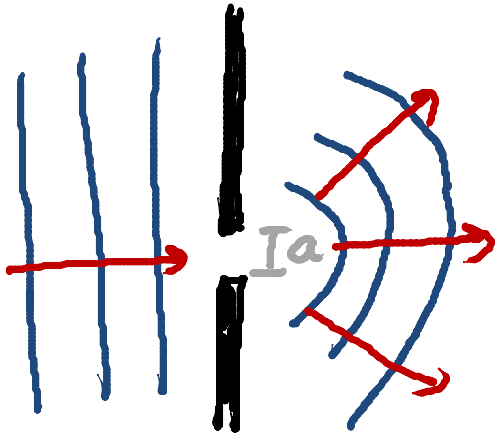
*~ 0.01° angular resolution
=> ~ 0.05 mm at near point
distance of 25 cm*

- A. ~1 mm
- B. ~0.5 mm
- C. ~0.05 mm**
- D. ~0.005 mm

Diffraction:

Wavefronts are 'bent' near edges & apertures.

Example: Diffraction of wave passing through a narrow slit:



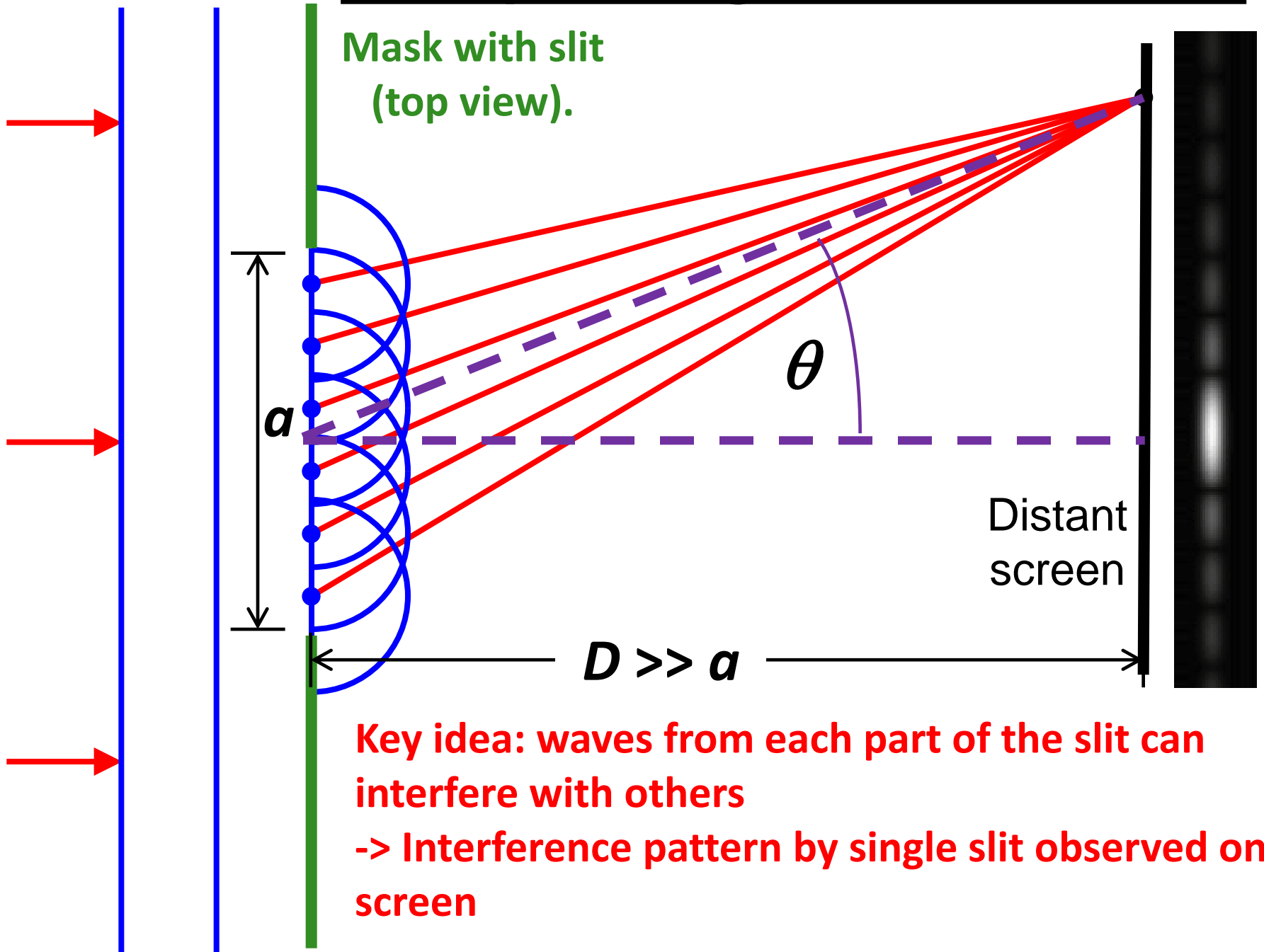
Recall: Huygens' Principle

All points on a wavefront
act as point sources

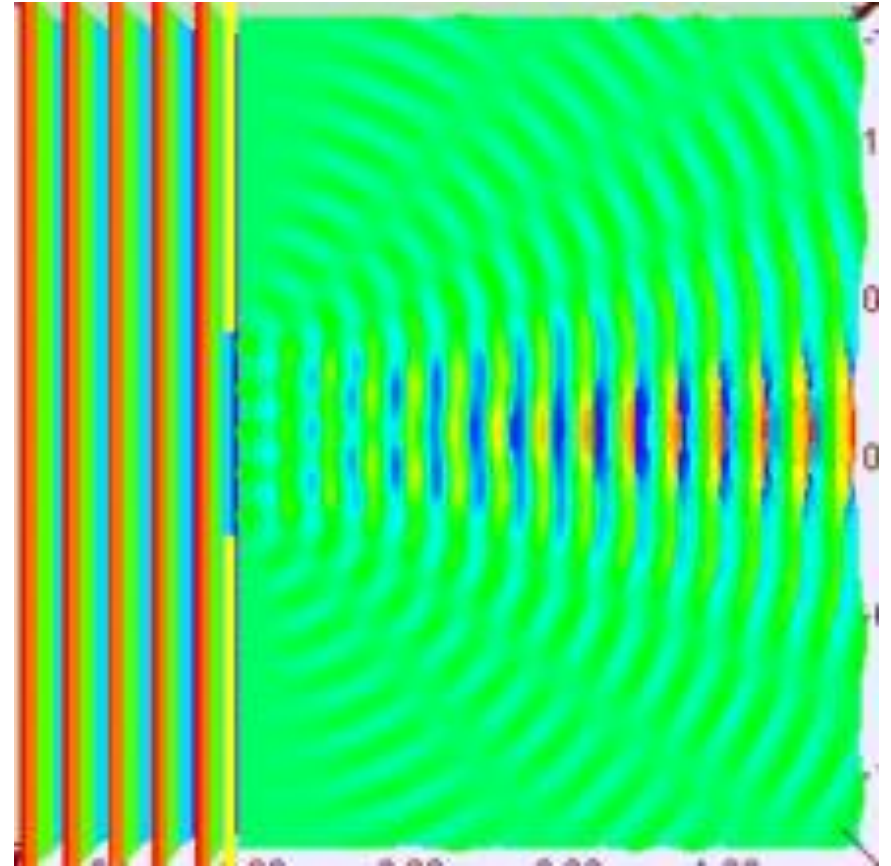
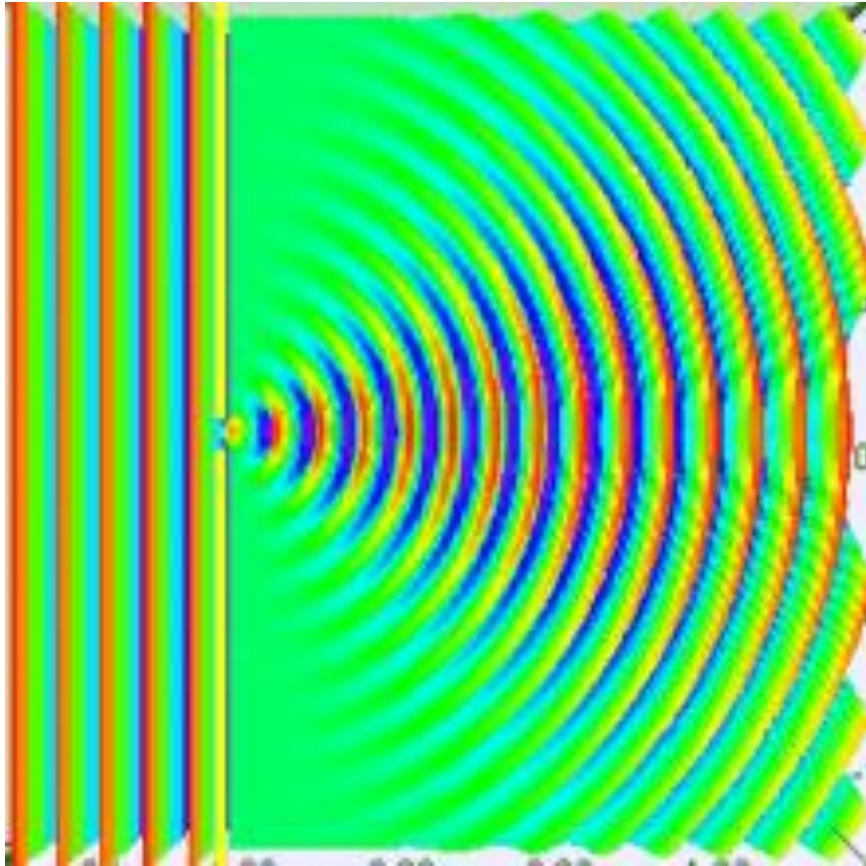
⇒ Light emerges in all directions
rather than just passing straight
through the narrow slit!

⇒ Important for apertures/objects/slits
of size $\sim \lambda$.

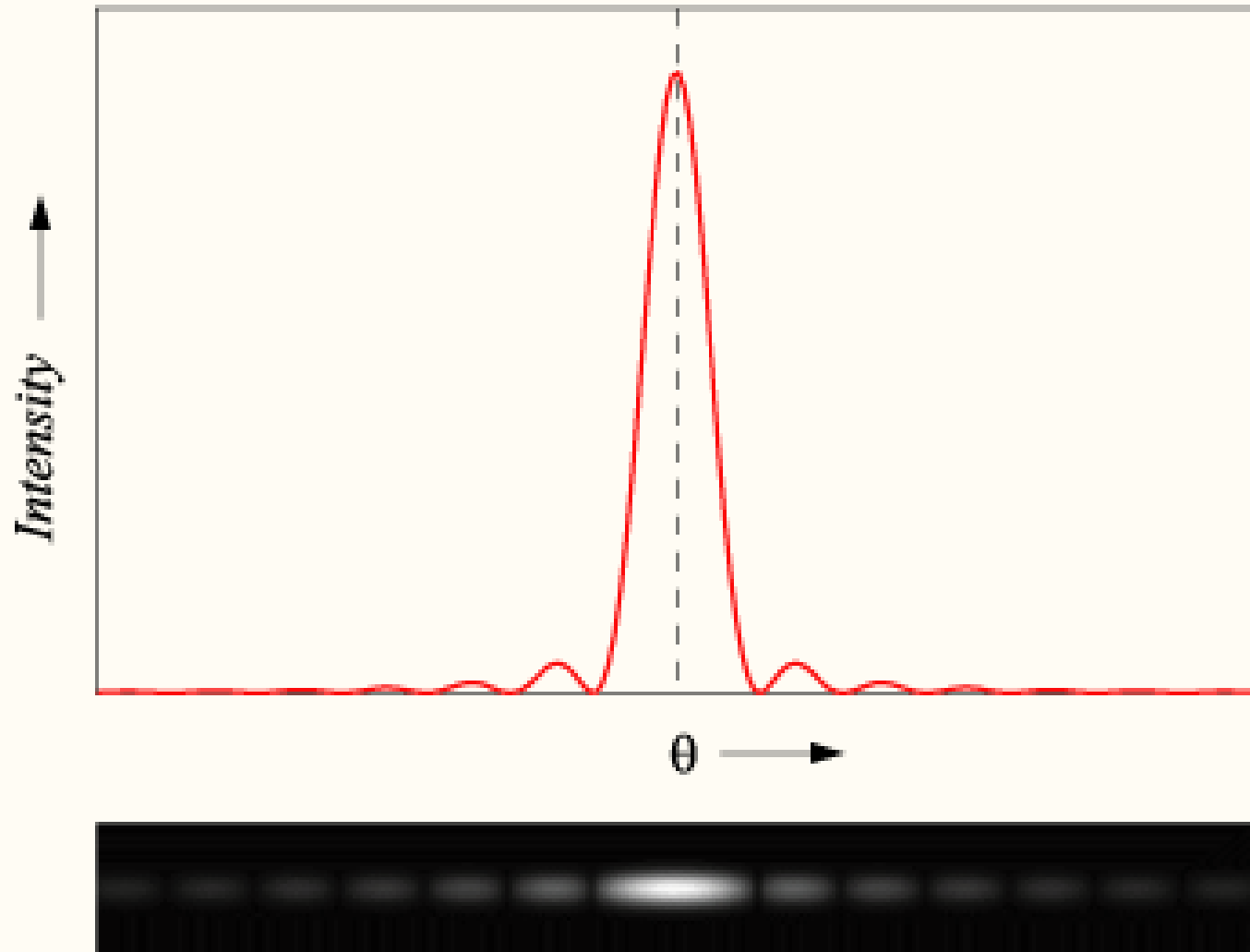
Example: Single-slit diffraction:



Single slit diffraction:

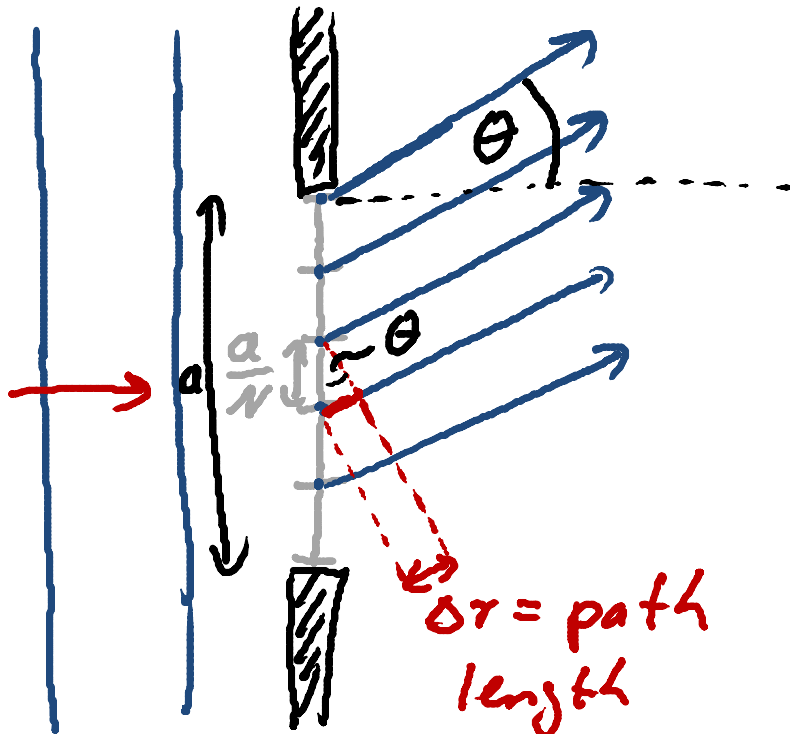


Single-slit diffraction pattern



Interference Pattern from a Single Slit:

slit



$\Delta r =$ path length difference between adjacent rays

- Break slit into N zones, each of width a/N
- Rays from different zones interfere on distant screen
Path length difference between two adjacent rays:

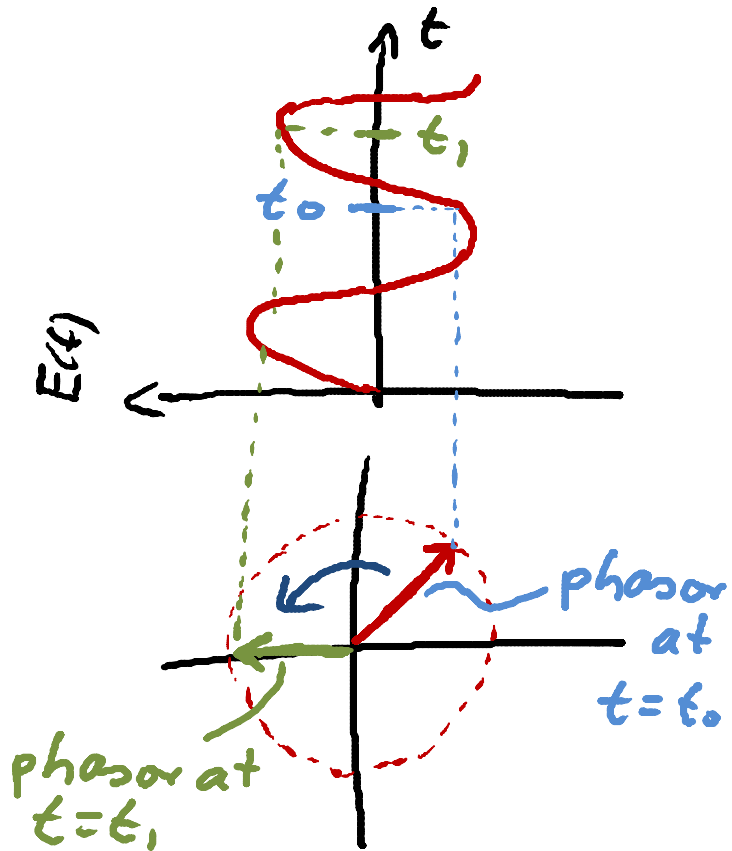
$$\Delta \text{path} = \frac{a}{N} \cdot \sin \theta$$

\Rightarrow phase difference between waves of adjacent rays:

$$\Delta \phi_{\text{adjacent rays}} = \frac{2\pi}{\lambda} \Delta \text{path} = \frac{2\pi}{\lambda} \frac{a}{N} \sin \theta$$

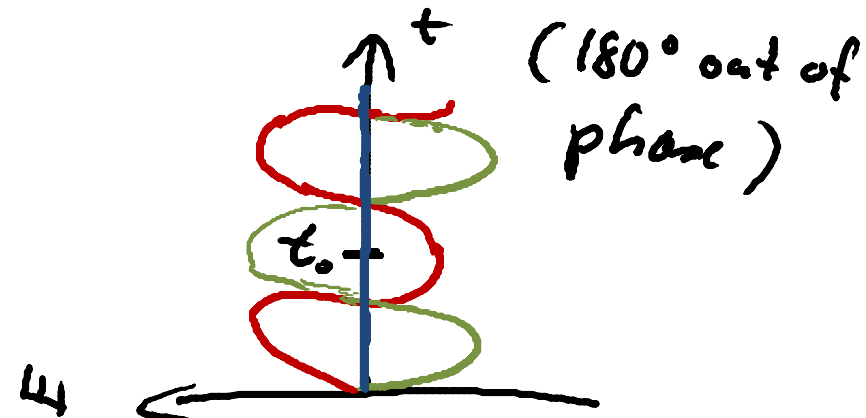
$\propto \sin \theta$

Side Note: Phasors

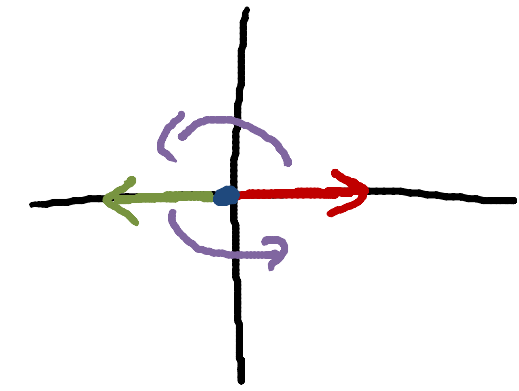


$$E(t) = E_{max} \cos(kx_0 - \omega t + \phi)$$

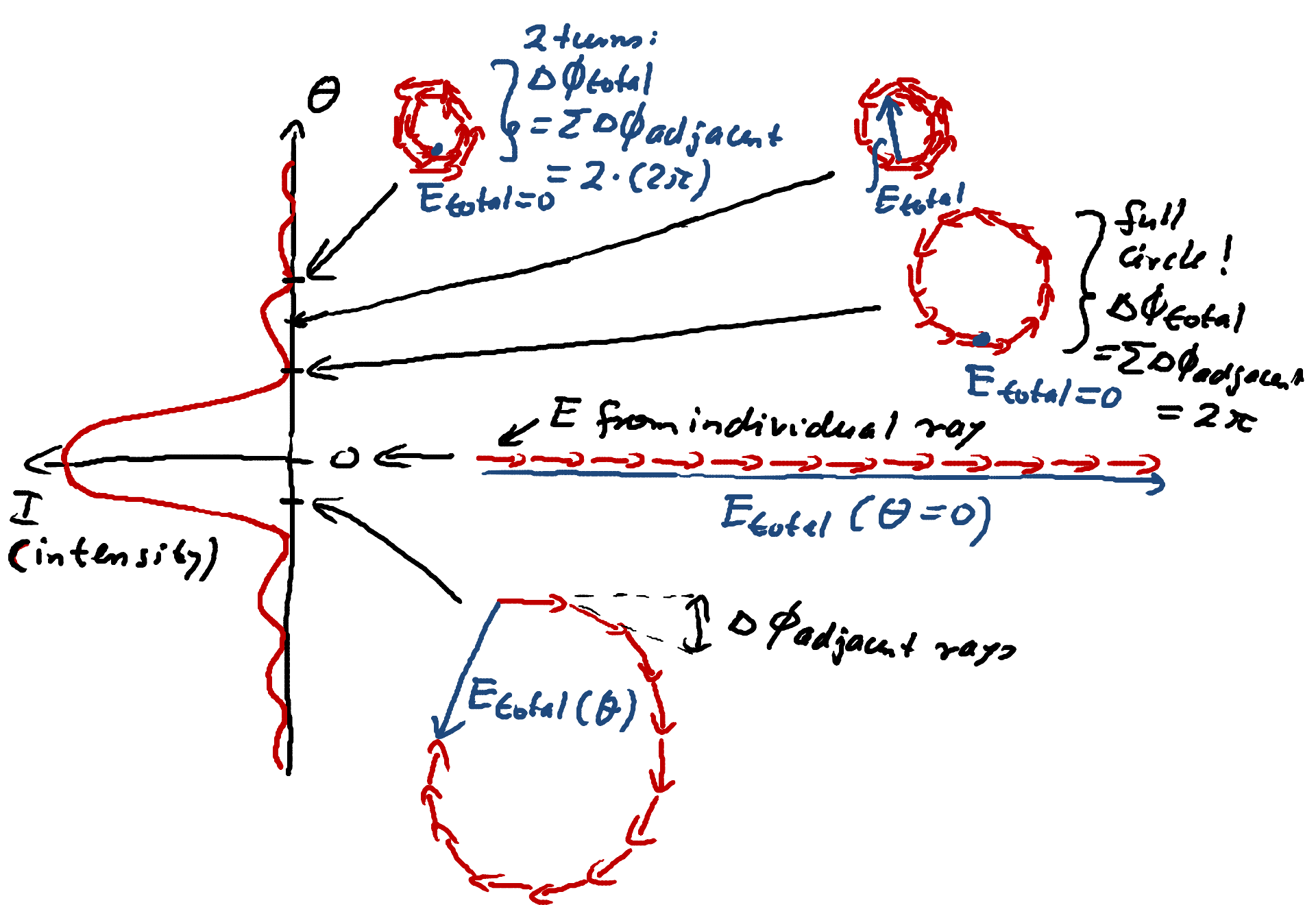
to add two waves:



at t = t_0



$$E_{sum} = 0 = E_1(t) + E_2(t)$$



⇒ Constructive Interference at highest intensity
at $\theta = 0$

⇒ Dark Fringes (intensity minima) each time
the electric field vectors of the individual waves
add up to zero:

$$\Rightarrow \text{for } \Delta\phi_{\text{total}} = \sum_{i=1}^N \Delta\phi_{i, \text{adjacent rays}} = m(2\pi)$$

$$m = \pm 1, \pm 2, \pm 3 \dots$$

but not $m = 0$

$$\Rightarrow \Delta\phi_{\text{total}} = N \cdot \Delta\phi_{\text{adjacent rays}}$$

$$= N \cdot \frac{2\pi}{\lambda} \frac{a}{N} \sin\theta = m(2\pi)$$

Minima for
single slit
diffraction

$$: \boxed{a \sin\theta = m\lambda}$$

↑
width of slit

$m = \pm 1, \pm 2, \pm 3 \dots$
(but not $m = 0$)

⇒ Also can find the single-slit diffraction intensity pattern: $I(\theta) \propto (E_{\text{total}})^2$

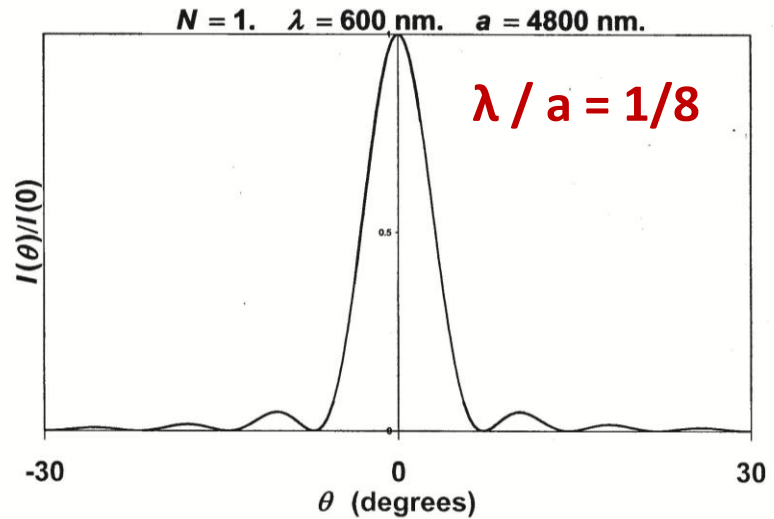
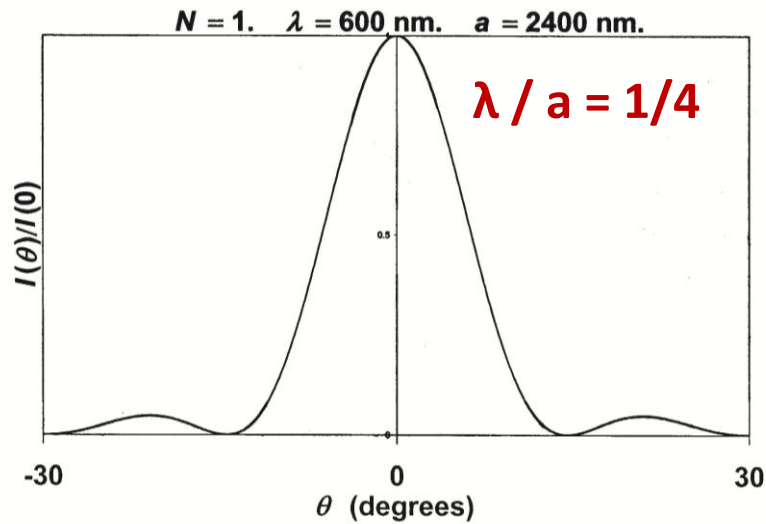
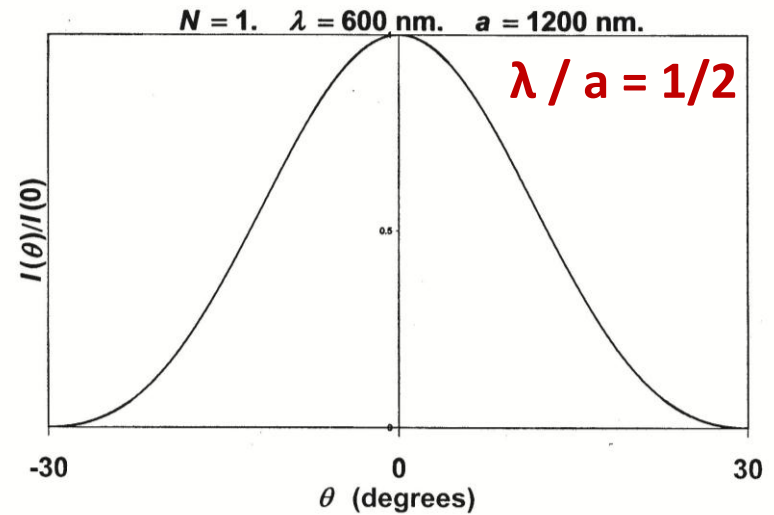
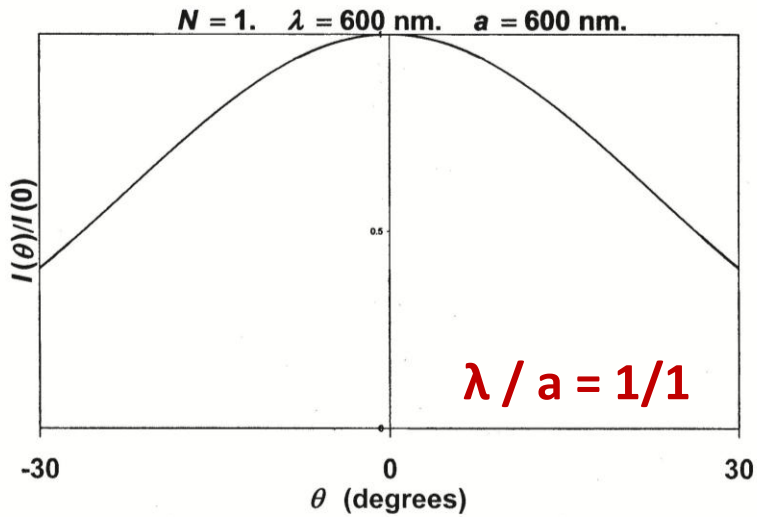
$$I(\theta) = I_{\text{max}} \left(\frac{\sin \alpha}{\alpha} \right)^2$$

where $\alpha = \frac{\pi a}{\lambda} \sin \theta$

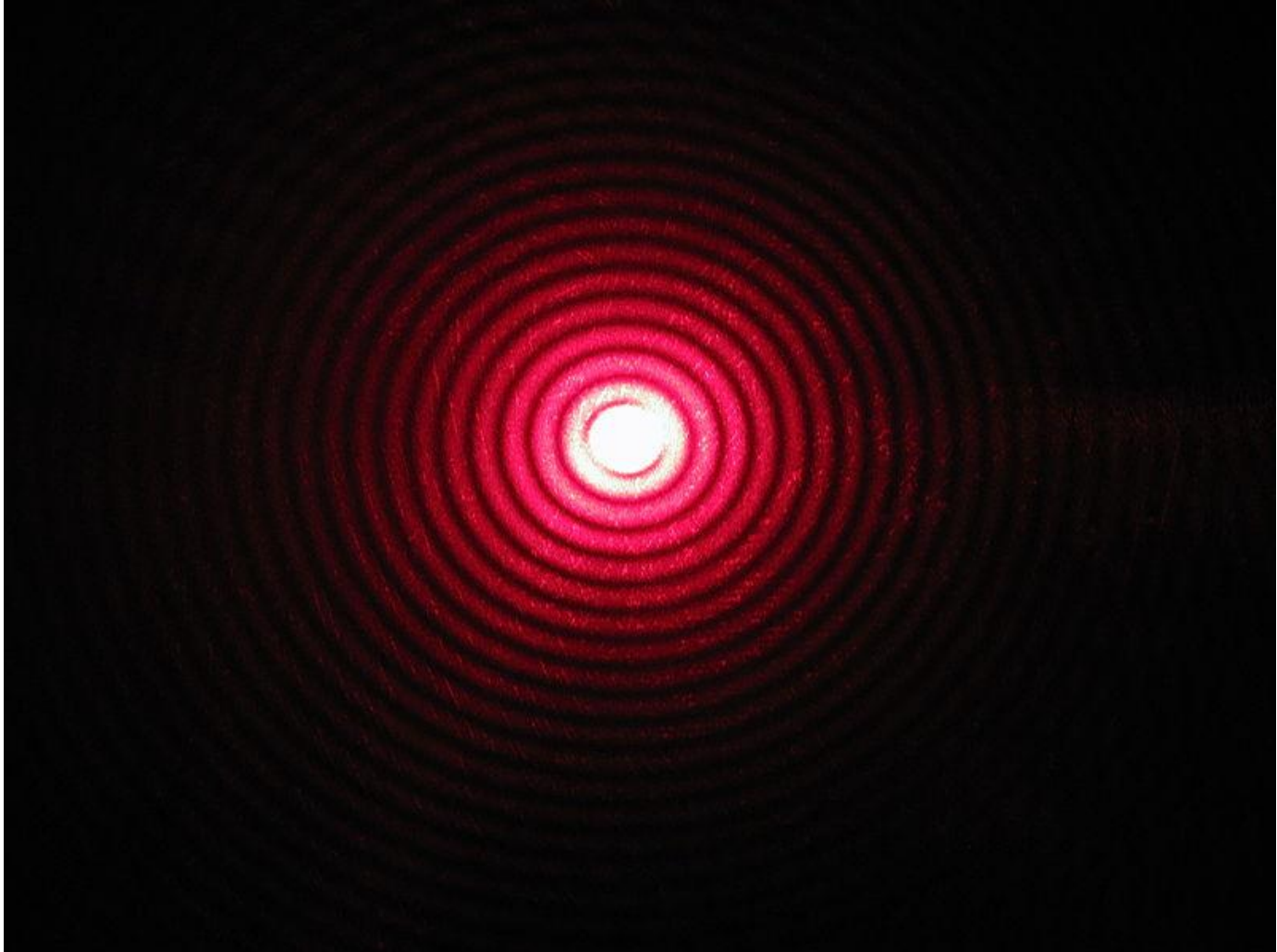
Note: wider slit (larger a) → more, sharper intensity maxima and minima

minima: $\sin \theta_{\text{min}} = m \frac{\lambda}{a} \leq 1$

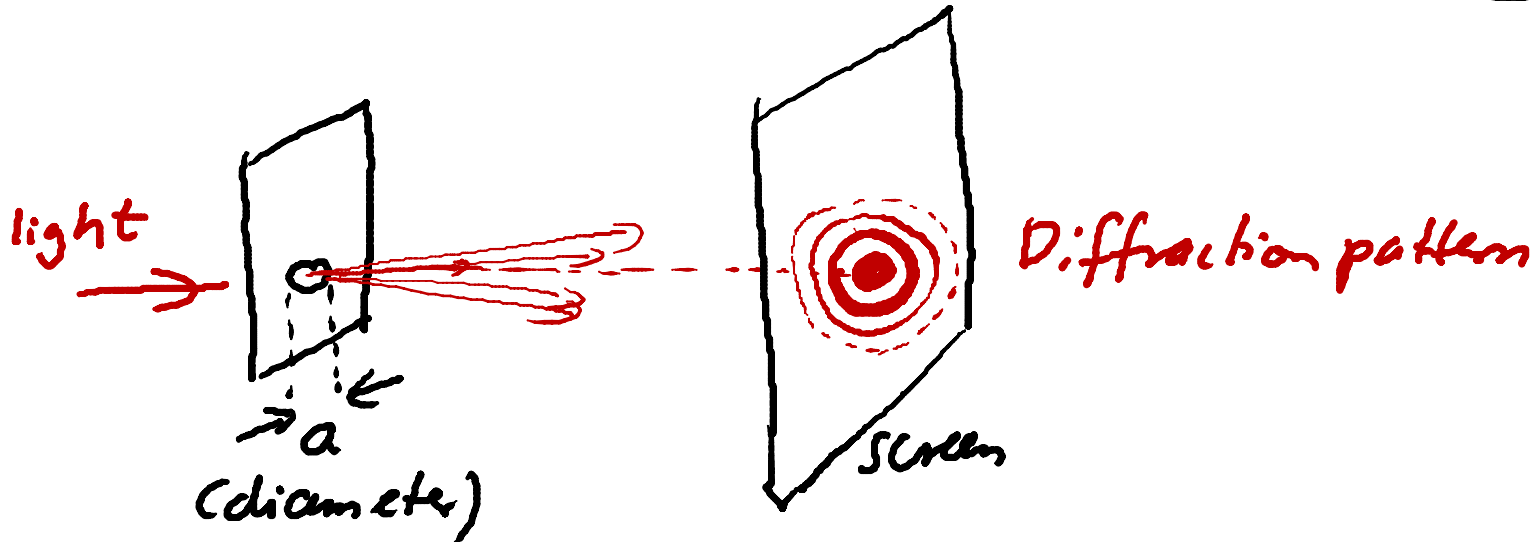
Single-slit diffraction pattern for different slit widths:



Diffraction of red laser beam on a Hole (Circular Aperture)



Diffraction by a circular Aperture



Interference of waves diffracted by the hole results in circular intensity pattern on the screen.

⇒ 1st intensity minimum at angle:

$$\sin \theta_1 = 1.22 \frac{\lambda}{a}$$

a = diameter of hole

