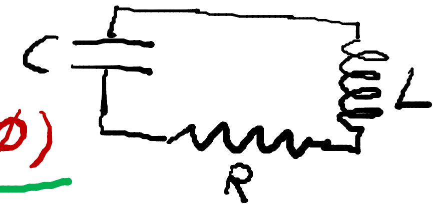
Recap I

- RLC – circuit:

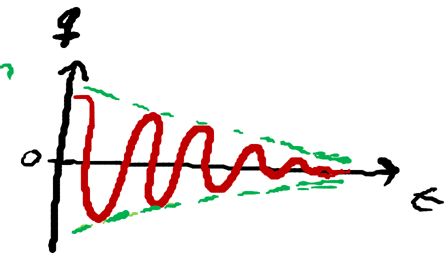


– undriven: charge: $q(t) = \underbrace{Q_0 e^{-t/\tau}} \underbrace{\cos(\omega' t + \phi)}$

damping with energy decay time constant $\tau = L/R$ due to power lost in resistor R

oscillation with $\omega' = \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2}$

$\approx \omega_0 = \frac{1}{\sqrt{LC}}$



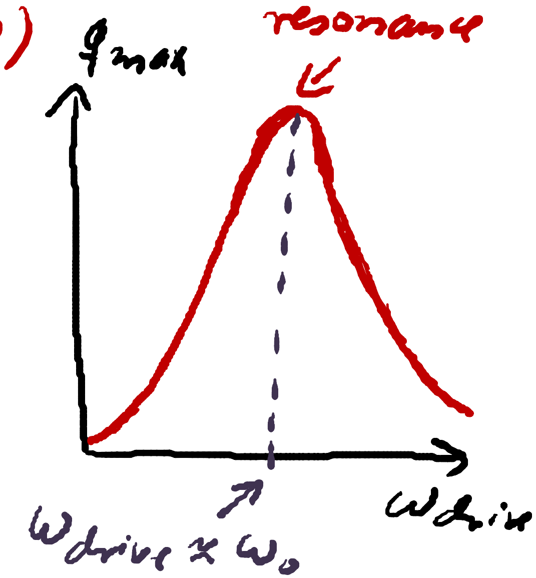
– driven:

charge: $q(t) = q_{max} \cos(\omega_{drive} t + \phi)$

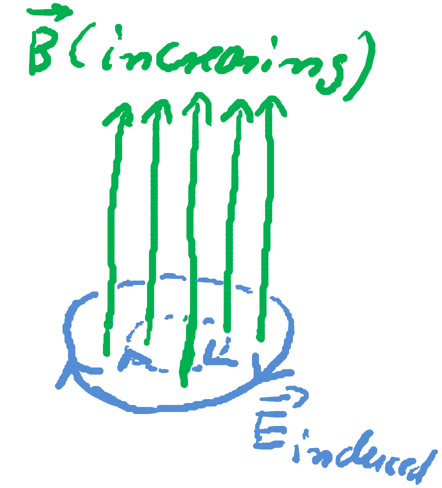
Always oscillates at the driving frequency!

Resonance when driven at

$\omega_{drive} \approx \omega_0 = \frac{1}{\sqrt{LC}}$



Recap II



- Faraday's Law:

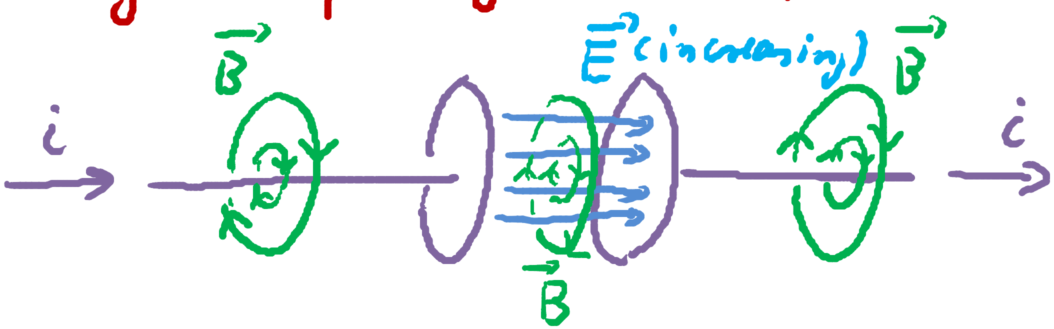
A changing magnetic field $\vec{B}$ produces an electric field $\vec{E}$.

$$\underbrace{\oint}_{\text{closed path}} \vec{E}\cdot d\vec{s} = -\underbrace{\frac{d\Phi_B}{dt}}_{\text{rate of change of magnetic flux through interior of path}}$$

- Ampère-Maxwell Law:

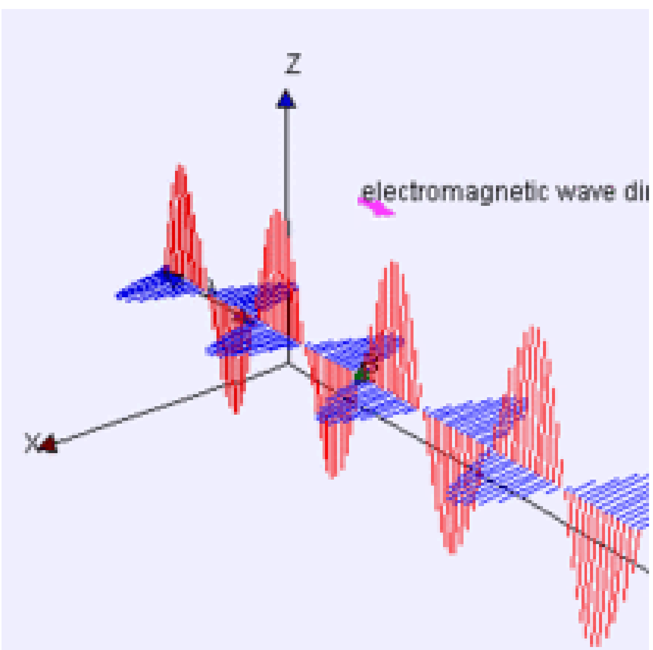
A changing electric field $\vec{E}$ and currents are both sources of a magnetic field $\vec{B}$

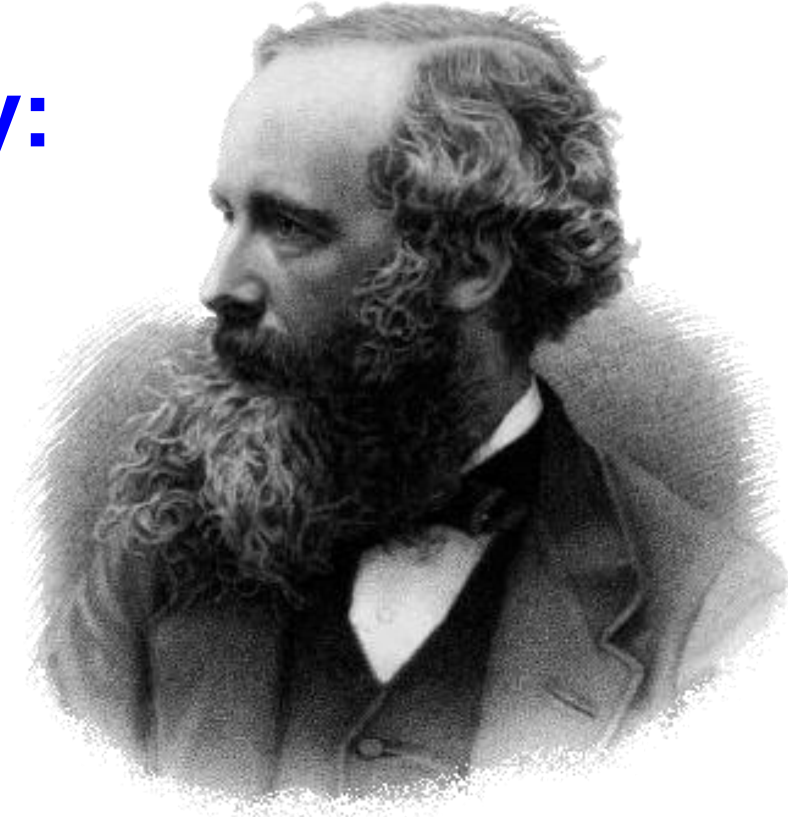
$$\underbrace{\oint}_{\text{closed path}} \vec{B}\cdot d\vec{s} = \mu_0 i_{enclosed} + \overbrace{\mu_0 \varepsilon_0 \underbrace{\frac{d\Phi_E}{dt}}_{\text{rate of change of electric flux through interior of path}}}^{\text{"displacement current"}}$$



Today:

- **Maxwell's equations**
- **Electromagnetic waves**
  - **Polarization**





$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \mathbf{B}\cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

What you have learned so far: The big Picture

Notice how powerful symmetry is in physics!

1) Electric charges produce electric fields. → Gauss' Law

2) Magnetic charges (monopoles) do not exist. → Gauss' law for magnetic fields

3) Changing magnetic fields induce/produce electric fields. → Faraday's Law

4) Electric currents produce magnetic fields.

5) Changing electric fields produce magnetic fields.

→ Ampère-Maxwell Law

The 4 Maxwell Equations:

Fundamental equations of electromagnetism

I) Gauss' Law for Electric Fields:

$$\boxed{\underset{\substack{\text{through a}\\\text{closed surface}}}{\Phi_{E,\,net}} = \underset{\substack{\text{closed}\\\text{surface}}}{\oint} \vec{E} \cdot d\vec{A} = \frac{Q_{net,\,inside}}{\varepsilon_0}}$$

Electric field lines are closed loops or start and stop on electric charges.

The electric flux through a closed surface is proportional to the net charge inside the surface.

II) Gauss' Law for magnetic Fields:

$$\boxed{\underset{\substack{\text{through a}\\\text{closed surface}}}{\Phi_{B,\,net}} = \underset{\substack{\text{closed}\\\text{surface}}}{\oint} \vec{B} \cdot d\vec{A} = 0}$$

Magnetic field lines are always closed loops since there are no magnetic charges (monopoles).

The magnetic flux through a closed surface is zero.

III) Faraday's Law:

$$\boxed{\oint_{\text{closed path}} \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}}$$

Changing magnetic fields are another source of electric fields (i.e. in addition to electric charges).

The circulation of the electric field around a closed path is equal to the negative of the rate of change of magnetic flux through the interior of the path.

IV Ampère-Maxwell Law:

$$\boxed{\oint_{\text{closed path}} \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enclosed}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}}$$

Both currents and changing electric fields are sources of magnetic fields.

The circulation of the magnetic field around a closed path is related to the enclosed current and to the rate of change of the electric flux through the interior of the path.

Electromagnetic (EM) Wave:

- Maxwell's equations in perfect vacuum (no charges, current):

$\oint \vec{E} \cdot d\vec{A} = 0$ (closed surface)

$\oint \vec{B} \cdot d\vec{A} = 0$ (closed surface)

$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ (closed path)

$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$ (closed path)

Notice: A changing magnetic field produces a (changing) electric field and vice versa!

$\Rightarrow$ Electric and magnetic field oscillations can sustain one another

$\Rightarrow$ "Electromagnetic wave" can exist in vacuum!

- from Maxwell's equations in vacuum:

$$\frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

differential equation for electric field of EM wave

"wave equation"

Solution of Wave Equation:

Example: Plane EM wave propagating in +x direction
(many other solutions exist...) $\omega = 2\pi f$

$$E_y(x,t) = E_{max} \sin(kx - \omega t)$$

$k = \frac{2\pi}{\lambda}$

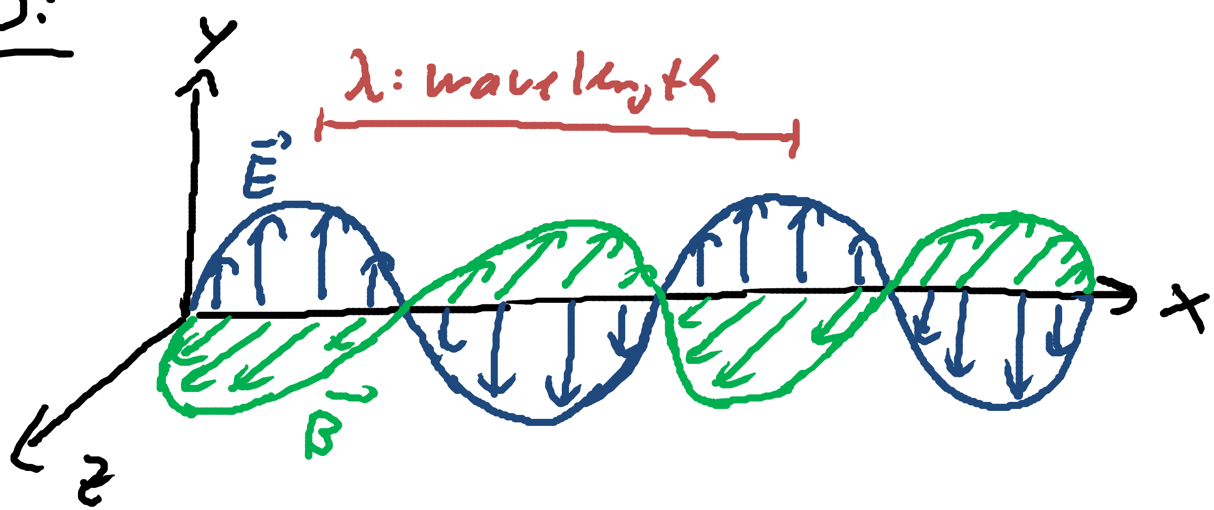
$\vec{E}$ points along y-direction ($E_x = 0$, $E_z = 0$)

$\Updownarrow$ Maxwell's equ.

$$B_z(x,t) = \underbrace{B_{max}}_{\text{wave amplitude}} \underbrace{\sin(kx - \omega t)}_{\text{wave that moves in +x direction}}$$

$\vec{B}$ points along z-direction ($B_x = 0$, $B_y = 0$)

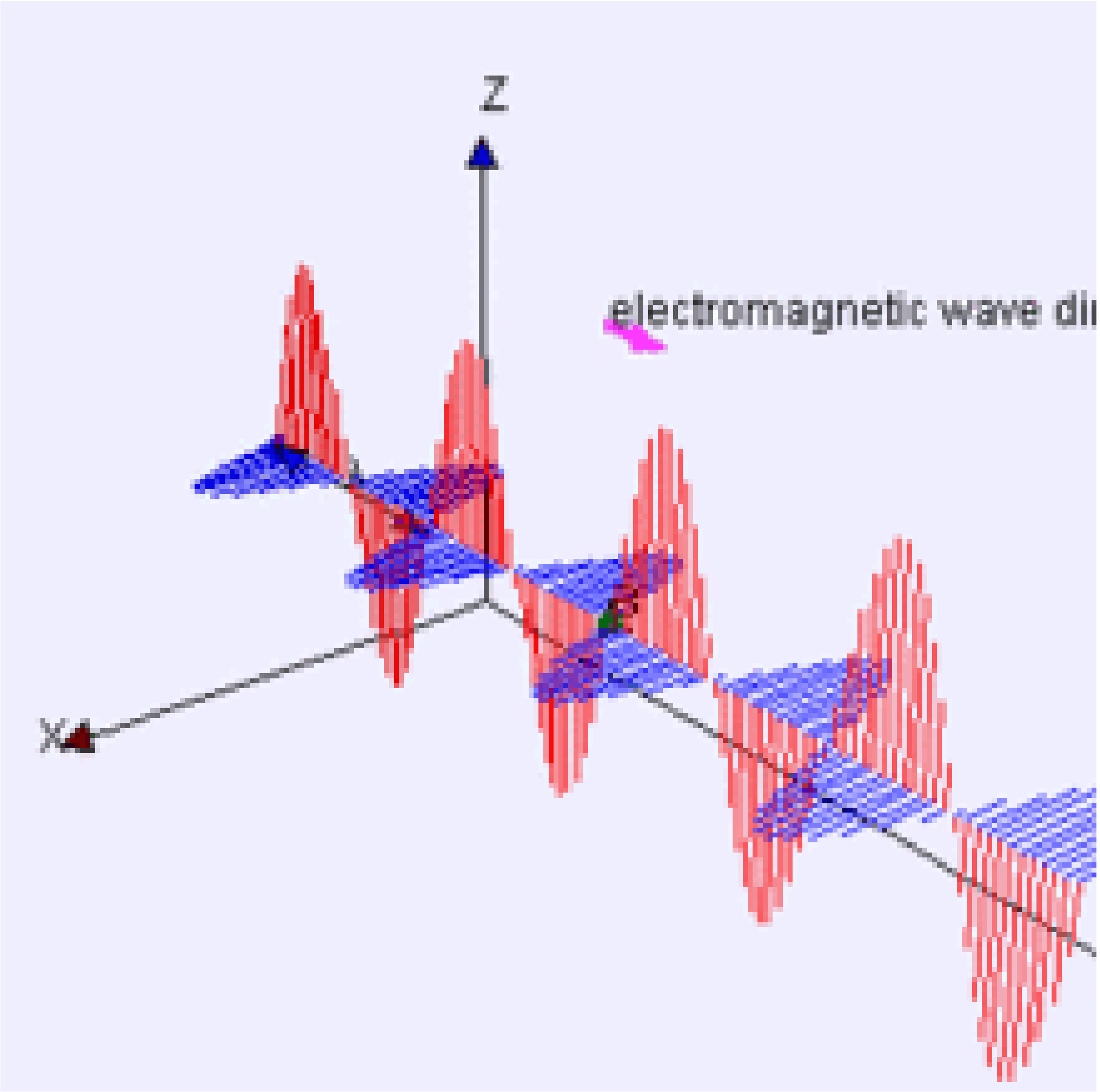
3D:



EM wave moves along this direction

$\longrightarrow$

v_{wave}


Z
electromagnetic wave di
X

Notes on EM waves:

I Wave speed:

EM waves propagate through vacuum (i.e. don't need a medium) with wave speed (in vacuum): $\omega = 2\pi f$

$$v_{wave} = \lambda f = \frac{\omega}{k} = \boxed{\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c} = \text{"speed of light"}$$

$$k = \frac{2\pi}{\lambda}$$

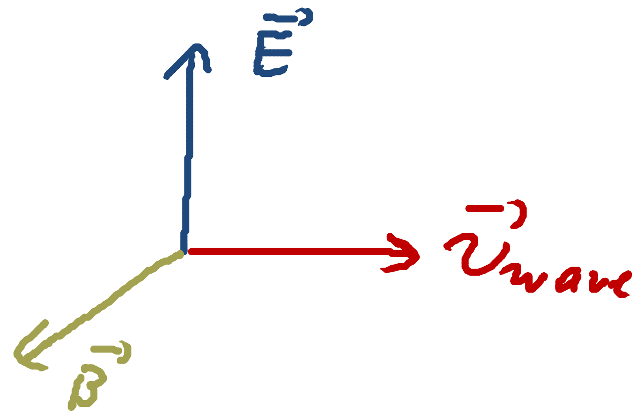
$$\boxed{c = 3.0 \cdot 10^8 \ m/s}$$

independent of frequency f of EM wave (in vacuum)

II Direction:

(a) The electric and magnetic fields ($\vec{E}$ and $\vec{B}$) are perpendicular to each other.

(b) The wave propagates in direction of the vector $\vec{E} \times \vec{B}$, i.e. to both $\vec{E}$ and $\vec{B}$



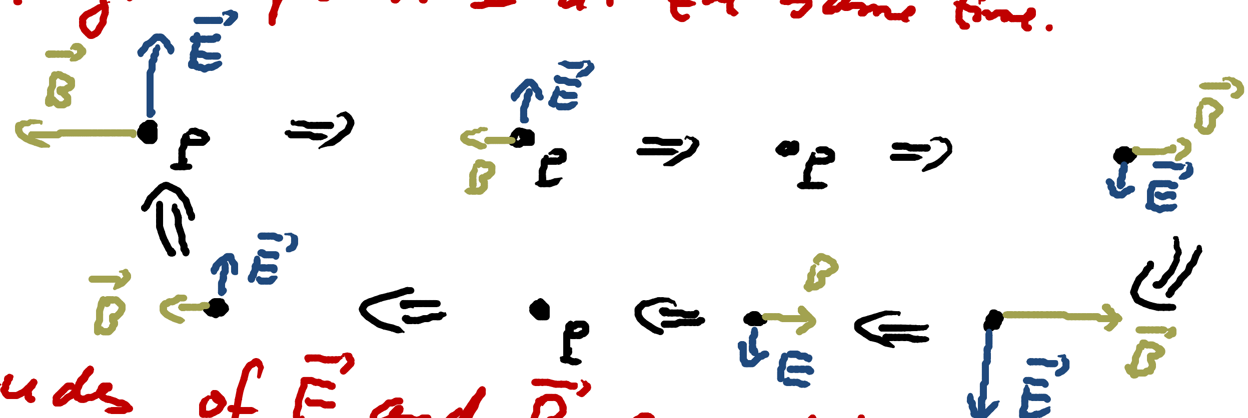
$\Rightarrow$ EM waves are <u>transverse</u> waves!

III Relation between $\vec{E}$ and $\vec{B}$ in EM wave:

$\vec{E}$ and $\vec{B}$ sustain one another $\Longrightarrow$ $\vec{E}$ and $\vec{B}$ are related to each other

(a) In a traveling, plane EM wave, $\vec{E}$ and $\vec{B}$ are in-phase with each other, i.e. reach max values at given point P at the same time.

at point P:



(b) Magnitudes of $\vec{E}$ and $\vec{B}$ are related by:

$$\boxed{\frac{E_{max}}{B_{max}} = c} = \text{"speed of light"} = \text{wave speed}$$

IV Polarization:

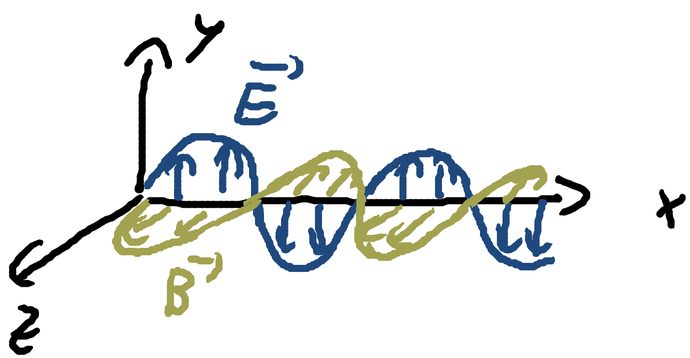
EM waves can be "polarized":

If the electric field $\vec{E}$ of an EM wave oscillates along one specific direction (e.g. y-direction), the wave is said to be plane-polarized!

Define:

$$\begin{pmatrix}\text{direction of} \\ \text{polarization}\end{pmatrix} = \begin{pmatrix}\text{Direction along which} \\ \vec{E} \text{ oscillates / points}\end{pmatrix}$$

Example:



wave is:

- polarized along y-direction
- propagates in +x direction

Which set of expressions describes the electric field of an EM wave that travels in the *-y* direction and is polarized along the *z* direction?

~~A~~. $E_y = E_m \sin(kz - \omega t),\ E_x = 0,\ E_z = 0.$

~~B~~. $E_y = E_m \sin(kz + \omega t),\ E_x = 0,\ E_z = 0.$

polarized in y-direction

moves in +y direction

~~C~~. $E_z = E_m \sin(ky - \omega t),\ E_x = 0,\ E_y = 0.$

moves in -y direction

D. $E_z = E_m \sin(ky + \omega t),\ E_x = 0,\ E_y = 0.$

polarized in z-direction

E. None of the above.

Which set of expressions describes the magnetic field of an EM wave whose electric field is given by

$$E_y = E_m \sin(kz + \omega t) \quad , E_x = 0 \quad , E_z = 0 \ ?$$

polarized along y moves in −z direction

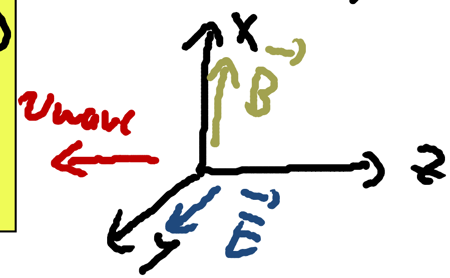
A. $B_x = \cancel{-}\dfrac{E_m}{c}\sin(kz + \omega t),\ B_y = 0,\ B_z = 0.$

B. $B_x = \dfrac{E_m}{c}\sin(kz + \omega t),\ B_y = 0,\ B_z = 0.$

C. $B_x = \dfrac{E_m}{c}\cancel{\cos}(kz + \omega t),\ B_y = 0,\ B_z = 0.$ } $\vec{E}$ and $\vec{B}$ are in phase!

D. $B_y = \dfrac{E_m}{c}\sin(kz + \omega t),\ B_x = 0,\ B_z = 0.$ } $\vec{E} \perp \vec{B}$ always

E. None of the above.

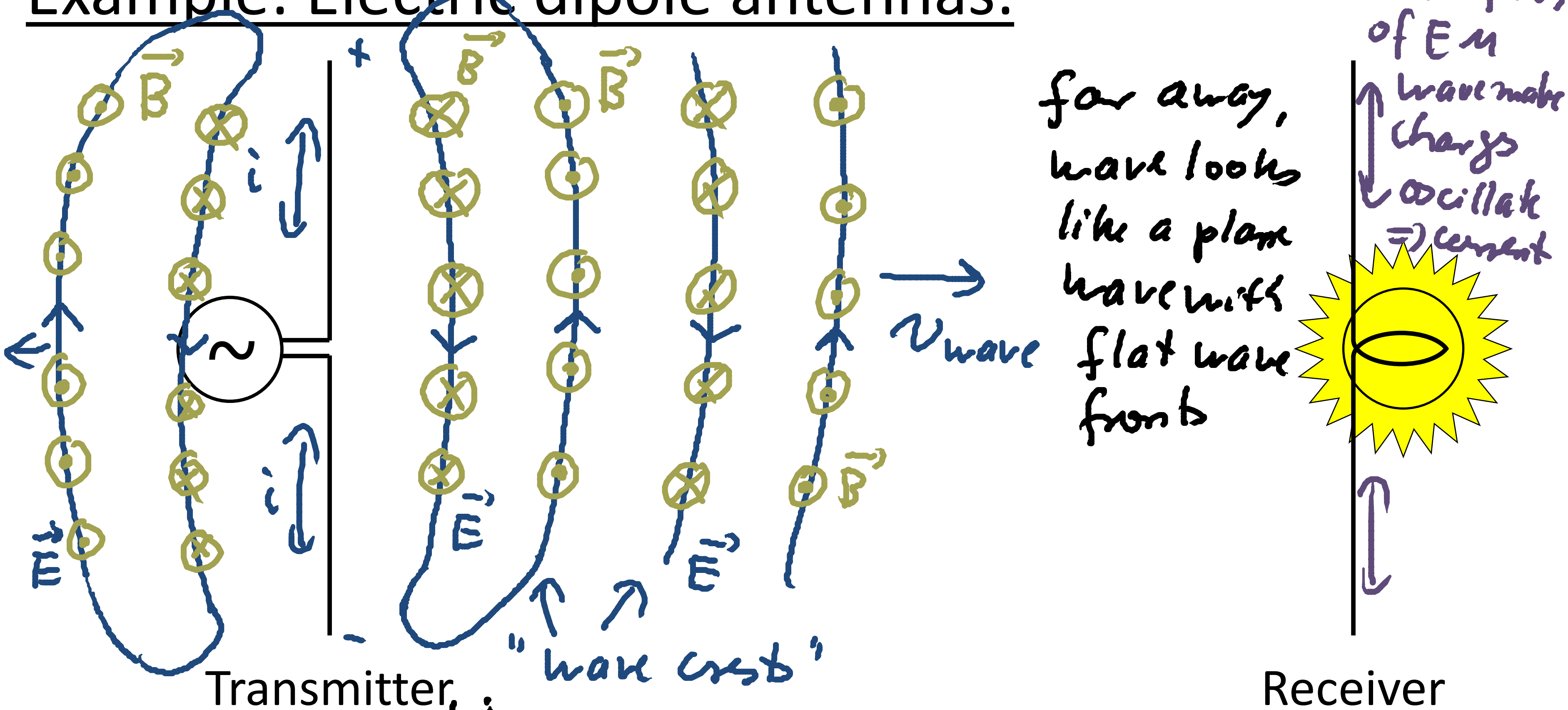


V: Sources of EM waves:

Accelerating charges (changing currents) radiate EM waves.

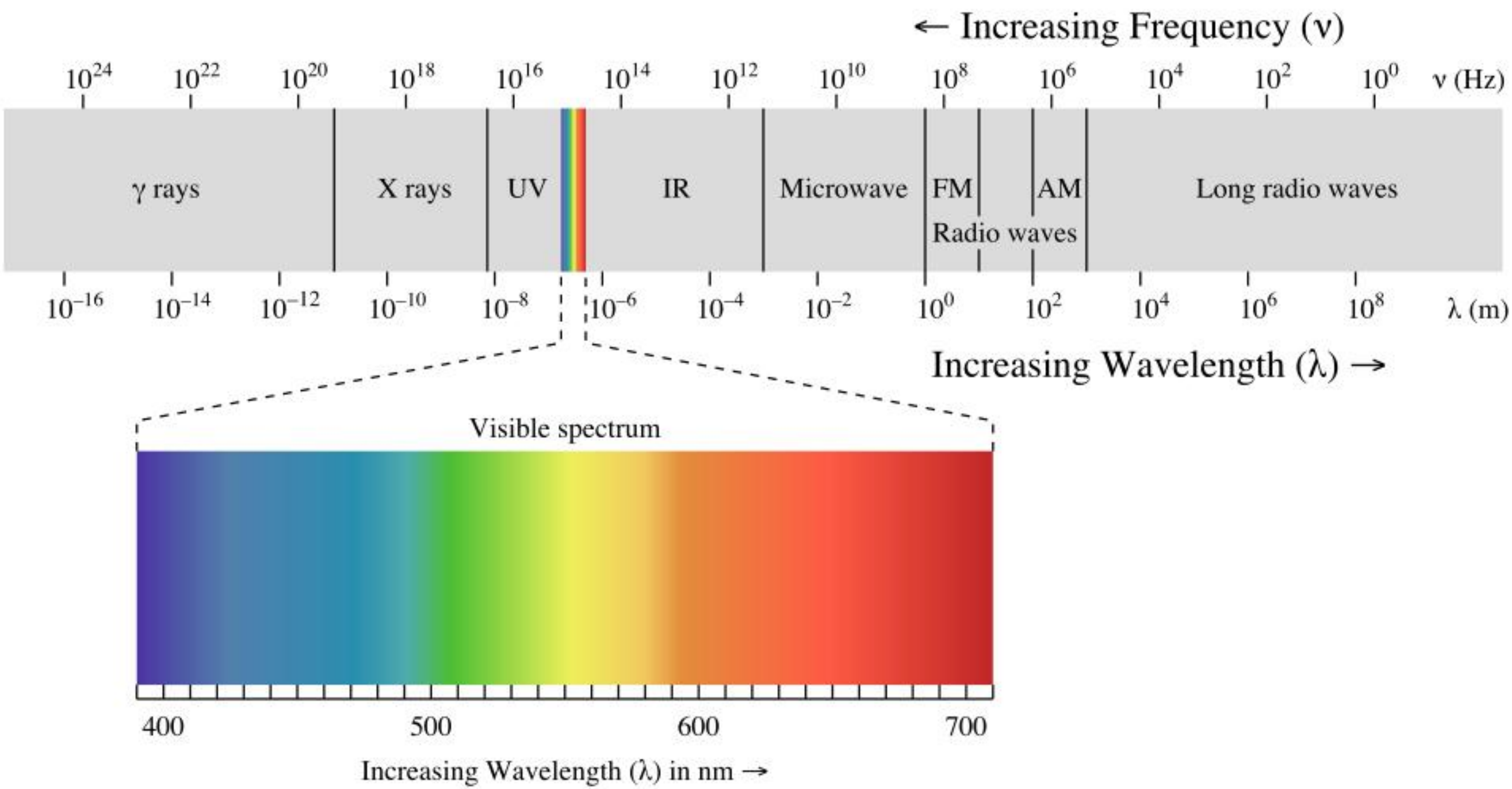
↑ no radiation this way

Example: Electric dipole antennas:



note: ↓ no radiation this way!

VI: Spectrum of EM waves:



Note: These are all electromagnetic waves! Only difference is frequency (wavelength)!