Recap I

Lecture 22

- **Inductors:**

  - produce a well defined magnetic field in a specific region
  - circuit symbol: 
  - Inductance L:

$$L = \frac{N\Phi_B}{i} = \frac{\text{flux linkage}}{\text{current}} \qquad [L] = \frac{Tm^2}{A} = \underline{\underline{H}} = \text{Henry}$$

  - for a solenoid:

# of turns

$$L = \mu_0 \frac{N^2 A}{\ell}$$

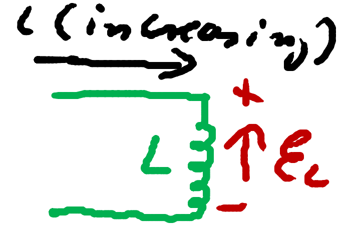
A ⇐ cross-sectional area

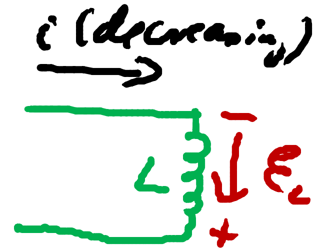
ℓ ⇐ length of solenoid

  - Self-induced emf:

    if current in a coil changes, an emf is induced in the coil:

The emf acts to oppose the change that produces it!



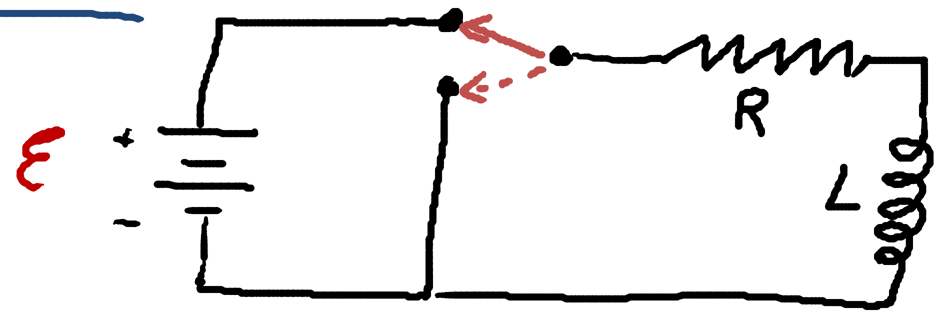


$$\mathcal{E}_L = \mathcal{E}_{self\text{-}induced} = -L\frac{di}{dt}$$

potential change over inductor: $|\Delta V_L| = \mathcal{E}_L$

Recap II

- RL circuits:



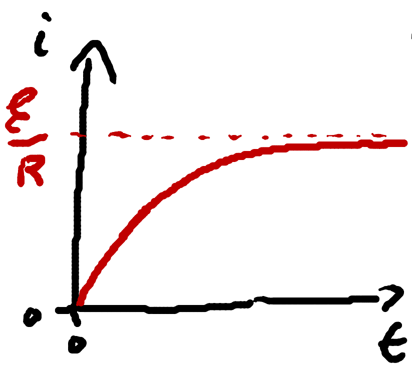
self induced emf opposes/slows down change in current

current rise:

$$i(t) = \frac{\mathcal{E}}{R}\left[1 - e^{-t/\tau}\right]$$

with inductive time constant

$$\tau = L/R$$

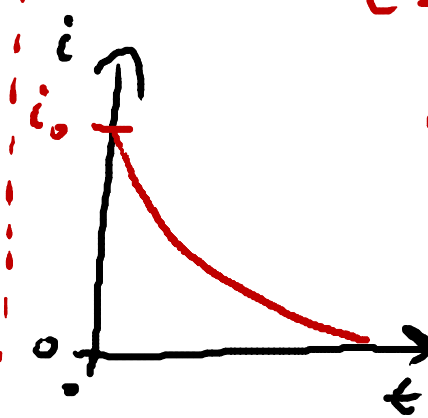


current decay:

$$i(t) = i_0\, e^{-t/\tau}$$

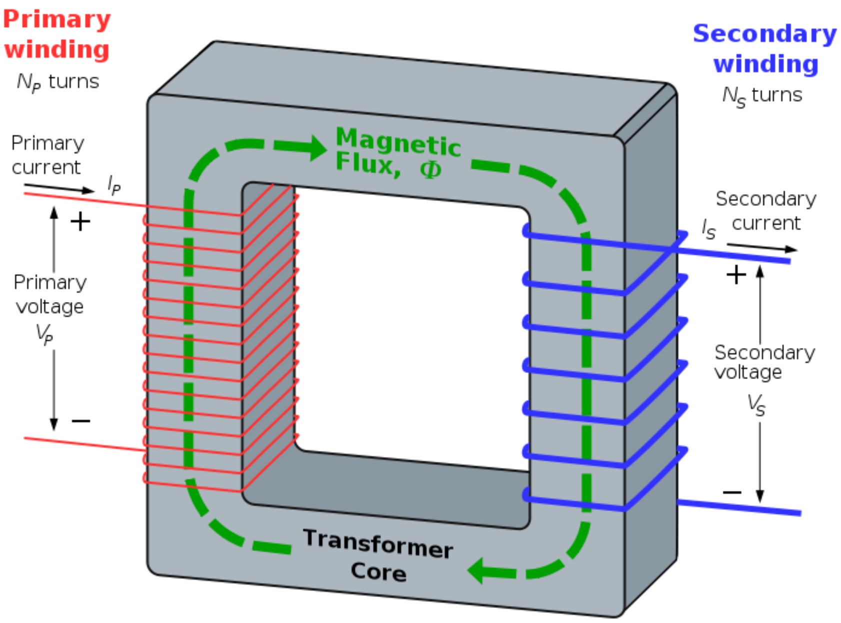
with inductive time constant

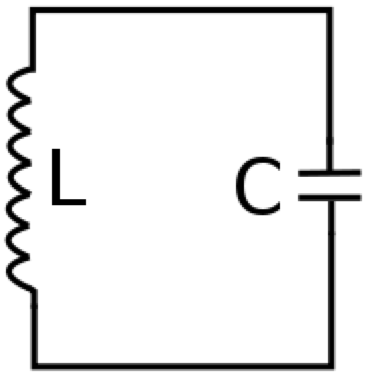
$$\tau = L/R$$



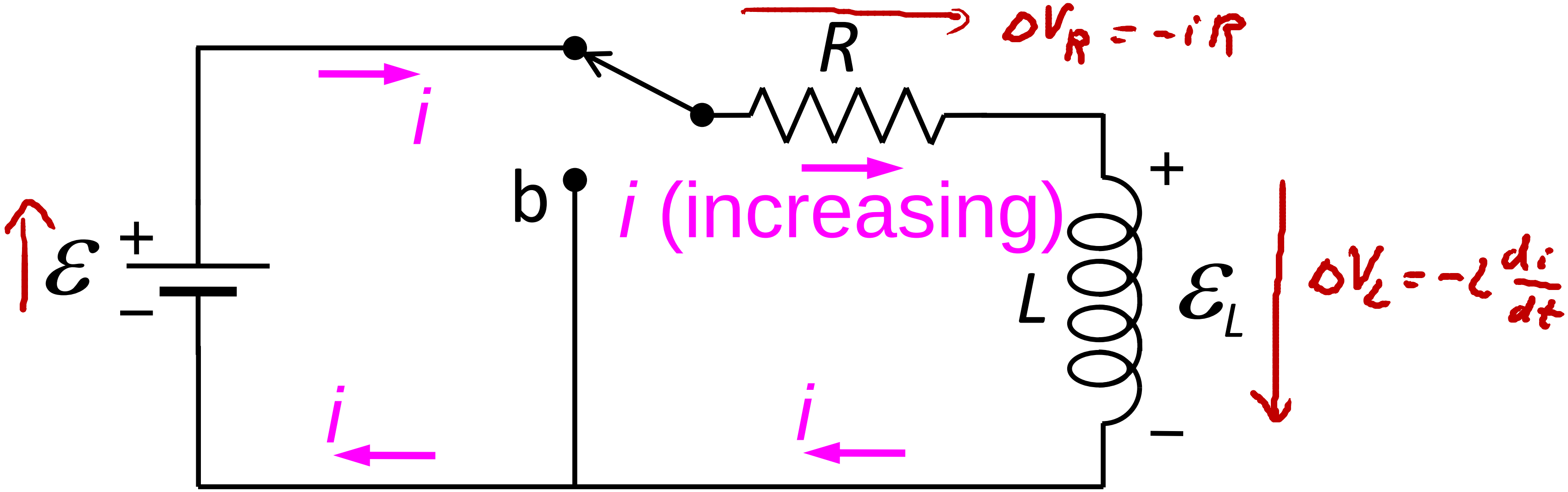
Today:

- **Energy density of a magnetic field**
- **Alternating current and power**
  - **Transmission lines and transformers**
- **Ideal LC circuit**





RL circuit:
Power supplied and dissipated in the circuit



Loop rule gives: $\mathcal{E} = iR + L\frac{di}{dt}$ $\left(\mathcal{E} - iR - L\frac{di}{dt} = 0\right)$

Multiply both sides by current i : $\underbrace{\mathcal{E}i}_{\text{power supplied by the battery}} = \underbrace{i^2R}_{\text{power dissipated by the resistor}} + \underbrace{iL\frac{di}{dt}}_{\text{this term must be the power delivered to the inductor!}}$

equation of conservation of energy/power in RL circuit

$\Rightarrow \left(\text{power delivered to inductor} \right) = \left(\text{rate at which the magnet potential energy } U_B \text{ is stored in the magnetic field of the inductor} \right) = \dfrac{dU_B}{dt}$

so $\dfrac{dU_B}{dt} = iL\dfrac{di}{dt} \Rightarrow dU_B = L\,i\,di$

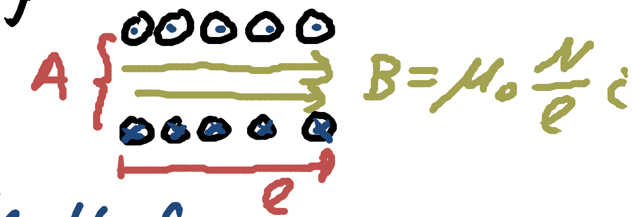
integrate: $\underline{\underline{U_B}} = \int_0^{U_B} dU_B = \int_0^{i} L\,i\,di = L\int_0^{i} i\,di = \underline{\underline{\frac{1}{2}L\,i^2}}$

$\Rightarrow$ Energy stored in the magnetic field of an inductor:

$$\boxed{U_B = \frac{1}{2}L\,i^2}$$

Magnetic fields have stored energy!

Energy density of a magnetic field:



energy density $= u_B = \dfrac{U_B}{\text{volume}} = \dfrac{\frac{1}{2} L i^2}{A\,\ell}$

for solenoid

A: cross-sectional area of solenoid

ℓ: length of solenoid

inductance L of solenoid: $L = \mu_0 \dfrac{N^2}{\ell} A$

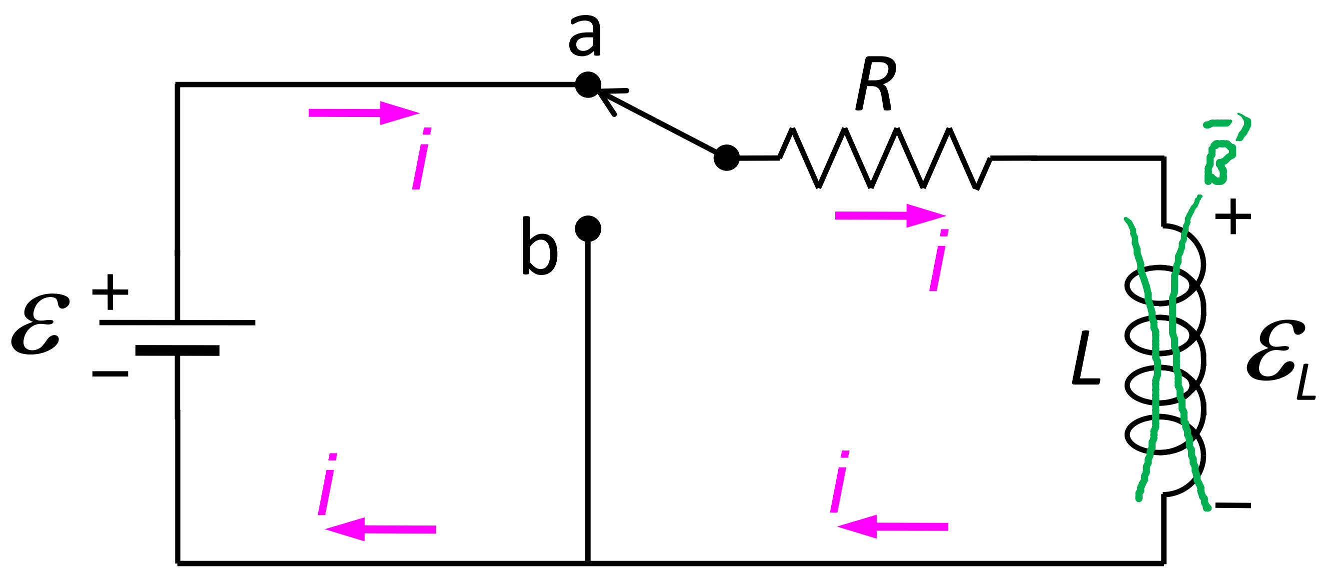
N: # of turns

$\Rightarrow u_B = \frac{1}{2} \mu_0 \dfrac{N^2}{\ell^2} i^2$; now use: $B = \mu_0 \dfrac{N}{\ell} i$

$\Rightarrow$ (energy density of a magnetic field) $= \boxed{u_B = \dfrac{1}{2} \dfrac{B^2}{\mu_0}} \propto B^2$ } holds for any magnetic field

recall:

(energy density of an electric field) $= \boxed{u_E = \frac{1}{2} \varepsilon_0 E^2} \propto E^2$



At time $t = 0$ the switch is moved to position a.

After $t = 0$, how does the power delivered to the inductor's magnetic field vary with time?

$P_{\text{to inductor}} = i L \frac{di}{dt}$

at $t=0$: $i=0 \Rightarrow P=0$
at $t \to \infty$: $i = \text{const} \Rightarrow P=0$

A. It starts low & steadily increases.
B. It starts high & steadily decreases.
C. It starts low, then increases until it reaches a peak, & then decreases.
D. It's constant. E. It oscillates.

Why is power transmitted at very high voltages in power transmission lines (several 100,000 volts)?

A. Because it reduce the energy lost in long-distance transmission

B. Because it maximized the power that can be transmitted

C. Because it is easier to generate high voltages

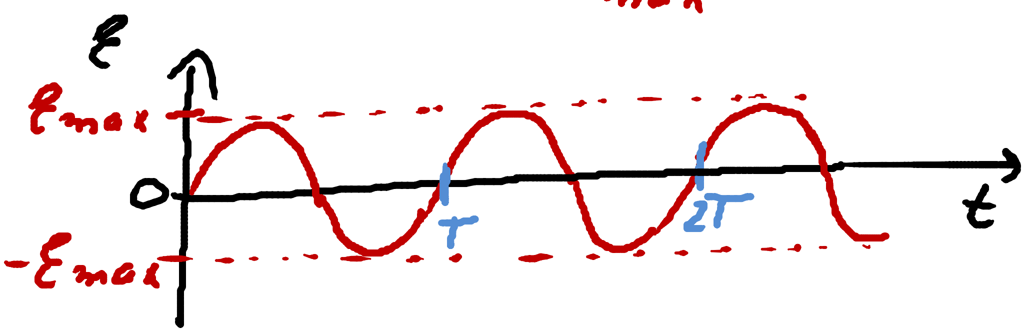
Alternating current (ac):

- Direct current (dc): flow of electric charge carriers is only only in one direction; i.e non-oscillating
- Alternating current (ac): movement of electric charge carriers periodically reverses direction
  → oscillating emf and current

Example: Household voltage in North America:

$$\mathcal{E}(t) = \mathcal{E}_{max} \sin(2\pi f t) = \mathcal{E}_{max} \sin(\omega t)$$

varies sinusoidally!

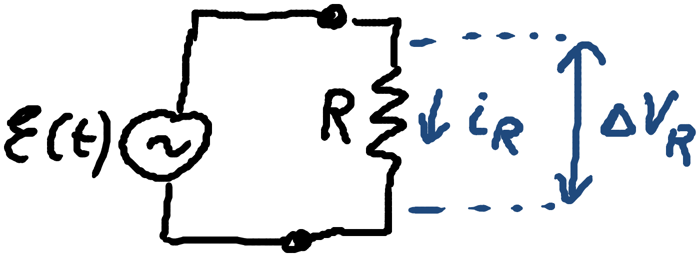


with $\mathcal{E}_{max} = 170\,V$

and $f = 60\,Hz$

$\Rightarrow T = 1/f = \frac{1}{60}\,s$

→ Consider circuit describing a device with resistance R plugged into a power outlet:



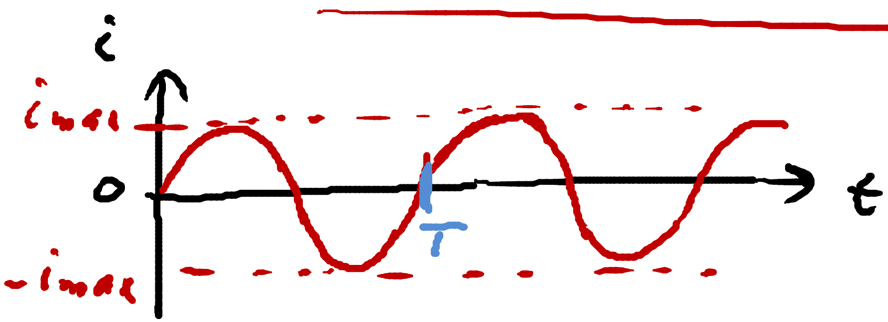
- loop rule: $\mathcal{E} - \Delta V_R = 0$

$$\Rightarrow \Delta V_R = \mathcal{E} = \mathcal{E}_{max} \sin(\omega t)$$

- since $R \equiv \Delta V_R / i_R$

$\Rightarrow$ current: $i_R = \dfrac{\Delta V_R}{R} = \dfrac{\mathcal{E}_{max}}{R} \sin(\omega t) = i_{max} \sin(\omega t)$

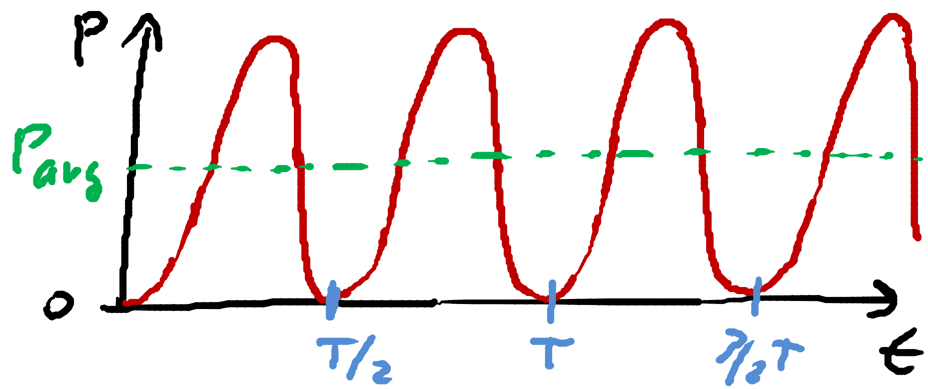
with $\boxed{i_{max} = \dfrac{\mathcal{E}_{max}}{R} = \dfrac{\Delta V_{R,max}}{R}}$ } for a resistor in an ac-circuit



Note: the potential drop ΔV_R and current i_R oscillate in phase for a resistor!

now: calculate power dissipated in the resistor R

from before: $P = i^2 R \Rightarrow$ for ac: $P = i_{max}^2 \sin^2(\omega t) R$



take time average:

$P_{avg} = (i^2)_{avg} R$

$= \frac{1}{2} i_{max}^2 R$

average of $\sin^2(\omega t) = 1/2$

$\Rightarrow$

$\left(\begin{array}{l}\text{Average power}\\ \text{dissipated in}\\ \text{resistor in AC circuit}\end{array}\right) = \underline{P_{avg}} = \frac{1}{2} i_{max}^2 R = \left(\frac{i_{max}}{\sqrt{2}}\right)^2 R = \underline{i_{RMS}^2 R}$

with "root-mean-square" (RMS) current: $\boxed{i_{RMS} \equiv \frac{i_{max}}{\sqrt{2}}}$

$\Rightarrow$ also can define rms voltage and rms emf:

$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}}$ $\qquad$ $\mathcal{E}_{rms} = \frac{\mathcal{E}_{max}}{\sqrt{2}}$

Note:

- in U.S.: $\mathcal{E}_{rms} = \frac{\mathcal{E}_{max}}{\sqrt{2}} = \frac{170V}{\sqrt{2}} = \underline{120V}$
- voltmeters, ammeters, etc... read rms values of ac currents
- Power transmission line:

Power transmission lines have resistance R

$\Rightarrow$ power lost in transmission

$$P_{avg,lost} = i_{RMS}^2 R$$

$\Rightarrow$ need to keep current low in lines!

Power delivered by power plant / transm. line:

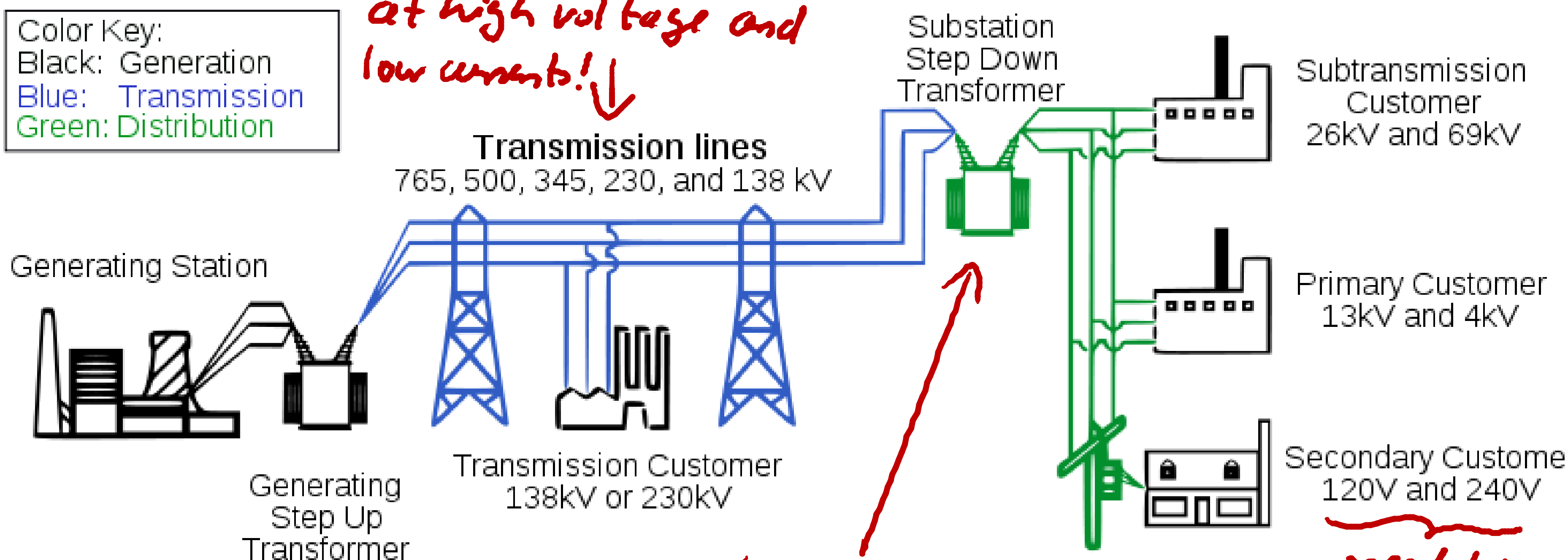
$$P_{avg,delivered} = \mathcal{E}_{rms}\, i_{rms} \Rightarrow P_{avg,lost} = \left(\frac{P_{avg,del}}{\mathcal{E}_{rms}}\right)^2 R$$

$\Rightarrow$ high voltage $\mathcal{E}_{rms}$ $\Rightarrow$ low current i_{rms}

$\Rightarrow$ reduce power lost in transmission line!

Electrical transmission system:

power is transmitted at high voltage and low currents!



need lower voltages

easy to do for ac, more difficult for dc ?

transformer:
transform ac input voltage to different output voltage, while keeping the product current × voltage ≈ constant