Recap I

- Torque on a Current Loop:

$$|\tau_{\text{on Loop}}| = \mu \, B \sin\theta$$

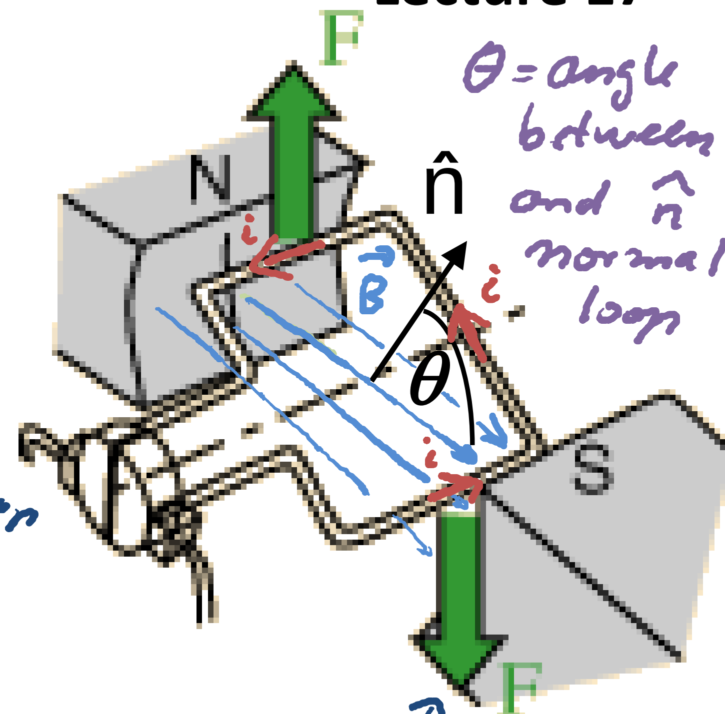
with magnetic dipole moment

$$\mu = A \, N \, i$$

A: area enclosed by the loop

N: number of turns of coil/loop

i: current in wire of loop



θ = angle between $\vec{B}$ and $\hat{n}$ normal to loop

force on straight wire:

$$\vec{F}_{\text{wire}} = i \, \vec{L} \times \vec{B}$$

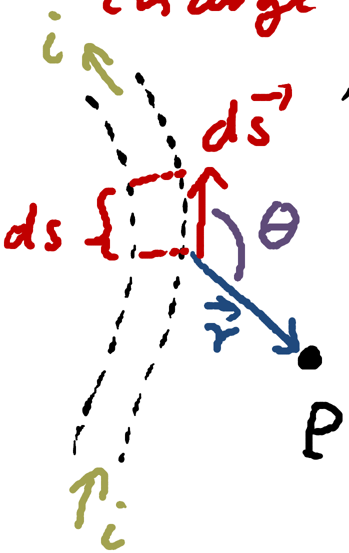
- Math:
  - dot product: $\vec{A} \cdot \vec{B} = A \, B \cos\theta = A_{\parallel} B = A \, B_{\parallel}$
  - cross product: $|\vec{A} \times \vec{B}| = A \, B \sin\theta = A_{\perp} B = A \, B_{\perp}$

θ = angle between $\vec{A}$ and $\vec{B}$

Recap II

- Magnetic Fields due to Currents:

Moving electric charge ⟹ produces a magnetic field around itself



Magnetic field at point P:

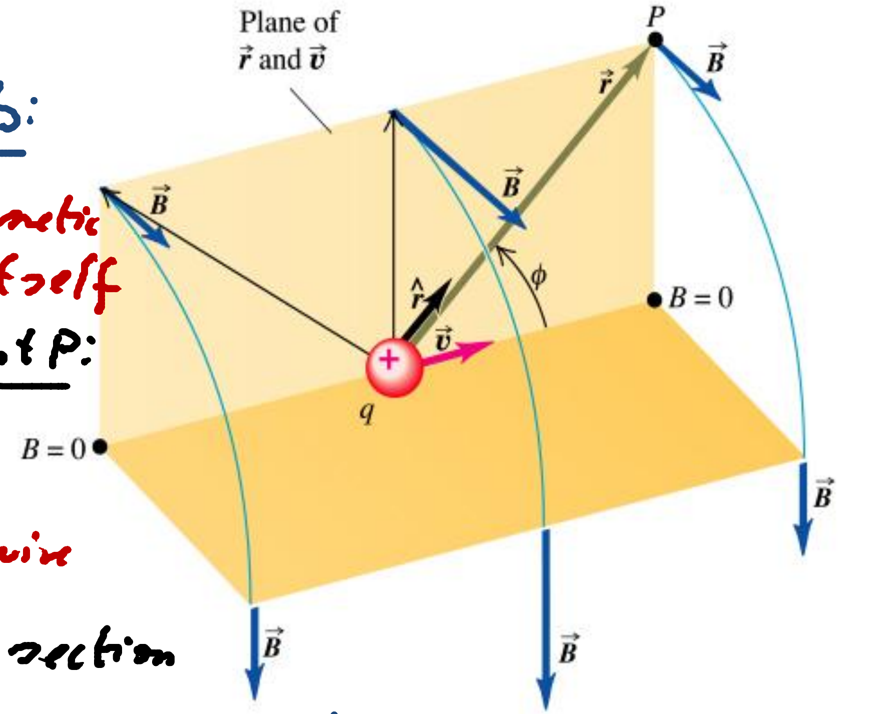
$$\vec{B}_P = \sum_{\text{all sections}} d\vec{B}_P = \int_{\text{along wire}} d\vec{B}_P$$

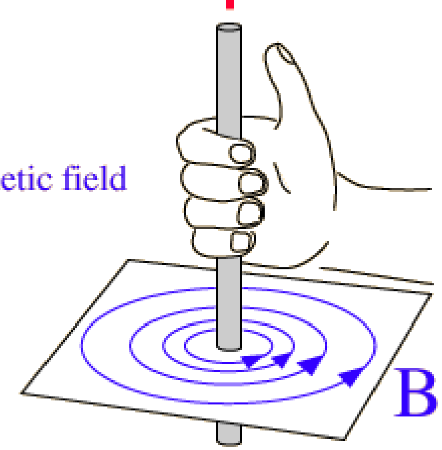
with field by given section $d\vec{s}$ of wire:

$$d\vec{B}_P = \frac{\mu_0}{4\pi}\, i\, \frac{d\vec{s} \times \hat{r}}{r^2}$$

unit vector $\hat{r} = \frac{\vec{r}}{|r|}$

μ_0 = permeability constant = $4\pi \cdot 10^{-7}$ T m/A

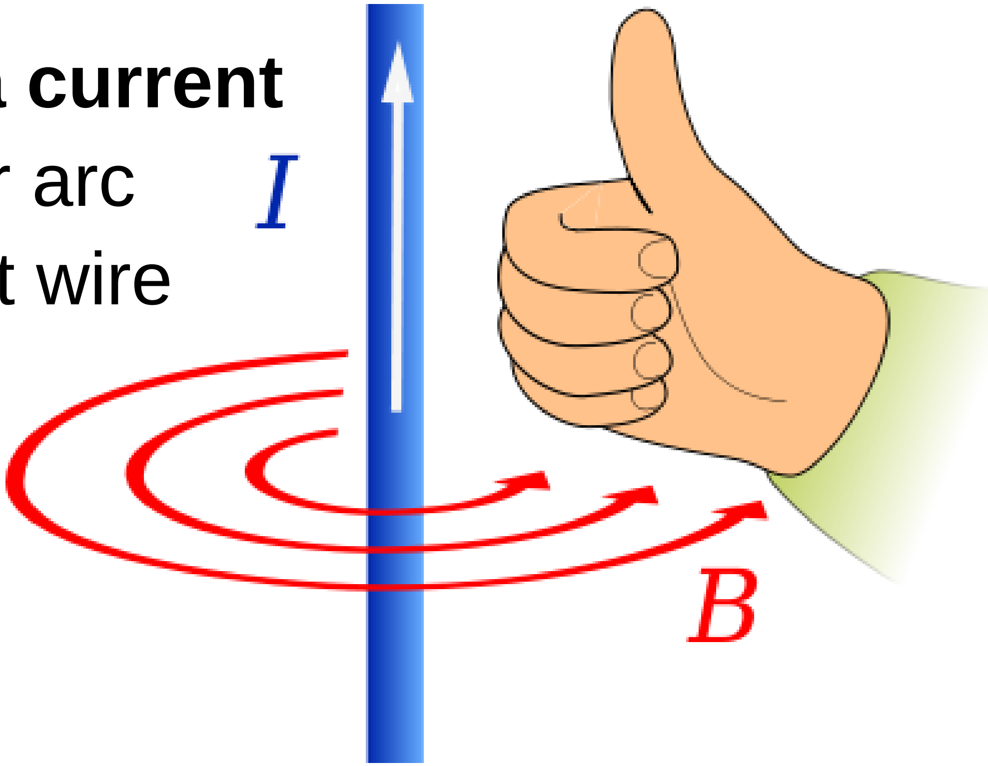


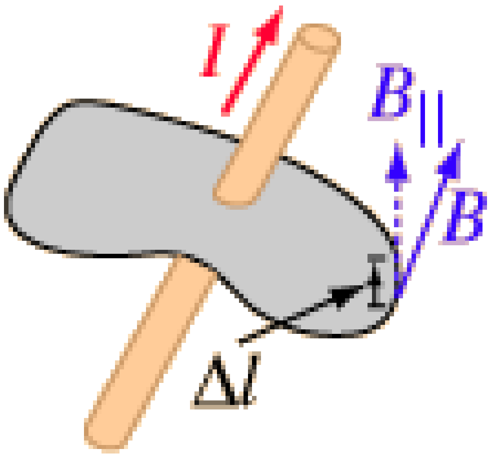


Right-hand rule: Point the thumb of your right hand in direction of the current. The fingers then reveal the B-field vector's direction.

Today:

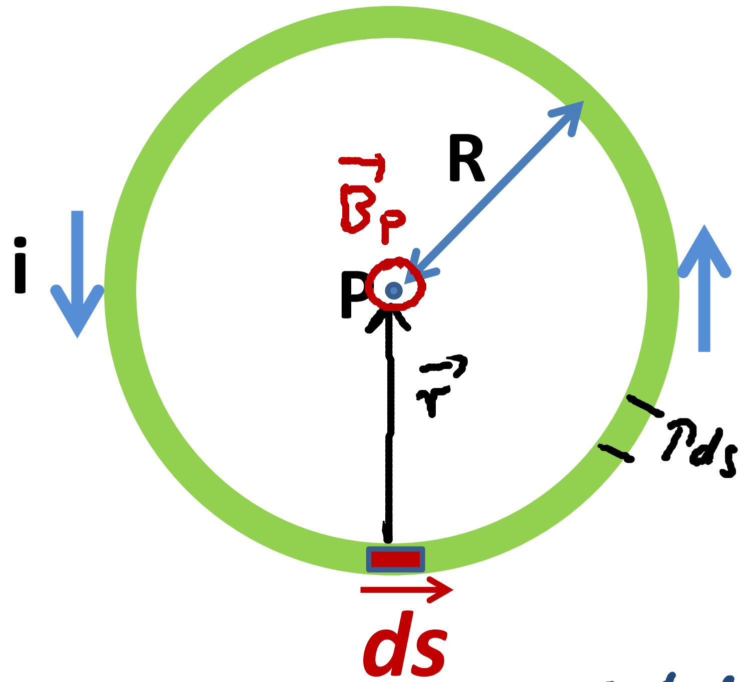
- **Magnetic field due to a current**
  - Field due to a circular arc
  - Field due to a straight wire
- **Ampere's law**





$$\sum B_{||} \Delta l = \mu_0 I$$

Example 1:
Magnetic field at Center of a Wire Loop due to a Current



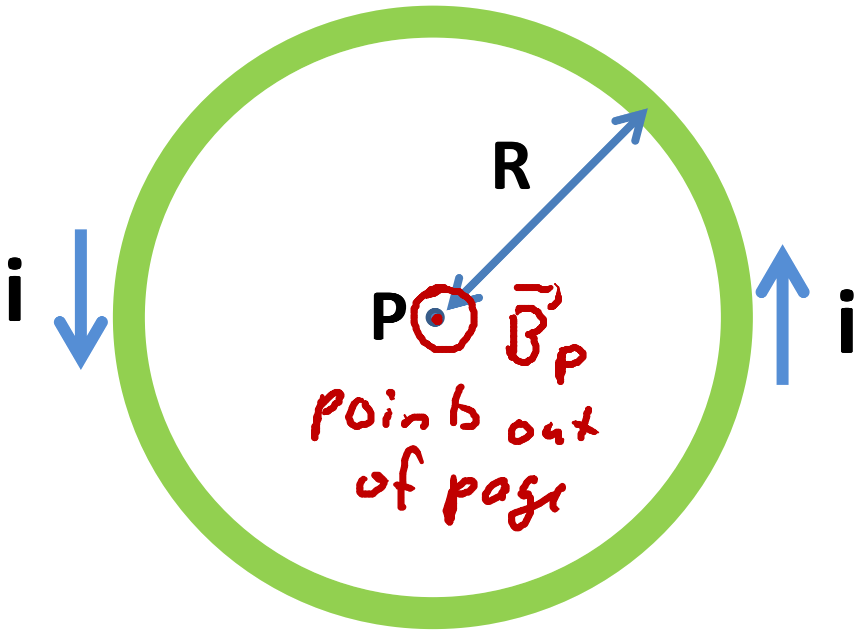
⇝ magnetic field at center P due to loop section $d\vec{s}$: angle between $\vec{r}$ and $d\vec{s}$ = 90° here

$$dB_p = \frac{\mu_0}{4\pi}\frac{i\,ds}{R^2}\underbrace{\sin\theta}_{=1}$$

⇝ field by each section ds of loop has this magnitude at point P, and points in same direction (out of the page)

⇒ total field at P = sum of differential fields by all sections around the loop

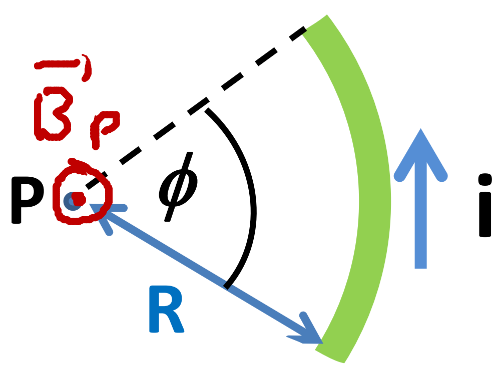
$$\Rightarrow \underline{B_p} = \int_{\text{around loop}} dB_p = \int_{\text{loop}} \frac{\mu_0}{4\pi}\frac{i}{R^2}\,ds = \frac{\mu_0 i}{4\pi R^2}\underbrace{\int ds}_{\substack{\text{length of}\\ \text{loop = circumference} = 2\pi R}} = \underline{\frac{\mu_0 i}{4\pi R^2}\,2\pi R}$$



Circumference= 2 π R

-> Magnetic field at Center of a Wire Loop:

$$\boxed{B_P = \frac{\mu_0 i}{2R}}$$



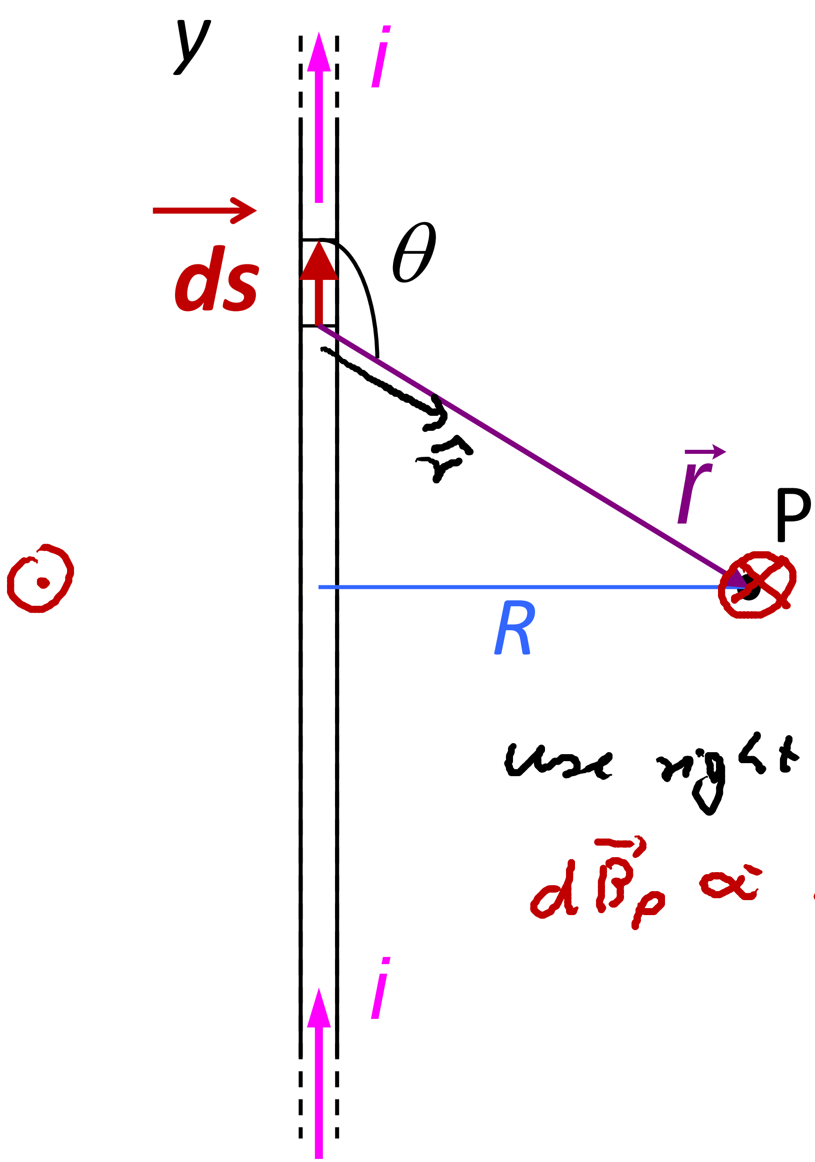
Arc length = ϕR

-> Magnetic field at Center of a circular Arc of Wire:

$$\boxed{B_P = \frac{\mu_0 i}{4\pi R}\phi}$$

in rad, not deg!

What is the direction of the magnetic field, $d\vec{B}_P$, at point P due to the current at $d\vec{s}$?



$d\vec{B}_p$ points into page

use right hand rule

$d\vec{B}_p \propto d\vec{s} \times \hat{r}$

A. ↑

B. ↓

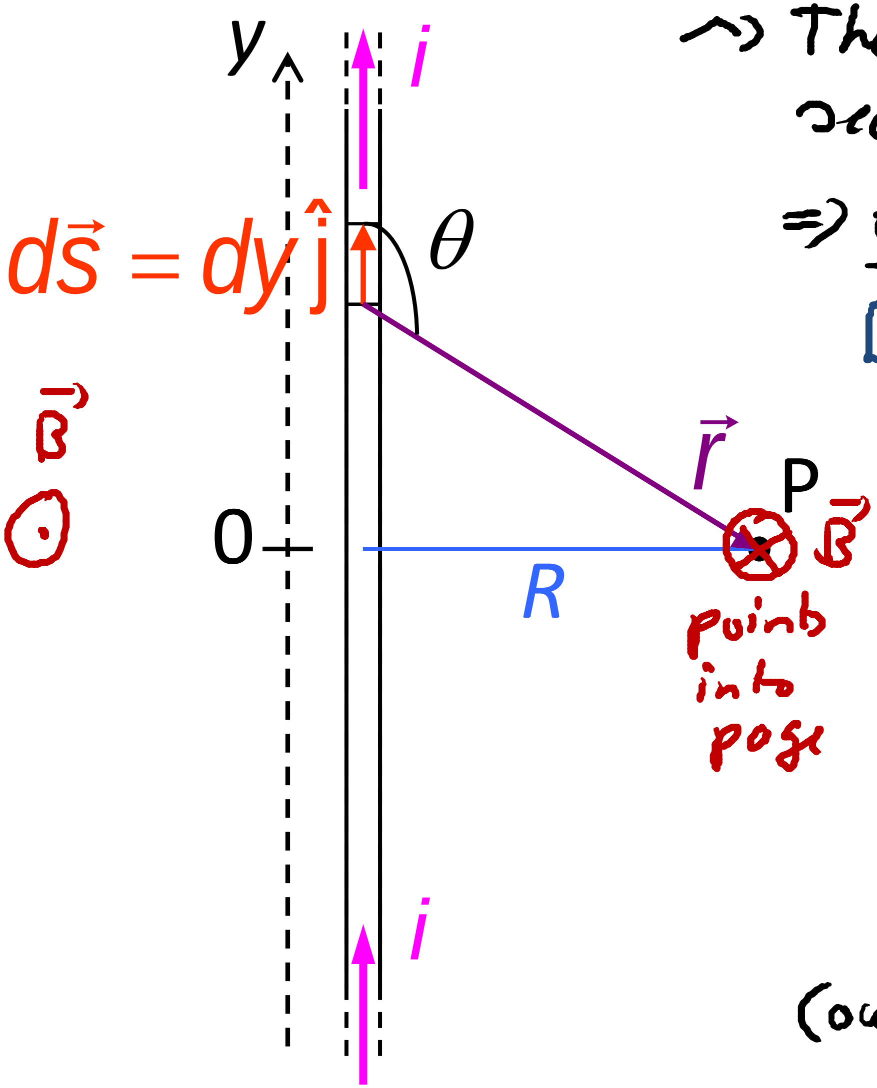
C. ⊙ (out of)

D. ⊗ (into)

E. Can't tell

Example 2:

Magnetic field due to a current in a long straight wire:



→ The direction of $d\vec{B}_P$ from any section $d\vec{s}$ in the wire is the same

⇒ total magnetic field at P:

$B_P = \int dB_P = \ldots$ math (see textbook)

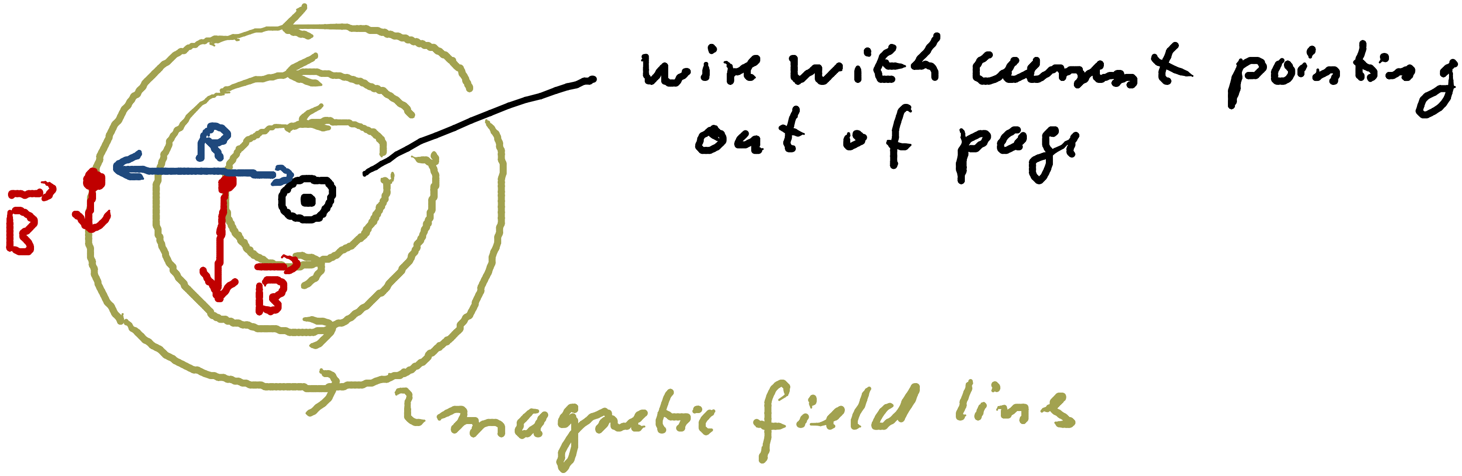
along infinite long straight wire from $-\infty$ to $+\infty$

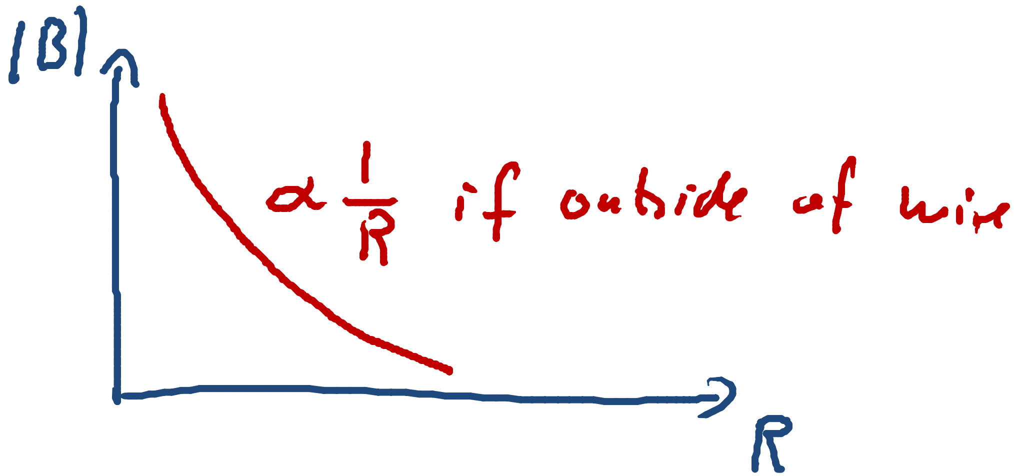
⇒ Result: for straight wire from $-\infty$ to $+\infty$

$$\boxed{B_P = \frac{\mu_0 i}{2\pi R} \propto \frac{1}{R}}$$

(outside of wire only)

Top view:


wire with current pointing out of page
R
$\vec{B}$
$\vec{B}$
magnetic field lines

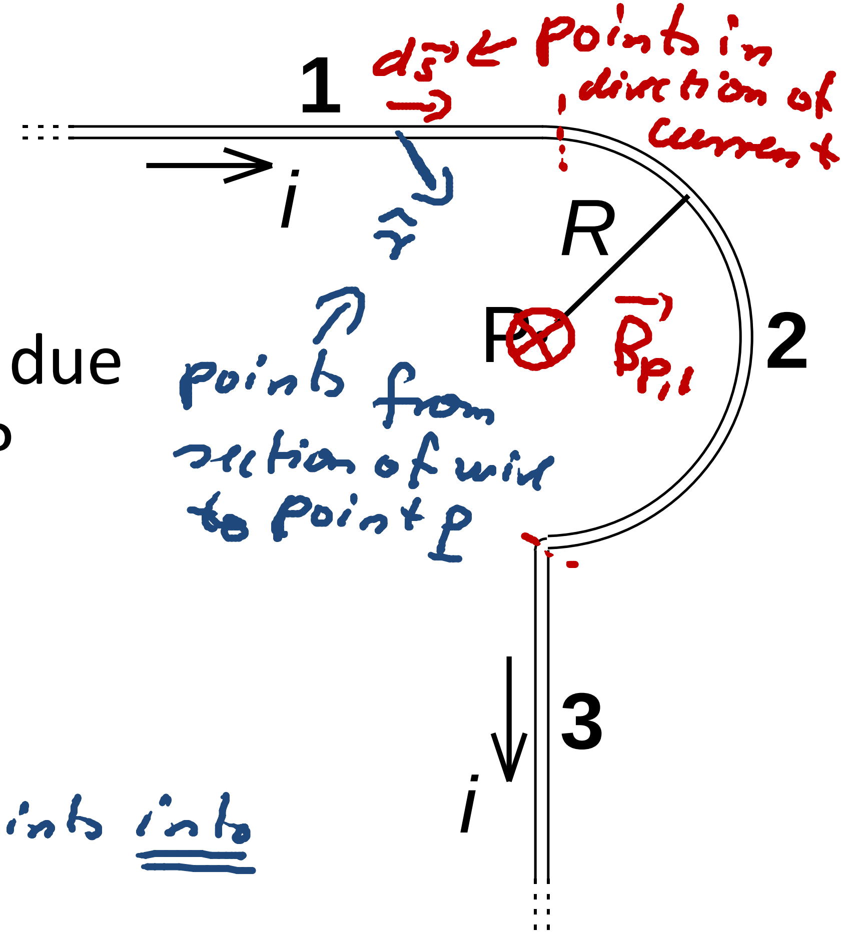

|B|
$\propto \frac{1}{R}$ if outside of wire
R

What is the direction of the magnetic field, $\vec{B}_{P,1}$, at point P due to the current in wire section **1**?

use Right-Hand-Rule

or $d\vec{B}_P = \frac{\mu_0}{4\pi} i \frac{d\vec{s} \times \hat{r}}{r^2}$

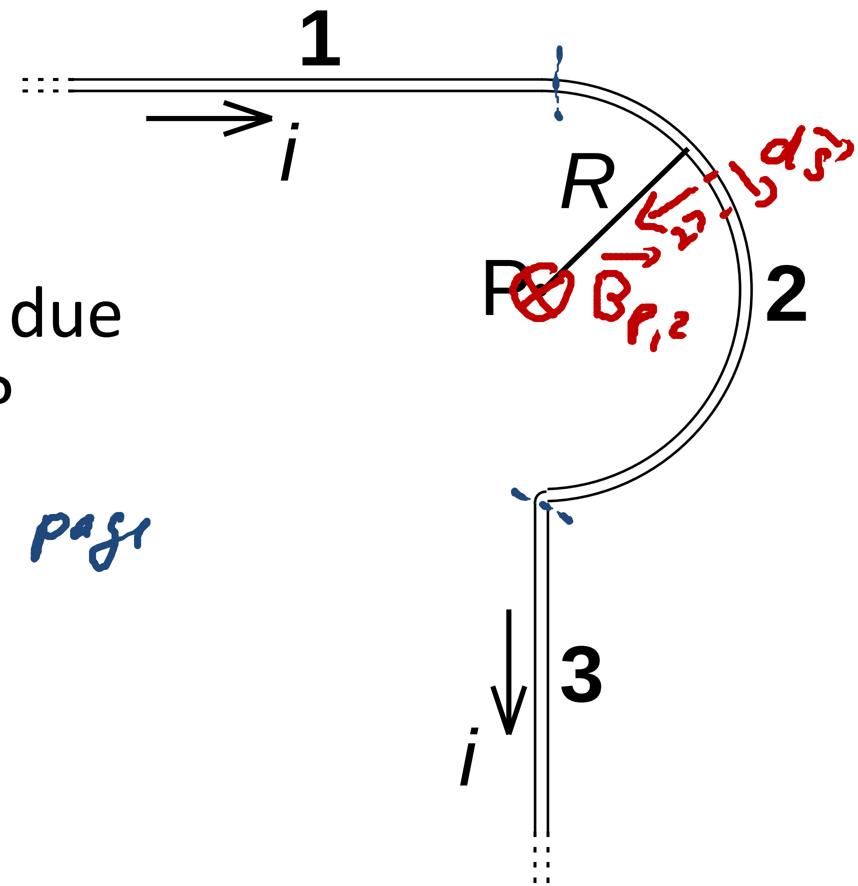
$d\vec{s} \times \hat{r}$: vector that points <u>into</u> the page



A. ⊙ (out of) B. ⊗ (into) C. ↑ D. ↓

E. No field at P due to section **1**.

What is the direction of the magnetic field, $\vec{B}_{\mathrm{P},2}$, at point P due to the current in wire section **2**?

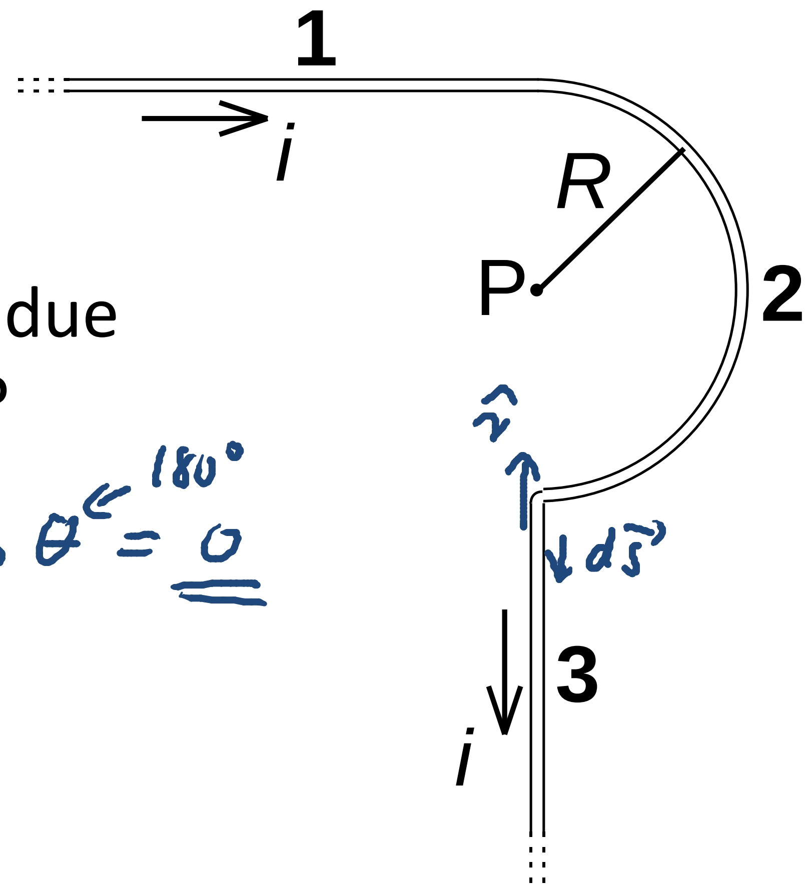


$d\vec{s}\,' \times \hat{r}$ points into the page

A. ⊙ (out of) B. ⊗ (into) C. ↑ D. ↓

E. No field at P due to section **2**.

What is the direction of the magnetic field, $\vec{B}_{\mathrm{P},3}$, at point P due to the current in wire section **3**?



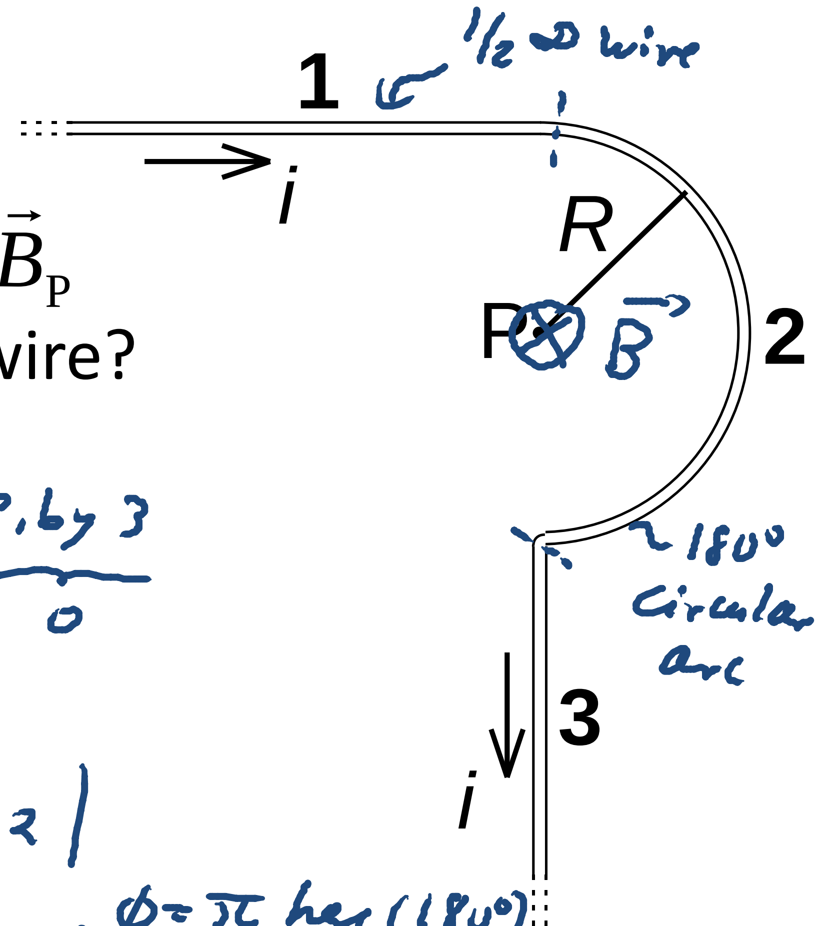
$$B_{P,3} \propto |d\vec{s} \times \hat{r}| = ds\, r \sin \overset{180^\circ}{\theta} = \underline{\underline{0}}$$

A. ⊙ (out of) B. ⊗ (into) C. ↑ D. ↓

E. No field at P due to section **3**.

Current-carrying wire:

What is the total magnetic field $\vec{B}_P$ at point P due to the current in wire?



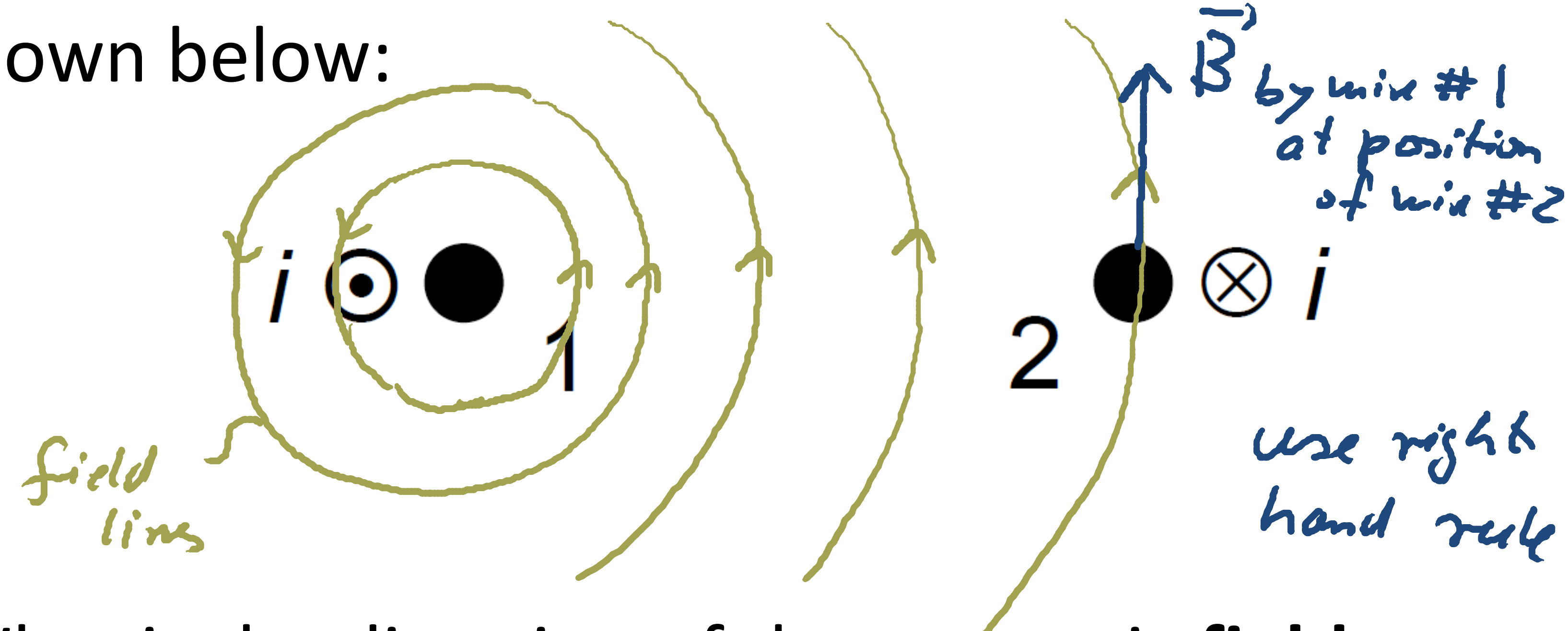
$$\vec{B}_p = \underbrace{\vec{B}_{p,\,by\,1} + \vec{B}_{p,\,by\,2}}_{\text{point in same direction (into page)}} + \underbrace{\vec{B}_{p,\,by\,3}}_{0}$$

$$\underline{|B_p|} = |B_{p,\,by\,1}| + |B_{p,\,by\,2}|$$

$\phi = \pi$ here (180°)

$$= \underbrace{\frac{1}{2}\frac{\mu_0 i}{2\pi R}}_{\text{by 1/2 }\infty\text{ wire}} + \frac{\mu_0 i}{4\pi R}\pi = \underline{\frac{\mu_0 i}{4R}\left(\frac{1}{\pi}+1\right)}$$

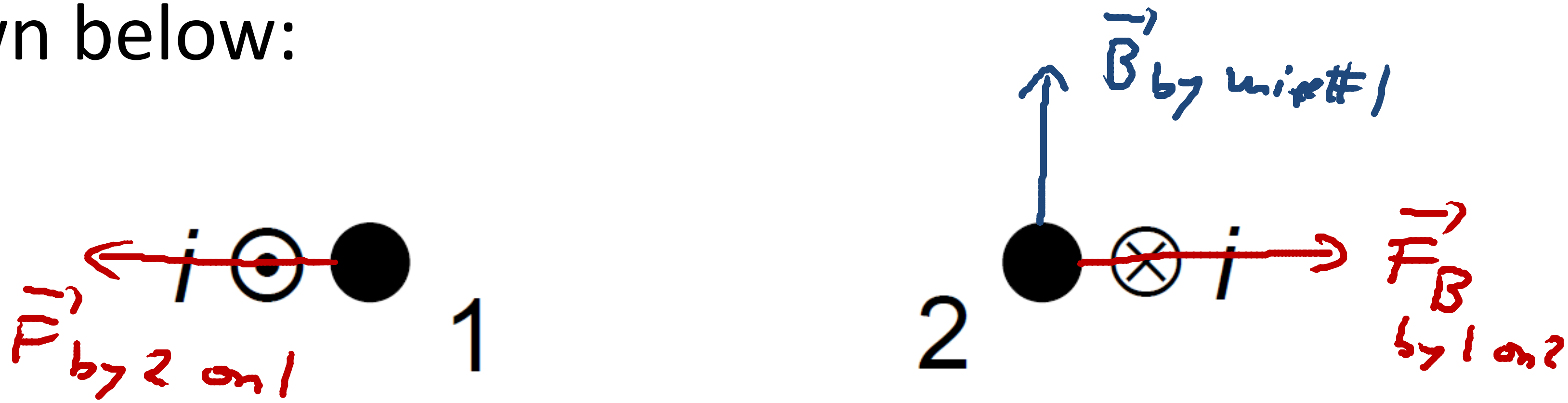
Consider two long wires running in parallel with current going through them in the directions shown below:



What is the direction of the magnetic **field at wire #2 due to the current in wire #1**?

A. ↑ B. ↓ C. ← D. →

Consider two long wires running in parallel with current going through them in the directions shown below:



$\vec{F}_{\text{on wire}} = i\vec{L} \times \vec{B}$

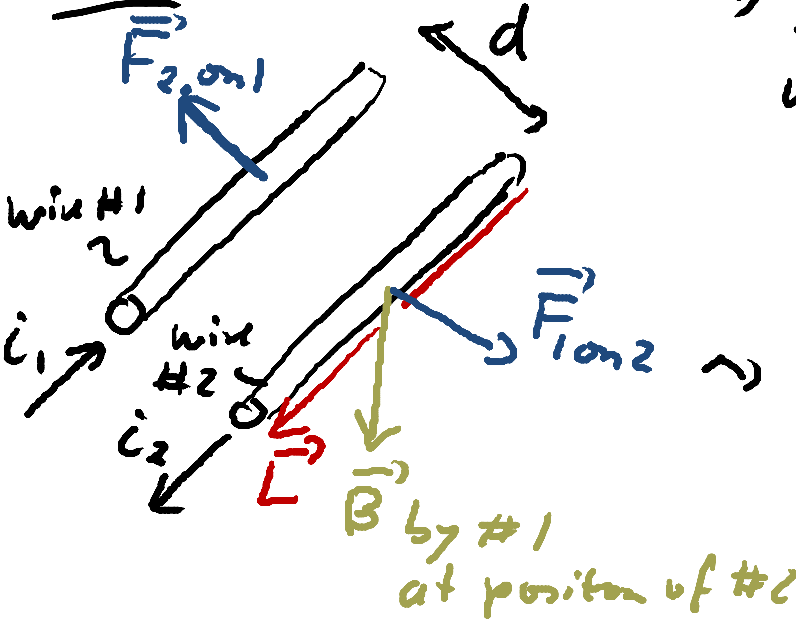
points along current direction

What is the direction of the **magnetic force on wire #2** by the field caused by wire #1?

A. ↑ B. ↓ C. ← D. →

Forces between two parallel wires/currents:

3D:



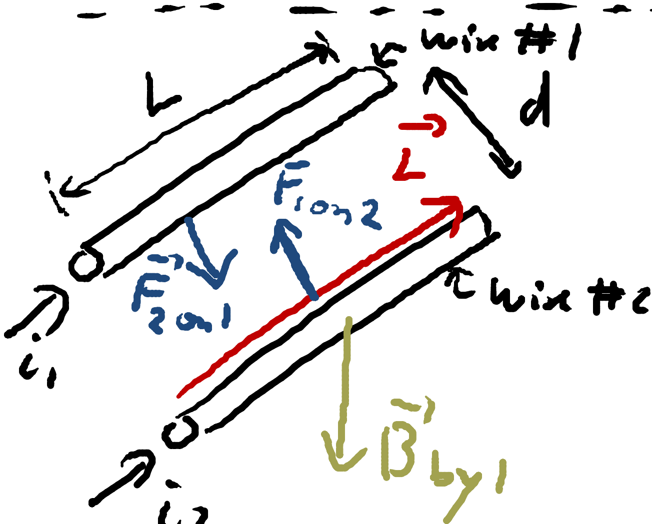
~> magnetic field at position of wire #2 by current in wire #1:

$$|B_{by\,1\,at\,2}| = \frac{\mu_0 i_1}{2\pi d}$$

~> resulting force on wire #2:

$$|F_{1\,on\,2}| = i_2 L B_{by\,1\,at\,2} \underbrace{\sin 90^\circ}_{=1}$$

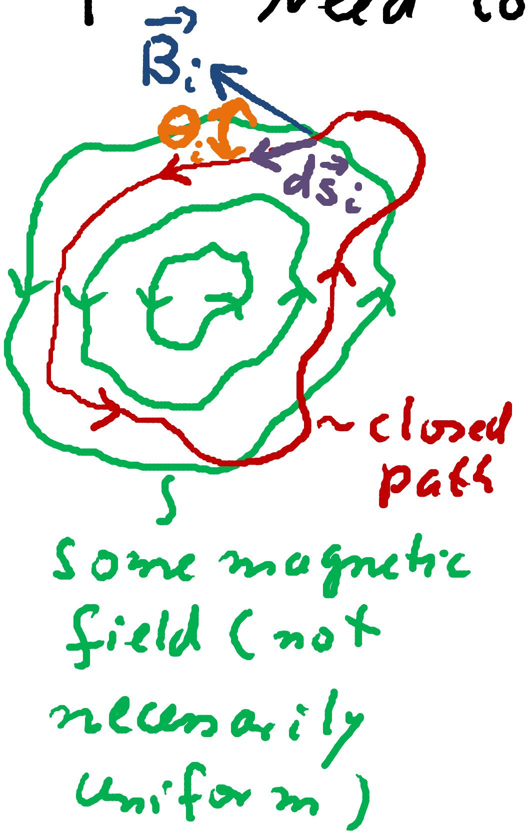
$$\Rightarrow \boxed{|F_{1\,on\,2}| = \frac{\mu_0 L i_1 i_2}{2\pi d}}$$



Direction: Parallel currents attract each other; and antiparallel currents repel each other!

Next: Ampere's Law:

• 1st: Need to define circulation Γ of a $\vec{B}$-field:



⇝ consider some imaginary closed path in a given magnetic field

⇝ Then "walk" along the closed path and integrate over (sum up) the magnetic field component $B_{\parallel}$ pointing along the direction of the path, for one full turn.