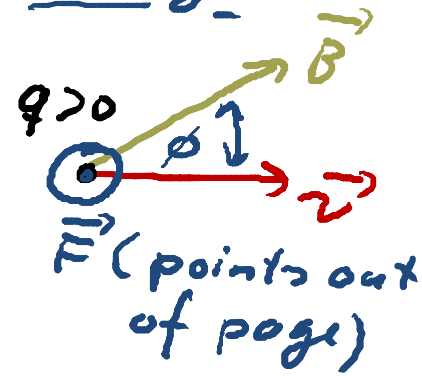
Recap I

Lecture 15

- Force by magnetic field on a moving charge:

$$\boxed{\vec{F}_B = q(\vec{v} \times \vec{B}) = q v B \sin\phi \, \vec{n}}$$

cross product of 2 vectors



- $\vec{F}_B$ is always $\perp$ to $\vec{v}$ and $\vec{B}$
- Direction is given by "Right-Hand-Rule". Watch out for sign of charge q!
- $\vec{F}_B$ never does any work on the charge: $W_{F_B} = 0$ (since it is always $\perp$ to the path)

- "Magnetic field lines":
  - used to indicate magnetic fields
  - Always form closed loops!
  - Emerge from "North pole" of a magnet and enter on the "South pole".

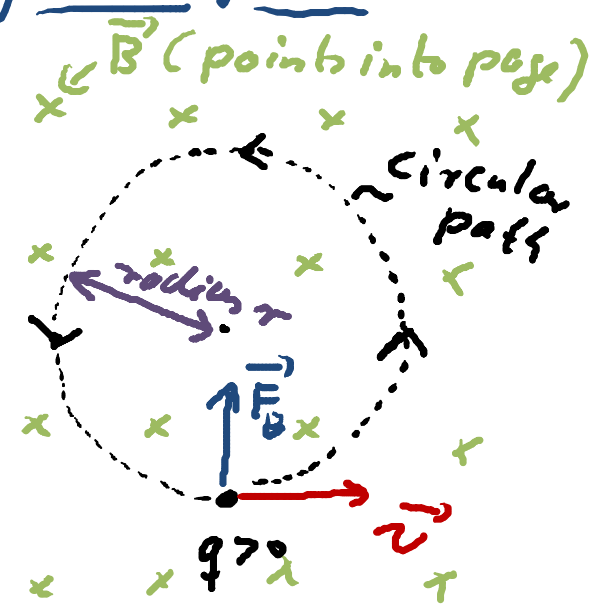
Recap II

- *Magnets:*
  - always have a south and north pole
    $\Rightarrow$ magnetic dipoles
  - Opposite magnetic poles attract each other
  - Like magnetic poles repel each other
- *Charge moving in a uniform magnetic field:*

$\Rightarrow$ Uniform circular motion:

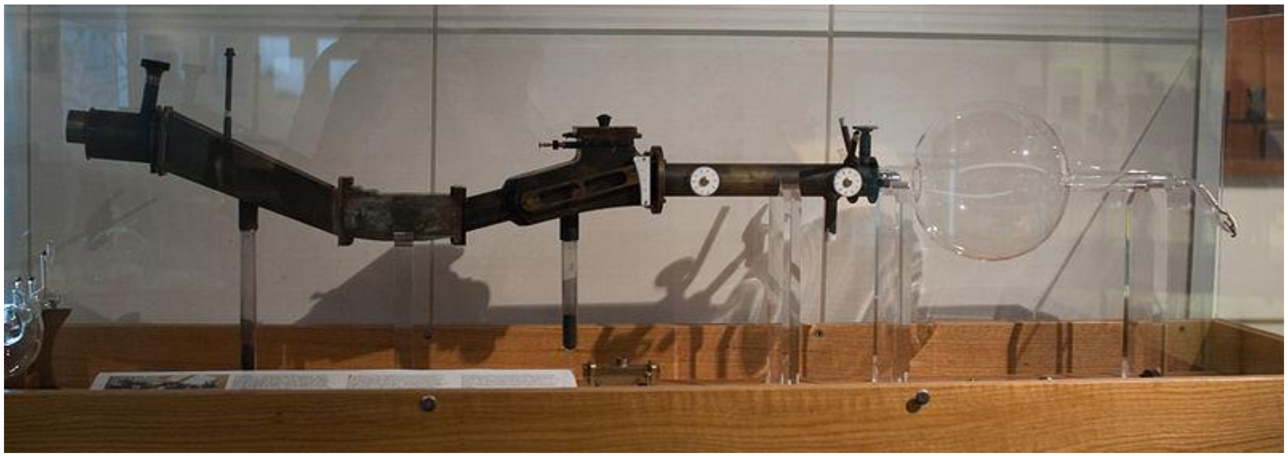
$$F_B = qBv \sin 90° = ma = m\frac{v^2}{r}$$

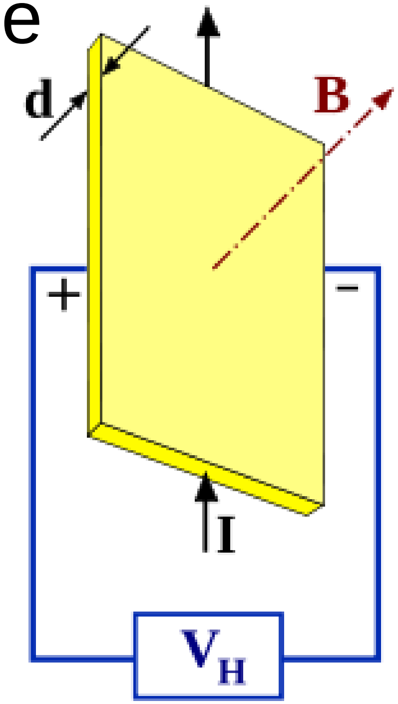
$$\Rightarrow \text{radius } r \text{ of path} = \frac{mv}{|q|\,B}$$



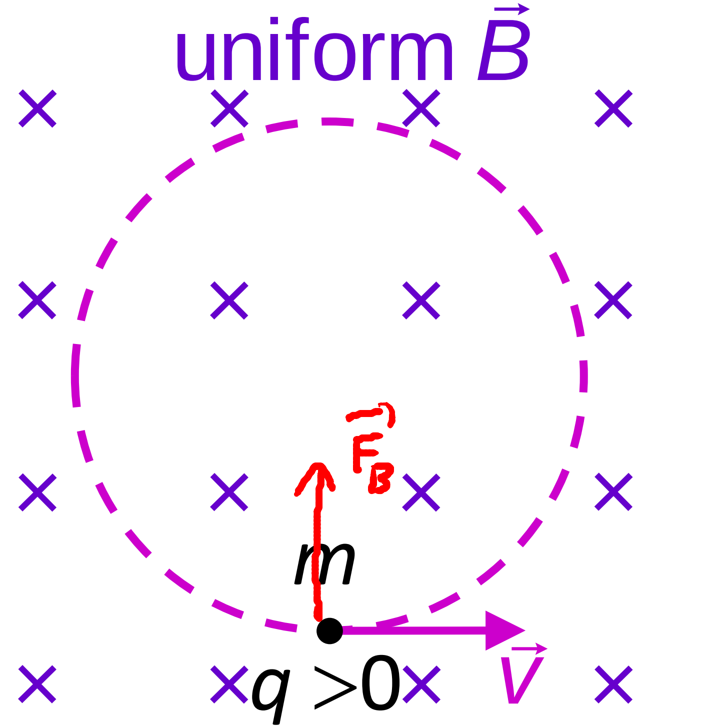
Today:

- Particle accelerators: The cyclotron and synchrotron
- Crossed electric and magnetic fields
  - Velocity selector
  - Hall effect
- Magnetic force on a current carrying wire
- Torque on a current loop





A particle of mass m and charge $q > 0$ is moving with speed $v \perp$ to a **uniform magnetic field** B.



How is the **period *T*** of the particle's orbit related to its speed v?

period $= T = \frac{2\pi r}{v} = 2\pi r\left(\frac{m}{|q|Br}\right) = \underline{\underline{\frac{2\pi m}{|q|B}}}$ indep. of v and r!

$\Rightarrow$ frequency $= f = \frac{1}{T} = \frac{|q|B}{2\pi m}$

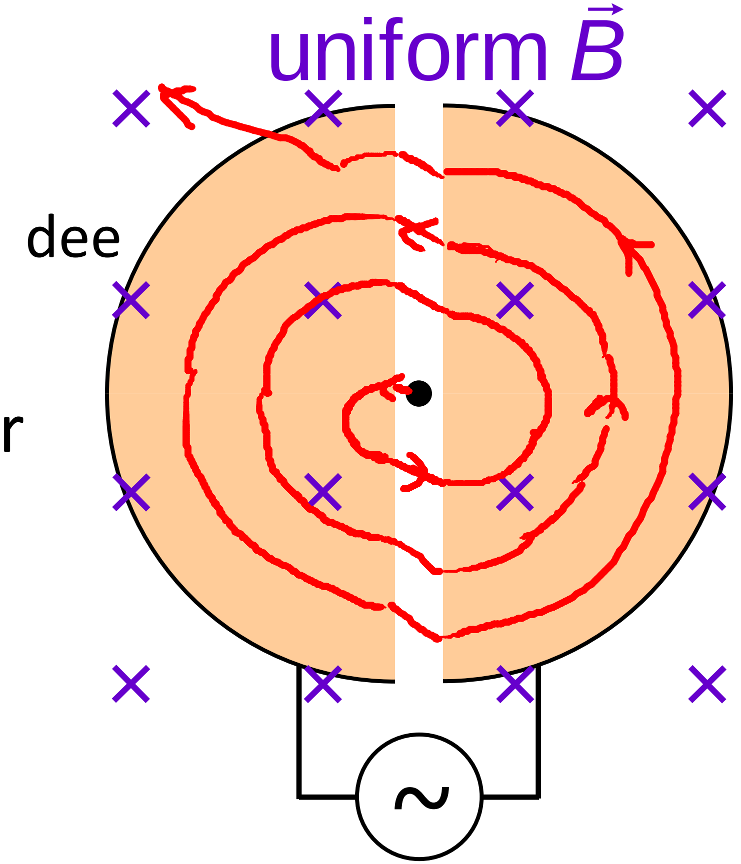
A. $T \propto v$ B. $T \propto v^2$ C. $T \propto v^{-1}$ D. $T \propto v^{1/2}$

E. T does not depend on v

Application: Particle Accelerators

The cyclotron:

- **Fixed magnetic field; changing orbit radius**
- composed of two hollow copper dees that are immersed in a uniform magnetic field & connected to an oscillating voltage source.
- Particles (e.g., protons), each of charge q & mass m, start at a source near the center of the dees.



oscillating voltage source of frequency f_{osc}

The Cornell Cyclotron

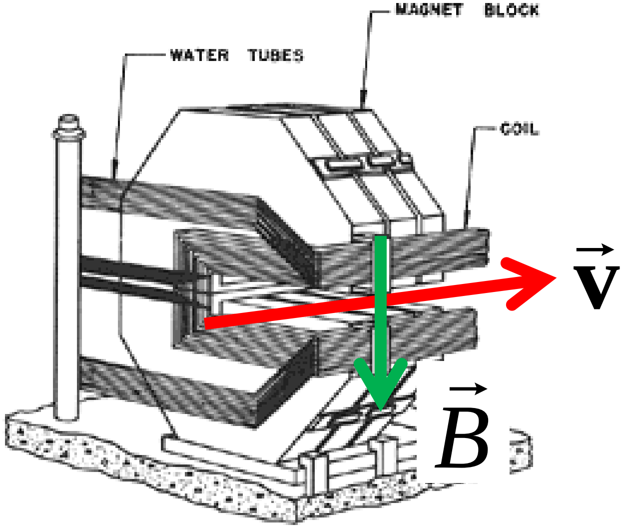
The Cornell cyclotron (2 MeV protons) was built about 1935 and decommissioned in 1956.

This photo with Assistant Professor Boyce D. McDaniel was taken in 1955.

Application: Particle Accelerators

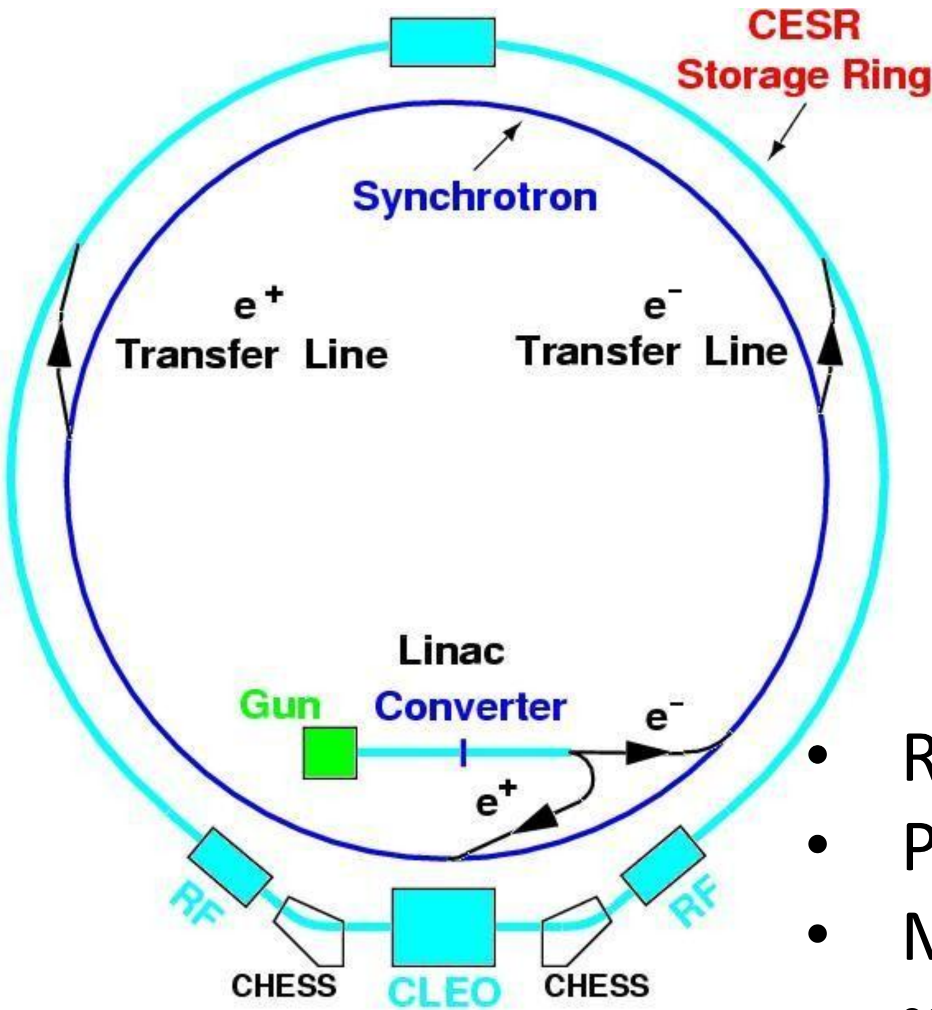
The synchrotron:

- **Fixed orbit radius; magnetic field adjusted for particle momentum/energy**
- "Dipole magnets" keep particles on fixed orbit.



$$Radius = \frac{p(t)}{qB(t)} = \text{const.}$$

The Cornell Synchrotron



- Radius = 122 m
- Particle energy: up to 5 GeV
- Magnetic bending fields: up to ~0.2 T (~3000* Earth's magnetic field)

Crossed Electric and Magnetic Fields

Consider charged particle moving through uniform magnetic and electric fields:

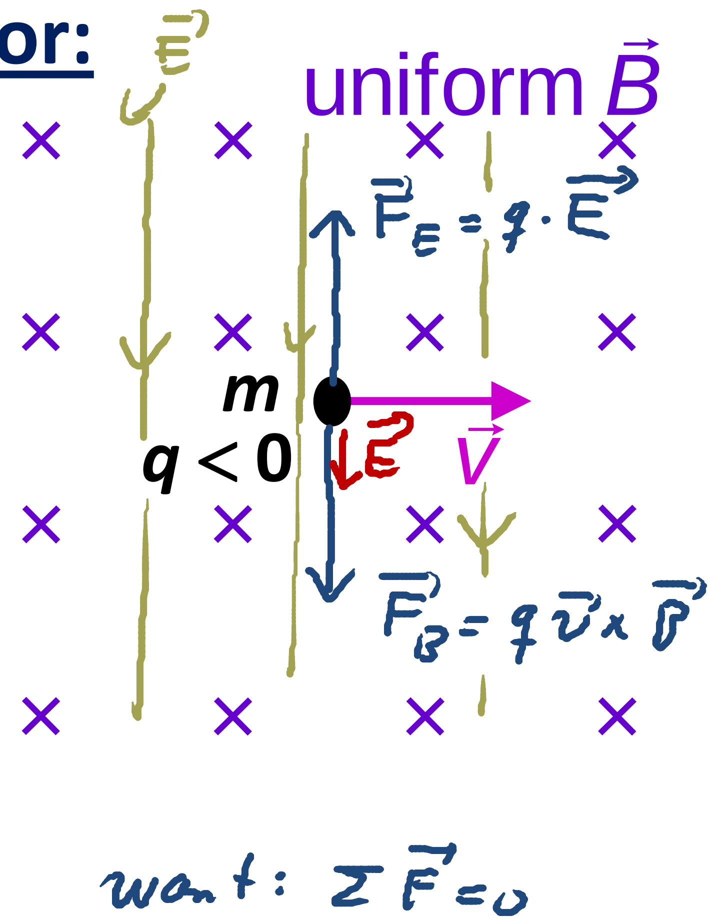
$$\Rightarrow \quad \vec{F}_E = q\vec{E} \qquad \vec{F}_B = q(\vec{v} \times \vec{B})$$

Examples:

- velocity selector
- Mass spectrometer
- Hall effect

Velocity Selector:

A particle of mass *m* and charge $q < 0$ is moving with speed $v \perp$ to a uniform **magnetic field *B***. By applying a uniform **electric field E** in the same region as the magnetic field, the particle can be made to **move in a straight line with constant speed *v***.



What should be the direction of the electric field?

A. ↑ B. ↓ C. ← D. ⊙ (out of) E. ⊗ (into)

Velocity Selector:

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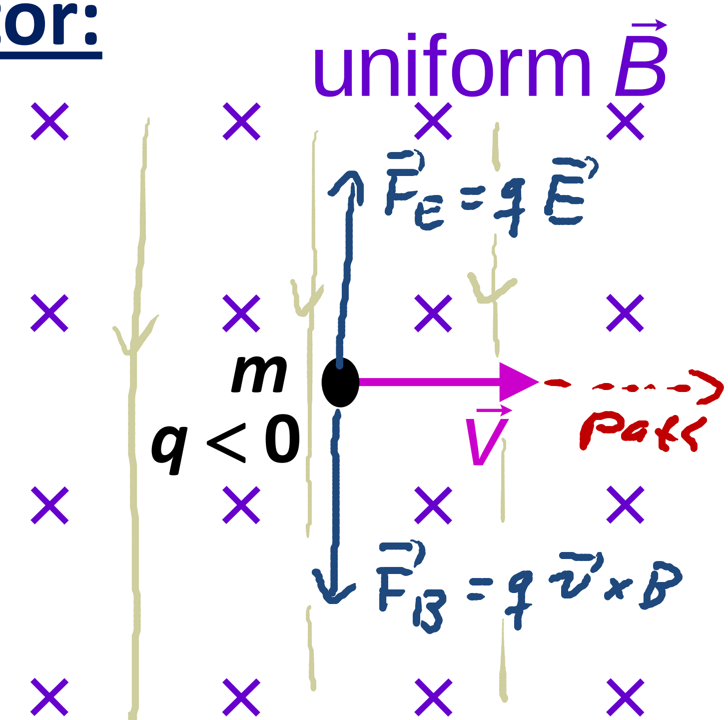


want $\Sigma \vec{F} = 0$

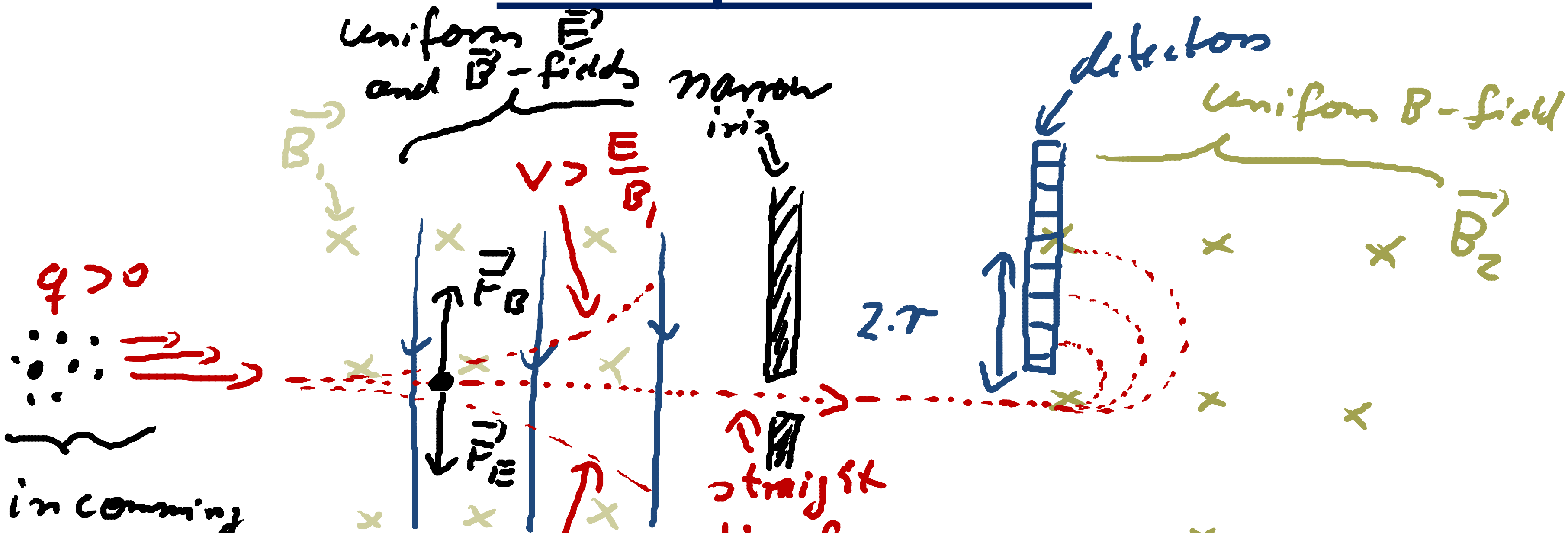
$\Rightarrow |F_E| = |F_B|$

$\Rightarrow |q|E = |q|vB\sin 90° \Rightarrow \boxed{E = vB}$

What should be the **magnitude of the electric field**?

A. $-qvB$ B. q^2vB C. vB D. $-mv/(qB)$ E. B

Mass Spectrometer



incoming particles with different velocities and masses

$v < \frac{E}{B_1}$

straight line for $v = \frac{E}{B_1}$

velocity selector

$$\boxed{v_{selected} = \frac{E}{B_1}}$$

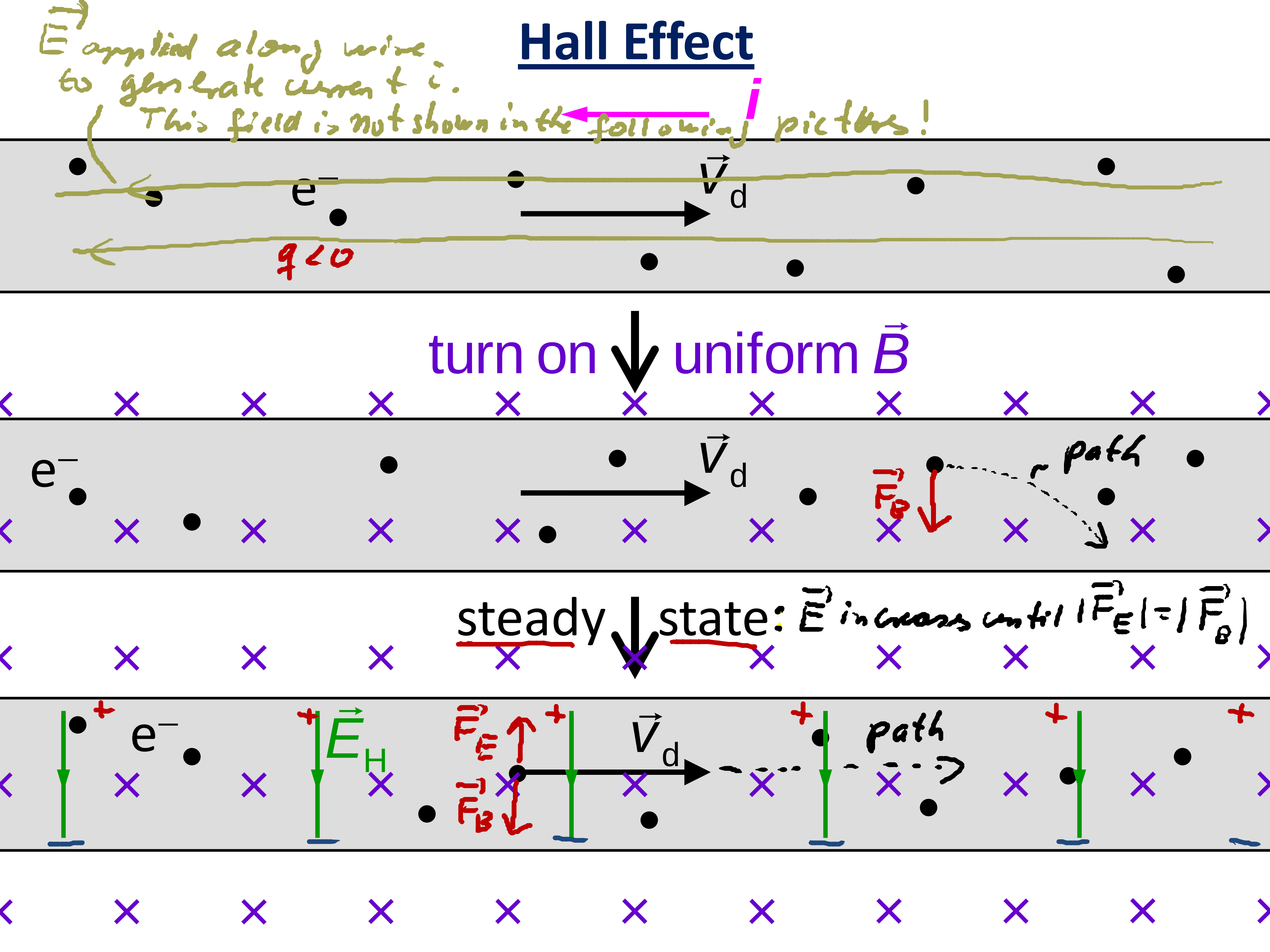
⇒ Bending radius r of path:

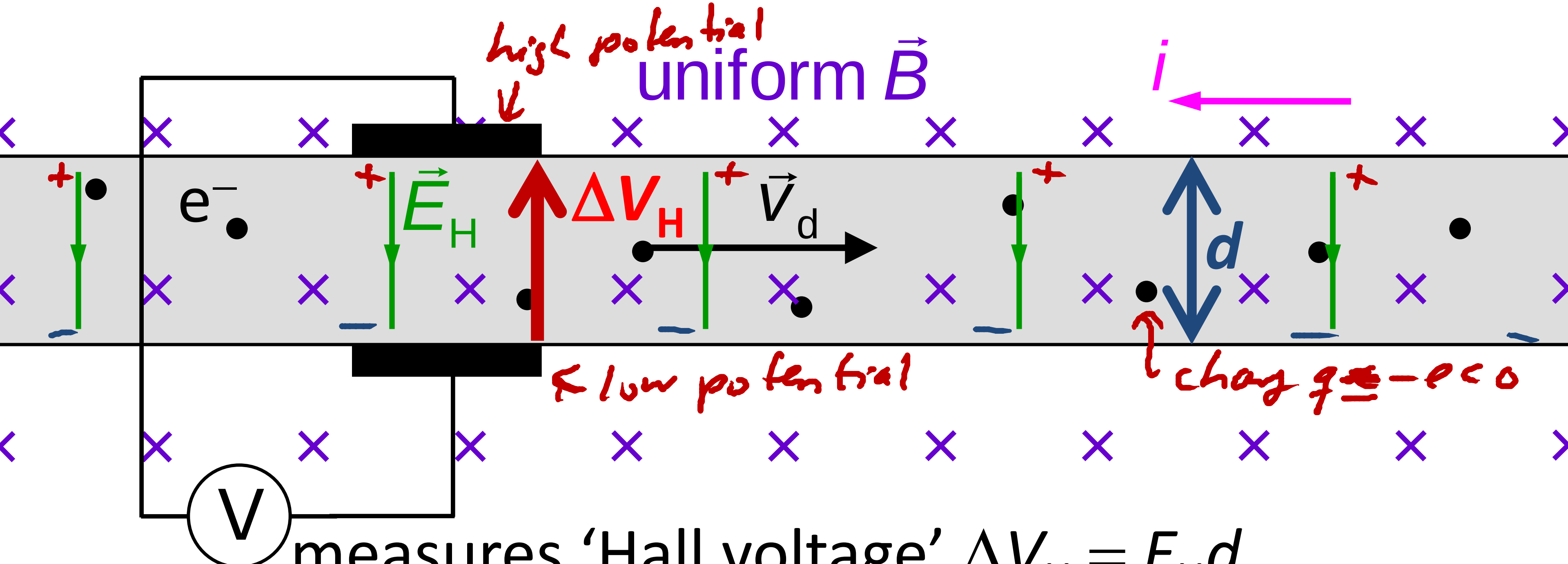
$$r = \frac{mv}{|q| B_2} \propto m \Rightarrow \frac{|q|}{m} = \frac{v}{r B_2}$$

⇒ can measure ratio q/m:

$$\frac{|q|}{m} = E/(B_1 B_2 r)$$

Hall Effect


$\vec{E}$ applied along wire to generate current i.
This field is not shown in the following pictures!
i
e^-
$\vec{V}_d$
$q<0$
turn on uniform $\vec{B}$
e^-
$\vec{V}_d$
$\vec{F}_B$
path
steady state: $\vec{E}$ increases until $|\vec{F}_E| = |\vec{F}_B|$
e^-
$\vec{E}_H$
$\vec{F}_E$
$\vec{F}_B$
$\vec{V}_d$
path
+
−



measures 'Hall voltage' $\Delta V_H = E_H d$

What is the drift speed v_d of the mobile electrons?

in steady state: $F_B = F_E$ } so that charge distribution on surfaces is fixed

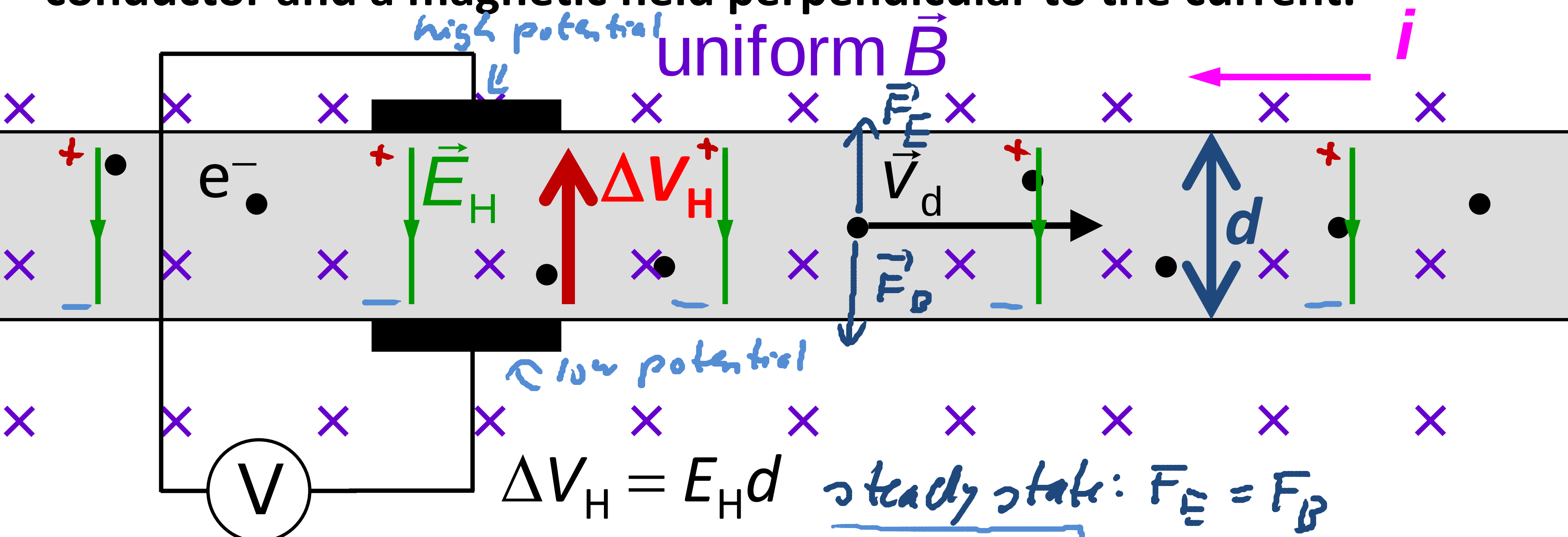
$\Rightarrow eE_H = ev_dB \Rightarrow v_d = \frac{E_H}{B} = \frac{\Delta V_H}{Bd}$ } can measure drift speed!

A. $\Delta V_H/(deB)$ B. $\Delta V_H/(de^2B)$ C. $\Delta V_H/(dB)$

D. None of the above

Hall Effect:

Production of a voltage difference (the Hall voltage) across an electrical conductor, transverse to an electric current in the conductor and a magnetic field perpendicular to the current.



$\Delta V_H = E_H d$

steady state: $F_E = F_B$

current density: $J = n e v_d \Rightarrow i = n e v_d A$

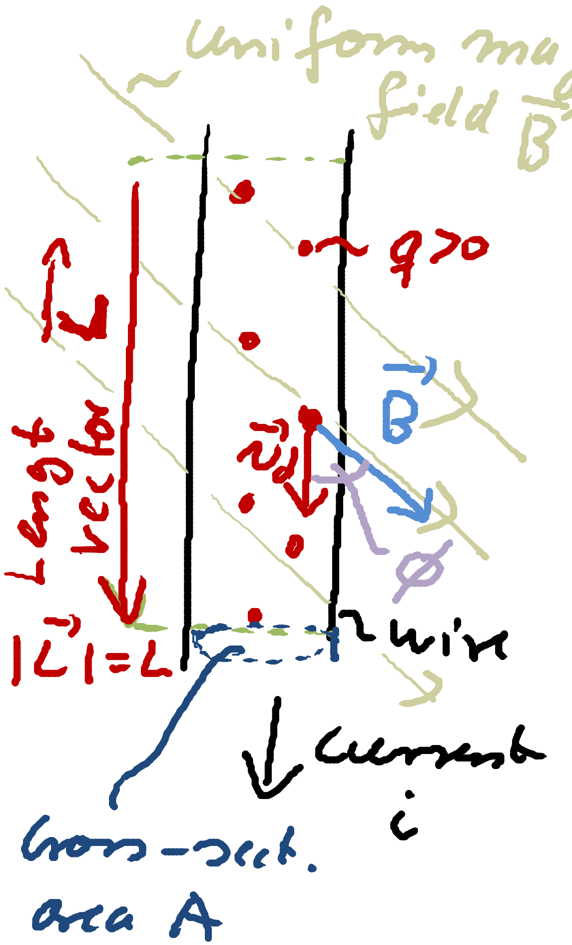
charge carriers per volume

cross-sectional area of conductor

$\Rightarrow v_d = \dfrac{\Delta V_H}{Bd} = \dfrac{E_H}{B}$

$\Rightarrow n = \dfrac{i}{e v_d A} = \dfrac{i B d}{e A \Delta V_H}$ } can find number of charge carriers per volume for measurable quantities!

Magnetic Force on a Current Carrying Wire:



⤳ Consider a straight wire with current density $\vec{J}$ in uniform magnetic field $\vec{B}$:

⇒ external force on a given single charge by the $\vec{B}$-field: $\vec{F}_B = q\,\vec{v}\times\vec{B}$

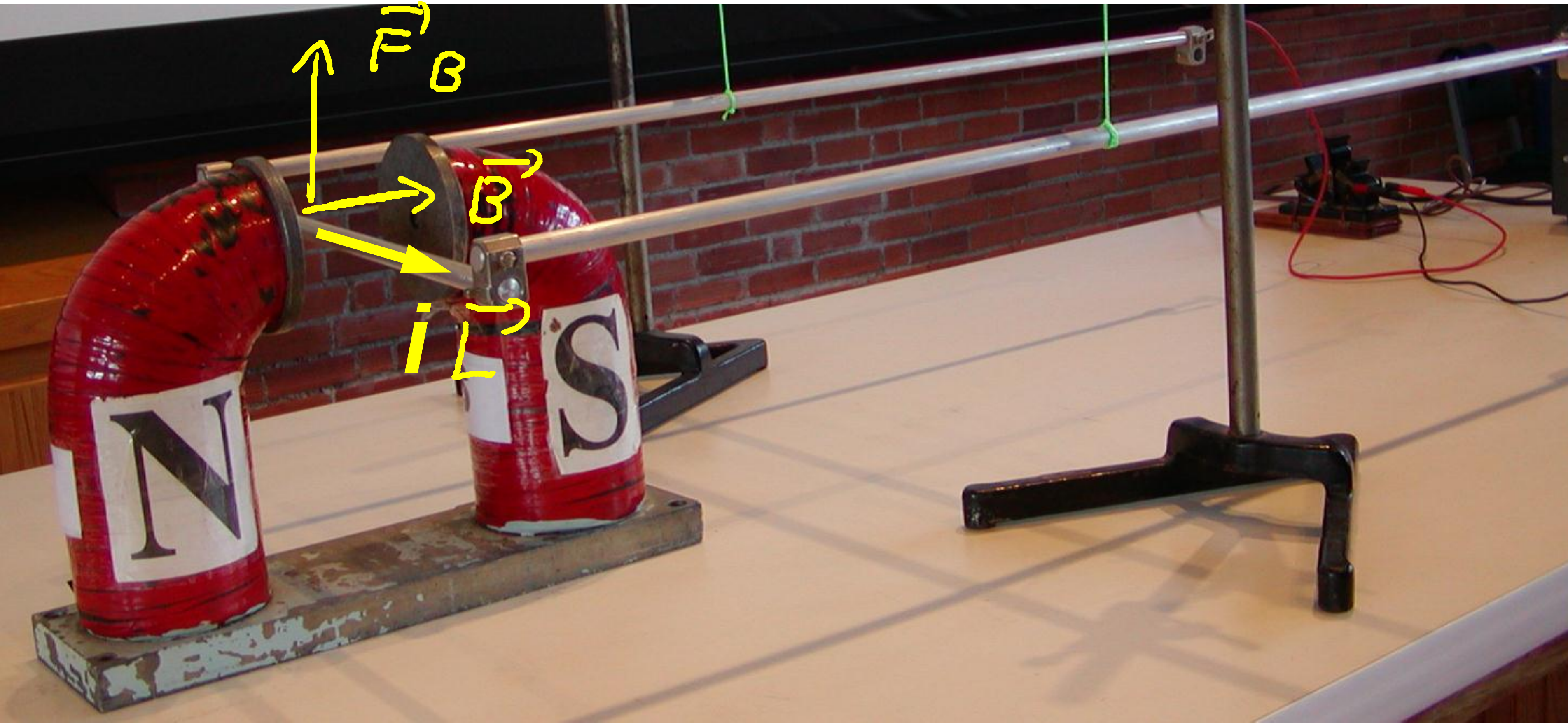
⇒ external force on <u>all</u> moving charges in wire of length L:

$$\vec{F}_{B,total} = Q_{total}\cdot\vec{v}\times\vec{B} = n\overbrace{LA}^{\text{volume}}q\,\vec{v}\times\vec{B}$$

$$= L\underbrace{A\vec{J}}_{A J = \text{current } i}\times\vec{B} \quad \text{since } \vec{J} = nq\,\vec{v}$$

⇒ $\boxed{\vec{F}_{B,\text{total on wire of length } L} = i\,\vec{L}\times\vec{B}}$ with length vector $\vec{L}$ pointing in direction of current, along wire

$|F_{total}| = iLB\sin\phi$ ⟸



If current is sent through the Aluminum bar that's between the magnet poles in the direction shown, which way will the bar move?

$\vec{F} = i\,\vec{L} \times \vec{B}$

A. ↑ B. ↓ C. That bar won't move!