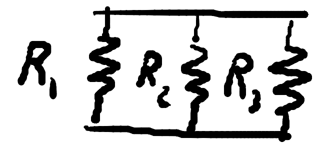
Recap

Lecture 13

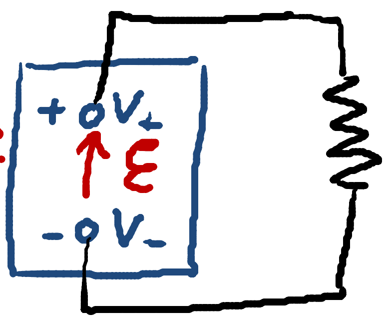
- Resistors in Series and Parallel

$$R_{eff,\,series} = \sum_{i=1}^{N} R_i$$



$$\frac{1}{R_{eff,\,parallel}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

- Emf device:

emf


$$emf = \mathcal{E} = \frac{dW}{dq} = V_+ - V_- = \left(\begin{array}{l}\text{work per unit}\\ \text{charge done by}\\ \text{emf device}\end{array}\right)$$

$$[\mathcal{E}] = \text{volt}$$

Power delivered by emf device $= P_{emf} = i\,\mathcal{E}$

Power "used" by circuit device $= P_{device} = i\,\Delta V_{over\ device}$

- Kirchhoff's rules:

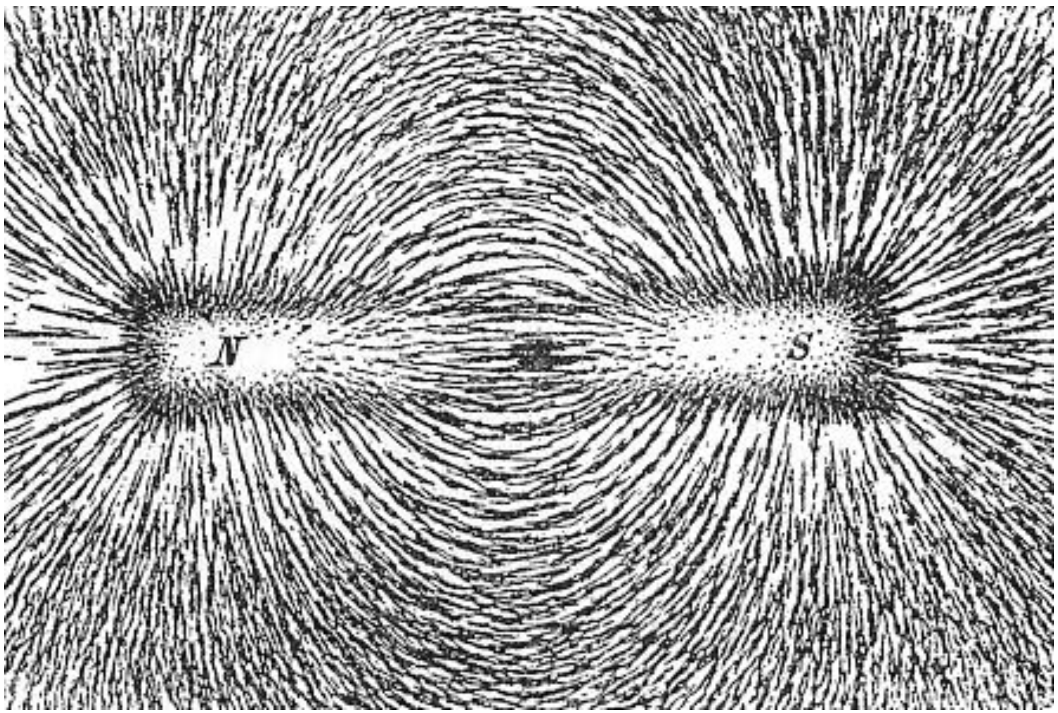
watch out for correct sign!!

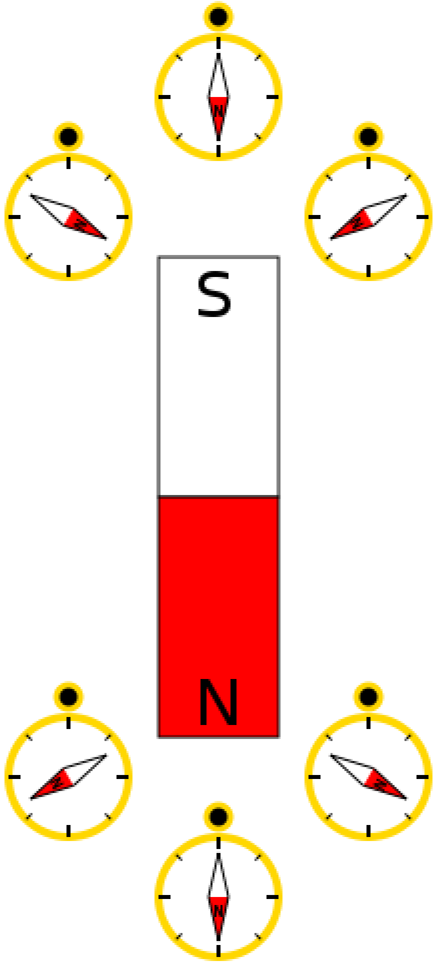
for closed loop: $\sum_{l=1}^{N} \Delta V_l = 0$

for junction: $\sum i_{in} = \sum i_{out}$

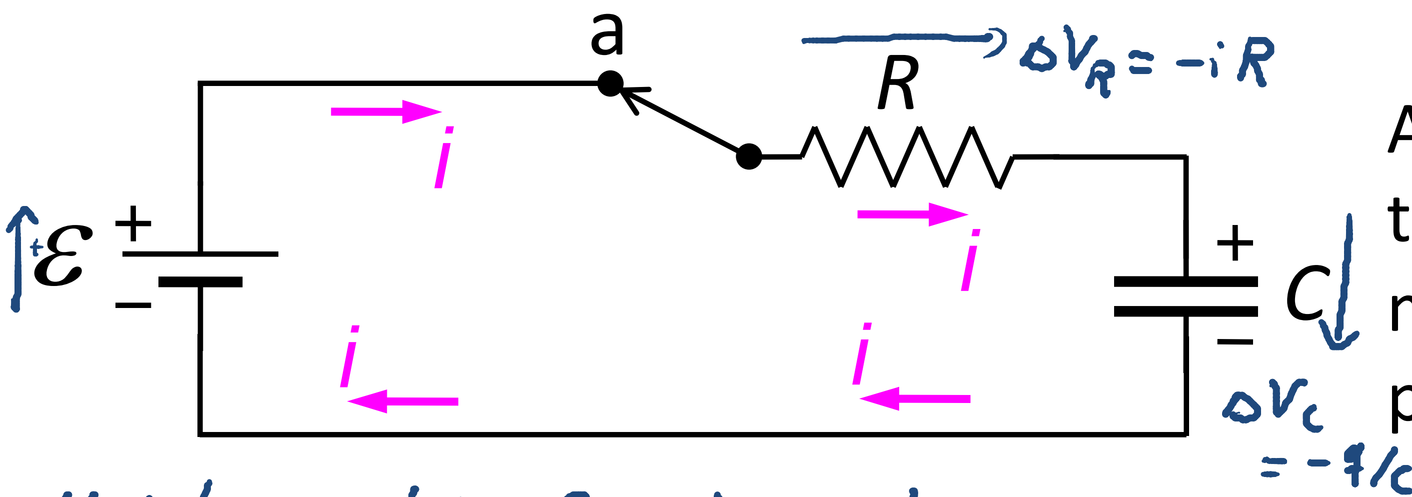
Today:

- More on RC circuits
- Magnets and magnetic field





RC circuit: Charging



At time $t = 0$ the switch is moved to position **a**.

Use loop rule: $\mathcal{E} + \Delta V_R + \Delta V_C = 0$

$\Rightarrow \quad \mathcal{E} - Ri - q/C = 0$ $\qquad$ use: $i = \dfrac{dq}{dt}$

$\Rightarrow \quad \mathcal{E} - R\dfrac{dq}{dt} - \dfrac{q}{C} = 0$ $\Big\}$ "differential equation" for charging

at $t=0$: $q(t=0) = 0$ $\Big\}$ "initial condition"

$\Rightarrow$ <u>Solution:</u> $\underline{\underline{q(t)}} = C\mathcal{E}\left[1 - e^{-t/RC}\right]$ ← exponential base $e = 2.718\ldots$

check: $q(t=0) = 0$ ✓ $\qquad \dfrac{dq}{dt} = \dfrac{\mathcal{E}}{R} e^{-t/RC}$

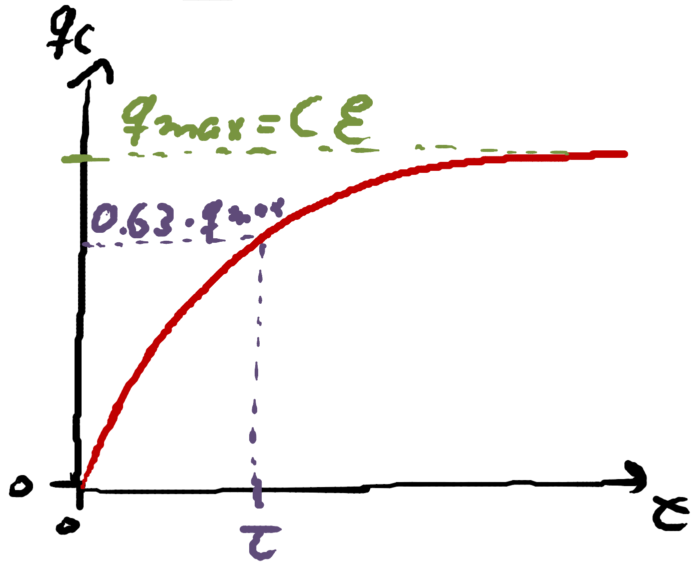
Charging of a Capacitor:

charge: $$\boxed{q_c(t) = C\,\mathcal{E}\left[1 - e^{-t/\tau}\right]}$$

with time constant $\boxed{\tau = RC}$ of exponential charging

recall: $e^{-1} = 0.37 = 37\%$

$\Rightarrow (1 - e^{-1}) = 0.63 = 63\%$

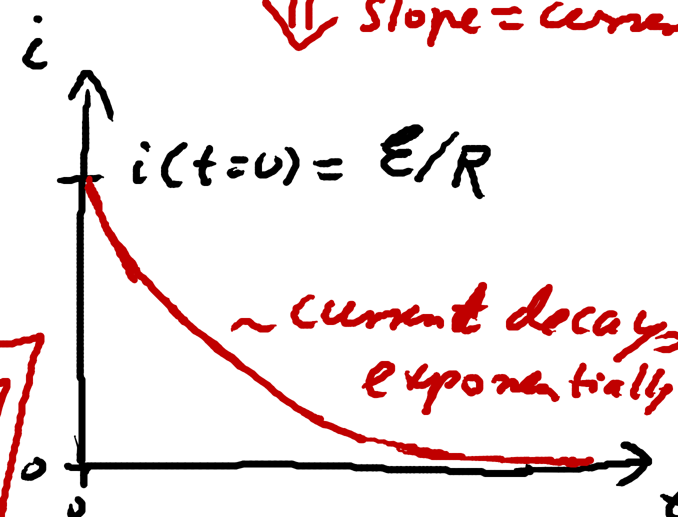


$\Downarrow$

$\Downarrow$ slope = current

current during charging $$\boxed{i(t) = \frac{dq}{dt} = \frac{\mathcal{E}}{R}\,e^{-t/\tau}}$$

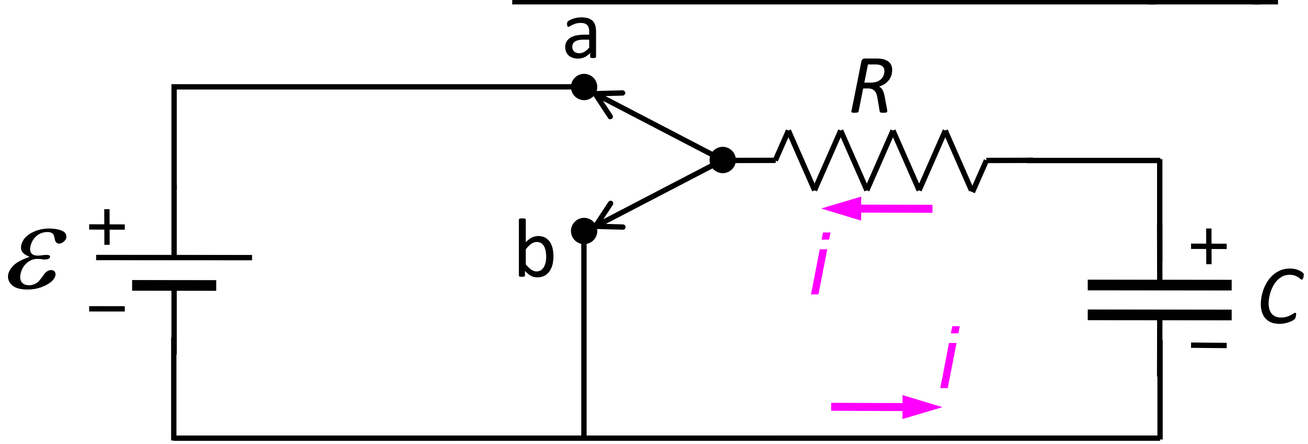
at $t=0$: $i(0) = \mathcal{E}/R$



potential change across capacitor $$\boxed{\Delta V_c = \frac{q}{C} = \mathcal{E}\left[1 - e^{-t/\tau}\right]}$$

at $t=0$: $\Delta V_c(0) = \mathcal{E}$

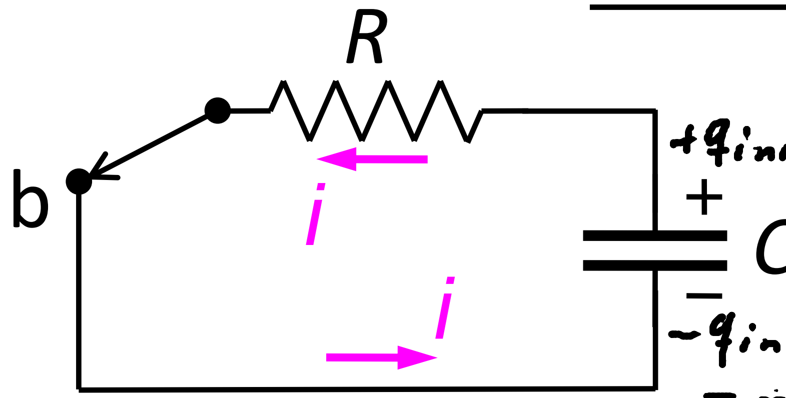
RC circuit: Discharging



- The switch has been at position **a** for a very long time.
- At time $t = 0$ move the switch to position **b**.
- Current *i* begins to flow to discharge the capacitor.

RC circuit: Discharging

$\Delta V_R = -iR$



$+q_{initial} = +q_0$

$-q_{initial} = -q_0$

At time $t = 0$ the switch is moved to position **b**.

$\Delta V_C = q/C$

use loop rule (in direction of current):

$\Delta V_C + \Delta V_R = 0 \Rightarrow \frac{q}{C} - iR = 0$ use: $\underbrace{i}_{>0} = - \underbrace{\frac{dq}{dt}}_{<0 \text{ discharging}}$?

$\Rightarrow \frac{q}{C} + \frac{dq}{dt} R = 0$ } differential equation for discharging

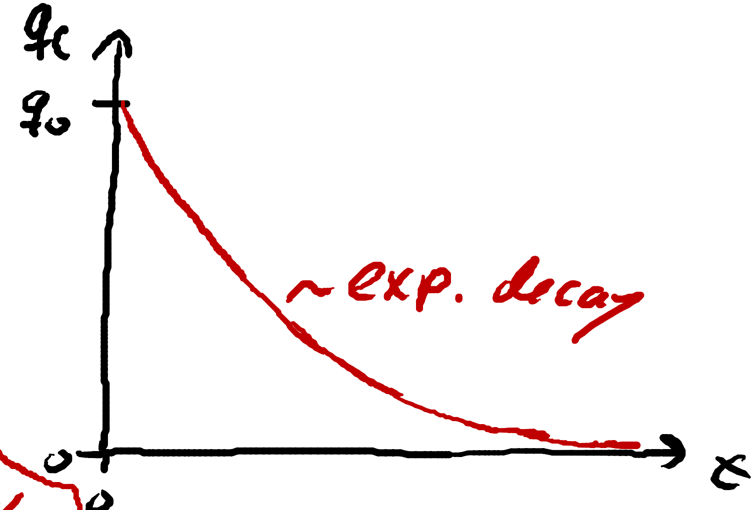
at $t=0$: $q_{initial} = q(t=0) = q_0$

Solution: $q(t) = q_0 \, e^{-t/RC}$

check: $q(t=0) = q_0$ ✓ $\frac{dq}{dt} = -\frac{q_0}{RC} e^{-t/RC}$

Discharging of a capacitor:

charge: $\boxed{q_c(t) = q_0 e^{-t/\tau}}$



with time constant $\boxed{\tau = RC}$
of exponential decay

$\Downarrow$

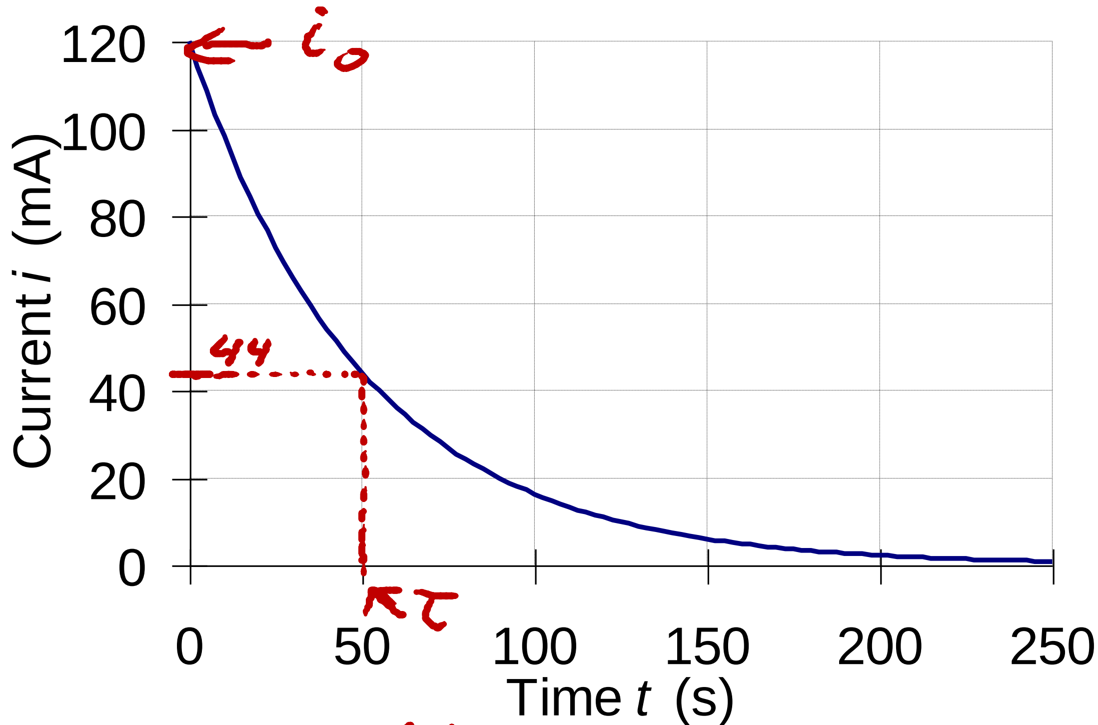
current during discharging $\boxed{i(t) = -\frac{dq}{dt} = \frac{q_0}{RC} e^{-t/\tau}}$

$\Downarrow$ – slope = current

$i(t=0) = q_0/RC$

potential change across capacitor $\boxed{\Delta V_c = \frac{q}{C} = \frac{q_0}{C} e^{-t/\tau}}$

What is the approximate value of the time constant τ for this decay of electric current from a discharging capacitor in a simple *RC* circuit?



$\tau = ?$

A. ~25 s

B. ~35 s

C. ~50 s

D. ~100 s

~~E.~~ ~250 s

$$i(t) = \underbrace{i_0}_{q_0/RC}\, e^{-t/\tau}$$

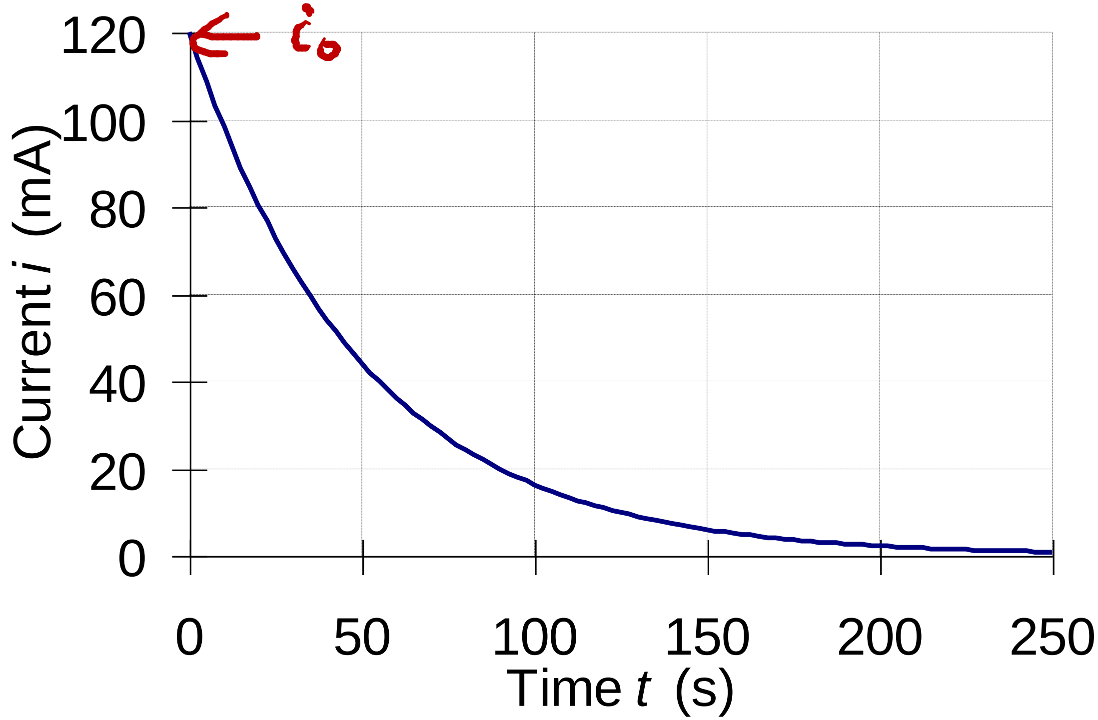
$$\Rightarrow i(t=\tau) = i_0\, e^{-1} = 120\,\text{mA} \cdot 0.37$$

$$= 44\,\text{mA}$$

$$\Rightarrow \tau = 50\,\text{s}$$

Approximately, what was the discharging capacitor's initial charge at time $t = 0$?

$q_0 = ?$



A. 1.2 C

B. 3.0 C

C. 6.0 C

D. 12 C

~~E.~~ 18 C

$i_0 = \frac{q_0}{RC} = \frac{q_0}{\tau} \Rightarrow q_0 = i_0 \cdot \tau$

$= 120\,mA \cdot 50\,s$

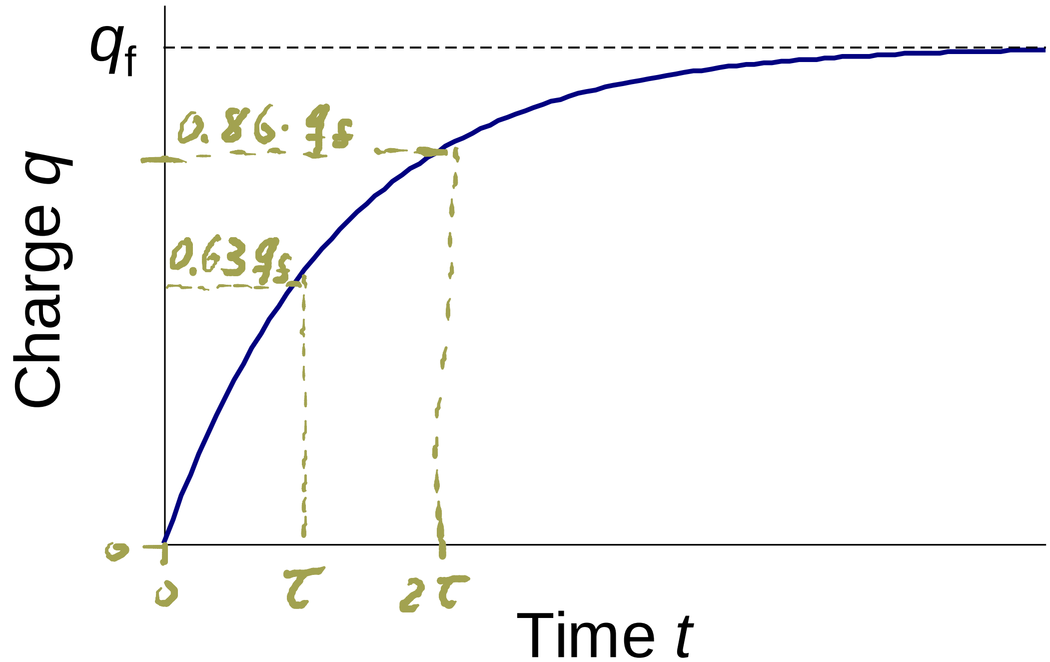
$= \underline{\underline{6\,C}}$

The graph shows the electric charge on a charging capacitor in a simple *RC* circuit.

At time $t = 2\tau$, how much charge is on the capacitor?

$q(t=2\tau)=?$

A. $0.14\ q_f$
B. $0.37\ q_f$
C. $0.63\ q_f$
D. $0.79\ q_f$
E. $0.86\ q_f$



$$q(t) = q_{final}\left(1 - e^{-t/\tau}\right)$$

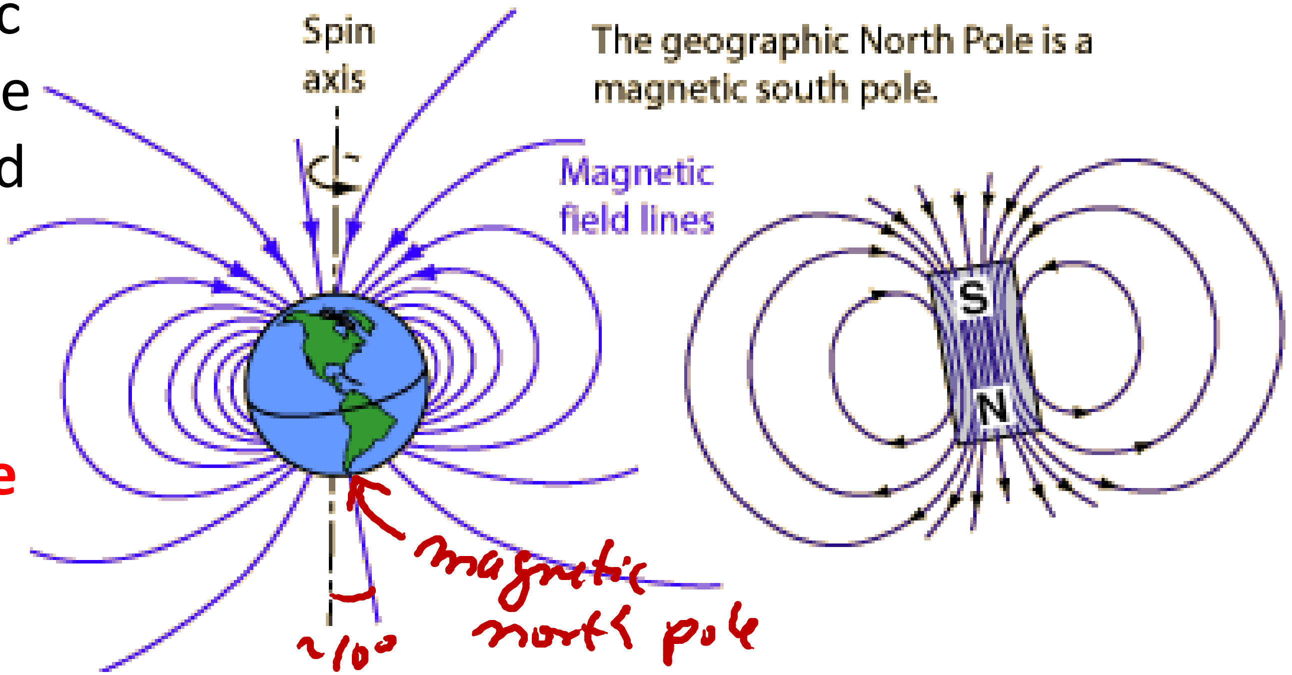
$$\Rightarrow q(t=2\tau) = q_{final}\left(1 - e^{-2\tau/\tau}\right)$$

$$= q_{final}\left(1 - e^{-2}\right)$$

$$= q_{final} \cdot \underline{\underline{0.86}}$$

Magnetic Fields and Forces

The Earth's magnetic field near the surface can be approximated by the field of a bar magnet. In which direction would the **magnetic north pole** of Earth's magnet point?



~~A.~~ To the geographic north pole

B. To a point near the geographic north pole

~~C.~~ To the geographic south pole

D. To a point near the geographic south pole

Magnetic Fields and Forces

- What produces magnetic fields $\vec{B}$?

(a) magnetic charges? (magnetic monopoles)

No! Never have been found (nobody knows why they do not exist...)

⇒ no individual "north" or "south" pole; always come in north-south pairs!!

(b) Electromagnet:

⇒ electric currents (moving charges) produce a magnetic field around them!

(c) Permanent magnet:

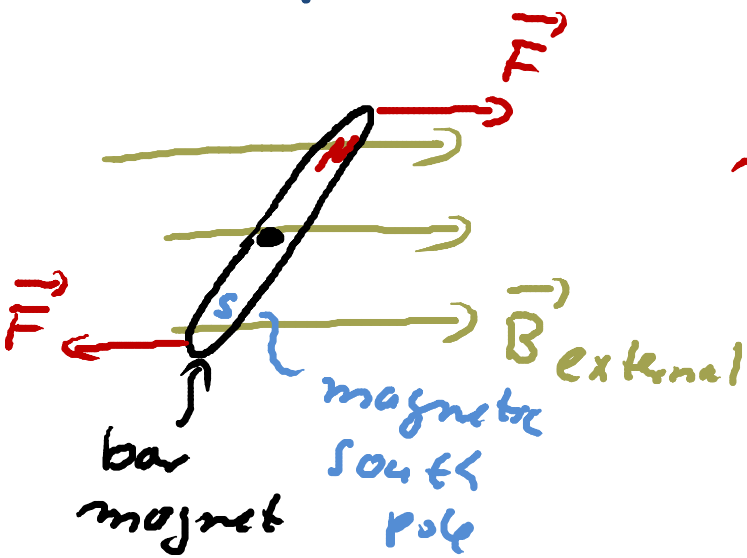
⇒ elementary particles have an intrinsic magnetic field around them ⇒ magnetic fields of particles add up in certain materials ⇒ net magnetic field around the material

How can we detect a magnetic field $\vec{B}$?

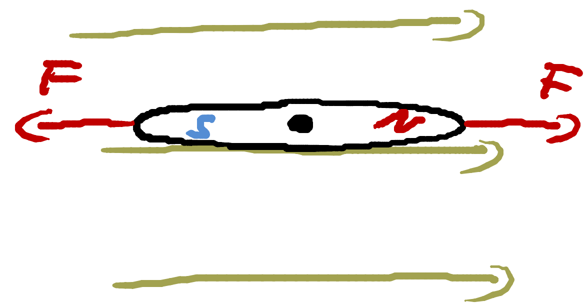
- Recall: for electric fields $\vec{E}$: generates force on test charge q_t $\quad \vec{F} = q_t \vec{E}$

- <u>for magnetic fields $\vec{B}$:</u>

(a) Torque τ on compass needle (bar magnet)

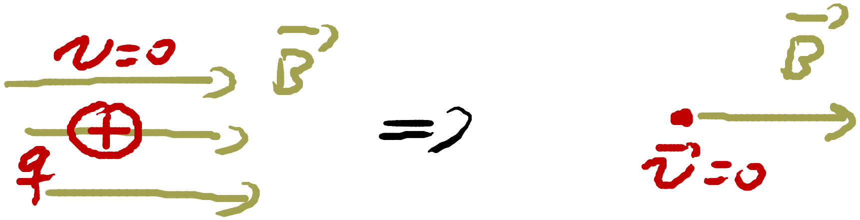


rotates! $\Rightarrow$

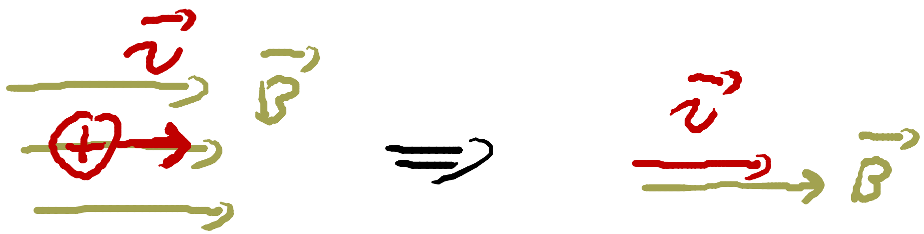


$\Rightarrow$ In a magnetic field, the torque on a bar magnet tends to align the magnet with the direction of the $\vec{B}$-field!

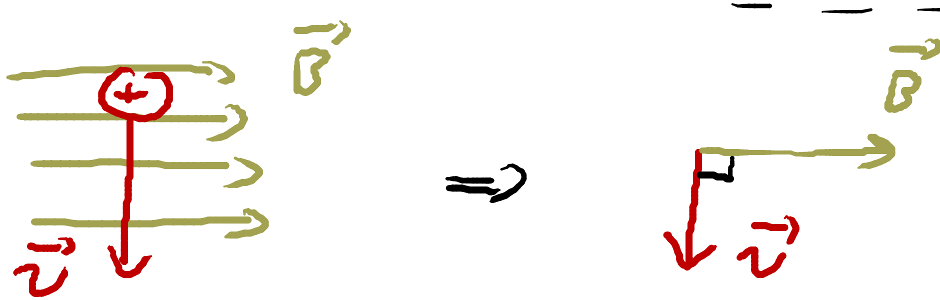
(b) by the <u>magnetic force</u> $\vec{F}_B$ exerted on a <u>moving</u> electric charge:



$\Rightarrow \vec{F}_B = 0$ } charge needs to move!



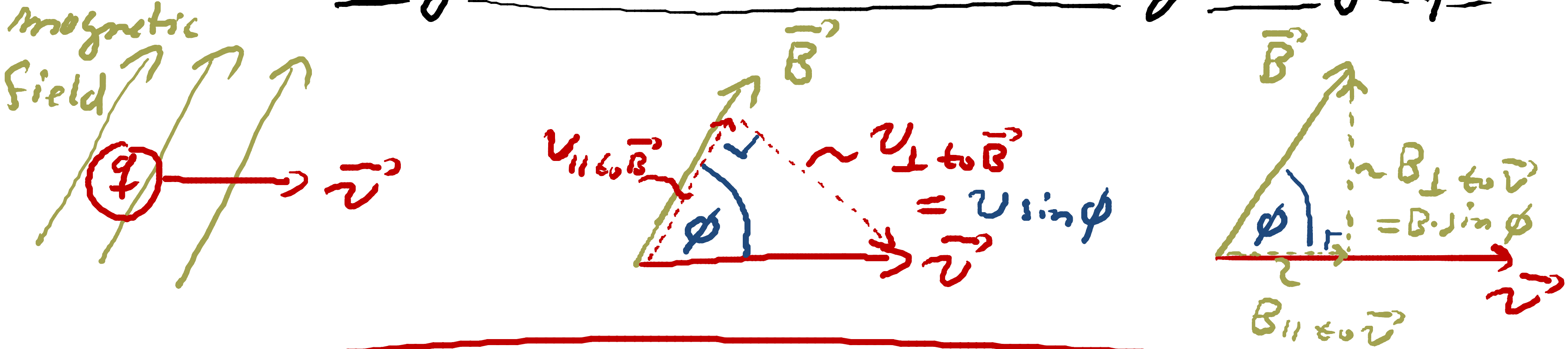
$\Rightarrow \vec{F}_B = 0$ } still no force if charge is moving in direction of $\vec{B}$



component of velocity $\vec{v} \perp$ to $\vec{B}$

$$\Rightarrow \boxed{|F_B| = |q|\, v_{\perp \text{ to } \vec{B}}\, B}$$

⇒ Magnetic Force on a moving charge q:



charge　　　　　　　　speed of moving charge

$$\boxed{|F_B| = |q|\, v_{\perp \text{ to } \vec{B}}\, B = |q|\, v\, B \sin\phi = |q|\, v\, B_{\perp \text{ to } \vec{v}}}$$

with ϕ: smallest angle between $\vec{v}$ and $\vec{B}$ $(0 \le \phi \le 180°)$

⇒ this equation defines the magnetic field B

Units: $[B] = \dfrac{[F]}{[q][v]} = \dfrac{N}{C\,\frac{m}{s}} = \dfrac{N}{A\,m} \equiv$ 1 tesla $= 1\,T$

$= 10^4$ gauss