Recap

Lecture 12

- *Electrical Resistance:*

Resistance: $R = \dfrac{\Delta V_{\text{over resistor}}}{\text{current } i}$

Resistivity/conductivity:

material property:

$$\vec{J} = \sigma \vec{E} = \frac{1}{\rho} \vec{E}$$

conductivity   resistivity

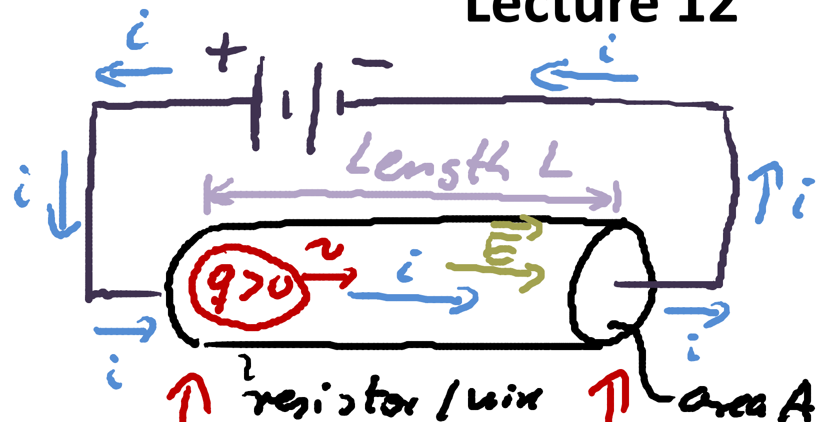
- temperature dependence:

$$\rho(T) = \rho(T_0)[1 + \alpha \Delta T]$$

temperature coefficient of resistivity

- for wire:

$$R_{\text{wire}} = \frac{\rho L}{A}$$



high potential → low potential

$$\Delta V = -iR$$

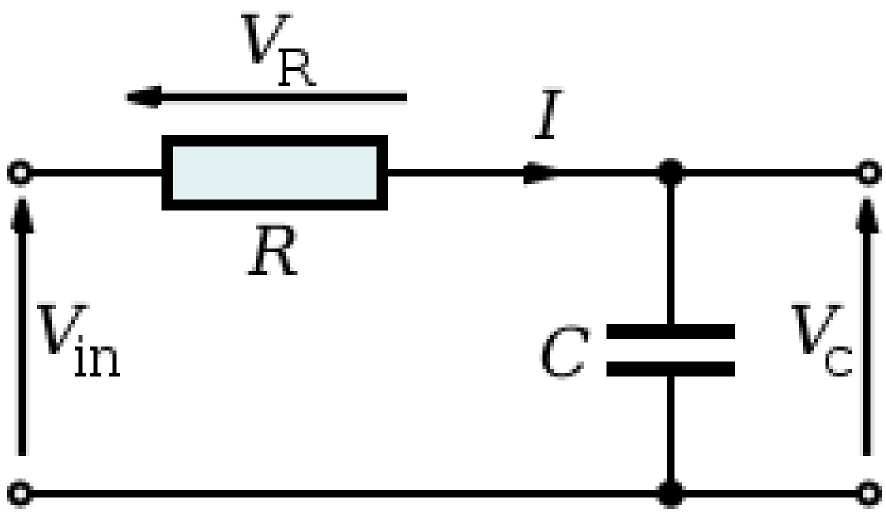
high pot. energy for charge $q>0$ → $\Delta U = q\Delta V$ → low potential energy for charge $q>0$

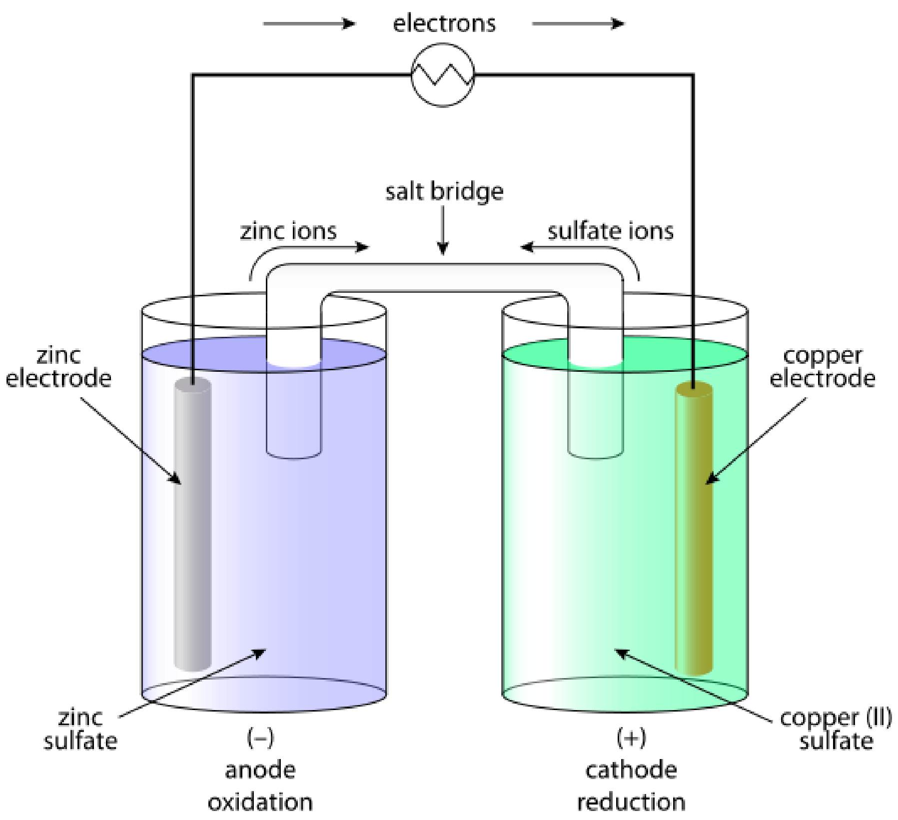
electric potential energy is transferred to other form of energy

$$P_{\text{by current}} = i\Delta V = iR^2 = \frac{\Delta V^2}{R}$$

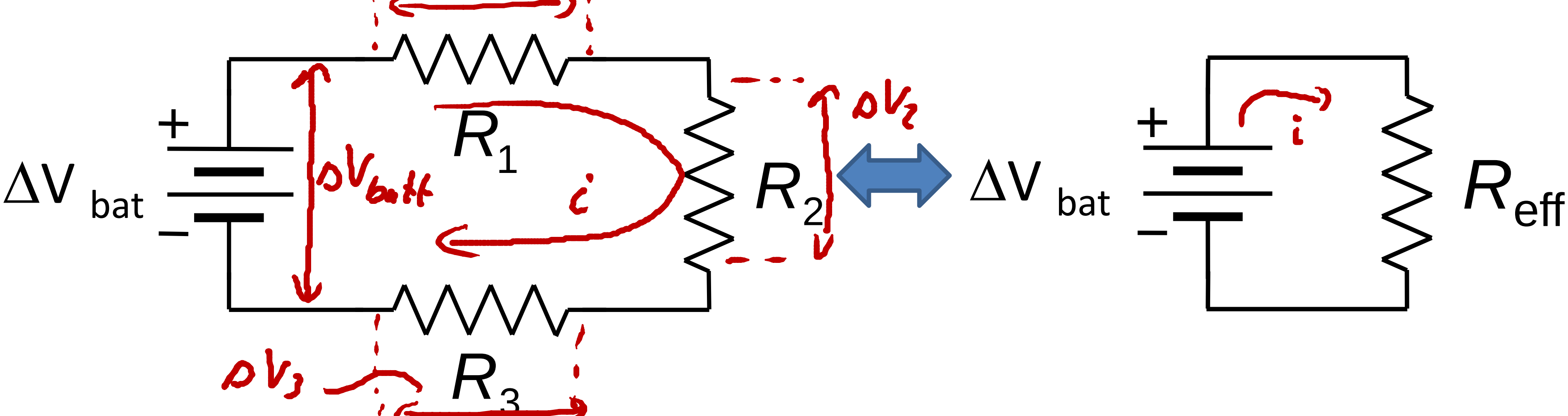
Today:

- "Pumping charges":emf
- RC circuits





Resistors in Series



What should be the value of R_{eff} in terms of R_1, R_2, & R_3 so that the same current flows in both circuits?

same: current: $i = i_1 = i_2 = i_3$

add: voltages: $\Delta V_{batt} = \Delta V_1 + \Delta V_2 + \Delta V_3$

$= R_1 i + R_2 i + R_3 i = (R_1 + R_2 + R_3) i$

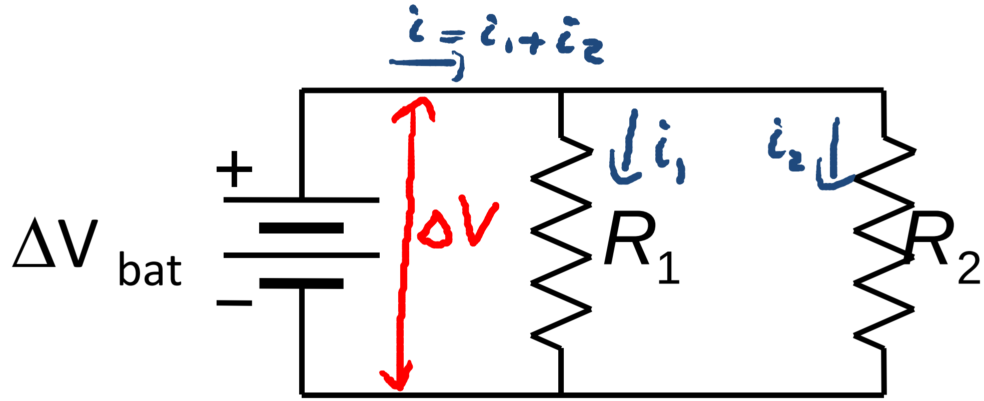
$= R_{eff}\, i$

with $\boxed{R_{eff} = \sum_{i=1}^{N} R_i}$ for resistors in series

Which resistor has the greater current going through it?

$$i = \frac{\Delta V}{R} \propto \frac{1}{R}$$

$\Delta V \leftarrow$ same



$R_1 > R_2$

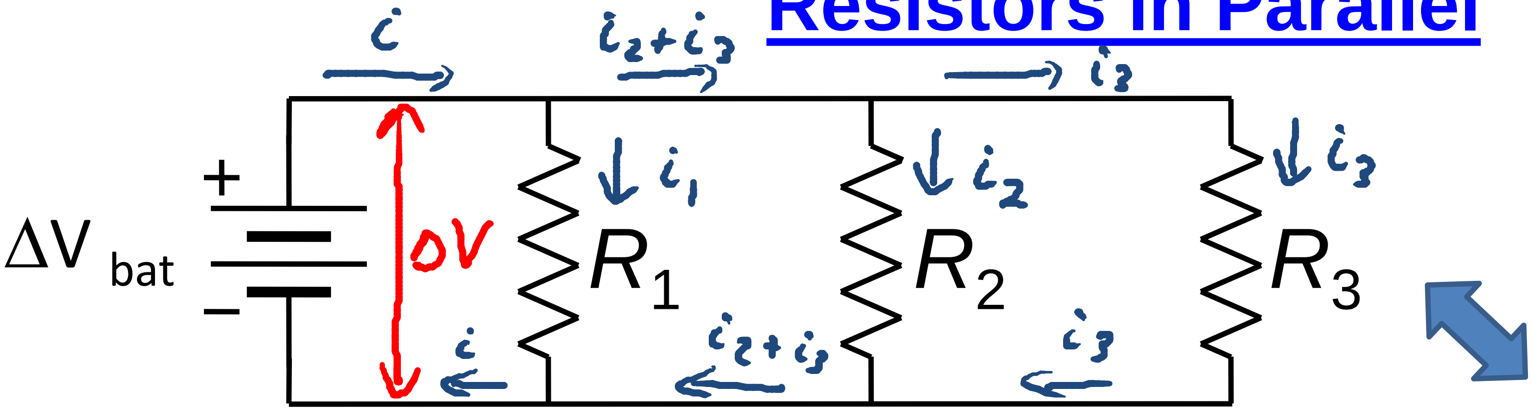
$\Rightarrow \quad i_1 < i_2$

A. R_1

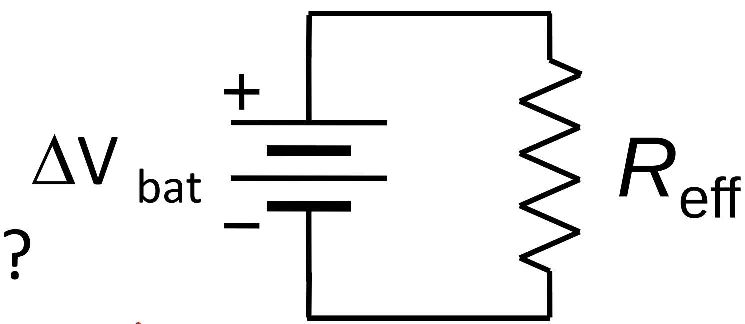
B. R_2

C. The current through both resistors is the same

Resistors in Parallel



What should be the value of R_{eff} in terms of R_1, R_2, & R_3 so that the same current flows in both circuits?



same: voltage $\Delta V_{batt} = \Delta V_1 = \Delta V_2 = \Delta V_3$

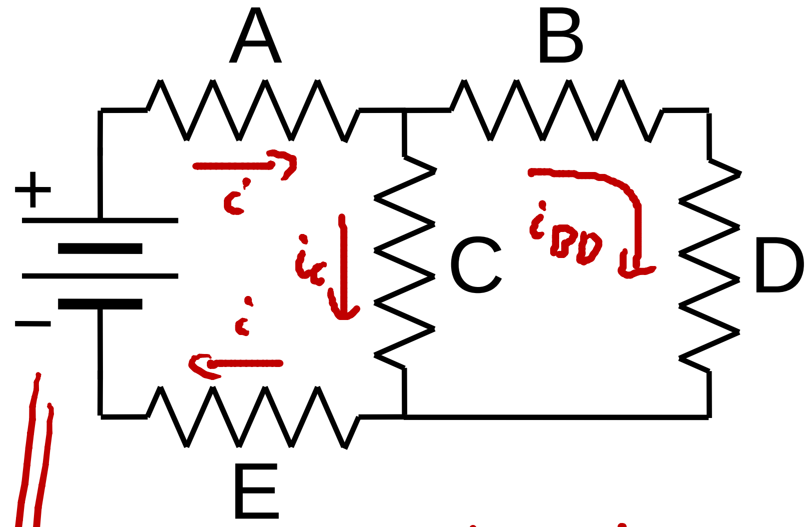
add: currents: $i = i_1 + i_2 + i_3 = \frac{\Delta V_{batt}}{R_1} + \frac{\Delta V_{batt}}{R_2} + \frac{\Delta V_{batt}}{R_3}$

$= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \Delta V_{batt} = \frac{1}{R_{eff}} \Delta V_{batt}$

with $\boxed{\frac{1}{R_{eff}} = \sum_{i=1}^{N} \frac{1}{R_i}}$ for resistors in parallel

Which resistors are in series?

Two resistors are in series if the same charge carrier must go through both resistors.

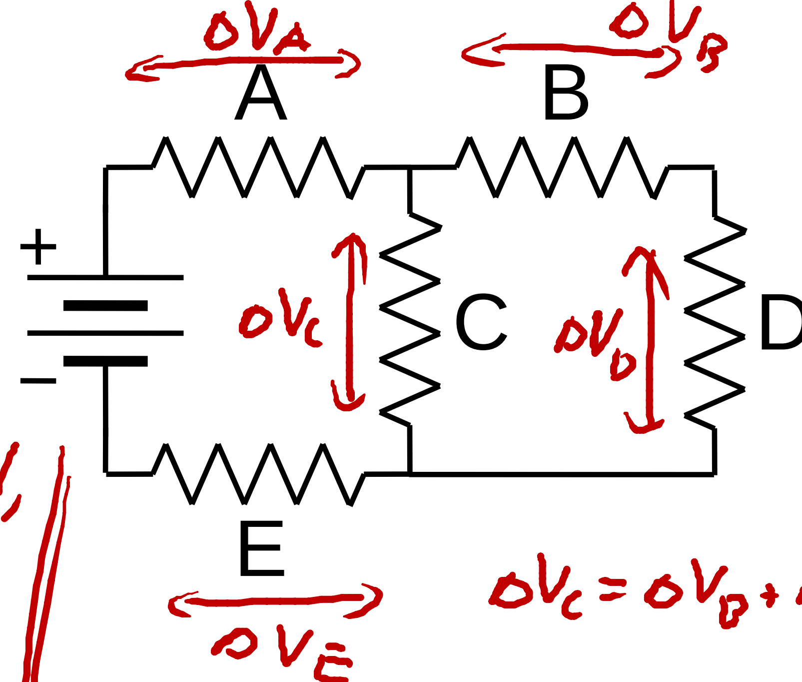


$i = i_c + i_{BD}$

A. A and B
B. A and C
C. A and E
D. B and D
E. Both answers C and D above

Which resistors are in parallel?

Two resistors are in parallel, if same potential difference ΔV is applied across both resistors!

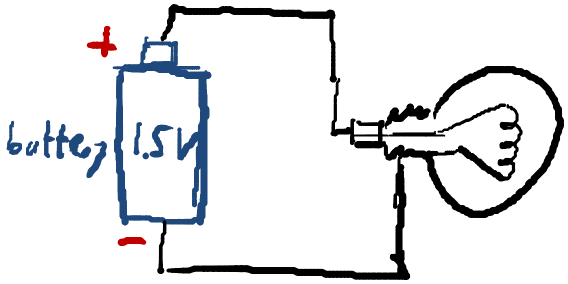


$\Delta V_C = \Delta V_B + \Delta V_D$

R_C is in parallel with $R_{BD} = (R_B + R_D)$ but not R_B or R_D alone.

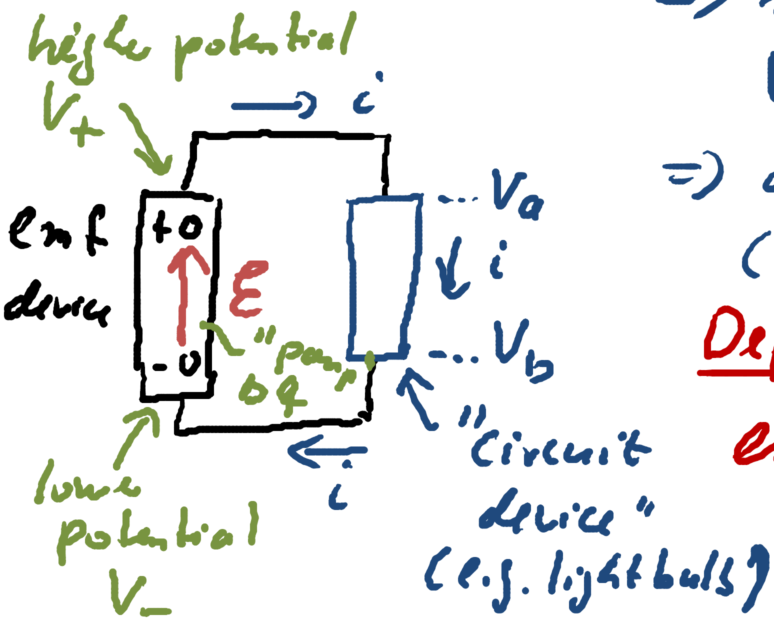
A. A and B
B. A and C
C. A and E
~~D.~~ C and D
E. No pair listed above

Circuits



⇓

Circuit diagram



$[\mathcal{E}] =$ volts

- **Emf devices**

(outdated name: "electromotive force")

– produces a steady flow of charges by "pumping" them to a higher electric potential energy

⇒ maintains a potential difference $V_+ - V_-$ between its terminals

⇒ converts some form of energy (chemical, sunlight...) into electrical energy

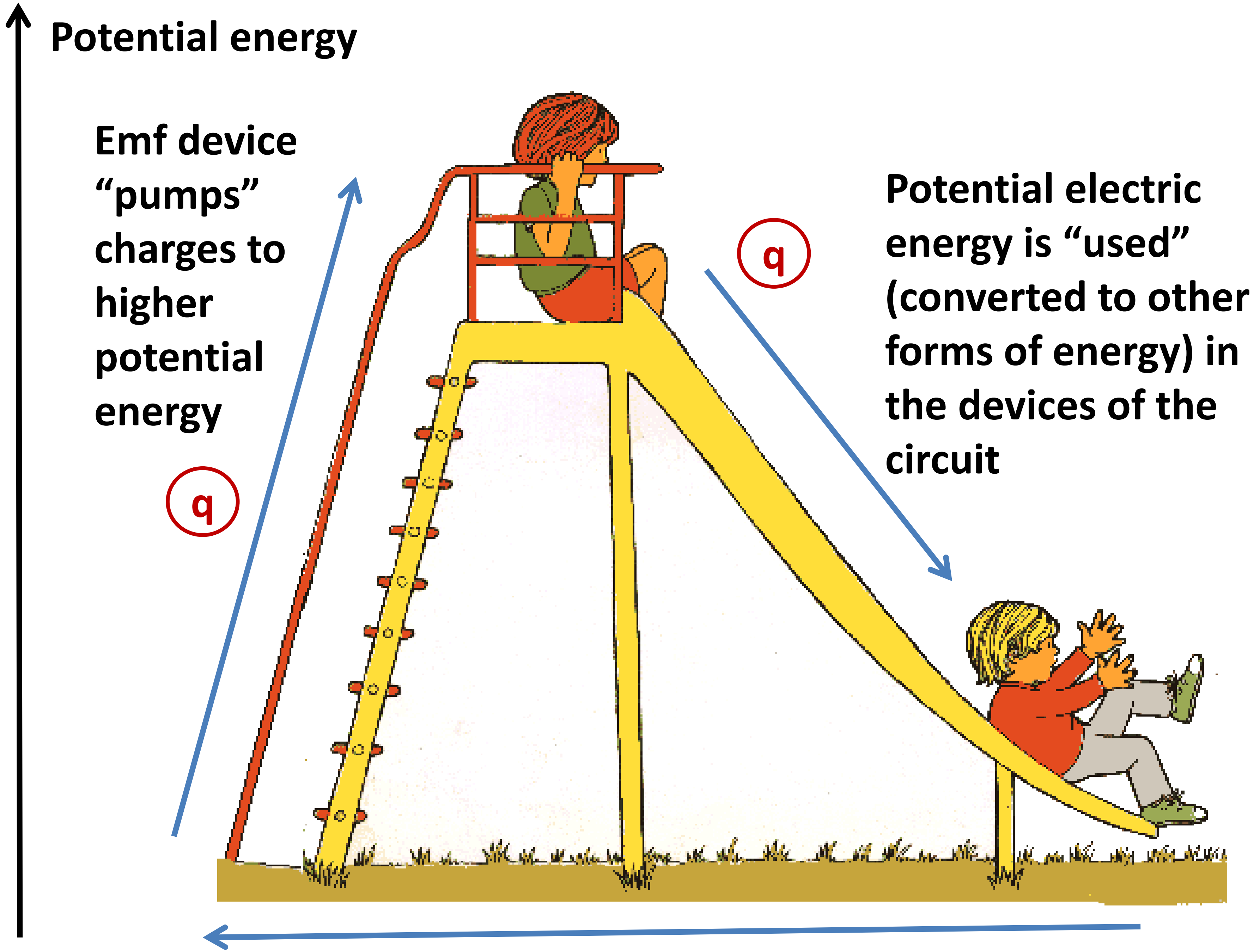
Define:

not ε_0!

$$\text{emf} = \mathcal{E} = \frac{\Delta W}{\Delta q} = \frac{dW}{dq} = \left(\begin{array}{l}\text{work per unit charge done by}\\ \text{the emf device to move charge}\\ \text{from low to high pot. energy terminal}\end{array}\right)$$

$$= \frac{dq\,(V_+ - V_-)}{dq}$$

$$= V_+ - V_-$$



Potential energy
Emf device “pumps” charges to higher potential energy
q
q
Potential electric energy is “used” (converted to other forms of energy) in the devices of the circuit

$$\mathcal{E} = \frac{dW}{dq} \Rightarrow \begin{pmatrix}\text{work done by emf} \\ \text{device to pump} \\ \text{charge } \Delta q\end{pmatrix} = \underline{\mathcal{E}\Delta q} = \frac{dW}{dq}\Delta q = \underline{\Delta W}$$

$\Rightarrow$ Power delivered by emf device:

$$\boxed{P_{emf} = \frac{\Delta W}{\Delta t} = \mathcal{E}\frac{\Delta q}{\Delta t} = \mathcal{E} i}$$

delivers energy in form of electric potential energy

$\Rightarrow$ This energy is "used"/converted into another form of energy in the electric circuit, i.e. by the circuit device: since $V_a > V_b$

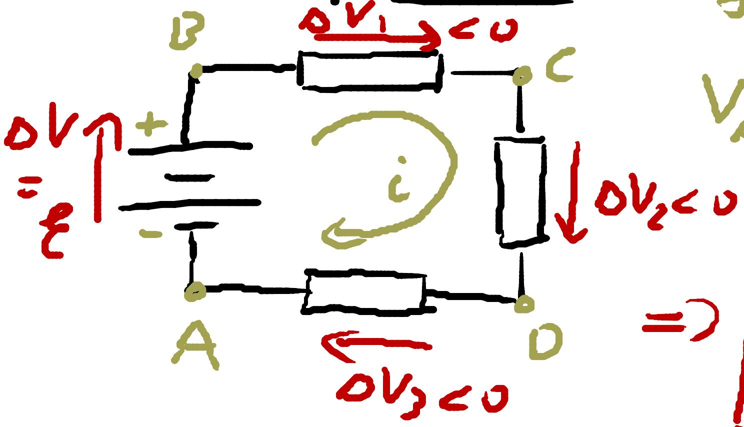
energy "used" in device $= \Delta q\,(V_a - V_b) = \Delta q\;\Delta V_{\text{over device}}$

$$\Rightarrow \begin{pmatrix}\text{power "used" by} \\ \text{circuit device}\end{pmatrix} = \underline{\underline{P_{device}}} = \frac{\text{energy used}}{\text{time interval}} = \frac{\Delta q}{\Delta t}\Delta V = \underline{\underline{i\,\Delta V_{\text{over device}}}}$$

$$[P] = \frac{C}{s}\frac{J}{C} = \frac{J}{s} = \underline{\text{Watt}}$$

Kirchhoff's circuit rules:

(a) Loop rule:

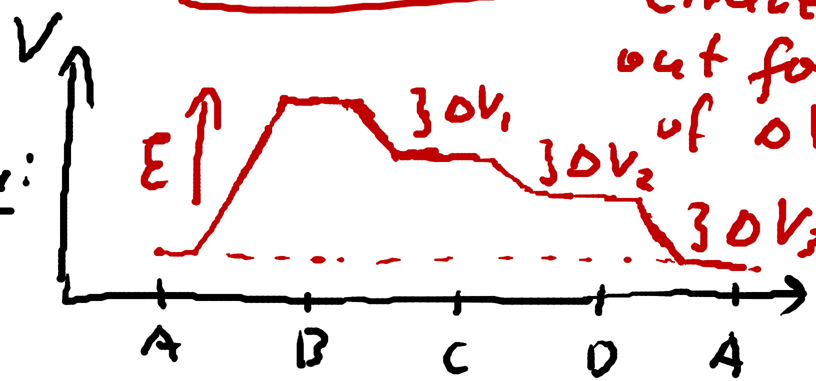


for closed loop:

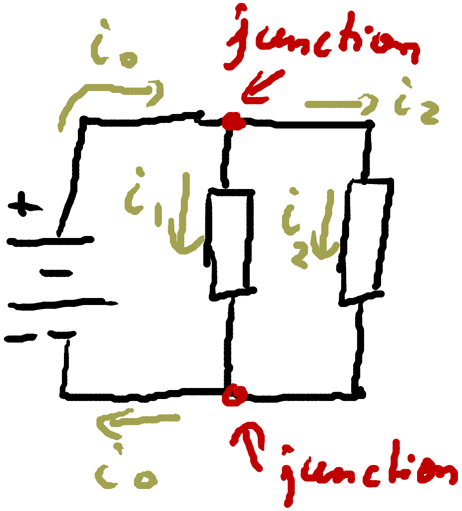
$$V_A + \underbrace{\mathcal{E} + \Delta V_1 + \Delta V_2 + \Delta V_3}_{=0} = V_A$$

$$\Rightarrow \boxed{\sum_{i=1}^{N} \Delta V_i = 0}$$

for sum of potential change in closed circuit loop; watch out for correct sign of ΔV!



(b) Junction rule:

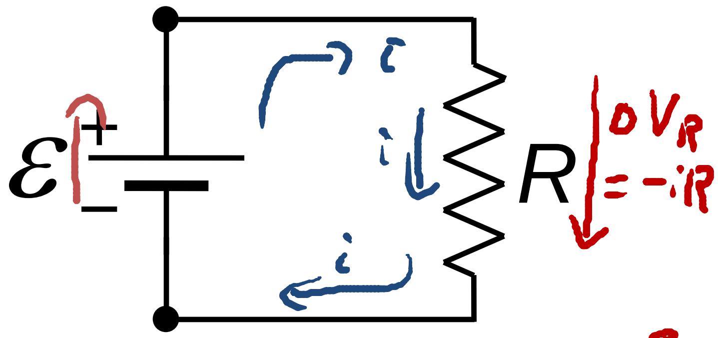


at junction: $i_0 = i_1 + i_2$

$$\Rightarrow \boxed{\sum i_{in} = \sum i_{out}}$$

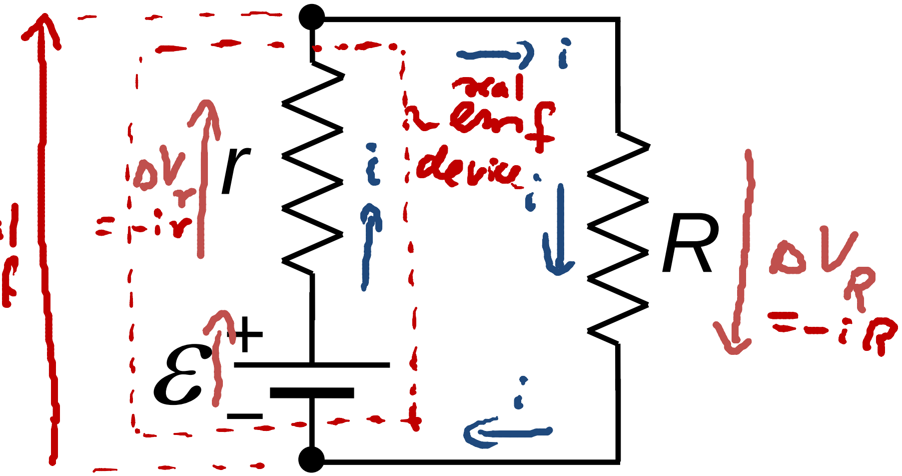
} charge is conserved

<u>Ideal</u> emf device – Has no internal resistance.



$\mathcal{E} + \Delta V_R = 0 = \mathcal{E} - iR \Rightarrow i = \frac{\mathcal{E}}{R}$

<u>Real</u> emf device – Has internal resistance *r*.



When a load resistance *R* is connected to the <u>real</u> emf device, what is the potential difference across its terminals?

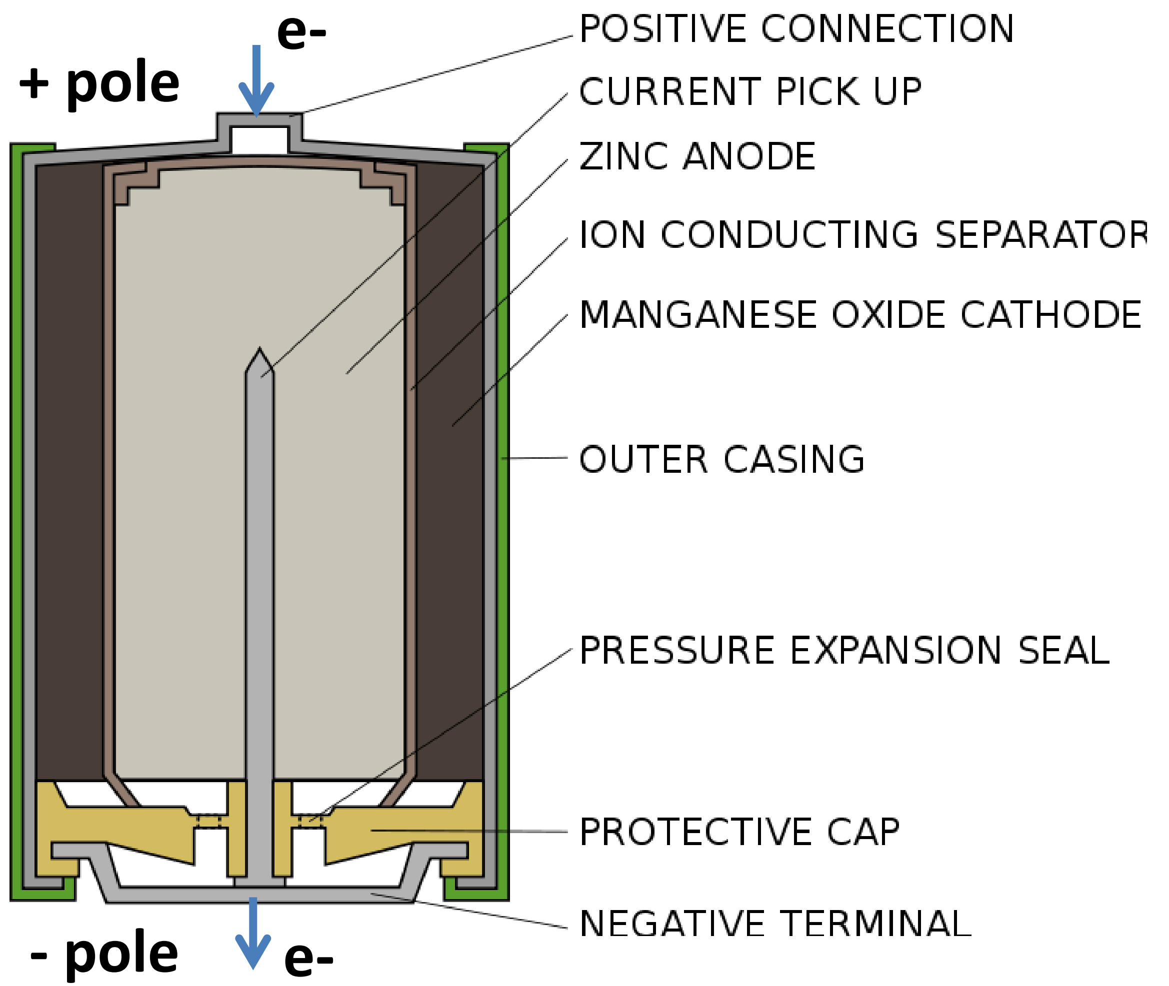
for real emf: $\mathcal{E} + \Delta V_r + \Delta V_R = 0 = \mathcal{E} - ir - iR = \mathcal{E} - i(r+R)$

$\Rightarrow \boxed{i = \frac{\mathcal{E}}{r+R}}$ $\quad \Delta V_{\text{real emf}} = \mathcal{E} + \Delta V_r = \mathcal{E} - ir = \mathcal{E} - \frac{\mathcal{E} r}{r+R} = \mathcal{E}\frac{R}{r+R} = -\Delta V_R$

A. $\mathcal{E}$ B. 0 C. $\mathcal{E}\left(\frac{r}{R}\right)$. D. $\mathcal{E}\left(\overbrace{\frac{R}{r+R}}^{<1}\right)$. E. $\mathcal{E}\left(\frac{r}{r+R}\right)$.

Standard Alkaline Batteries:

- Converts chemical energy into electrical energy
- Anode (negative terminal) is made of zinc powder
- Cathode (positive terminal) is composed of manganese dioxide
- Electrolyte is potassium hydroxide



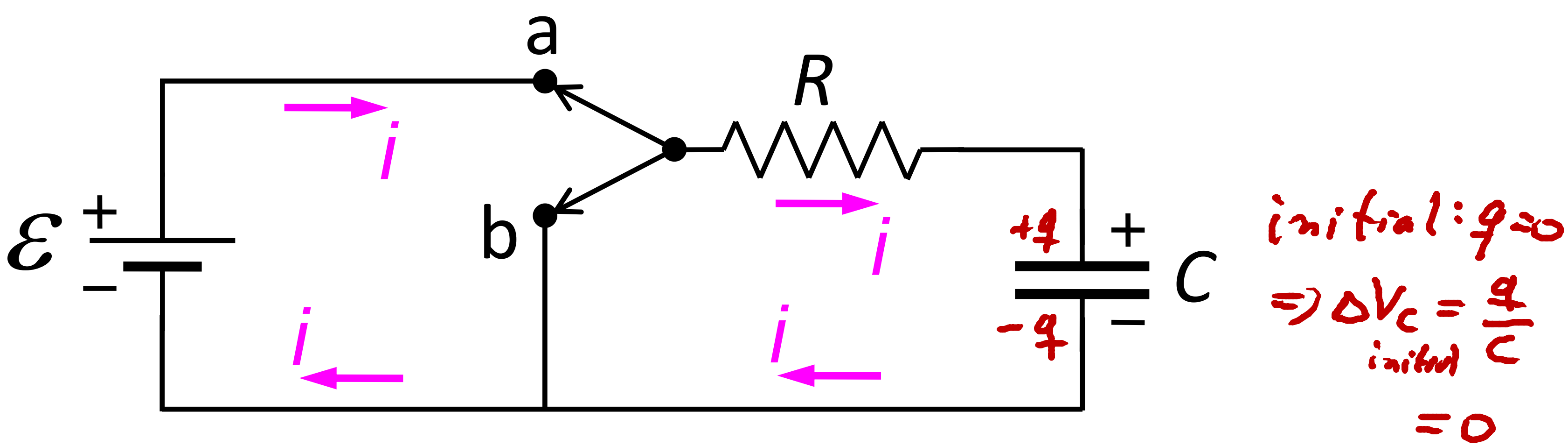
$$Zn + 2\,OH^- \longrightarrow ZnO + H_2O + 2\,e^-$$

At potential V ~ 1.5V

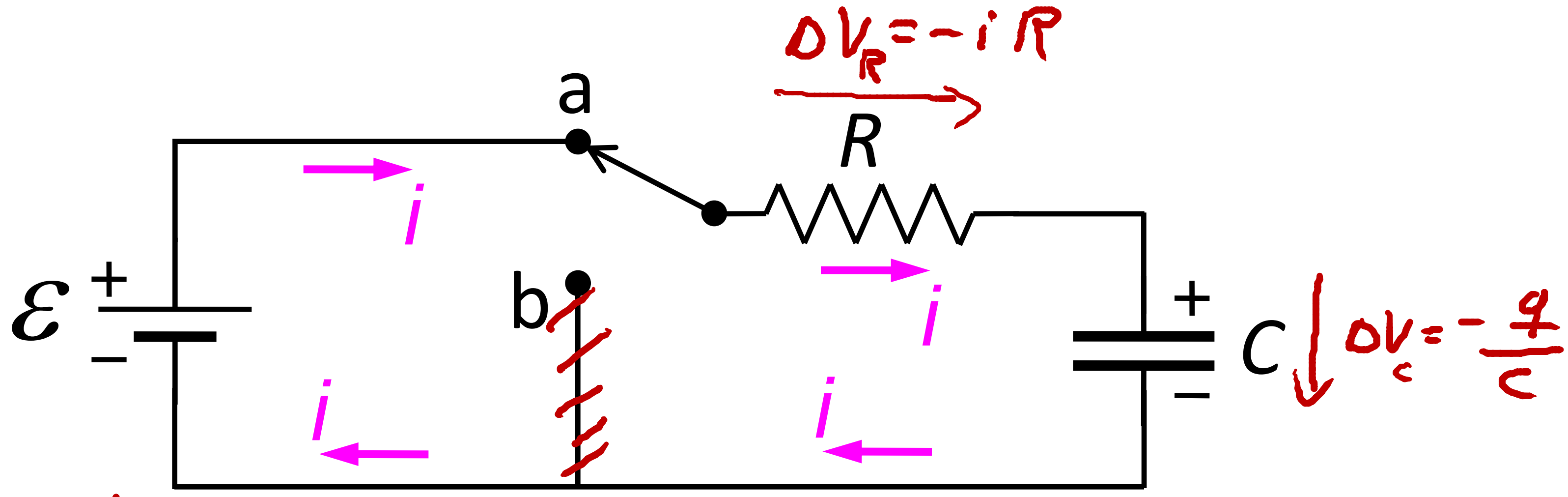
At potential V ~ 0V

$$2\,MnO_2 + H_2O + 2\,e^- \longrightarrow Mn_2O_3 + 2\,OH^-$$

RC circuit: Charging and discharging of a capacitor



- At time $t = 0$ move the switch to position **a**.
- Current *i* begins to flow to **charge** the capacitor.
- *i* into the upper plate of the capacitor always equals *i* out of the lower plate even though no charge flows across the gap between the plates.



loop rule: $\mathcal{E} + \Delta V_R + \Delta V_c = 0$

At time $t = 0$ the switch is moved to position **a**.

After a very long time what will be the voltage on the capacitor? after long time: $i \to 0 \Leftrightarrow$ fully charged capacitor

$\Rightarrow \Delta V_R \to 0 \Rightarrow |\Delta V_c| = \mathcal{E}$

A. 0

B. iR

C. $\mathcal{E}$

D. $\rightarrow\infty$ V, the voltage will keep increasing as long as the switch is at position a.