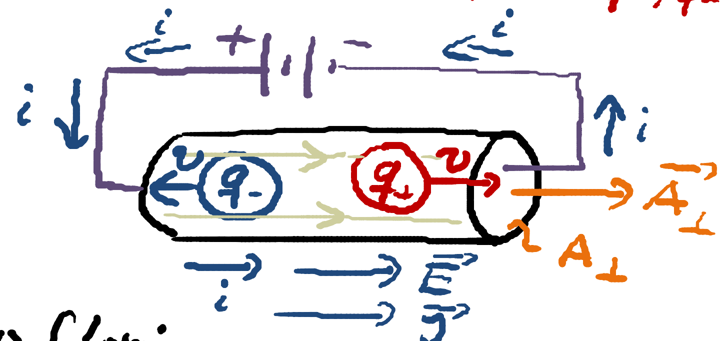
Recap

- Current density: $\vec{J}$

# of positive charge carriers per volume

charge per positive charge carrier

$$|\vec{J}| = \frac{\text{current}}{\text{area} \perp \text{to current flow}} = \frac{i}{A_\perp} = n_+ q_+ |v_{drift,q_+}| + n_- |q_-| |v_{drift,q_-}|$$

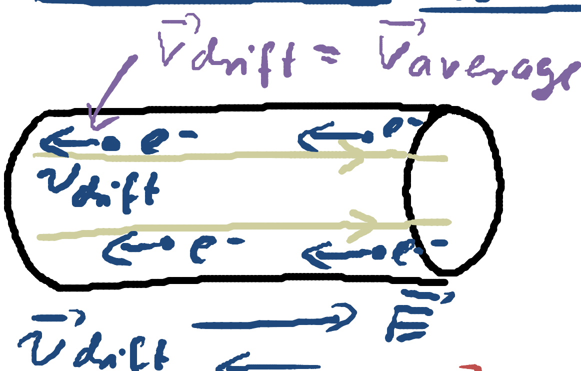


$\vec{J}$ always points in direction positive charges would move, i.e. in direction of $\vec{E}$!

$\Rightarrow$ total current through area A $\perp$ to flow:

$$i = J A_\perp = \int_{\text{area}} \vec{J} \cdot d\vec{A}$$

- Electric currents in metals:



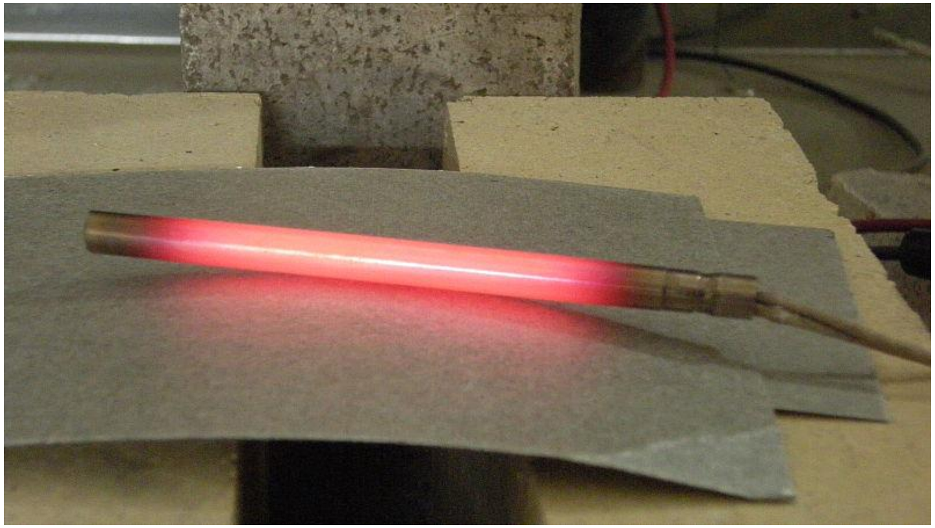
$$\vec{J} = n(-e)\,\vec{v}_{drift}$$

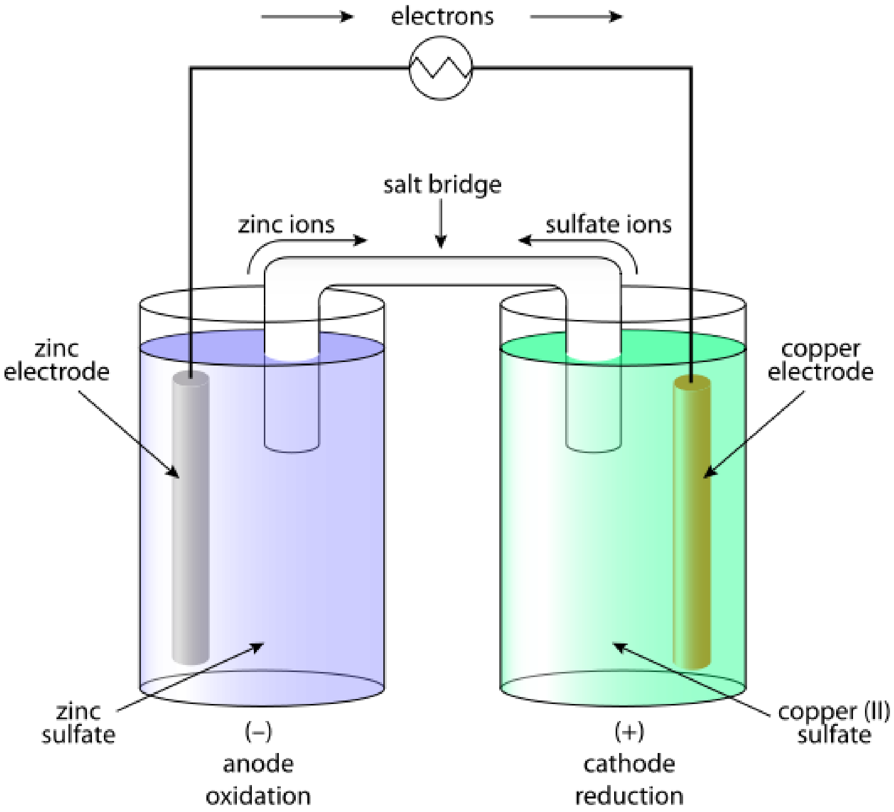
- charge carriers: electrons $q_{e^-} = -e$
- $n_{free\ e^-} \approx 10^{29}/m^3$
- move randomly with $v_{random} \approx 10^6 \frac{m}{s}$
- drift in electric field very slowly

$v_{drift} = v_{average} \approx 10^{-5}$ to 10^{-3} m/s

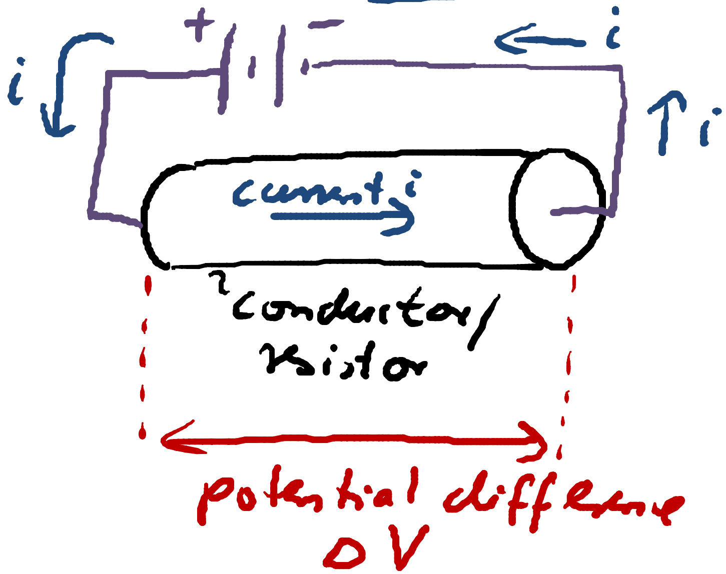
Today:

- Electrical resistance
  - Resistivity and conductivity

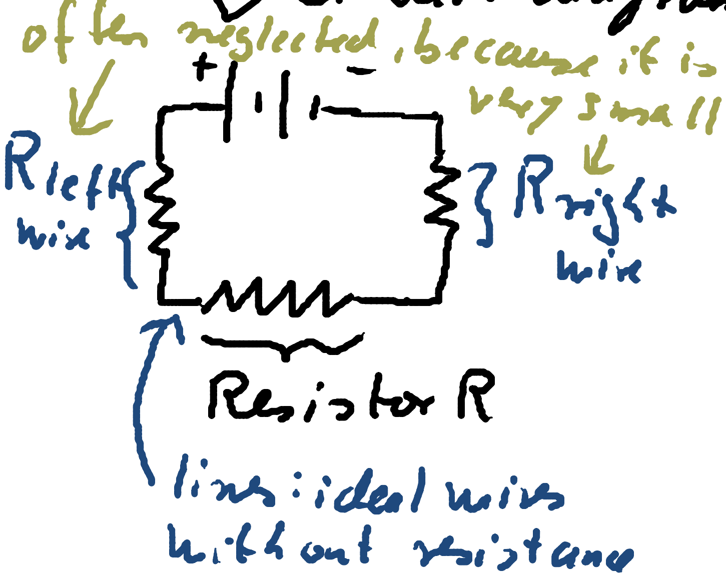




Electrical Resistance / Resistor



⇓ Circuit diagram



- Conductor provides resistance to current flow (collisions!)

$\Rightarrow$ $v_{drift,avg} = \text{const}$ even though electric force is acting on charges

$\Rightarrow$ $\left| W_{\text{by electric field on charges}} \right| = \left| W_{\text{lost due to resistance}} \right|$

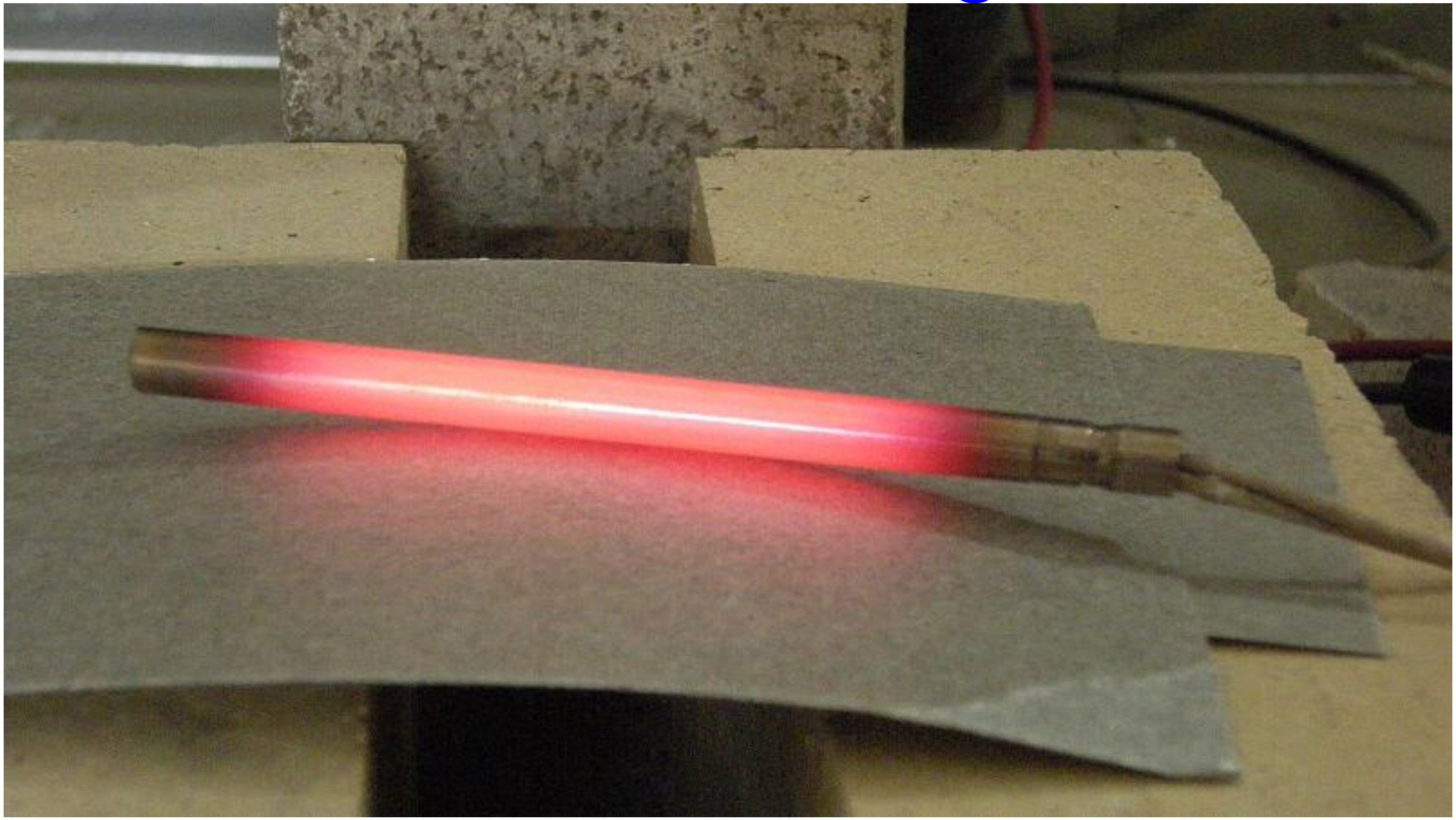
$\Rightarrow$ electrical energy usually dissipated into thermal energy ("Joule heating")

$\Rightarrow$ Define electrical Resistance R:

$$R \equiv \frac{\Delta V_{\text{over resistor}}}{\text{current } i} \iff \boxed{\Delta V = R\, i}$$

Units: $[R] = \frac{\text{volt}}{\text{ampere}} \equiv \text{ohm} = 1\,\Omega$

Joule Heating:



Running current through a resistance creates heat, in a phenomenon called Joule heating. In this picture, a cartridge heater, warmed by Joule heating, is glowing red hot.

Notes:

① High resistance → low current i for given potential difference ΔV

② Resistor: conductor whose function in a circuit is to provide a specific resistance

typically: $R = 1\,\Omega \ldots$ many $M\Omega$

③ Resistance R depends on:

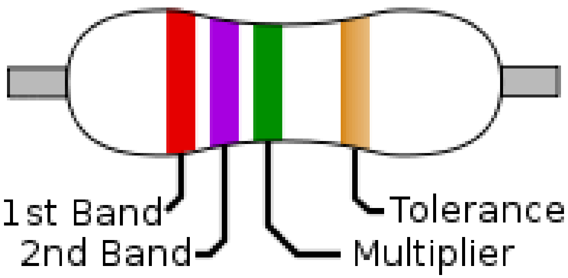
- Geometry of resistor (length, diameter...)
- Resistivity ρ of the conducting material used (see next slide)
- usually varies with temperature, since $\rho = \rho(T)$
- sometimes changes with applied potential difference

"Ohm's Law"

$R = \frac{\Delta V}{i} = \text{const}$, i.e. indep. of ΔV for special kind of resistors

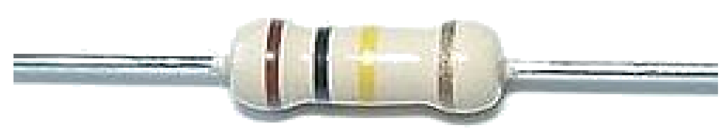
Electronic color code

- Used to indicate the values of electronic components
- very commonly for resistors, but also for capacitors, inductors



Color	Significant figures	Multiplier	Tolerance		Temp. Coefficient (ppm/K)	
Black	0	$\times 10^0$	–		250	U
Brown	1	$\times 10^1$	±1%	F	100	S
Red	2	$\times 10^2$	±2%	G	50	R
Orange	3	$\times 10^3$	–		15	P
Yellow	4	$\times 10^4$	(±5%)	–	25	Q
Green	5	$\times 10^5$	±0.5%	D	20	Z
Blue	6	$\times 10^6$	±0.25%	C	10	Z
Violet	7	$\times 10^7$	±0.1%	B	5	M
Gray	8	$\times 10^8$	±0.05% (±10%)	A	1	K
White	9	$\times 10^9$	–		–	
Gold	–	$\times 10^{-1}$	±5%	J	–	
Silver	–	$\times 10^{-2}$	±10%	K	–	
None	–	–	±20%	M	–	

- band #1 is first significant figure of component value (left side)
- band #2 is the second significant figure
- band #3 is the decimal multiplier
- band #4 if present, indicates tolerance of value in percent (no color means 20%)



100 kOhm = $10*1\times10^4$

Resistivity / conductivity of Materials

→ for many materials : $\left(\text{current density}\right) \propto \left(\text{electric field applied}\right)$

$$\Rightarrow \quad \boxed{\vec{J} = \sigma \vec{E} = \frac{1}{\rho}\vec{E}} \qquad \boxed{\sigma = 1/\rho}$$

with σ : conductivity of the material $\quad [\sigma] = \frac{A/m^2}{V/m} = \frac{1}{\Omega m}$

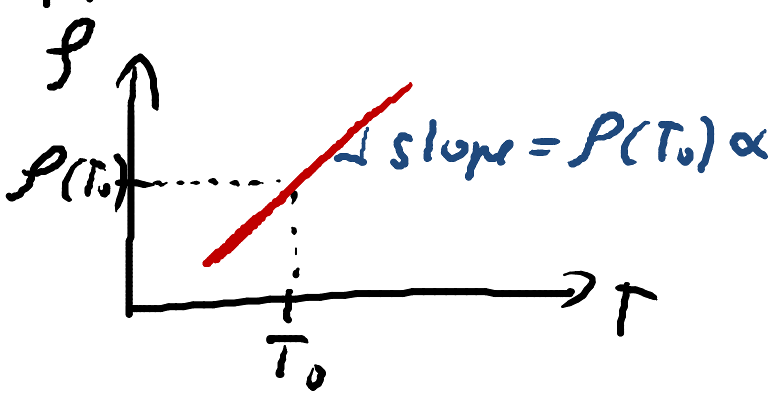
ρ : resistivity of the material $\quad [\rho] = \Omega m$

e.g. $\rho_{copper} = 1.7 \cdot 10^{-8}\,\Omega m$

→ Variation with temperature:

good approximation:

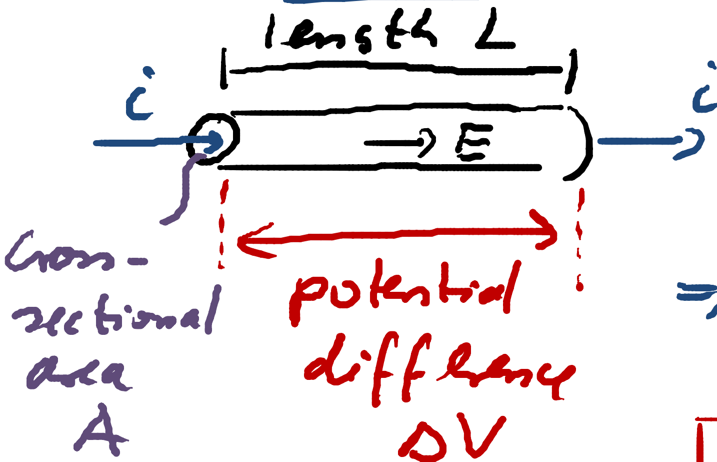
$$\rho(T) = \underset{\substack{\uparrow \\ \text{initial temp}}}{\rho(T_0)} + \rho(T_0)\,\alpha\,(\underbrace{T - T_0}_{\Delta T})$$



α : temperature coefficient of resistivity

$[\alpha] = 1/\text{Kelvin}$

Example: Wire with constant diameter



have: $\Delta V = EL \qquad i = JA$

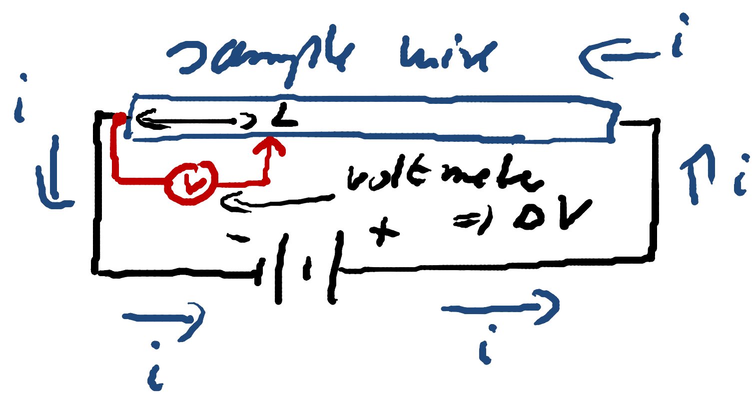
and $E = \rho J$

$$\Rightarrow R_{wire} = \frac{\Delta V}{i} = \frac{EL}{JA} = \frac{\rho J L}{JA} = \frac{\rho L}{A}$$

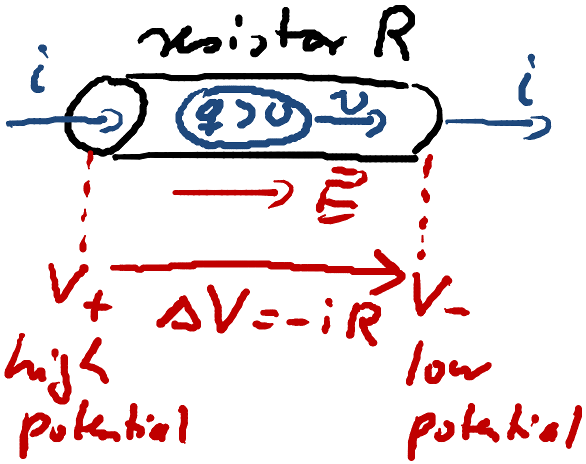
$$\Rightarrow \boxed{R_{wire} = \frac{\rho L}{A}} \propto L$$

resistivity of material (not density!)

for $i = \text{const}$



Resistive Power Dissipation:



- $\Delta V = V_- - V_+ = -iR < 0$
- charge Δq moves through resistor in some time Δt

$\Rightarrow$ decrease in electric potential energy:

$$\Delta U = \Delta q \cdot \Delta V = \underbrace{i \cdot \Delta t}_{\Delta q}\, \Delta V < 0$$

$\Rightarrow$ This energy is lost in a resistors to other forms of energy (e.g. heat)

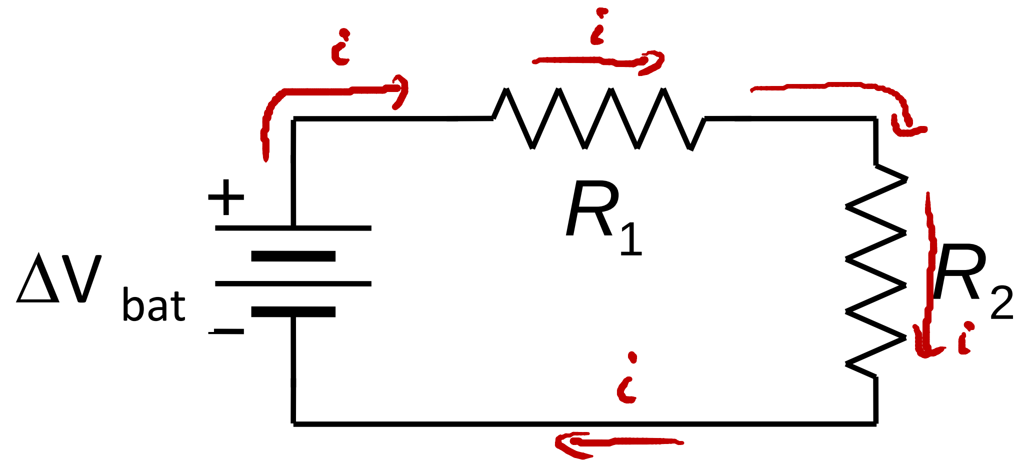
$$\begin{pmatrix}\text{rate of energy} \\ \text{loss/transfer}\end{pmatrix} = \left|\frac{\Delta U}{\Delta t}\right| = \left|\frac{dU}{dt}\right| = \text{Power} = \boxed{P = i|\Delta V|}$$

for resistor:

$$\boxed{P_{\text{dissipated by current}} = i\,|\Delta V| = i^2 R = \frac{V^2}{R}}$$

since $R = \dfrac{|\Delta V|}{i}$

Which resistor has the greater current going through it?



$R_1 > R_2$

charge is conserved:

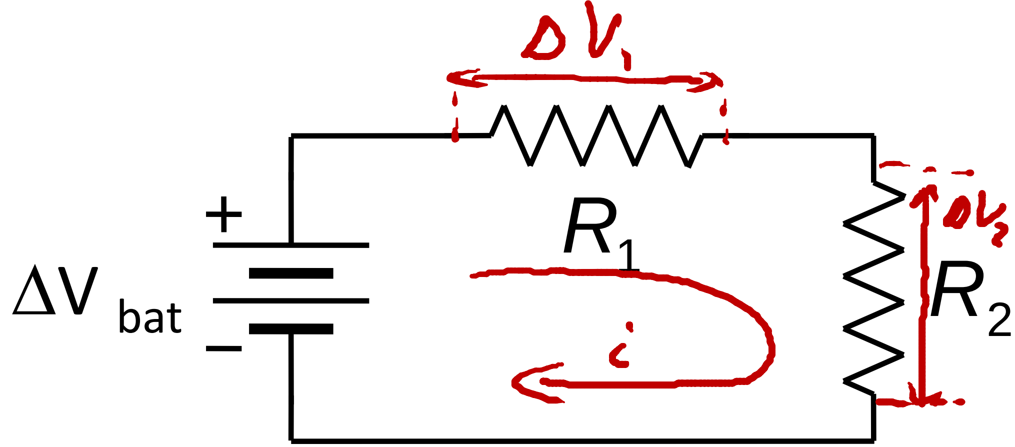
$i_1 = i_2$

A. R_1

B. R_2

C. The current through both resistors is the same

Which resistor has the greater voltage (magnitude of potential difference) across it?



$R_1 > R_2$

$\Delta V = i R \propto R$

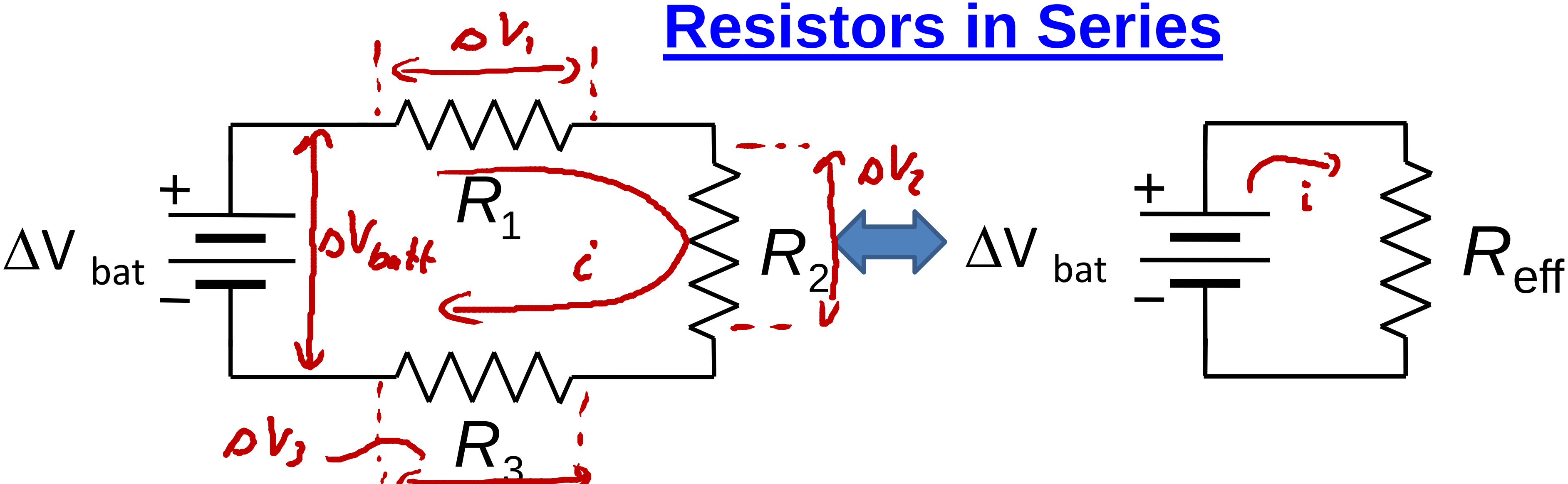
↑ same current

$\Delta V_1 > \Delta V_2$

A. R_1 B. R_2

C. The voltage across both resistors is the same

Resistors in Series



What should be the value of R_{eff} in terms of R_1, R_2, & R_3 so that the same current flows in both circuits?

same: current: $i = i_1 = i_2 = i_3$

add: voltages: $\Delta V_{batt} = \Delta V_1 + \Delta V_2 + \Delta V_3$

$$= R_1 i + R_2 i + R_3 i = (R_1 + R_2 + R_3) i$$

$$= R_{eff}\, i$$

with $\boxed{R_{eff} = \sum_{i=1}^{N} R_i}$ for resistors in series