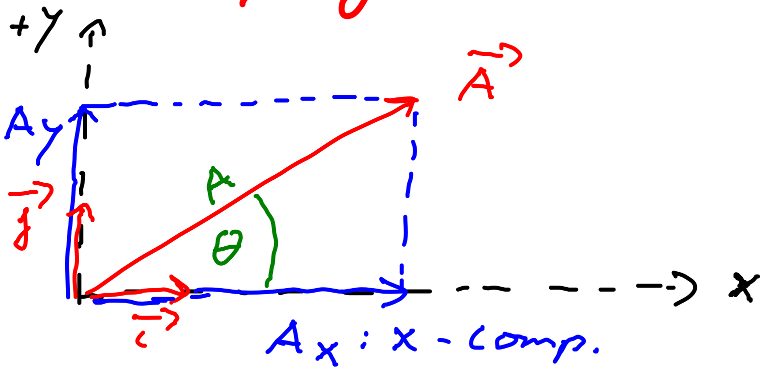
Recap: Vectors

Lecture 6

- have magnitude + direction: $\vec{r}, \vec{v}, \vec{a}, \vec{F}, \ldots$
- Specifying vectors:



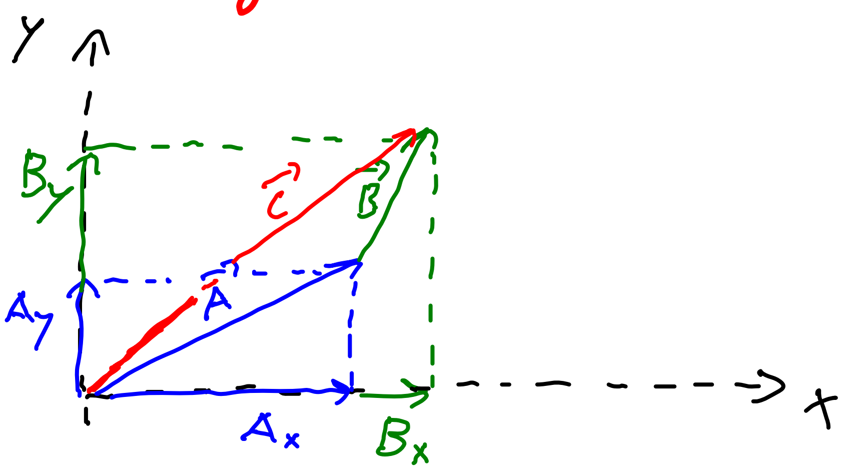
① by magnitude A and angle θ

② by components A_x, A_y

$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$

$$\vec{A} = A_x\vec{i} + A_y\vec{j}$$ ← unit vector

- Adding vectors:



① "tail to tip", then "tail to tip"

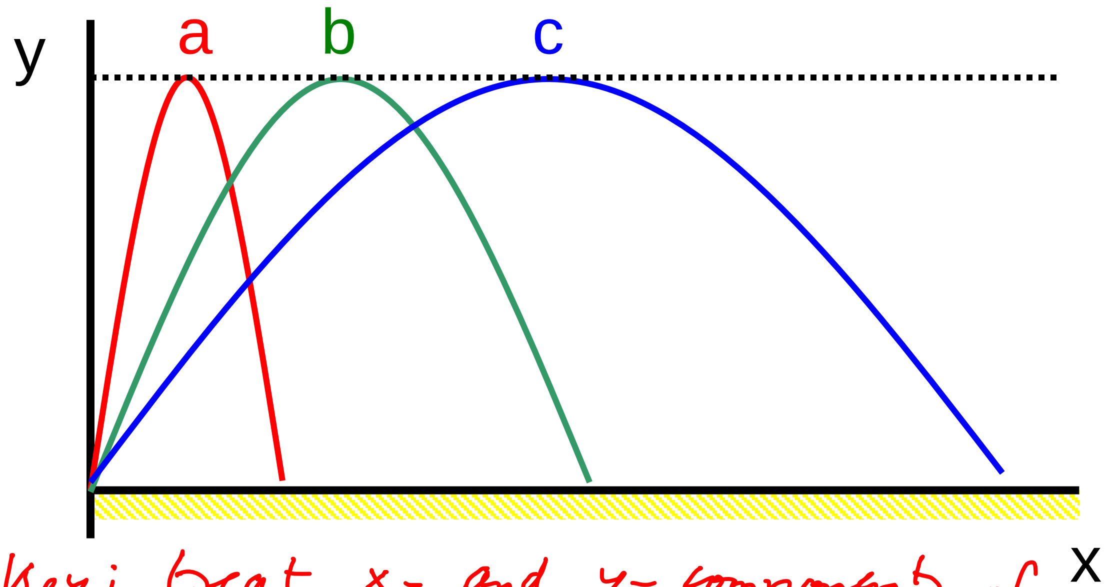
② by components:

$$\vec{C} = \vec{A} + \vec{B} = (A_x + B_x)\vec{i} + (A_y + B_y)\vec{j}$$

Never mix x and y components!

Three trajectories for a thrown ball are shown below.

Which trajectory has the longest time of flight?



A. a

B. b

C. c

D. all the same

Key: treat x- and y-components of 2-D motion independently!

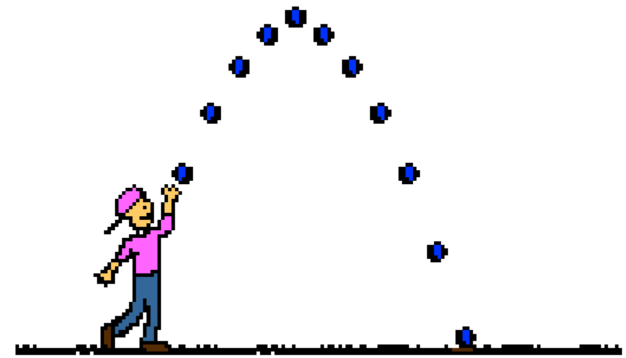
⇒ same height → same vertical motion

→ "time of flight" is determined by y-motion here

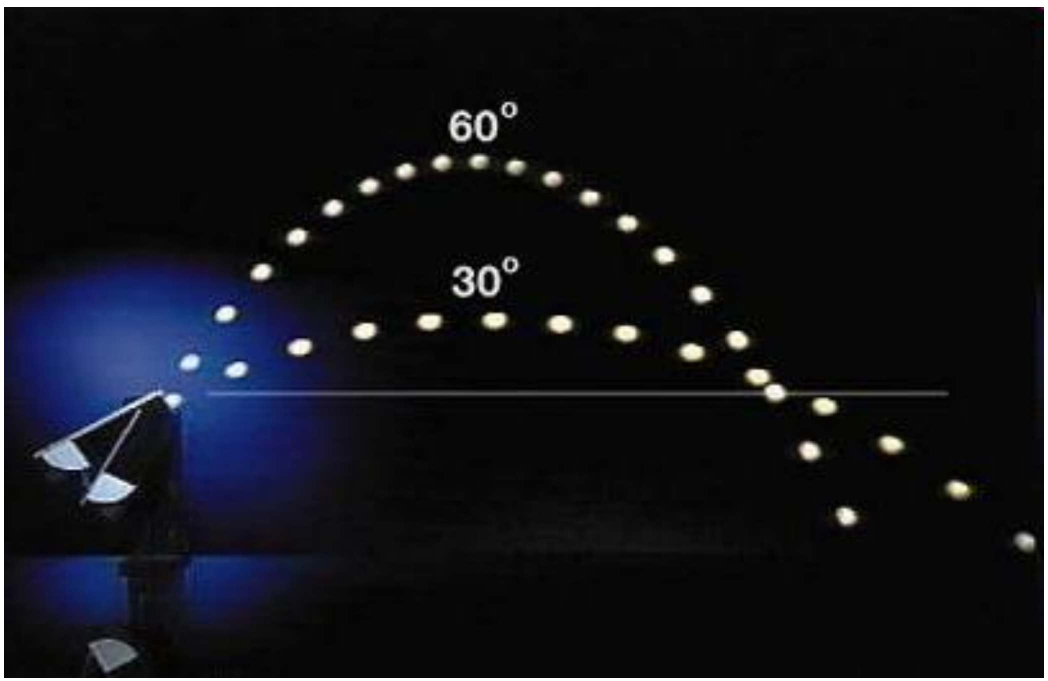
⇒ same Δt!

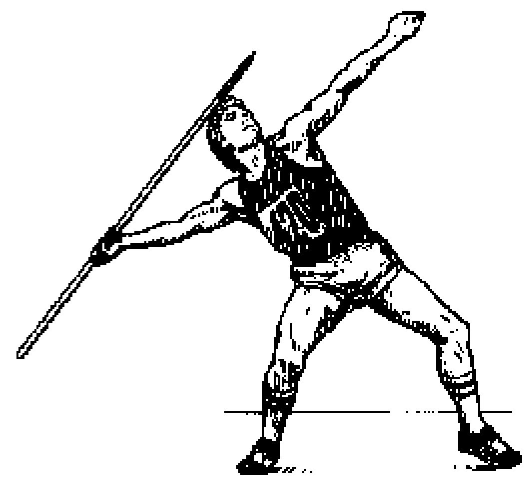
Today:

- **Motion in a plane (2-D motion)**
  - **Position $\vec{r}(t)$, velocity $\vec{v}(t)$, acceleration $\vec{a}(t)$**
  - **Projectile motion**



A projectile often moves horizontally as it moves upward and/or downward.

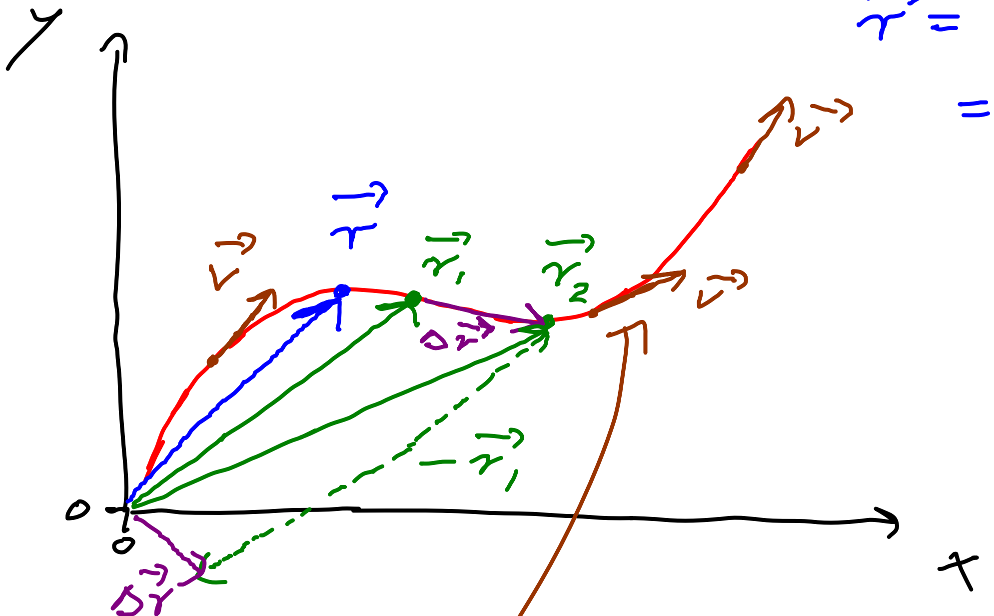




$\vec{r}$ = position vector = $\vec{r}(t)$

$= x(t)\,\vec{i} + y(t)\,\vec{j}$

separate out components of motion!



displacement:

$$\Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$$

velocity:

direction of $\vec{v}$ points along path always! (tangent to path)

• average velocity: $\vec{v}_{avg} = \dfrac{\overrightarrow{\text{displacement}}}{\Delta t} = \dfrac{\Delta \vec{r}}{\Delta t} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$

slope of x-t graph

• inst. velocity: $\vec{v}(t) = \dfrac{d\vec{r}}{dt} = \dfrac{dx}{dt}\vec{i} + \dfrac{dy}{dt}\vec{j}$

slope of y-t graph

$= v_x \vec{i} + v_y \vec{j}$

The direction of $\vec{v}(t)$ is tangent to the path at $\vec{r}(t)$.

• average acceleration: $\vec{a}_{avg} = \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$

• inst. acceleration:

slope of v_x-t graph

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\vec{i} + \frac{dv_y}{dt}\vec{j}$$

slope of v_y-t graph

$$= a_x \vec{i} + a_y \vec{j}$$

$$= \frac{d^2x}{dt^2}\vec{i} + \frac{d^2y}{dt^2}\vec{j}$$

$\Rightarrow$ x- and y- components of motion can be treated independently!

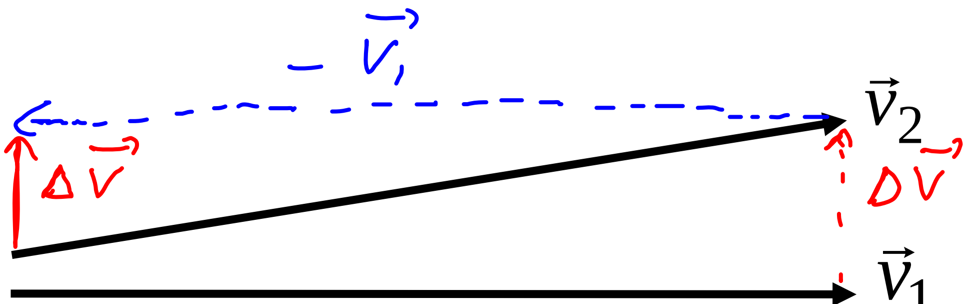
$$x(t) \underset{\text{area} = \Delta x}{\overset{\text{slope}}{\rightleftarrows}} v_x(t) \underset{\text{area} = \Delta v_x}{\overset{\text{slope}}{\rightleftarrows}} a_x(t)$$

$$y(t) \underset{\text{area} = \Delta y}{\overset{\text{slope}}{\rightleftarrows}} v_y(t) \underset{\text{area} = \Delta v_y}{\overset{\text{slope}}{\rightleftarrows}} a_y(t)$$

$\Rightarrow$ 2D motion = two 1-D motions!

The velocity $\vec{v}$ of a particle at two different times t_1 and t_2 is shown below ($t_2 > t_1$).

Which vector best represents the **average acceleration of the particle between these two times?**

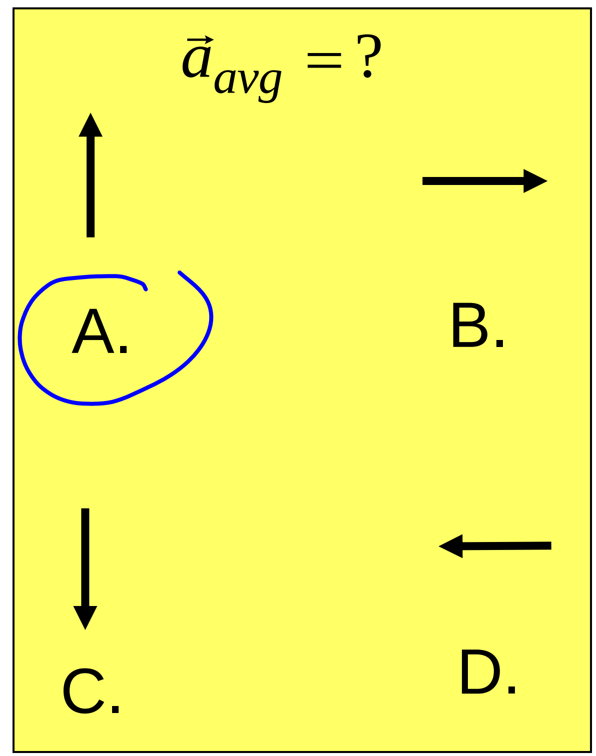


$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$

same direction

$\Delta\vec{v}$ = later − earlier = $\vec{v}_2 - \vec{v}_1$

$= \vec{v}_2 + (-\vec{v}_1)$



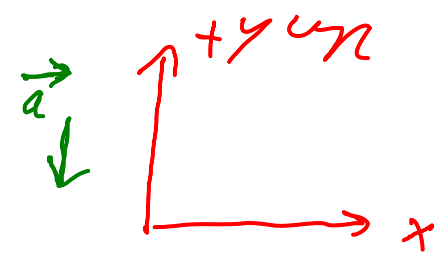
Example of 2-D motion: **Projectile motion**

① Object is given initial $\vec{v}_0 = v_{0x}\,\vec{i} + v_{0y}\,\vec{j}$ and moves in vertical plane.

② 2D motion in the vertical plane defined by $\vec{v}_0$ and direction of gravity $\vec{g}$.

③ Only acting force / acceleration is gravity (neglect air)

$$\vec{a}(t) = 0\,\vec{i} - g\,\vec{j}$$



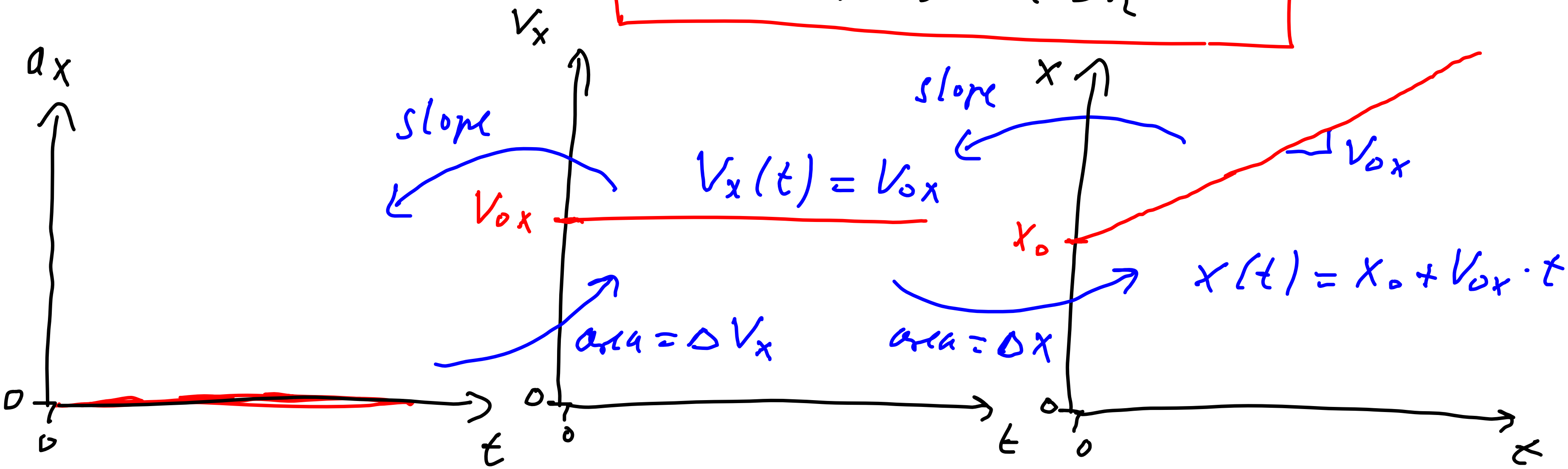
$g = +10\ m/s^2$

④ "Time of flight" (and Δx, Δy) determined by either x- or y-motion.

⇒ analyze horizontal and vertical motions separately!

- <u>Horizontal Motion:</u>

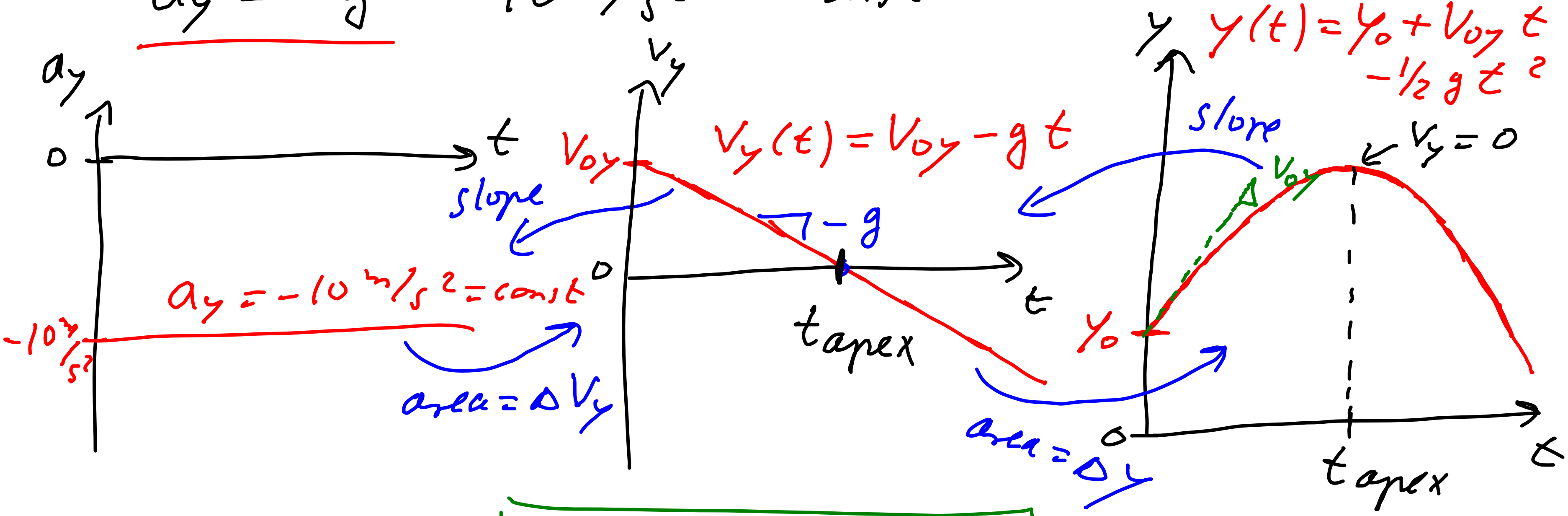
$a_x = 0 \Rightarrow \boxed{V_x(t) = \text{const} = V_{0x} \text{ in } x\text{-direction}}$



• Vertical Motion:

+y up

$$a_y = -g = -10\,m/s^2 = \text{const}$$



$\Rightarrow$ $\boxed{\text{Free fall in } y!}$

A student sits on a cart moving at constant speed, and tosses a ball upward.

If the student wants to catch the ball in his/her lap on the way down, in what direction ***relative to his body*** should he toss the ball?

⇒ x and y motions are independent!

⇒ think about "frame of reference" in moving cart!

A. Throw behind.

B. Throw straight up.

C. Throw ahead.

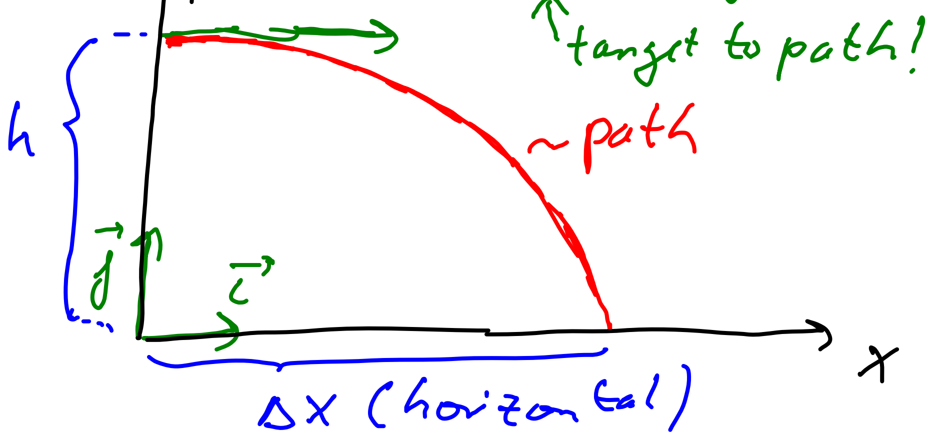
both, ball and cart are moving with the same horizontal speed.

Special cases:

① Horizontal projectile motion from height h

$\vec{v}_0 = v_{0x}\,\vec{i} + 0\,\vec{j}$

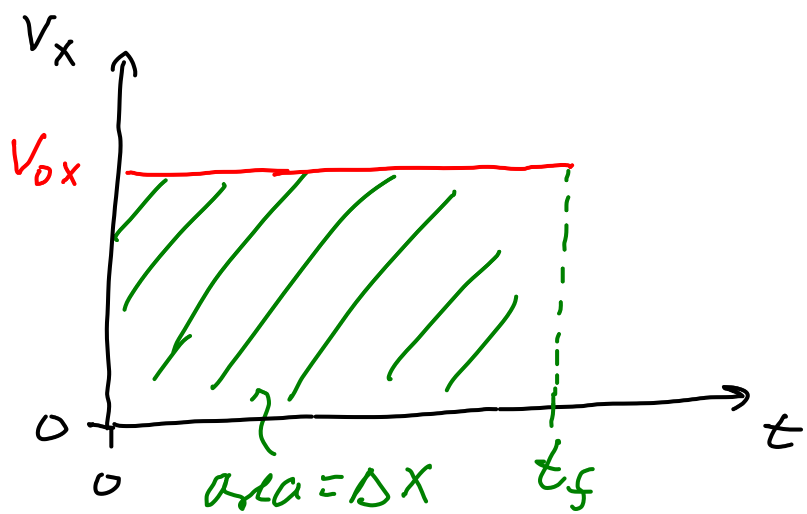
↑ tangent to path!



"time of flight" (t_f)

determined by y-motion here

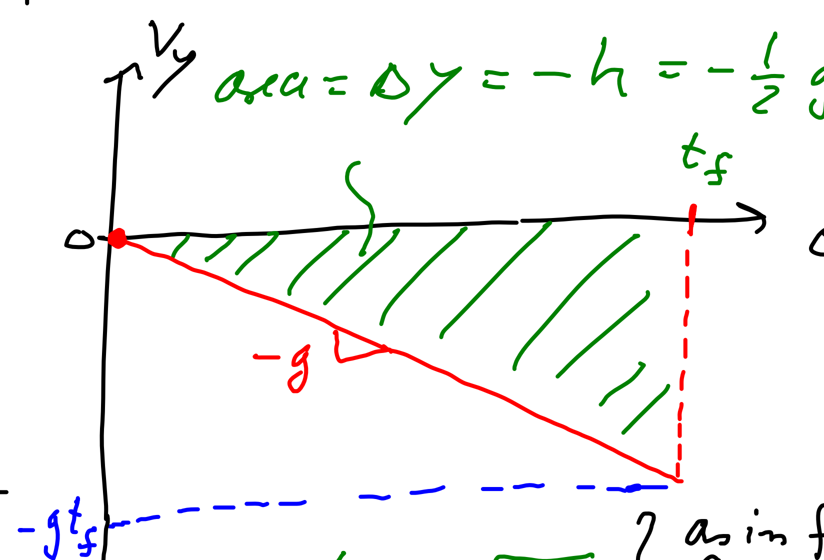
$$h \xrightarrow[\text{y-motion}]{} t_f \xrightarrow[\text{x-motion}]{} \Delta x$$



area $= \Delta x$

$= v_{0x} \cdot t_f$

area $= \Delta y = -h = -\frac{1}{2} g t_f^2$



$\Rightarrow t_f = \sqrt{\frac{2h}{g}}$ } as in free fall from height h!