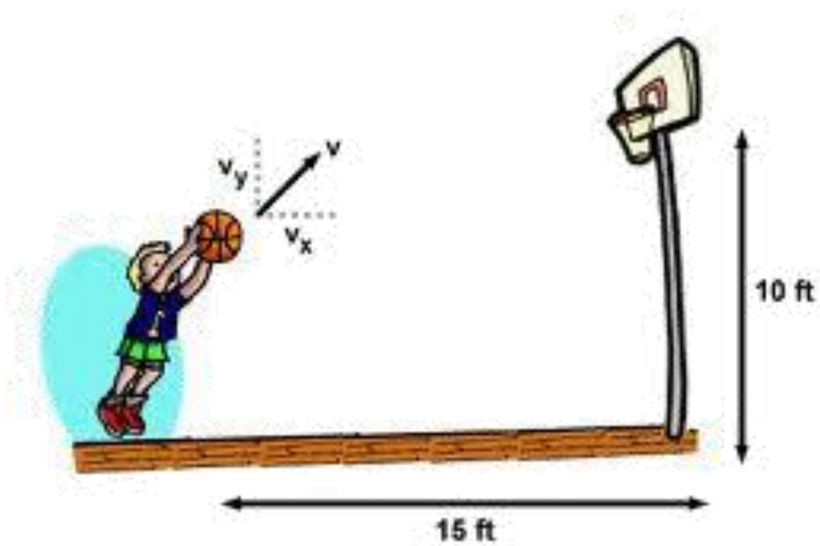
Recap: Motion along a line

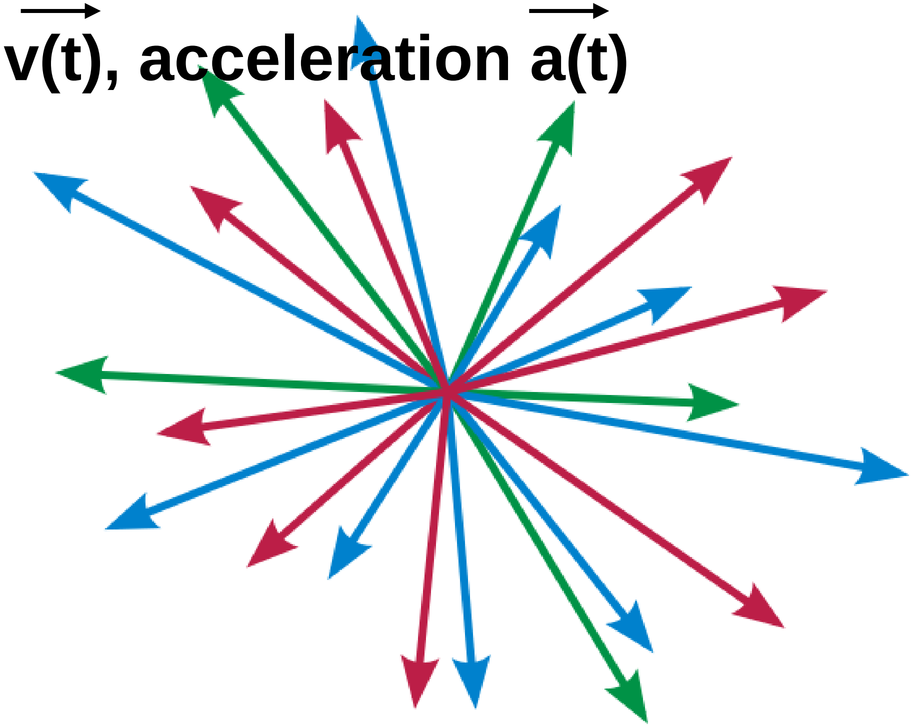
Lecture 5

- relevant variables: x, v, a
  - → magnitude : $|x|$, $|v|$, $|a|$ $\quad$ $|\ |$ = "absolute value"
  - → direction : $\pm$ sign
- Complicated motion:
  - apply a = const analysis to each a = const interval
  - use Δv = area "under" a-t graph to get v-t graph
- Solving physics problems:
  - I Identify type of problem
  - II Draw + collect information
  - III Solve (use equations
  - IV Check your answer
    - order of magnitude reasonable?
    - # of significant figures?
    - <u>units</u> correct?

Today:

- **Motion in a plane (2-D motion)**
  - The key to 2-D (and 3-D) motion
  - Specifying vectors
  - Vector addition and subtraction
  - Position $\overrightarrow{r(t)}$, velocity $\overrightarrow{v(t)}$, acceleration $\overrightarrow{a(t)}$

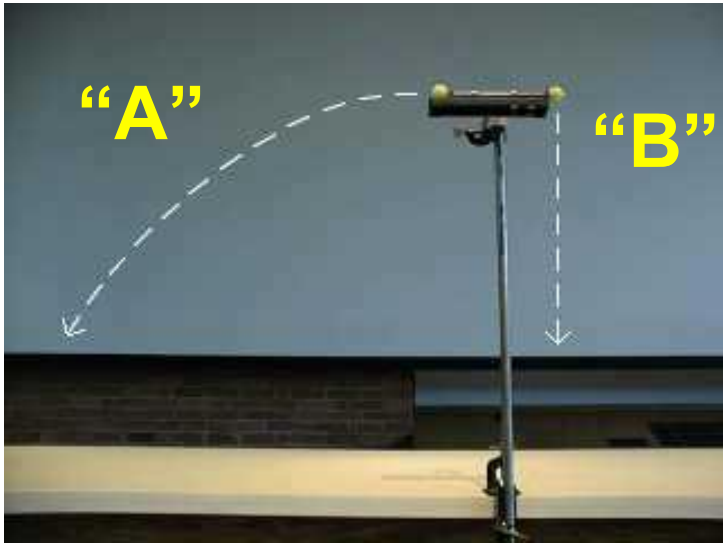




Motion in a Plane (2-D Motion)

Two balls are launched simultaneously from the same height, one starting from rest and dropping straight down, the other given an initial horizontal velocity.

Which ball hits the ground first?



y

x

A. Ball "A"

B. Ball "B"

C. Both at same time

- same height
- same y-motion, indep. of x-motion
- y motion determines "time of flight" here → same Δt

Motion in a Plane (2-D Motion)

- The x- and y-components of a 2-D motion can be treated independently!
- No mixing of x and y-components!
- 2D motion problems become two 1-D motion problems!

How to specify/describe 2-D Motion?

magnitude :- easy (e.g. speed)
direction ?

Vectors: have magnitude and direction

examples: $\vec{r}$, $\vec{v}$, $\vec{a}$, $\vec{F}$ ← "vector"

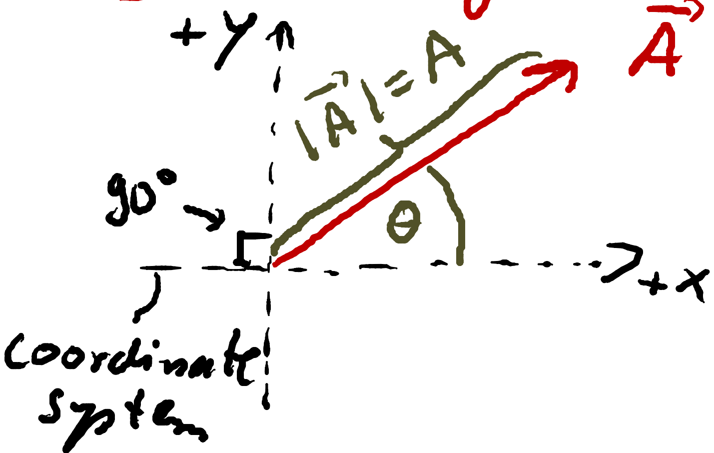
Scalars: have magnitude only

example: m, speed, T, t, E ...

Energy

Specifying Vectors:

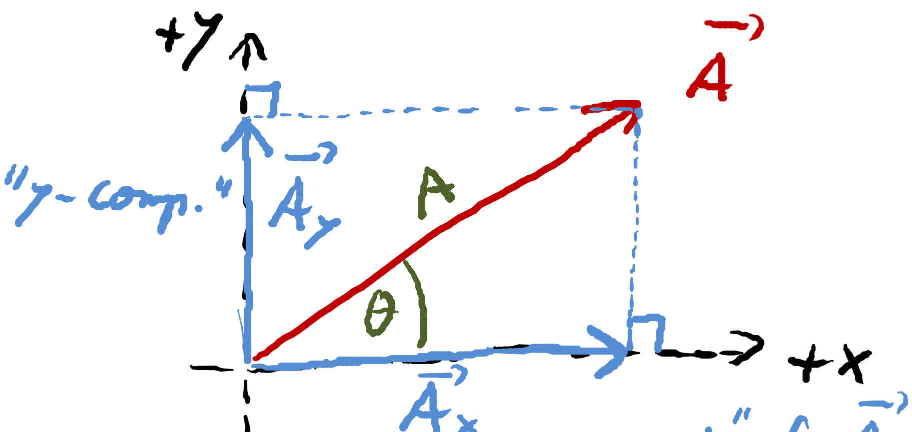
① Polar: Magnitude + direction



$|\vec{A}| = A =$ magnitude of $\vec{A}$

$\angle \vec{A} = \theta =$ gives direction

② Components:



"x-component" of $\vec{A}$
= component along x-direction

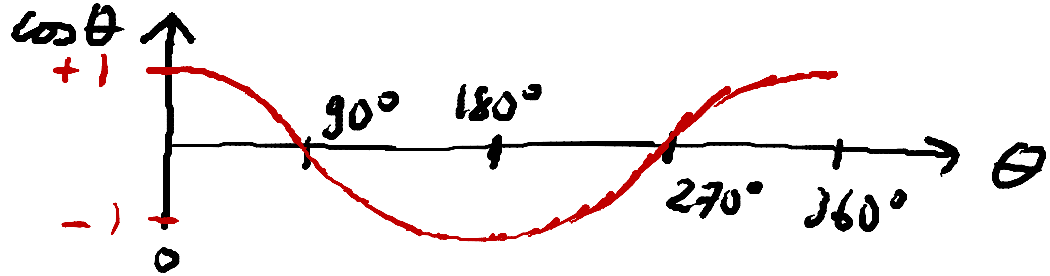
$$\vec{A} = \vec{A}_x + \vec{A}_y$$

$$A_x = A \cdot \cos\theta$$

$$A_y = A \cdot \sin\theta$$

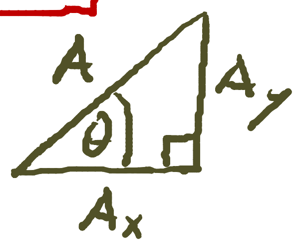
$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$



$$\boxed{\begin{array}{l} |\vec{A}| = A = \sqrt{A_x^2 + A_y^2} \\ \theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right) \end{array}}$$

inverse tan



Pythagoras: $A^2 = A_x^2 + A_y^2$

$(c^2 = a^2 + b^2)$

$$\sin\theta = \frac{A_y}{A} \qquad \cos\theta = \frac{A_x}{A}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{A_y}{A_x}$$

Unit vector:

$\vec{i}$, $\hat{i}$ = unit vector along +x

$\vec{j}$, $\hat{j}$ = unit vector along +y

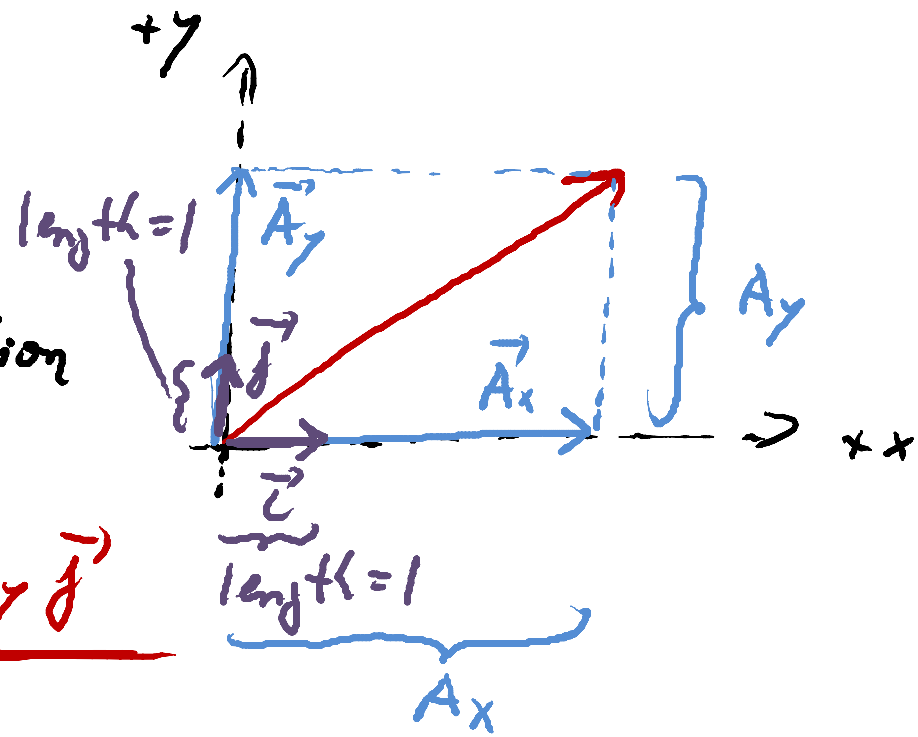
have magnitude = 1, no units

⇒ only indicate direction

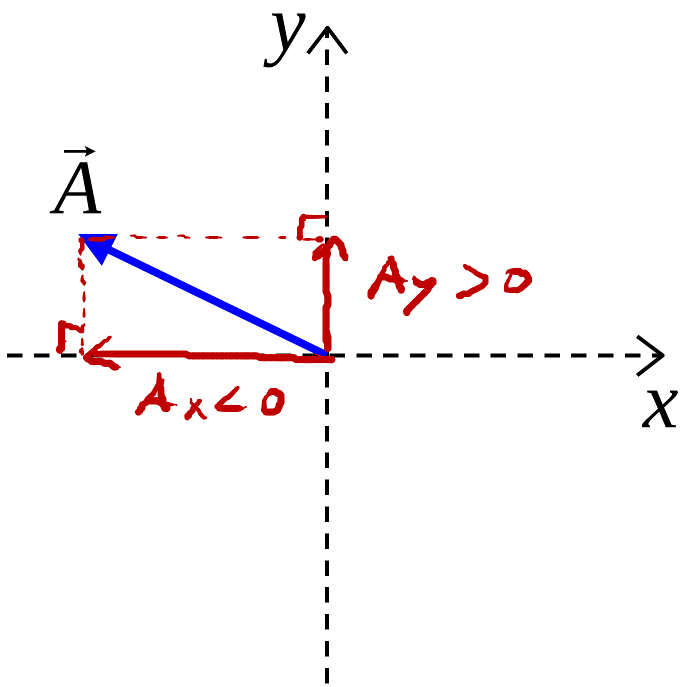
$$\Rightarrow \vec{A}_x = A_x \vec{i}$$

$$\vec{A}_y = \underbrace{A_y}_{\text{magnitude, includes sign, units}} \underbrace{\vec{j}}_{\text{direction}}$$

$$\underline{\underline{\vec{A} = \vec{A}_x + \vec{A}_y = A_x \vec{i} + A_y \vec{j}}}$$



Which correctly describes the components of $\vec{A}$?



A. $A_x > 0$ **and** $A_y > 0$

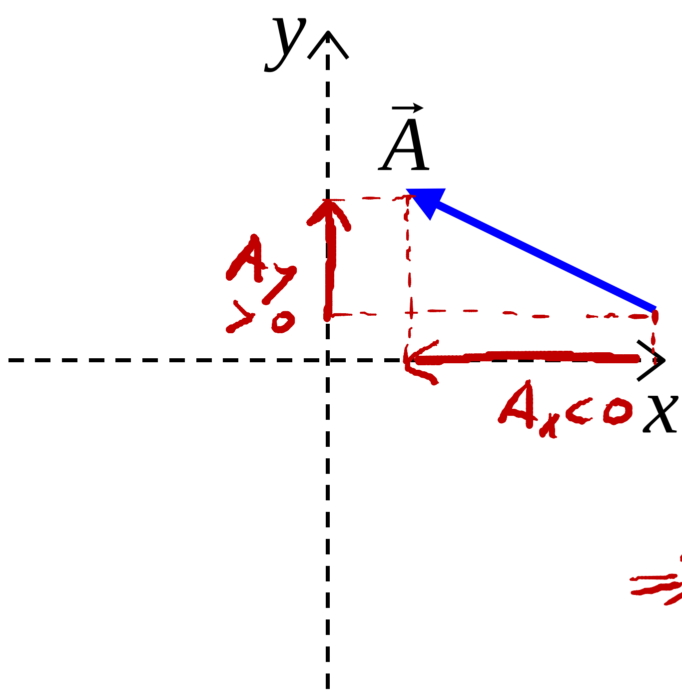
B. $A_x > 0$ **and** $A_y < 0$

C. $A_x < 0$ **and** $A_y > 0$

D. $A_x < 0$ **and** $A_y < 0$

E. None of the above.

Which correctly describes the components of $\vec{A}$?



A. $A_x > 0$ **and** $A_y > 0$

B. $A_x > 0$ **and** $A_y < 0$

C. $A_x < 0$ **and** $A_y > 0$

D. $A_x < 0$ **and** $A_y < 0$

E. None of the above.

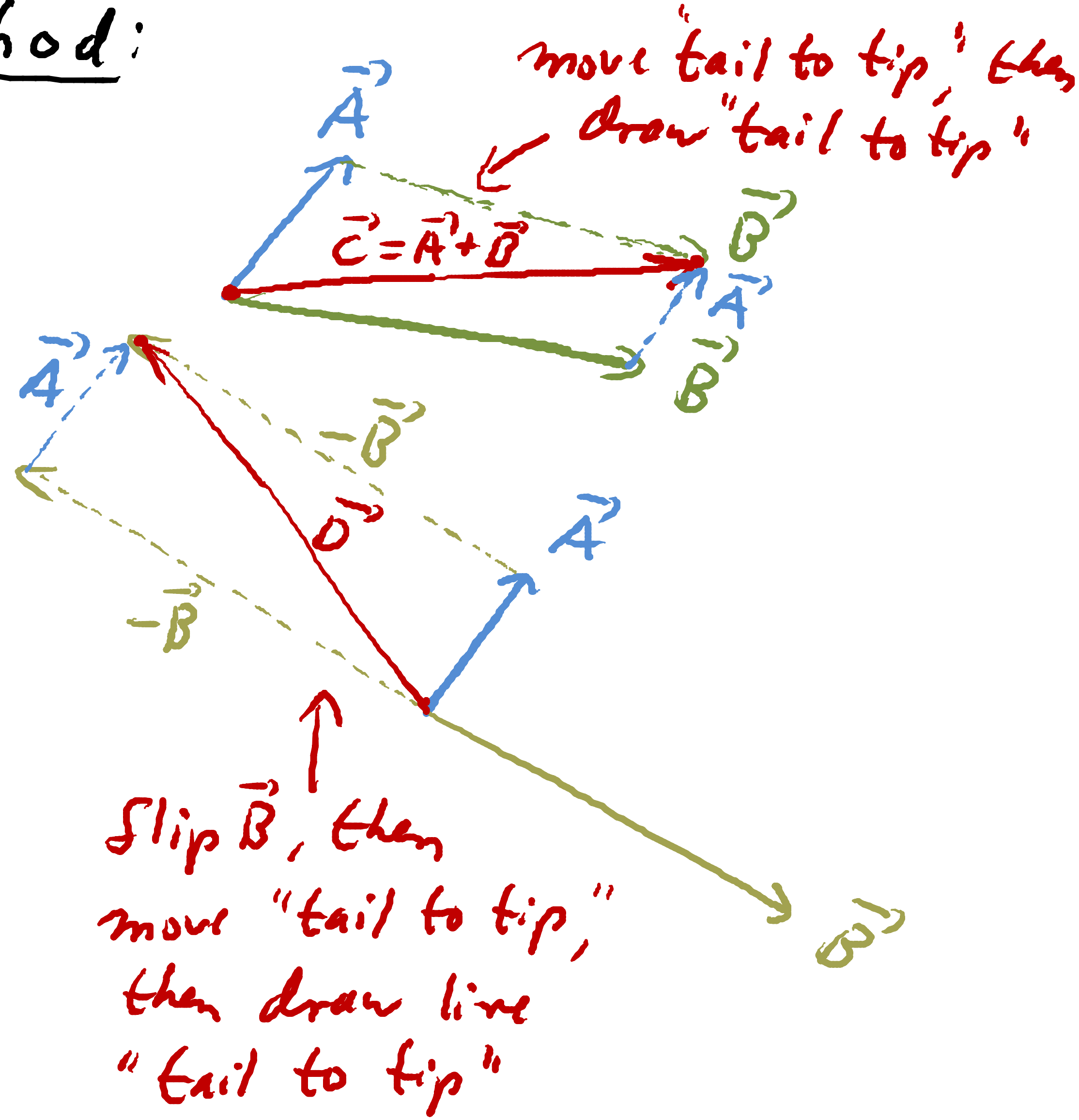
$\Rightarrow$ vectors don't care where they are in the plane!

Vector Addition and Subtraction:

① Graphical Method:

$$\vec{C} = \vec{A} + \vec{B}$$
$$= \vec{B} + \vec{A}$$

$$\vec{D} = \vec{A} - \vec{B}$$
$$= \vec{A} + (-\vec{B})$$
$$= (-\vec{B}) + \vec{A}$$



② Using Components:

$\boxed{\text{Never mix } x\text{- and } y\text{- components}}$

$$\vec{A} = A_x \vec{i} + A_y \cdot \vec{j}$$

$$\vec{B} = B_x \vec{i} + B_y \vec{j}$$

$$\vec{C} = \vec{A} \pm \vec{B} = (\underbrace{A_x \pm B_x}_{\text{add } x\text{- components}})\vec{i} + (\underbrace{A_y \pm B_y}_{\text{sum of } y\text{- components}})\vec{j}$$