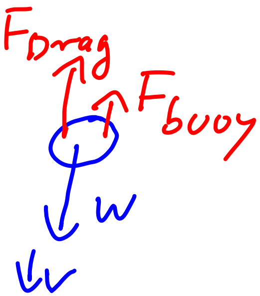
Recap

- **Terminal Speed of a Sphere:**



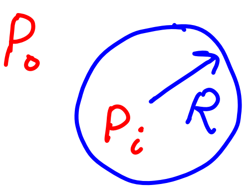
$$V_{t,turb} = \sqrt{\frac{16}{3}\frac{(\rho_{sp}-\rho_f)}{\rho_f} g r} \quad \text{or} \quad V_{t,visc} = \frac{2}{9}\frac{\rho_{sp}-\rho_f}{\eta} g r^2$$

- **Surface Tension** $= \gamma = \dfrac{\text{energy}}{\text{area}}$ required to create a liquid-gas interface

$= \dfrac{\text{Force}}{\text{length}}$ acting ⊥ to edge of a surface to keep it from collapsing

$$[\gamma] = \frac{J}{m^2} = \frac{N}{m}$$

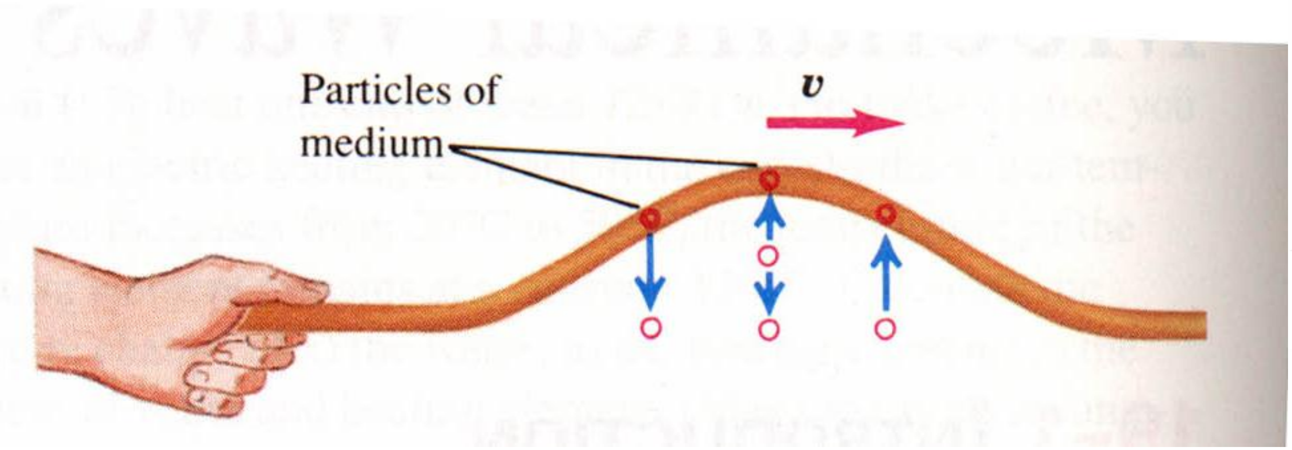
- **Bubbles:** Need $P_i > P_o$ to prevent bubbles from collapsing under the surface tension!



2-sided: $\Delta P = P_i - P_o = \dfrac{4\gamma}{R}$ 1-sided: $\Delta P = \dfrac{2\gamma}{R}$

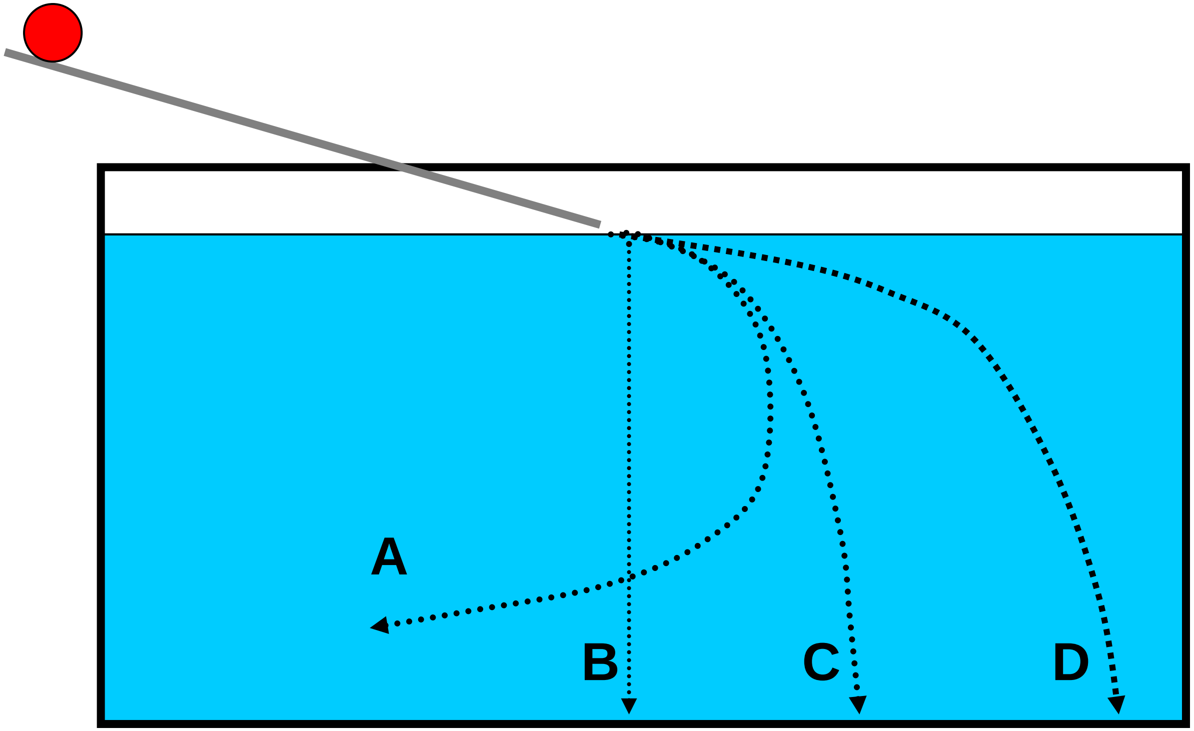
Today:

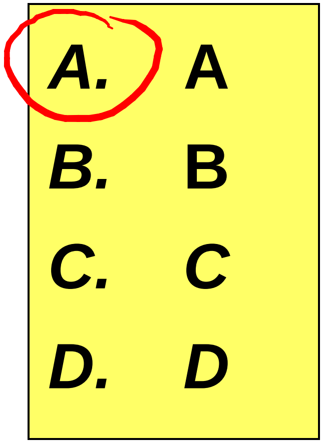
- **The Magnus force**
- **Vortex (smoke) rings**
- **Mechanical Waves**
- **Traveling transverse waves**



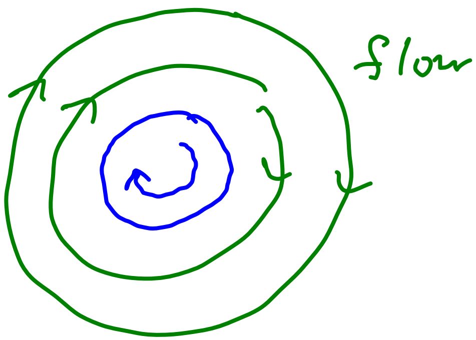
A cylinder with $\rho > \rho_w$ rolls down a ramp into a water-filled aquarium.

Which path best describes the motion of the cylinder in the water?



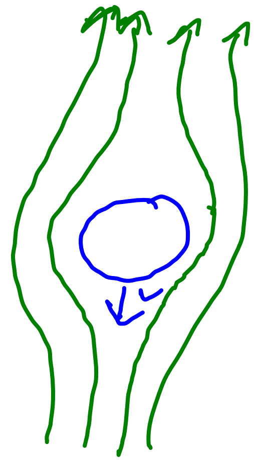


Magnus force:



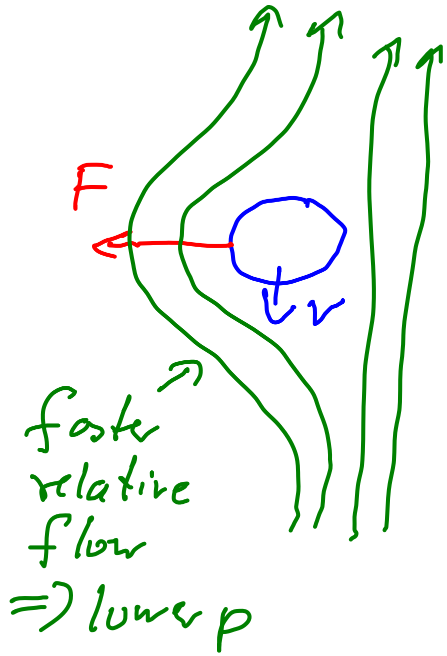
Create fluid flow due to rotation (spinning)

+



Create flow due to translation (symmetric)

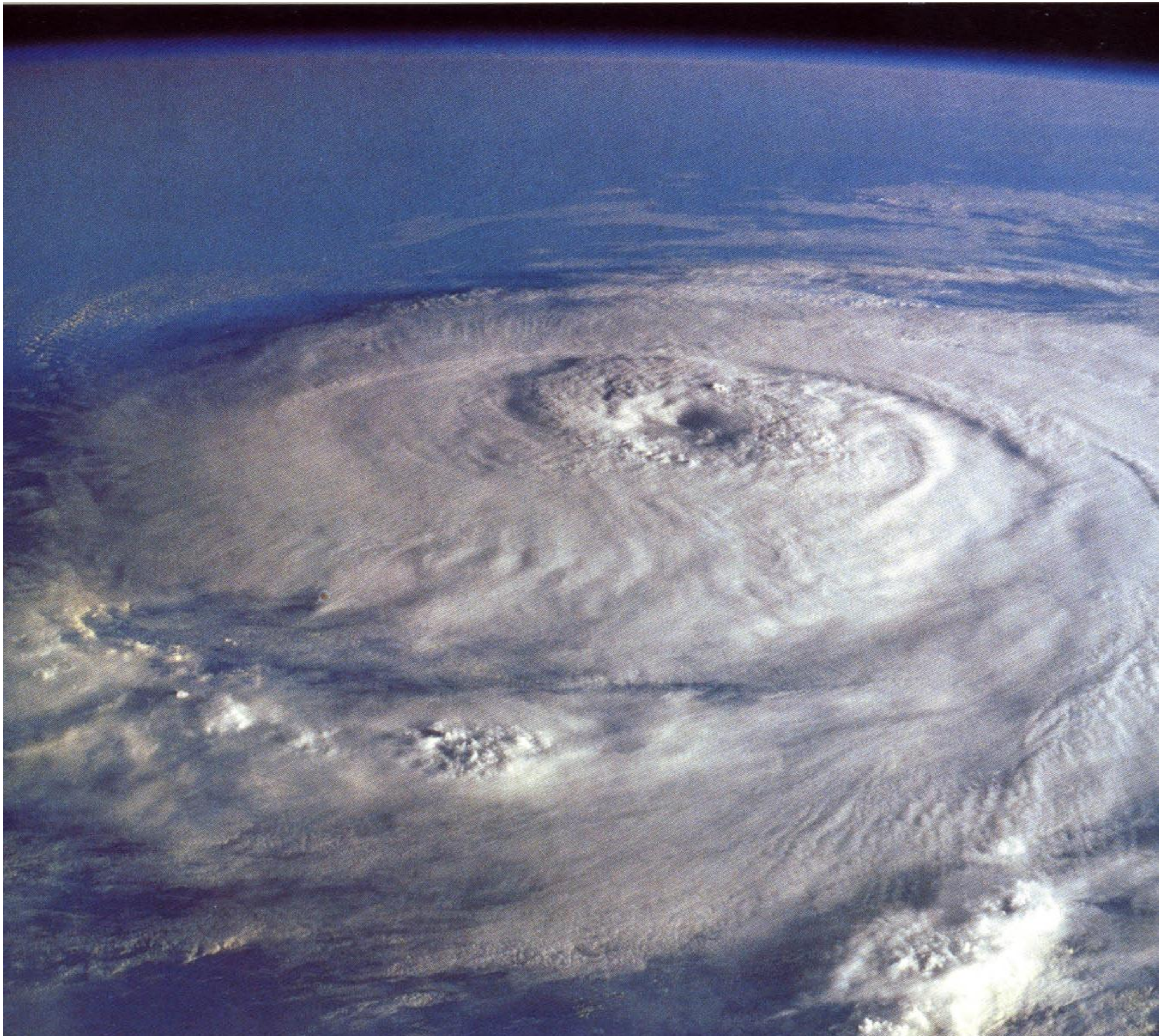
=



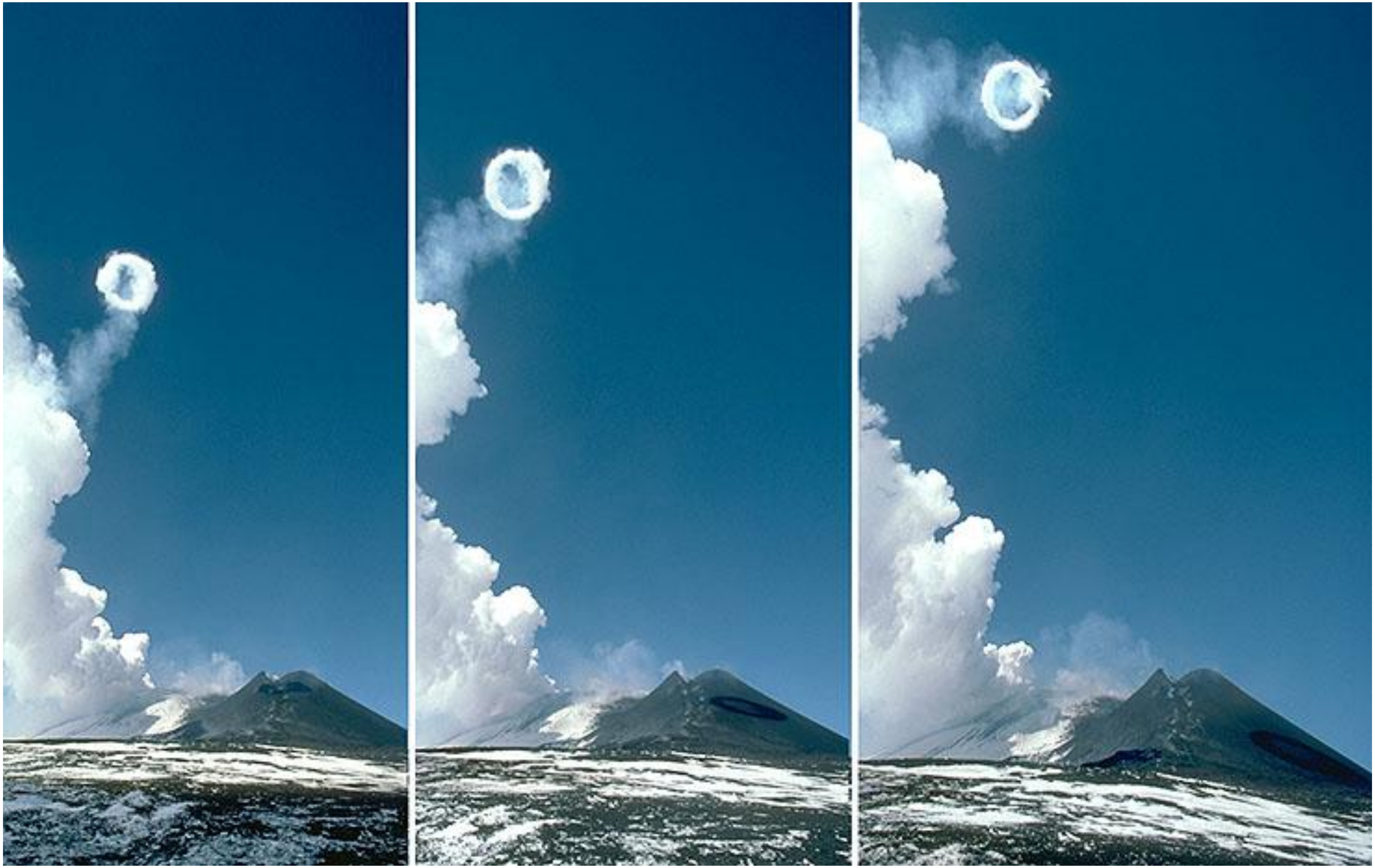
Force from Δp ⊥ to path

Vortices in the turbulent wake of a moving cylinder:

Vortex on the tip of an airplane wing:



Mount Etna, Italy



(Mechanical) Waves:

- Disturbance that propagate (through a medium) at velocity v_{wave} (that's determined by the medium)
- Energy and Momentum propagate
- No propagation of mass!
- waves are reflected at boundaries when v_{wave} changes, i.e. when medium changes

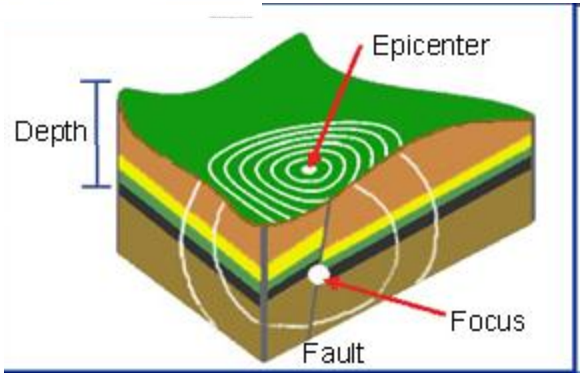
Examples:

- waves on string, spring

- sound waves

- water waves

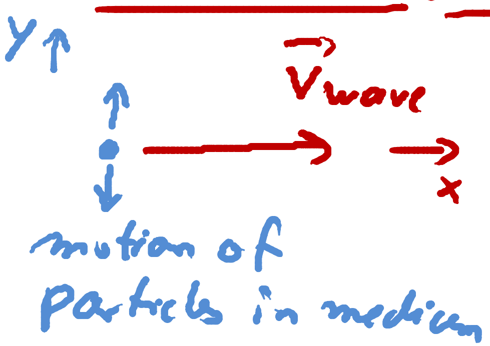
- seismic waves



- electromagnetic waves (no medium required!)

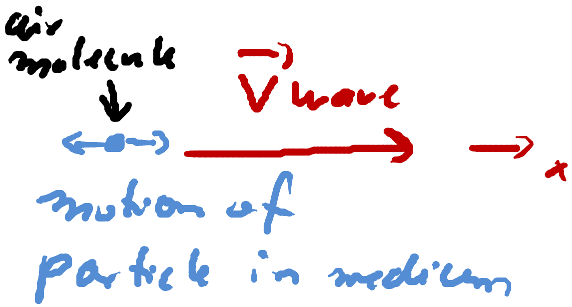
→ Two basic wave types:

• Transverse Wave:



- motion/displacement of particles of medium is ⊥ to wave velocity, i.e. ⊥ to direction of wave propagation
- e.g. wave on violin string

• Longitudinal Wave:



- motion/displacement of particles of medium is ∥ to wave velocity, i.e. along direction of wave propagation
- e.g. sound waves

Examples of Travelling Waves:

• Transverse wave: wave on string

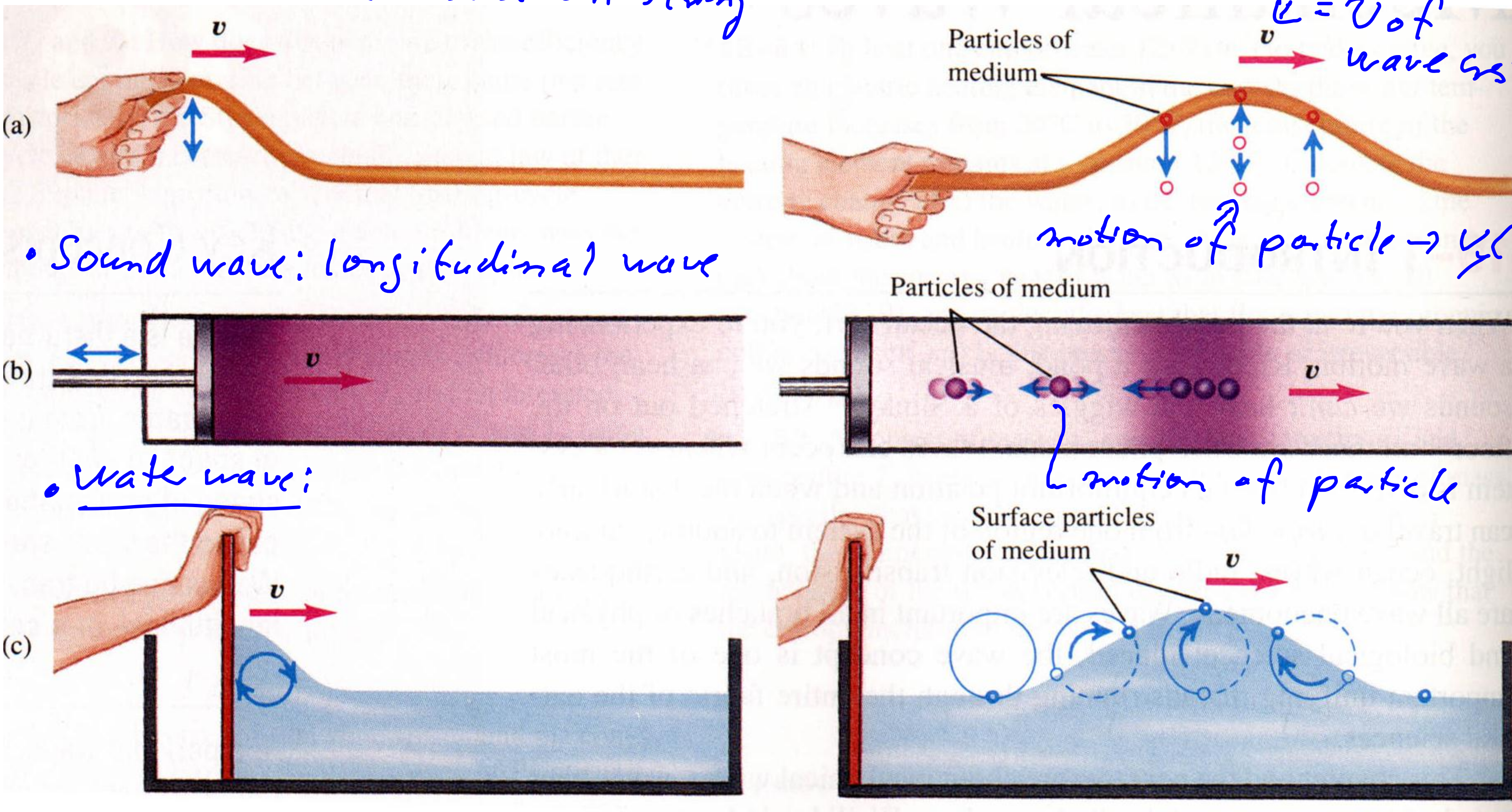
v_{wave} = v of wave crst

motion of particle → $v_y(t)$

• Sound wave: longitudinal wave

motion of particle

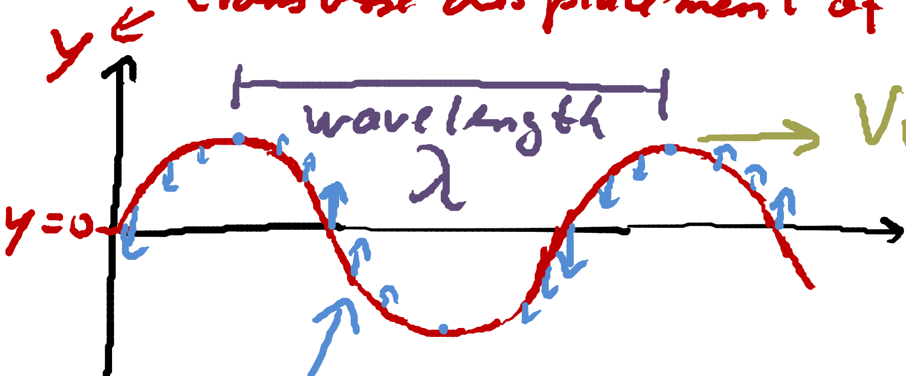
• Water wave:



→ Traveling Transverse Waves:

Sinusoidal wave: Why? Any wave can be expressed as a sum of sine waves of different frequencies and wavelengths!

Snapshot at some time t:



each particle of the medium undergoes transverse motion

v_{wave} = wave velocity; independent of time

x ↑ direction of wave propagation

= velocity of propagation of disturbance

= "speed of wave crest"

not the same thing!

⇒ transverse velocity $v_y(t)$ of particles, time dependent!

⇒ Oscillate in time ⇒ **SHM**

"Snapshots" of displacement y versus position x at fixed time t_0:

Transverse Waves:

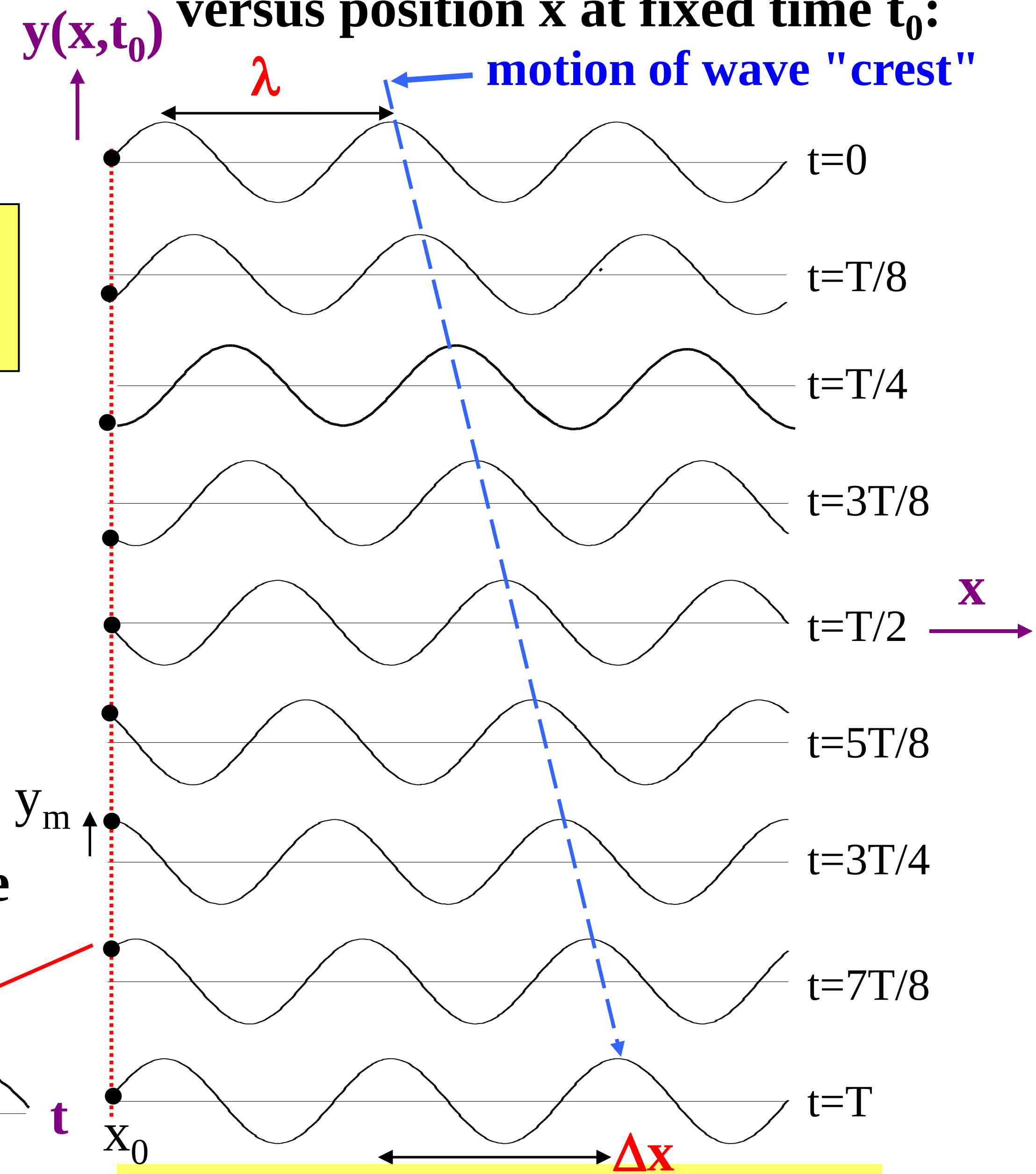
$$y(x,t) = y_m \sin[2\pi(x/\lambda - t/T)] = y_m \sin[kx - \omega t)]$$

position of nth crest:

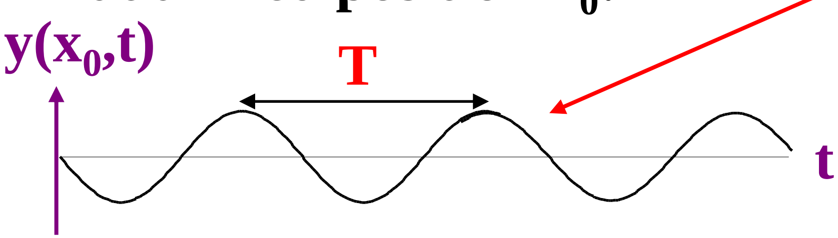
$x/\lambda - t/T = \pi/2 \; (+ 2n\pi)$

At fixed $t=t_0$,

$$y(x,t) = y_m \sin[2\pi(x/\lambda - t_0/T)] = y_m \sin[2\pi(x/\lambda - \phi_0)]$$



Displacement y versus time at a fixed position x_0:



wave velocity $v = \Delta x/\Delta t = \lambda/T = \lambda f$

Math Description:

⊥ displacement of particle in medium (y); wave amplitude (y_m)

$$y(x,t) = y_m \sin[kx \mp \omega t] = y_m \sin\left[2\pi\left(\frac{x}{\lambda} \mp \frac{t}{T}\right)\right]$$

(x: space; t: time; sin: sin-wave; k: wave number; ω: angular frequ.; λ: wavelength; T: period of oscillation)

$$= y_m \sin\left[\frac{2\pi}{\lambda}(x \mp v_{wave} \cdot t)\right]$$

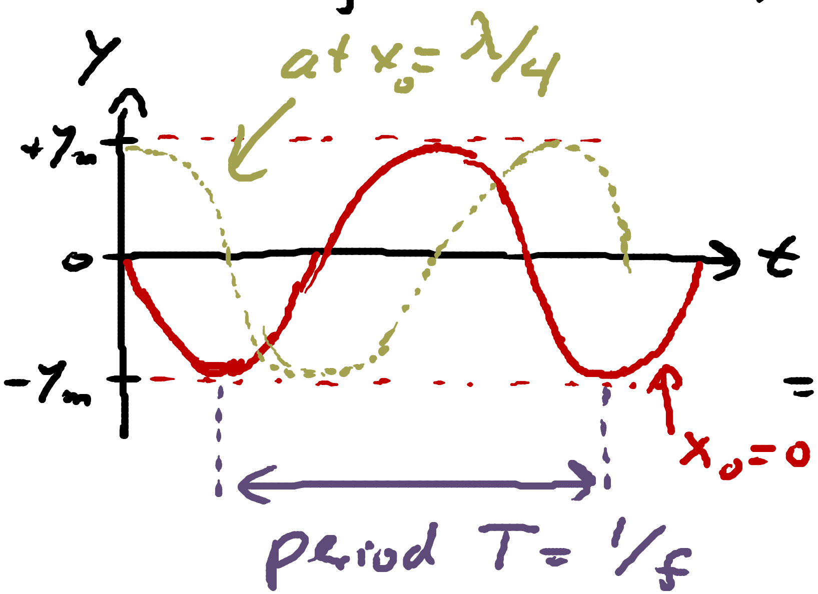
"−" ⇒ wave moves in +x direction

"+" ⇒ waves move in −x direction

- λ = wave length = "period in space" = distance from crest to crest
- T = Period of oscillation in time at a given x, as in SHM
- $k = \frac{2\pi}{\lambda}$ = wave number $[k] = \frac{1}{m}$
- $\omega = 2\pi f = 2\pi/T$ = angular frequency $[\omega] = \frac{\text{"rad"}}{s}$
- v_{wave} = wave speed $= \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$

→ Motion of a particle in the medium at $x = x_0$

fix x at x_0, and plot y vs. t



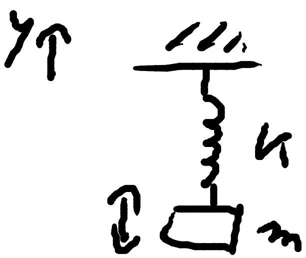
$$y(x_0, t) = y_m \sin[k x_0 - \omega t]$$

$\sin(-b) = -\sin(b)$

$$= -y_m \sin[\omega t + \phi_0]$$

with $\phi_0 = -k x_0 = -\frac{2\pi}{\lambda} x_0$

recall SHM:



$$y(t) = y_m \sin(\omega t + \phi)$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

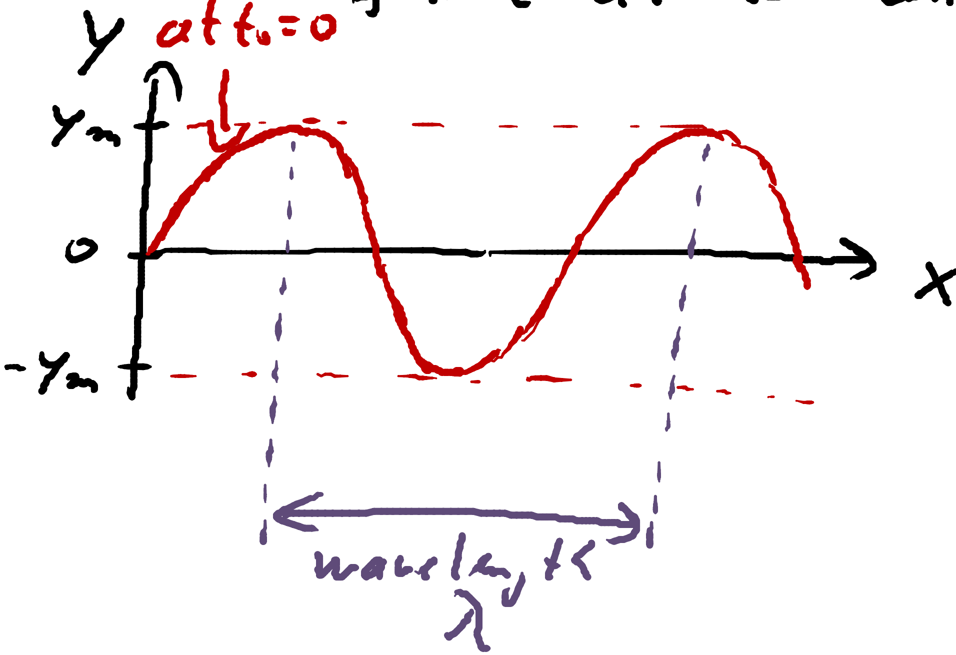
- each particle undergoes SHM!
- same amplitude y_m
- same frequency ω
- <u>but:</u> phase $\phi_0 = -k x_0$ determined by position!
- transverse velocity:

$$v_y = \left.\frac{dy}{dt}\right|_{x = x_0} \qquad v_{y,max} = \omega y_m$$

$$a_{y,max} = \omega^2 y_m$$

→ "Snapshot" of the wave:

fix t at t_0 and plot y vs. x



"−" ⇒ moves in +x direction

$$y(x, t_0) = y_m \sin[kx - \omega t_0]$$

$$= y_m \sin[kx + \phi_0]$$

with $\phi_0 = -\omega t_0 = -\frac{2\pi}{T} t_0$