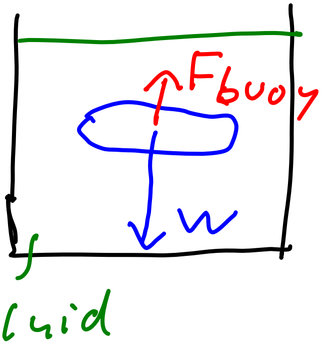
Recap:

Static Liquid:

- Buoyant Force:



$F_{buoy\ on\ object} = \rho_f g V_{fluid\ displaced}$

= Weight of fluid displaced by object

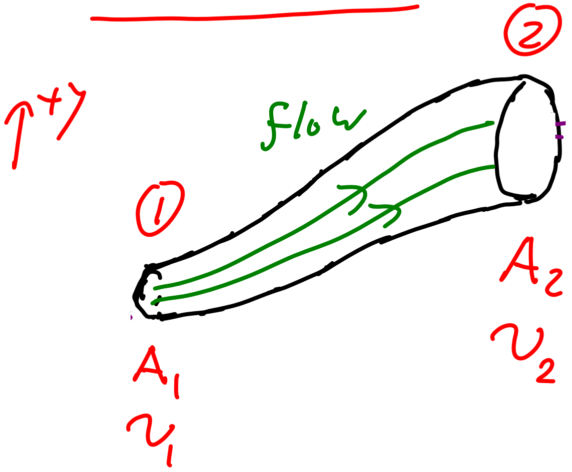
= net force on object from fluid pressure on its surface

Note: F_{buoy} is a consequence of pressure variation with depth h in a fluid!

Recap:

- **Ideal Fluid:** no fluid friction; ρ = const; laminar flow

- **Ideal Flow:**



- Volume flow rate:

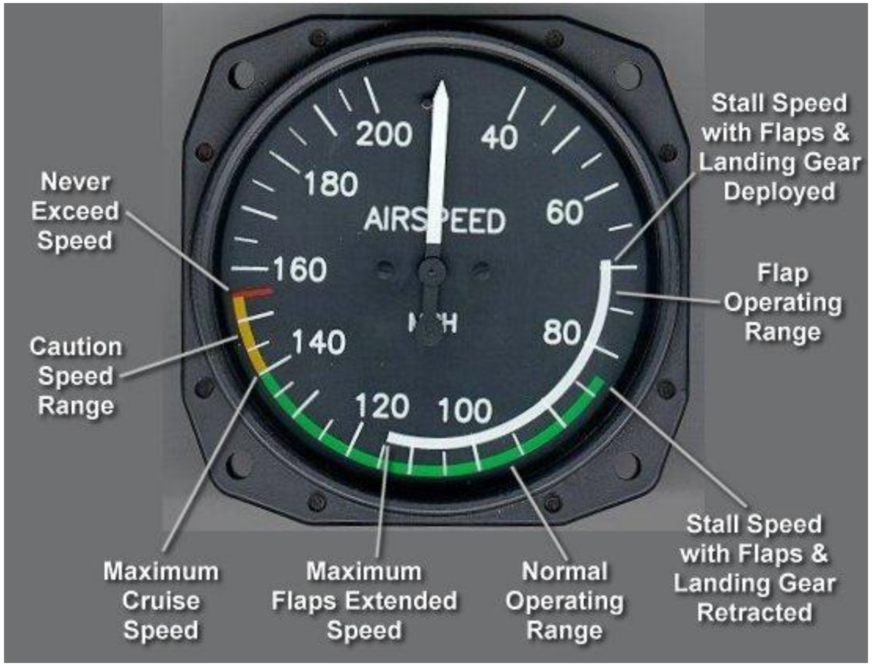
$$R = \frac{\Delta \text{Volume passing}}{\Delta t} = \overset{\substack{\text{x-sectional}\\\text{area}}}{A} \; \underset{\text{speed of flow}}{v}$$

- Continuity:

$$R_1 = R_2 \Rightarrow A_1 v_1 = A_2 v_2$$

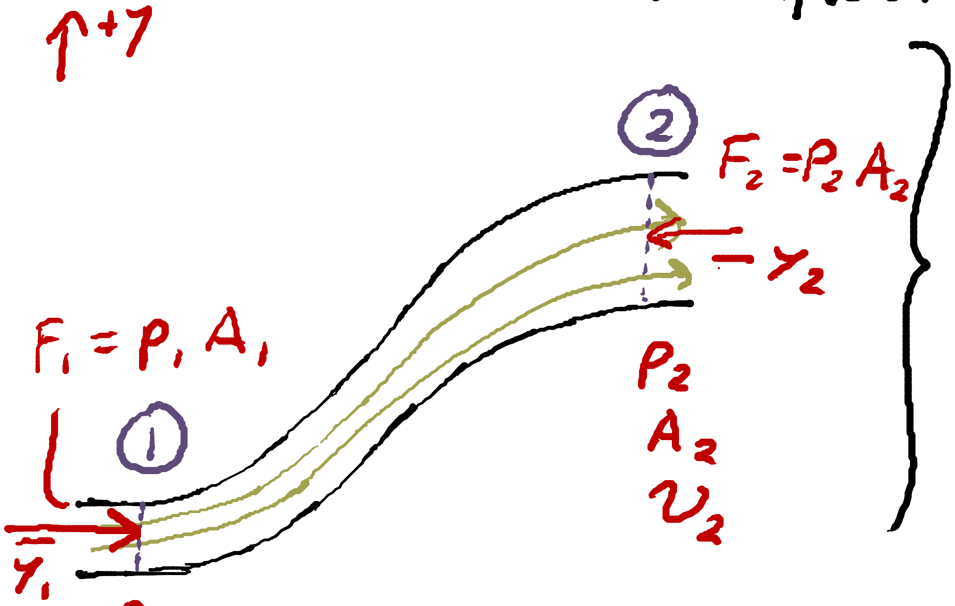
Today:

- **Fluids in motion**
- **Bernoulli's Equation**
- **Measuring air speed**



Bernoulli's Equation

→ consider a tube with flow:



P_1 (pressure)

A_1 (x-sect. area)

v_1 (speed of flow)

- continuity : $R_1 = R_2$

$$\Rightarrow \underbrace{A_1 v_1}_{\frac{\Delta V_1}{\Delta t}} = \underbrace{A_2 v_2}_{\frac{\Delta V_2}{\Delta t}}$$

- Use Work - Kinetic Energy Theorem:

$$\Delta K = W_{\text{on fluid by forces}}$$

$$= \underbrace{W_{\text{gravity}}}_{= -\Delta U_g} + \underbrace{W_{\text{on fluid by applied forces}}}_{\text{by pressure}}$$

Note: for ideal fluid,
i.e. no friction here!

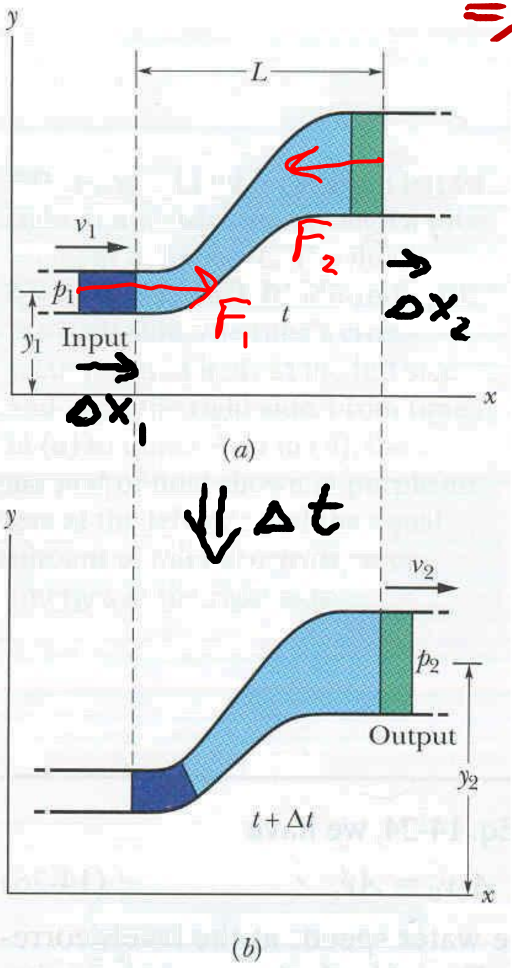


FIG. 14-20 Fluid flows at a steady rate through a length L of a tube, from the input end at the left to the output end at the right. From time t in (a) to time $t + \Delta t$ in (b), the amount of fluid shown in purple enters the input end and the equal amount shown in green emerges from the output end.

$$\Rightarrow W_{\text{on fluid by pressure}} = \Delta U_g + \Delta K = \Delta E \quad \Big\}\ \text{during time } \Delta t$$

- pressure difference between ① and ② drives flow:

$$W_{\text{on fluid by } \Delta p} = F_1 \Delta x_1 - F_2 \Delta x_2$$

$$= P_1 \underbrace{A_1 \Delta x_1}_{\Delta V_1} - P_2 \underbrace{A_2 \Delta X_2}_{\Delta V_2}$$

$$\Delta V_1 = \Delta V_2 = \Delta V$$

$$\Rightarrow W_{\text{on fluid by } \Delta p} = (P_1 - P_2)\Delta V \qquad (A)$$

- $\Delta U_g = U_2 - U_1 = m_2 g y_2 - m_1 g y_1$

$$m_2 = m_1 = m \ (\Delta V_1 = \Delta V_2)$$

$$\Rightarrow \Delta U_g = mg(y_2 - y_1) = \rho \Delta V g (y_2 - y_1) \qquad \rho = \text{const} \qquad (B)$$

- $\Delta K = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

$$= \frac{1}{2} \rho \Delta V (v_2^2 - v_1^2) \qquad (C)$$

insert (A), (B), (C) into:

$$W_{\text{on fluid by } \Delta p} = \Delta U_g + \Delta K \quad \Big\} \text{ during flow in some } \Delta t$$

$$\Rightarrow (p_1 - p_2)\Delta V = \rho \Delta V (y_2 - y_1) + \frac{1}{2}\rho \Delta V (v_2^2 - v_1^2)$$

$$\neq \Delta v^2$$

$$\boxed{\underbrace{(P_1 - P_2)}_{\frac{\text{work}}{\text{volume}}} = \underbrace{\rho g (y_2 - y_1)}_{\frac{\Delta U_g}{\text{volume}}} + \underbrace{\frac{1}{2}\rho (v_2^2 - v_1^2)}_{\frac{\Delta K}{\text{volume}}}}$$

$$[p] = \frac{J}{m^3}$$

Bernoulli's Equation

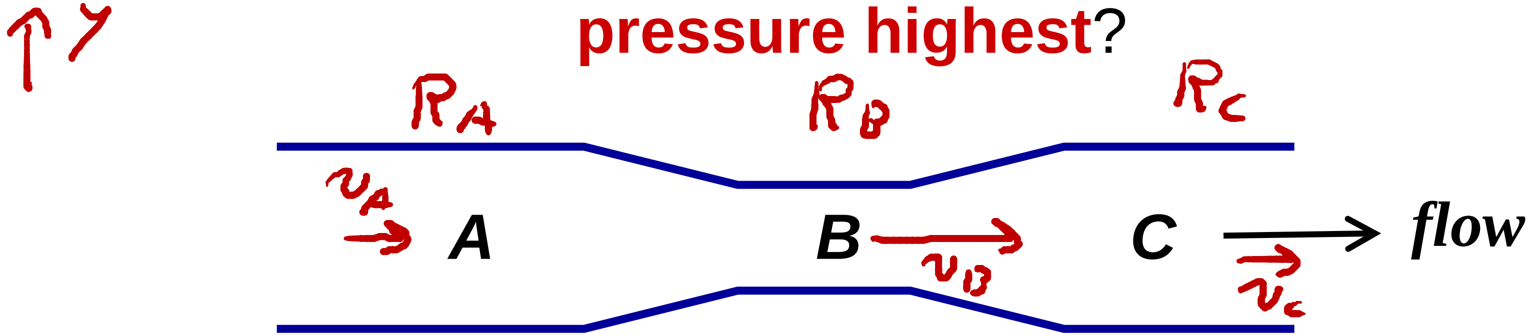
↝ Relates change in pressure in flowing fluid to change in height and change in flow speed

$\Rightarrow$ for <u>ideal</u> fluid flow:

$$R = \frac{\Delta \text{Volume}}{\Delta t} = v_1 A_1 = v_2 A_2 = \text{const}$$

$$P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2 \overset{!}{=} \text{const}$$

Air flows through a tube with a constriction. Where is the **pressure highest**?



Continuity:

$v_A A_A = v_B A_B = v_C A_C$

$A_A = A_C > A_B$

$\Rightarrow v_A = v_C < v_B$

$y_A = y_B = y_C$: constant height

$\Rightarrow P_A + \frac{1}{2} \rho v_A^2 = P_B + \frac{1}{2} \rho v_B^2$

$= P_C + \frac{1}{2} \rho v_C^2$

$\Rightarrow P_A = P_C > P_B$ ⇒ for constant height: when flow speed is large → p is small!

$\Delta P = P_A - P_B > 0$ to accelerate fluid from v_A to $v_B > v_A$

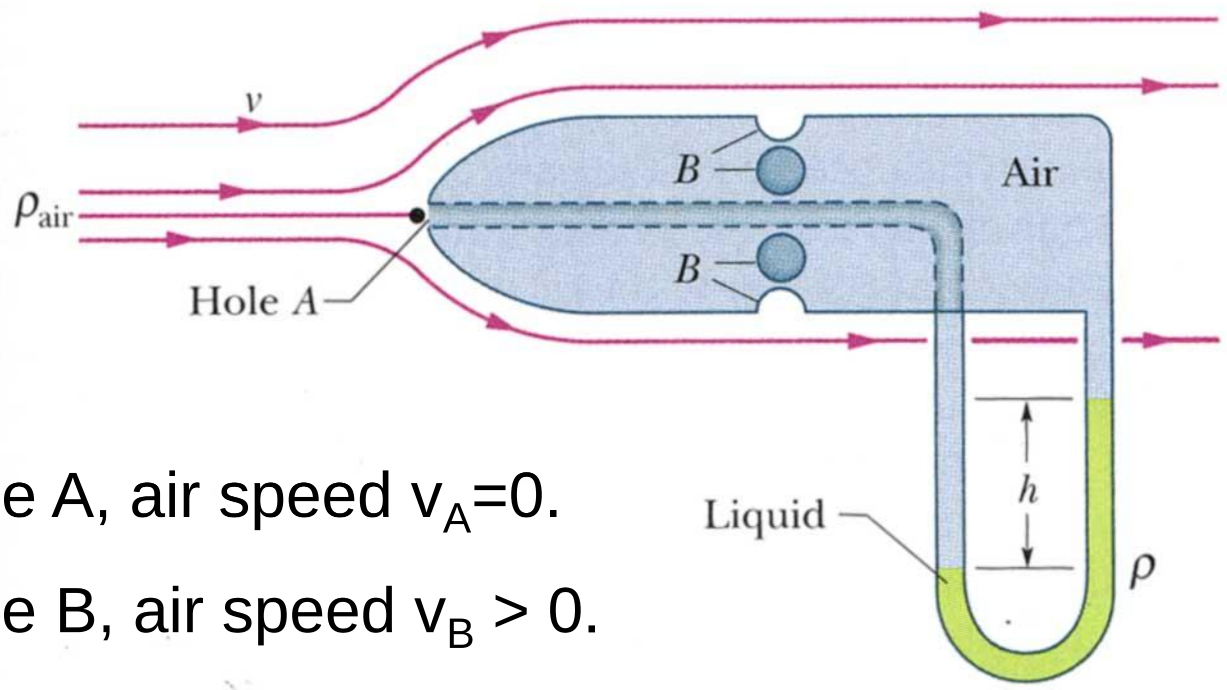
- **A.** at *A*
- **B.** at *B*
- [✓ **C.** same at both *A* and *C*]
- **D.** not enough information

Bernoulli:

For flow at *constant height*, **if v ↑, p ↓**

Some Applications

Measuring Air Speed



At Hole A, air speed $v_A=0$.

At Hole B, air speed $v_B > 0$.

By Bernoulli's equation, $p_A + 1/2\ \rho\ v_A^2 = p_B + 1/2\ \rho\ v_B^2$

$$\Rightarrow \mathbf{v_B = [2(p_A - p_B)/\ \rho\ ]^{1/2}}$$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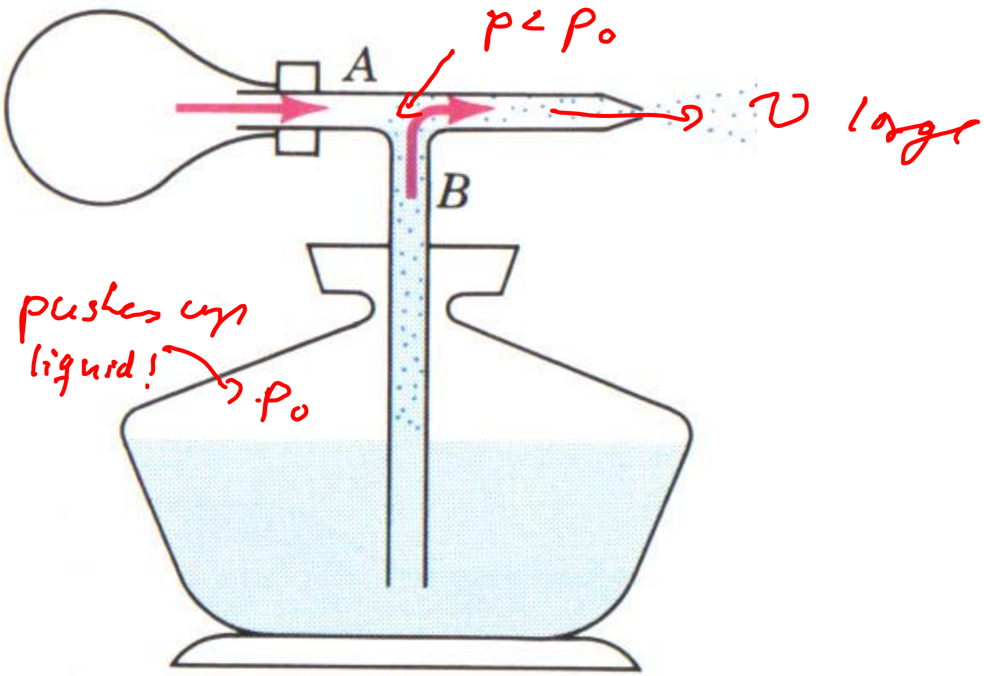
Perfume atomizers

Flowing air creates Δp that **pushes** fluid out of container.

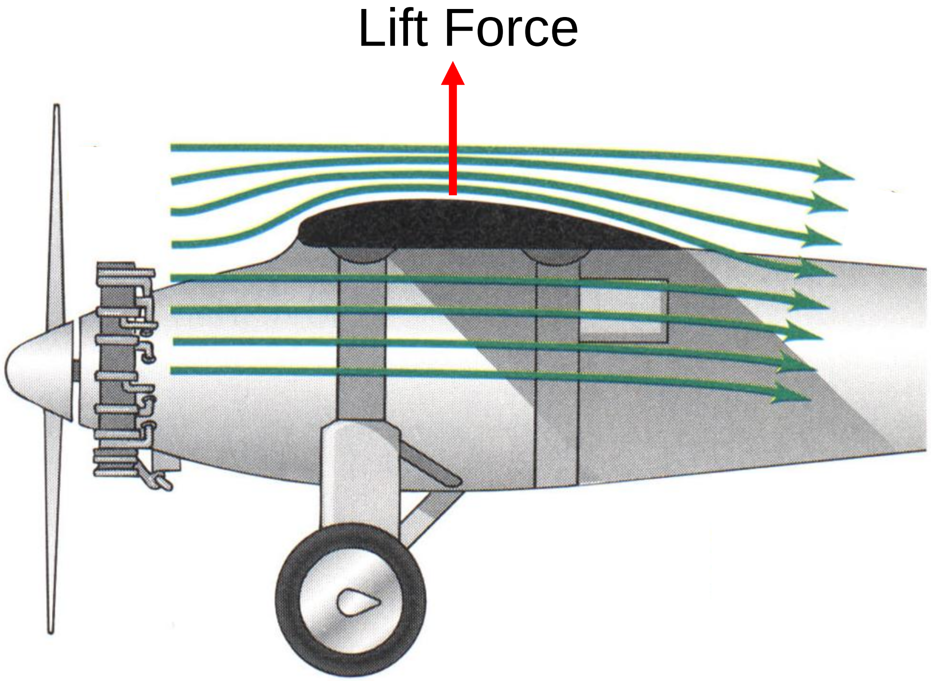


Airfoils in aircrafts?

Air must travel a **larger distance** over the top of an airfoil than over the bottom.

$\Rightarrow$ air velocity $\mathbf{v_{top} > v_{bot}}$, $\mathbf{p_{top} < p_{bot}}$

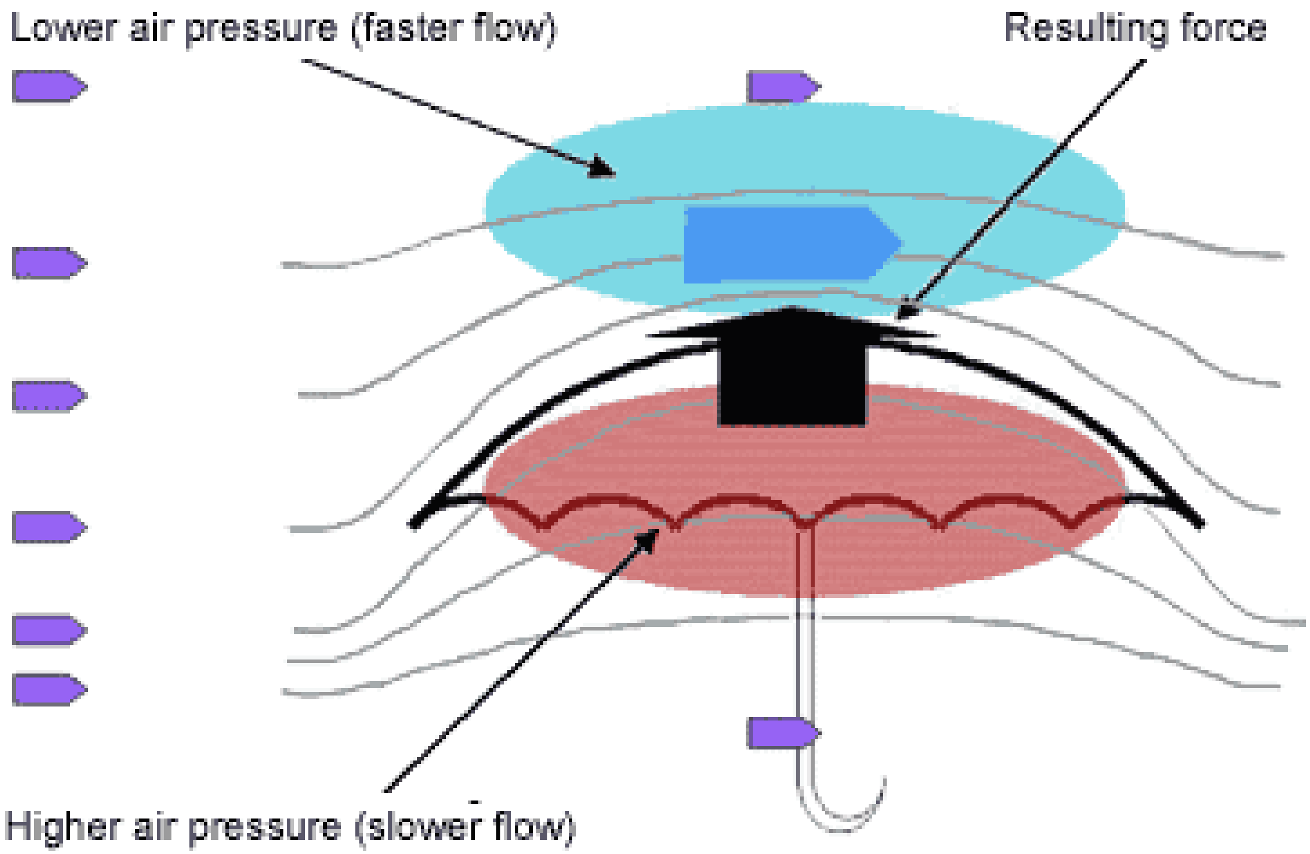
$\Rightarrow$ **Lift Force $F_L \sim \Delta p\ A_{wing}$**



But:

- **Bernoulli's equation is for laminar flow only!!**
- **The flow of air is highly turbulent here!**

The Bernoulli principle acting on an umbrella



⇒ Same principle used in sailing ⇒ can go faster than wind!

Wind damage to buildings

v_{inside} ~0, v_{outside} >>0 ⇒ **$\Delta p = p_{\text{in}} - p_{\text{out}} >> 0$**

⇒ **building "explodes“!**

E.g.:

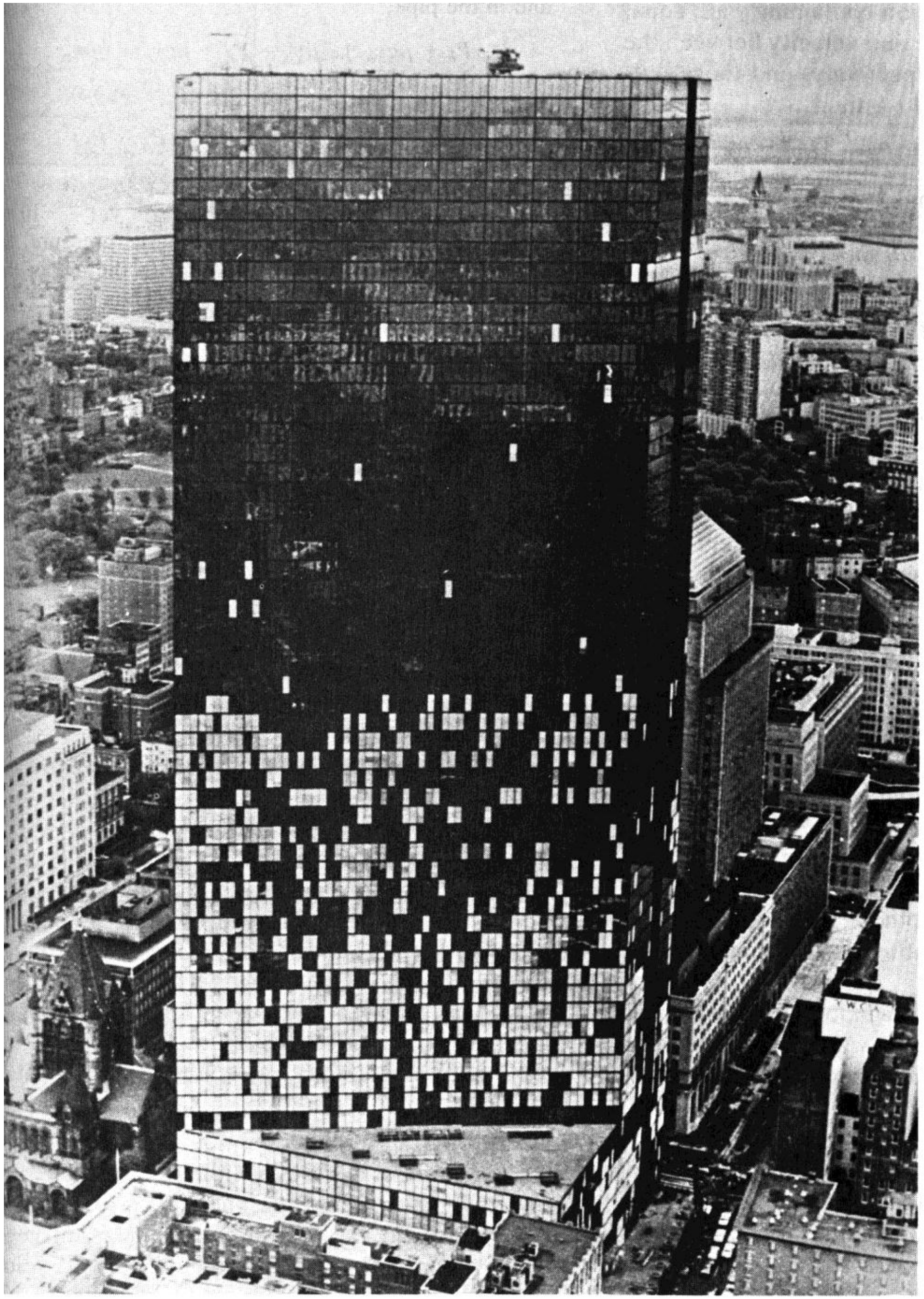
$v_{\text{outside}} = 360$ km/h (~220 mi/h)

$\Delta p = (1/2)\, \rho_{\text{air}}\, v_{\text{out}}^2 \sim 6000$ Pa (~**0.06 p_{atm}**)

⇒ Upward force on 80 m^2 house roof:

$\Delta p\, A \sim 5 \times 10^5$ N ~ **50 tons!**

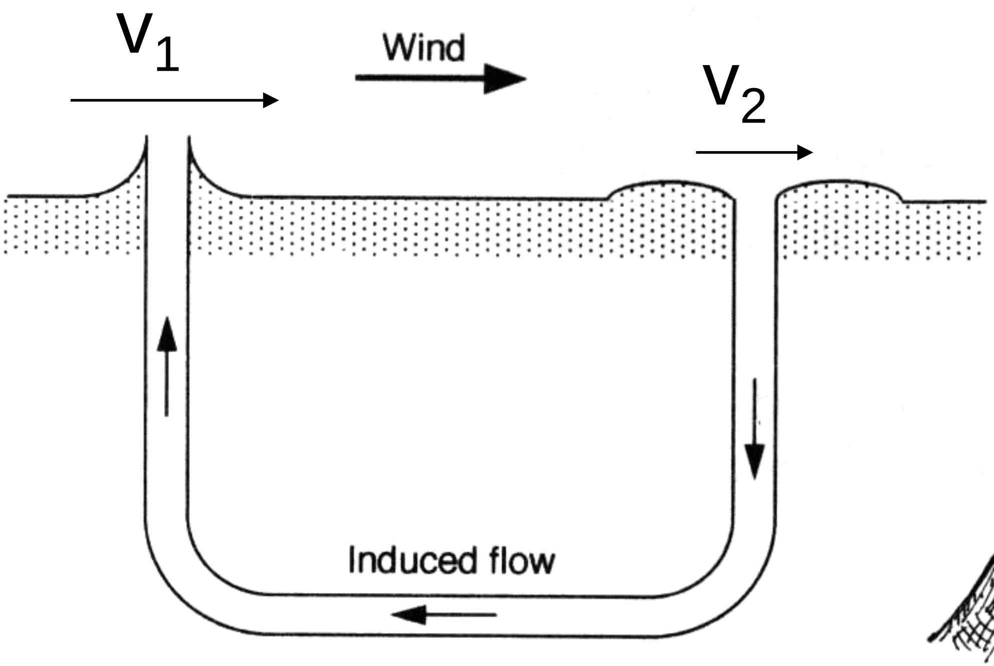
Hancock Building (Boston, 1973):



Ventilation of prairie dog burrows

$v_1 > v_2$

$\Rightarrow p_1 < p_2$

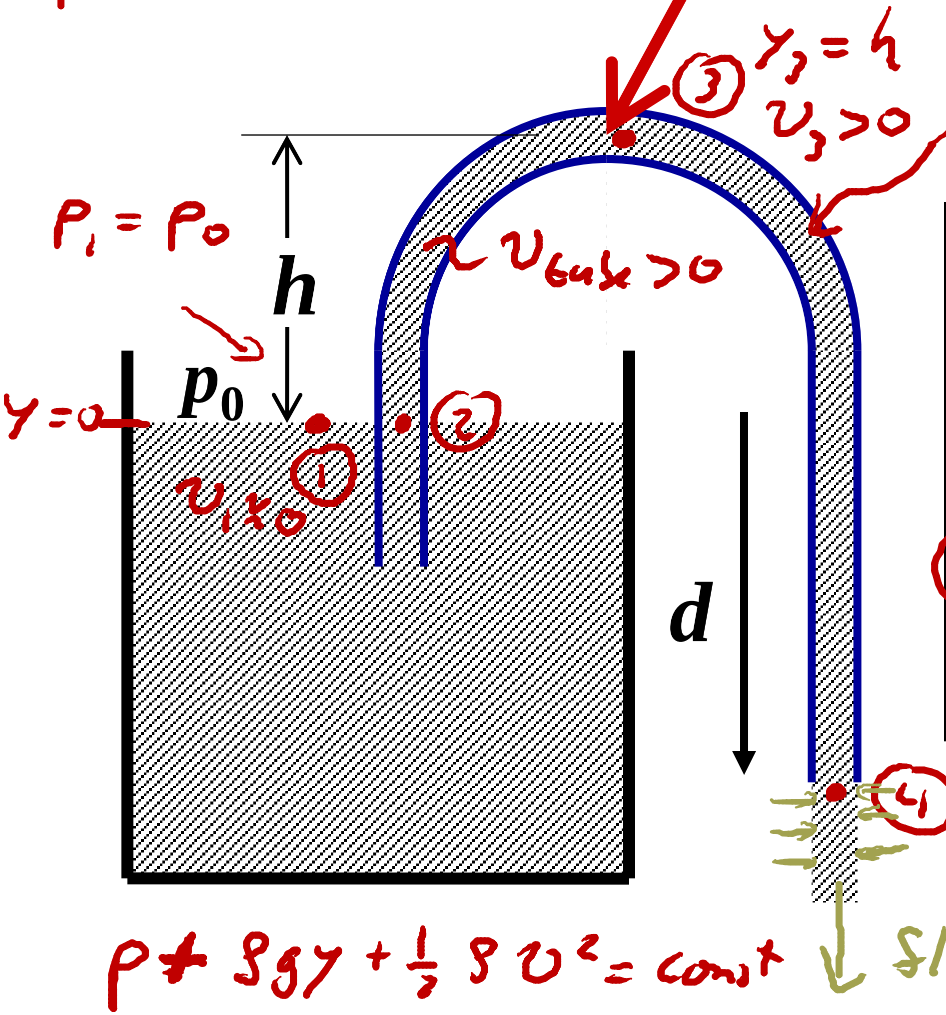


assume: $A_{tank} >> A_{tube}$

$\uparrow +y$

What is the pressure at the top of the siphon?

in tube: $R = vA = const$
$A = const$ $\Big\}$ $v = const$
$v_2 = v_3 = v_4$



A. p_0

B. $p_0 + \rho gh$

C. $p_0 - \rho gh$ } static case! (no flow)

[✓ D.] less than $p_0 - \rho gh$ } with flow

E. greater than $p_0 + \rho gh$

$P_4 = P_0$

outlet must be at P_0 !

$P + \rho g y + \frac{1}{2} \rho v^2 = const$

$\Rightarrow P_1 > P_2 > P_3$

$P_4 = P_1 = P_0$

→ at point ①:

$$p_1 = p_0 \;,\; y_1 = 0 \;,\; v_1 \approx 0 \quad (A_{tank} \gg A_{tube})$$

→ at point ②

$$\underbrace{p = p_2}_{p_2 < p_1} \;,\; y_2 = 0 \;,\; v_2 > 0$$

} pressure difference from 1 → 2 produces change in speed of flow ($\Delta y = 0$)

→ at point ③

$$\underbrace{p = p_3}_{p_3 < p_2} \;,\; y_3 = h \;,\; v_3 = v_2 > 0$$

} work done by $\Delta p_{2\to 3}$ to raise fluid by h while $\Delta K = 0$

→ at point ④

$$p_4 = p_0 \;,\; y = y_4 = -d \;,\; v_4 = v_3 = v_2$$

outlet must be at atm. pressure! (p_0)

⇒ use Bernoulli's equation for ① and ③

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_3 + \rho g y_3 + \frac{1}{2}\rho v_3^2$$

$$\Rightarrow P_0 + 0 + 0 = P_3 + \rho g h + \frac{1}{2}\rho v_3^2$$

$$\Rightarrow \underline{\underline{P_3}} = P_0 - \rho g h - \underbrace{\underline{\underline{\frac{1}{2}\rho v_3^2}}}_{>0}$$ ⇒ answer D!

- to find $v_3 = v_2 = v_4 = v_{tube}$: use points ① and ④

$$\Rightarrow P_0 + 0 + 0 = P_0 - \rho g d + \frac{1}{2}\rho v^2$$

$$\Rightarrow v^2 = 2gd \Rightarrow v = \sqrt{2gd}$$

⇒ as expected from 1-D motion for free fall from height d!