Recap: Static Fluids

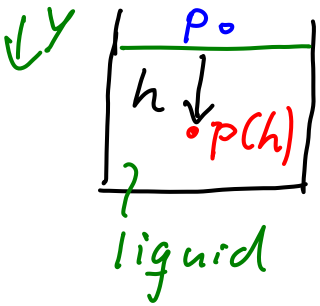
- In a Static liquid:

$$\boxed{p(h) = p_0 + \rho_L g h}$$

$\leftarrow$ depth, $+h$: down $\downarrow$ } static only!

p_0: pressure at surface

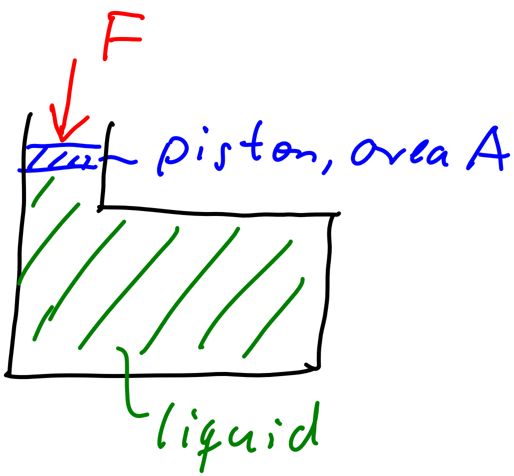
ρ_L: density of liquid



$\rightarrow$ in a static liquid, p depends only on depth

(same y-position $\Rightarrow$ same p)

$\rightarrow$ p at given h must support the weight/area of everything above it.

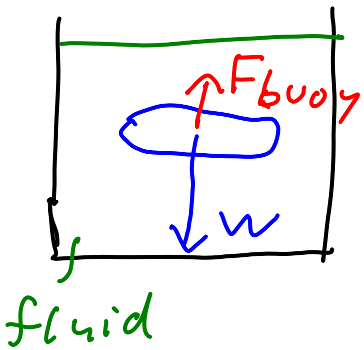


$\rightarrow$ Pascal's principle: $p_0 = \frac{F}{A}$

$\Rightarrow \Delta p = \frac{\Delta F}{A}$ } everywhere in liquid!

Recap: Static Fluids

- **Buoyant Force:**



$F_{buoy \text{ on object}} = \rho_f \, g \, V_{\text{fluid displaced}}$

= Weight of fluid displaced by object

= net force on object from fluid pressure on its surface

Note: F_{buoy} is a consequence of pressure variation with depth h in a fluid!

Today:

- More on Buoyancy
- Fish in unstable equilibrium
- An in-lecture question most physicist get wrong…
- Fluid flow

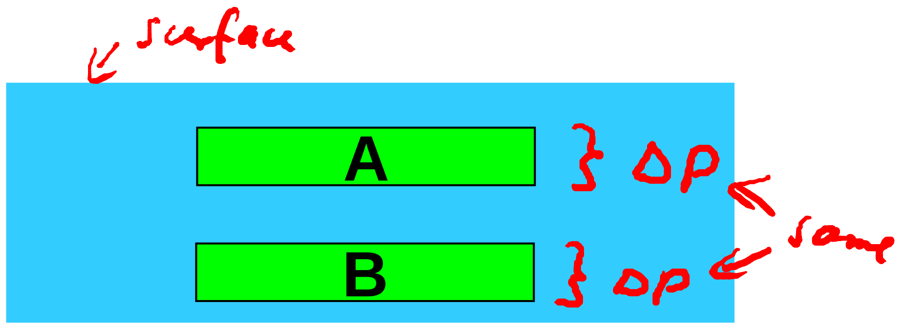
Two identical bricks are held under water. **Brick *A*** is just beneath the surface of the water, while **brick *B*** is at a greater depth.

The force needed to hold brick *B* in place is

A. larger

B. the same as

C. smaller



than the force required to hold brick *A* in place.

W_{brick} : same

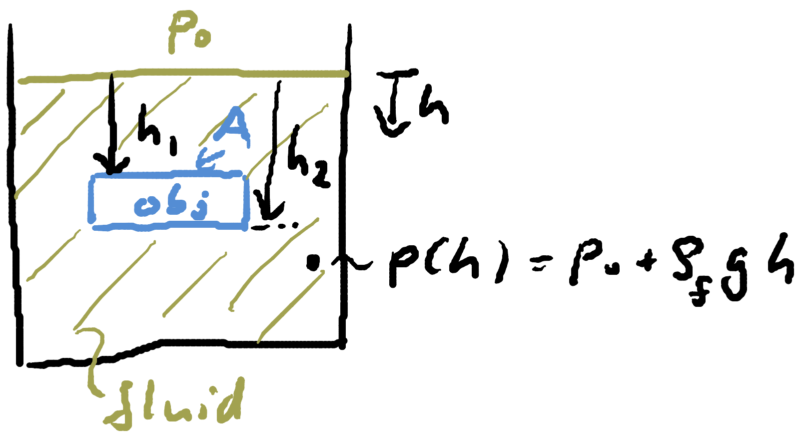
F_{buoy} : same

$F_{to\ hold\ in\ place} = W - F_{buoy}$ } same!

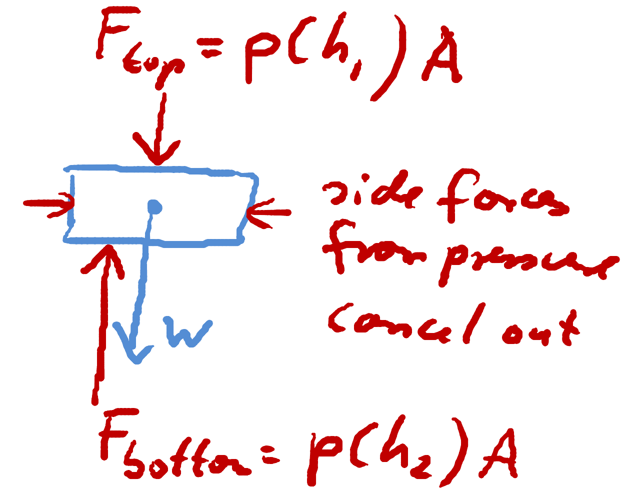
$F_{buoy} = |W_{fluid\ displaced}|$

$= \rho_f\, g\, V_{displ} \Leftarrow$ same!

Example:



FBD:



$F_{buoy} = F_{\text{net from pressure on surface of object}} = F_{bottom} - F_{top} = p(h_2)A - p(h_1)A$

$= \big(\underbrace{p(h_2) - p(h_1)}_{\Delta P}\big)A$

$= \rho_f\, g\, (\underbrace{h_2 - h_1}_{})A$

$V_{object} = V_{\text{fluid displaced by object}}$

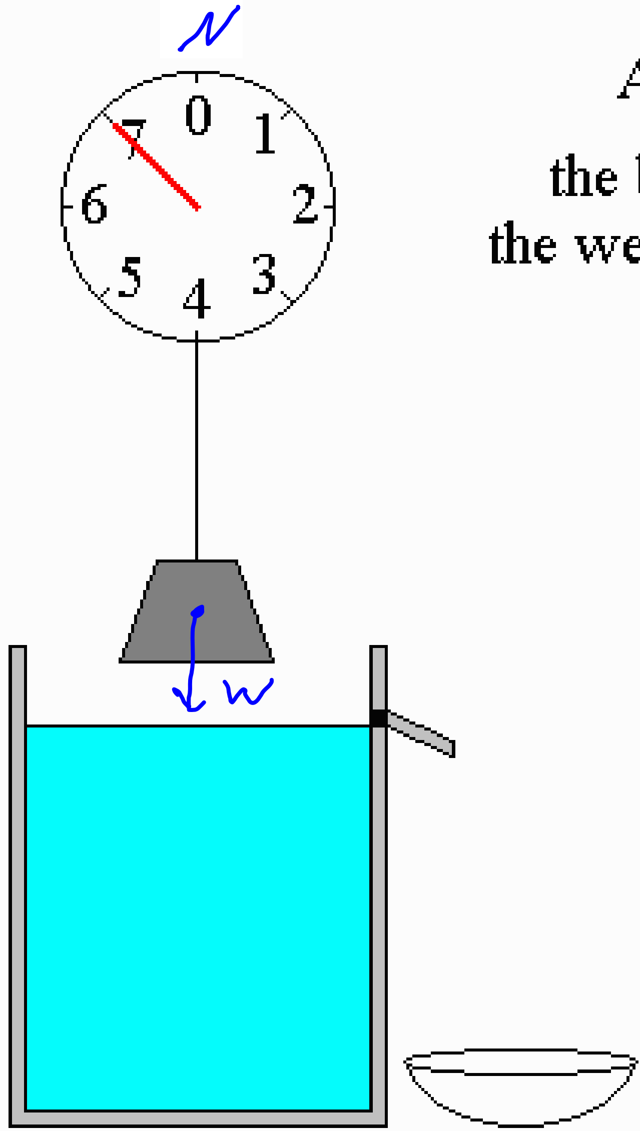
$\Rightarrow \underline{F_{buoy}} = \rho_{fluid}\, g\, V_{\text{fluid displ.}}$

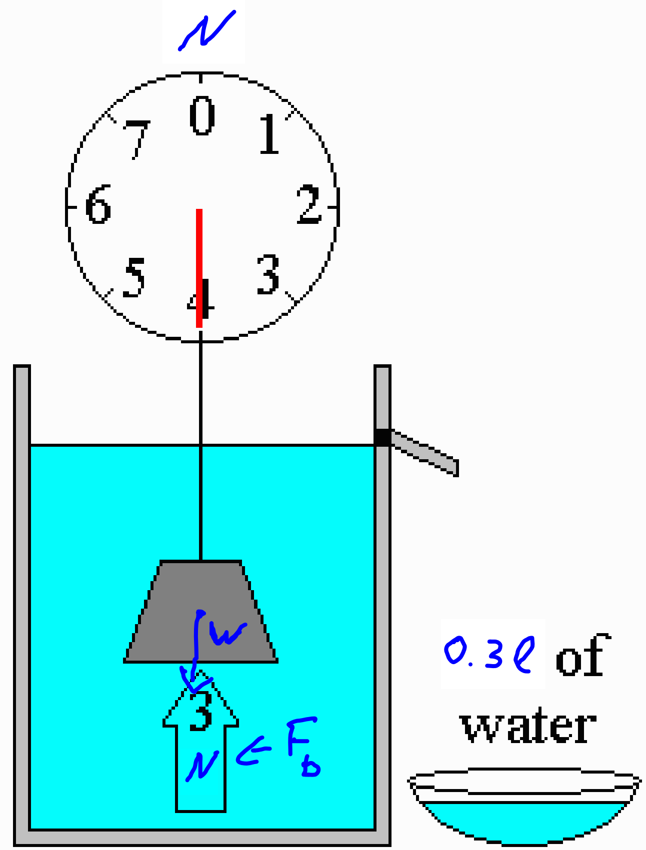
$= m_{\text{fluid displ.}}\, g$

$\underline{= |W_{\text{fluid displaced by object}}|}$

Archimedes' Principle

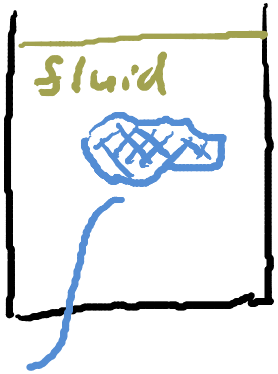
the buoyant force is equal to
the weight of the displaced water





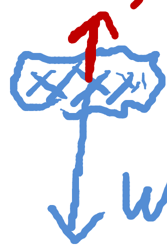
water displaced has $W = 3N$!

a) Submerged Object



FBD:

$F_{buoy} \neq |W_{obj}|$



$(\vec{a} \neq 0$ in general$)$

$V_{fluid\ displaced} = V_{object}$

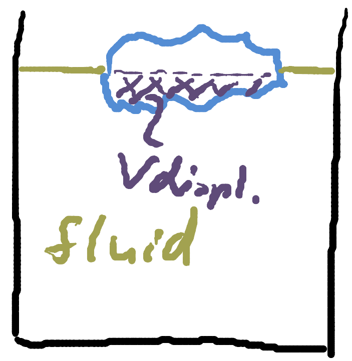
Apparent weight:

$$\underline{W_{app}} = |W_{obj}| - |F_{buoy}|$$

$$= |W_{obj}| - |W_{fluid\ displaced}|$$

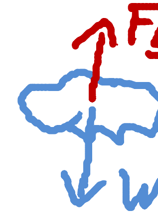
$$= (\rho_{obj} - \rho_{fluid})\, g\, V_{obj}$$

b) Floating Object:



object floats:

$\vec{a} = 0 \Rightarrow \Sigma \vec{F} = 0$



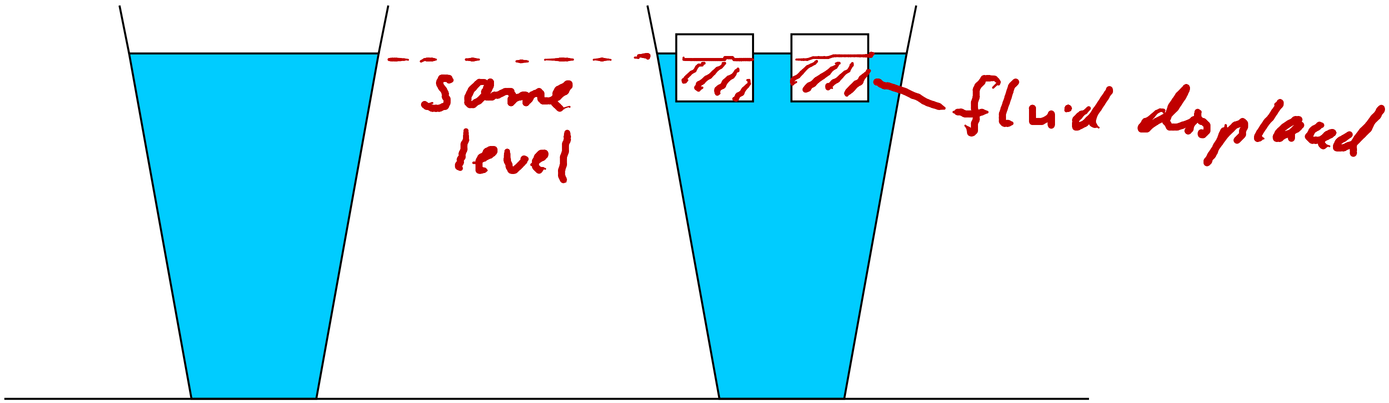
note: $V_{fluid\ displaced} \leq V_{object}$

$$F_{buoy} \stackrel{!}{=} |W_{obj}|$$

$$\Rightarrow \rho_{fluid}\, g\, V_{fluid\ displaced} = \rho_{obj} \cdot g\, V_{obj}$$

$$\Rightarrow \boxed{\frac{\rho_{fluid}}{\rho_{obj.}} = \frac{V_{obj}}{V_{fluid\ displ.}}} \geq 1$$

Two identical glasses are filled to the same level with water. One of the two glasses has ice cubes floating in it.



Which **weighs** more?

Ice cubes are floating!

$$\Rightarrow |W_{\text{ice cubes}}| = |F_{\text{buoy}}|$$

$$= |W_{\text{fluid displaced}}|$$

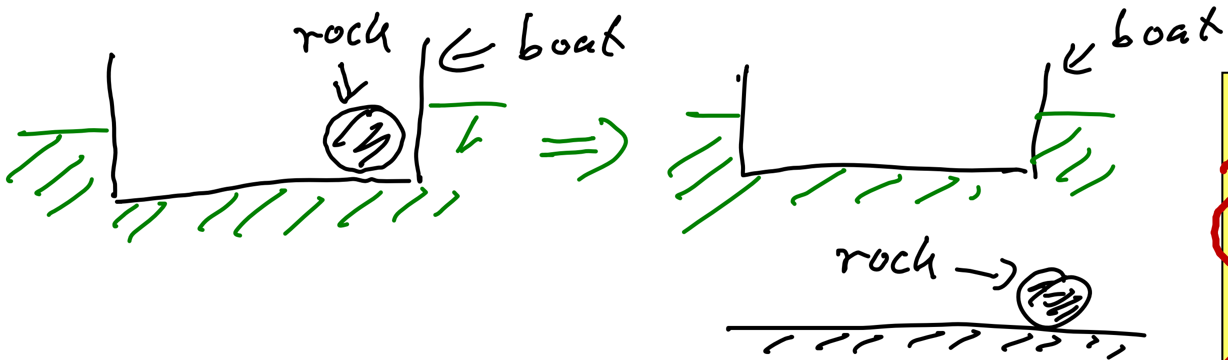
A. the glass without ice cubes

B. the glass with ice cubes

C. both weigh the same

A boat carrying a **boulder** is floating on a lake. The boulder is **thrown overboard and sinks**.

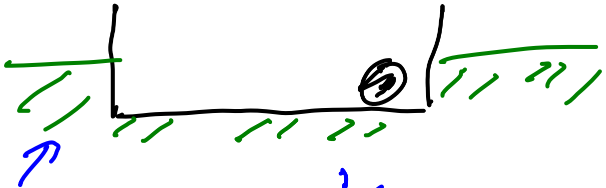
The **water level in the lake** (with respect to the shore):



- ~~**A.**~~ **rises**
- **B. drops**
- ~~**C.**~~ **stays the same**
- **D. not enough info**

→ water level is determined by water displaced by rock and boat!

before



$$F_{buoy} = \rho_{water}\, g\, V_{displaced,\ total}$$

$$= |W_{total}| = |W_{boat}| + |W_{rock}|$$

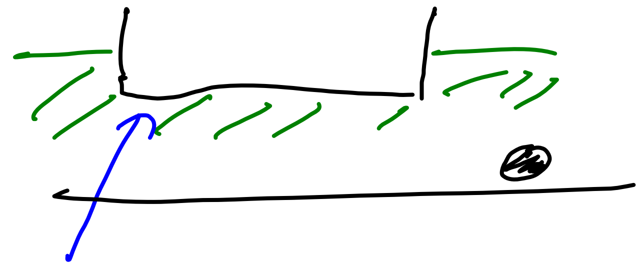
$$= m_{boat}\, g + m_{rock} \cdot g$$

$$\Rightarrow V_{displaced,\ total} = \frac{m_{boat}}{\rho_{water}} + \frac{m_{rock}}{\rho_{water}}$$

$$= \frac{m_{boat}}{\rho_{water}} + \underbrace{\frac{\rho_{rock}}{\rho_{water}}}_{>>1} V_{rock}$$

⇒ boat "helps" rock to displace enough water to float rock!

after:



$$F_{buoy,\ boat} = \rho_{water}\, g\, V_{\text{displaced to float boat}}$$

$$= |W_{boat}| = m_{boat}\, g$$

$$\Rightarrow V_{\text{displ. to float boat}} = \frac{m_{boat}}{\rho_{water}}$$

$$V_{\text{displ by rock}} = V_{rock}$$

$$\Rightarrow V_{\text{displ., total}} = \frac{m_{boat}}{\rho_{water}} + 1 \cdot V_{rock}$$

⇒ less than before

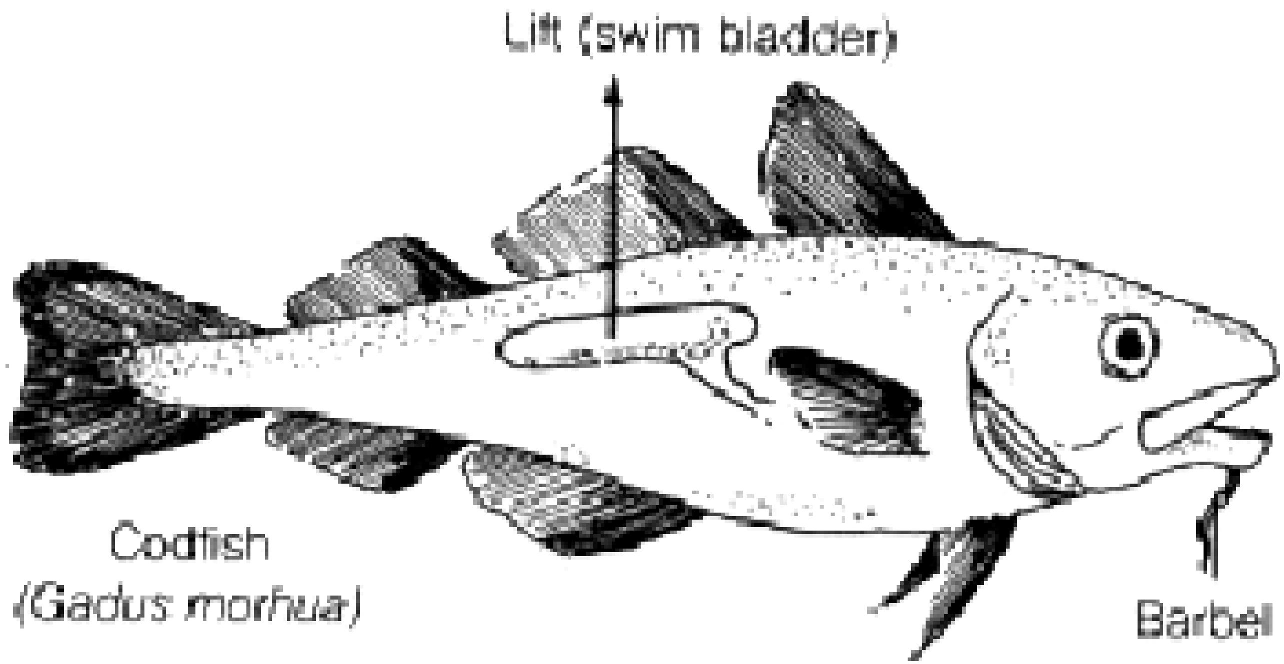
⇒ level drops!

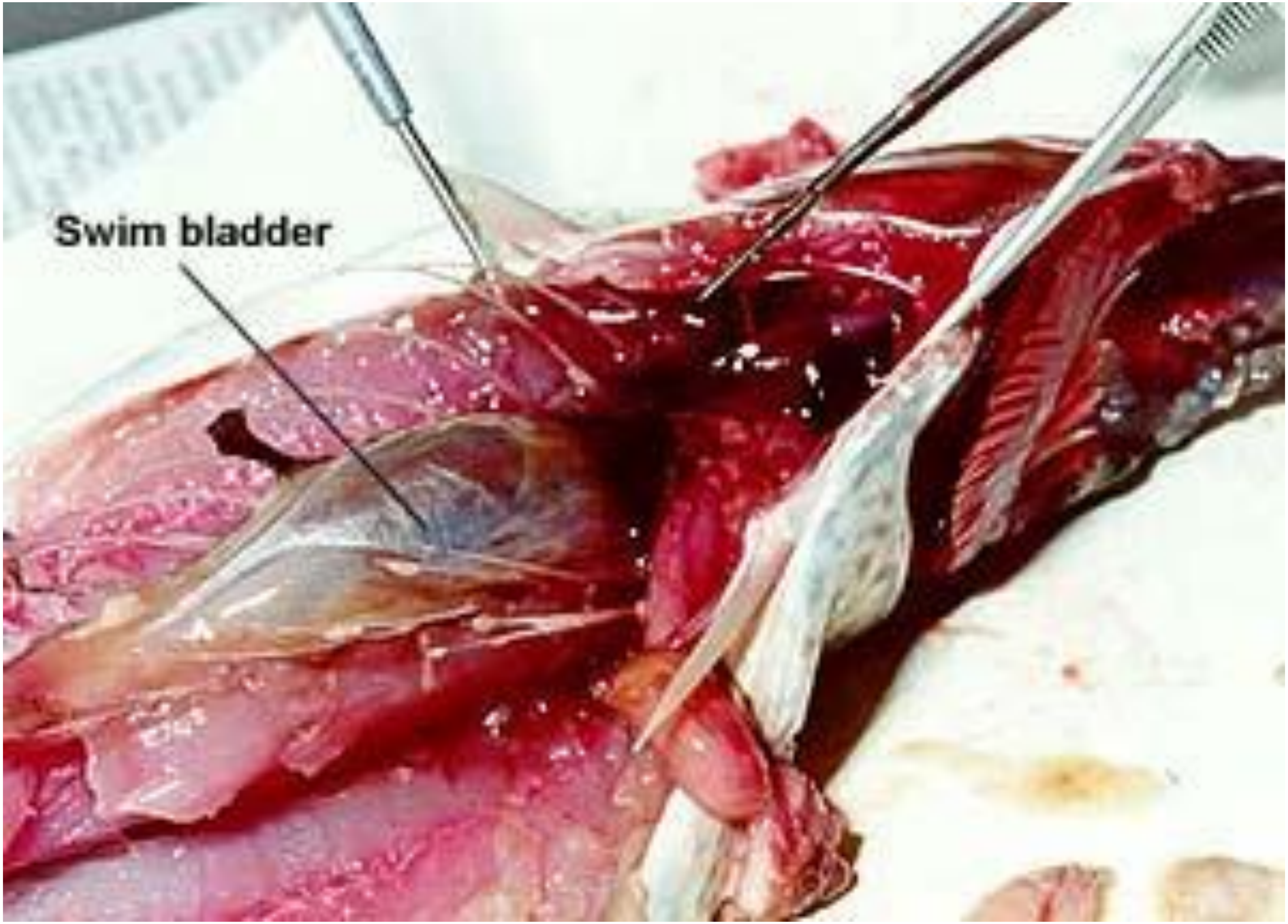
Buoyancy and Fish

Fish adjust their density ρ_{fish} so that $\rho_{fish}=\rho_{water}$ and **$F_B=W_{fish}$ ("neutrally buoyant"). How?**

Teleost Fish use a Swim Bladder:

- flexible, membrane-enclosed bag of gas
- fish secretes gas into bag, changing V_{fish} and ρ_{fish}.




Swim bladder

What happens if an initially neutrally buoyant fish goes a little deeper (i.e., h↑) ?

$p(h)\uparrow \quad \Rightarrow \quad V_{bladder}\downarrow \quad \Rightarrow \quad V_{fish}\downarrow \quad \Rightarrow \quad \rho_{fish}\uparrow$

$\rho_{fish}\uparrow \quad \Rightarrow \quad F_B < W \quad \Rightarrow$ **fish sinks!**

If it goes a little higher, it rises.

$\therefore$ Equilibrium is **unstable**, and fish must constantly adjust gas in bladder!

Cuttlefish use a <u>Cuttlebone</u>:

Cuttlefish use a Cuttlebone:

- **Rigid**, porous bone filled with gas and liquid

  $\Rightarrow$ Does not compress

- Fish secretes gas into bone, changing ρ_{fish}, but V_{fish} stays constant, regardless of h and p(h).

$\therefore$ Can maintain neutral buoyancy when ascending or descending without adjusting gas in cuttlebone (**stable**).

http://video.google.com/videoplay?docid=5053807934424522294

→ So far: Static fluids

Now → **Ideal Fluids in Motion:**

- Ideal fluid:

① No fluid friction (viscosity = 0)

② Fluid is incompressible

⇒ ρ_{fluid} = const in the flow

– true for liquids

– mostly true for flowing gases because Δp's usually small ⇒ $\Delta \rho$'s small

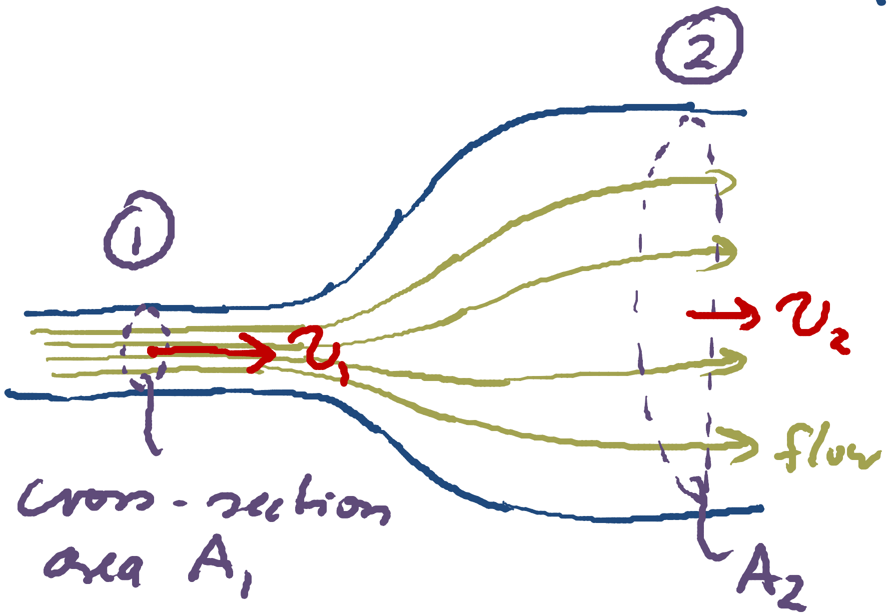
③ Steady and non-turbulent (laminar) flow

⇒ V_{fluid} at any fixed point does not change with time

• Continuity:

tube with flow:

– true for any steady fluid flow



In time Δt, the volume ΔV_1 of fluid entering at ① must equal the volume ΔV_2 of fluid leaving at ②

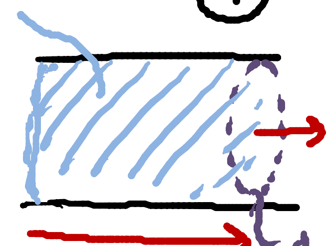
$\Rightarrow \Delta V_1 = \Delta V_2$

$\Rightarrow \dfrac{\Delta V_1}{\Delta t} = \dfrac{\Delta V_2}{\Delta t} = R$

v_1, v_2: speed of flow at ① and ②

$\Rightarrow$ in Δt:

$$\Delta V_1 = \Delta V_2$$



$$\Rightarrow A_1 \underbrace{v_1 \Delta t}_{\Delta x_1} = A_2 \underbrace{v_2 \Delta t}_{\Delta x_2}$$

$$\Rightarrow \boxed{A_1 v_1 = A_2 v_2 = \text{const in pipe}} \quad \Big\} \text{ equation of continuity}$$

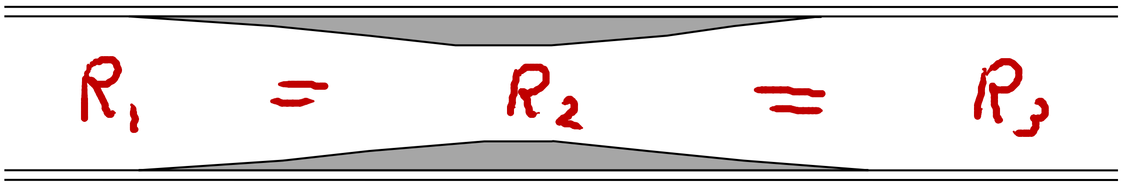
$\Rightarrow$ define volume flow rate R:

$$\boxed{R = \frac{\Delta(\text{volume})}{\Delta t} = A_1 v_1 = A_2 v_2 = \text{const in pipe}}$$

$$[R] = \frac{m^3}{s}$$

Blood flows through an artery that is partially blocked by deposits along the artery wall.

Through **which part** of the artery is the **volume flow rate *R* the largest**?



$R_1 = R_2 = R_3$ $\longrightarrow$ Slow

$R = \frac{\Delta V}{\Delta t} = Av$

$=$ const in pipe

A. **the narrow part**

B. **the wide parts**

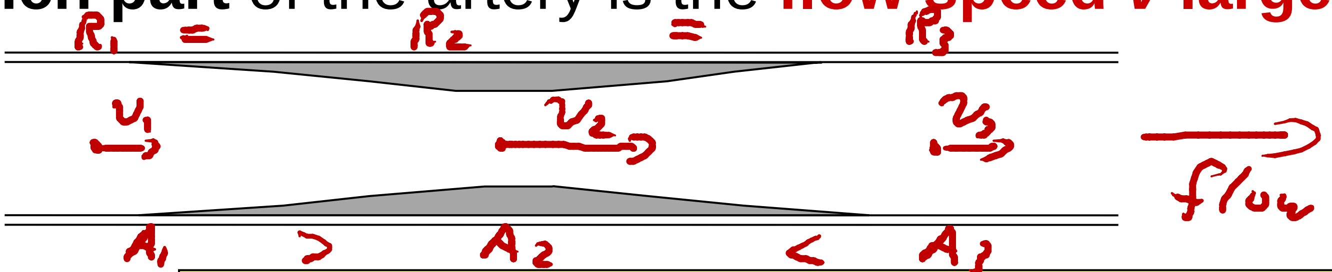
C. **the part upstream of the blockage**

D. **the part downstream of the blockage**

E. **same volume flow rate everywhere**

Blood flows through an artery that is partially blocked by deposits along the artery wall.

Through **which part** of the artery is the **flow speed *v* largest**?



$R = R_1 = R_2 = R_3$

$= v_1 A_1 = v_2 A_2$

$= v_3 A_3$

$A \downarrow \Rightarrow v \uparrow$

A. the narrow part

B. the wide parts

C. the part upstream of the blockage

D. the part downstream of the blockage

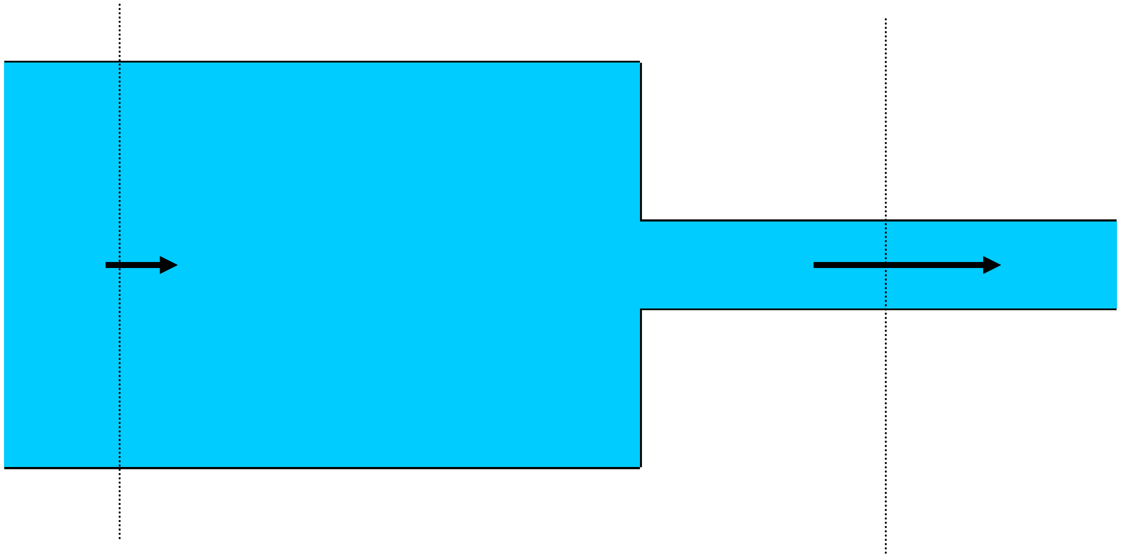
E. same volume flow rate everywhere

Continuity and Gorges

Volume flow rate R = vA=const!

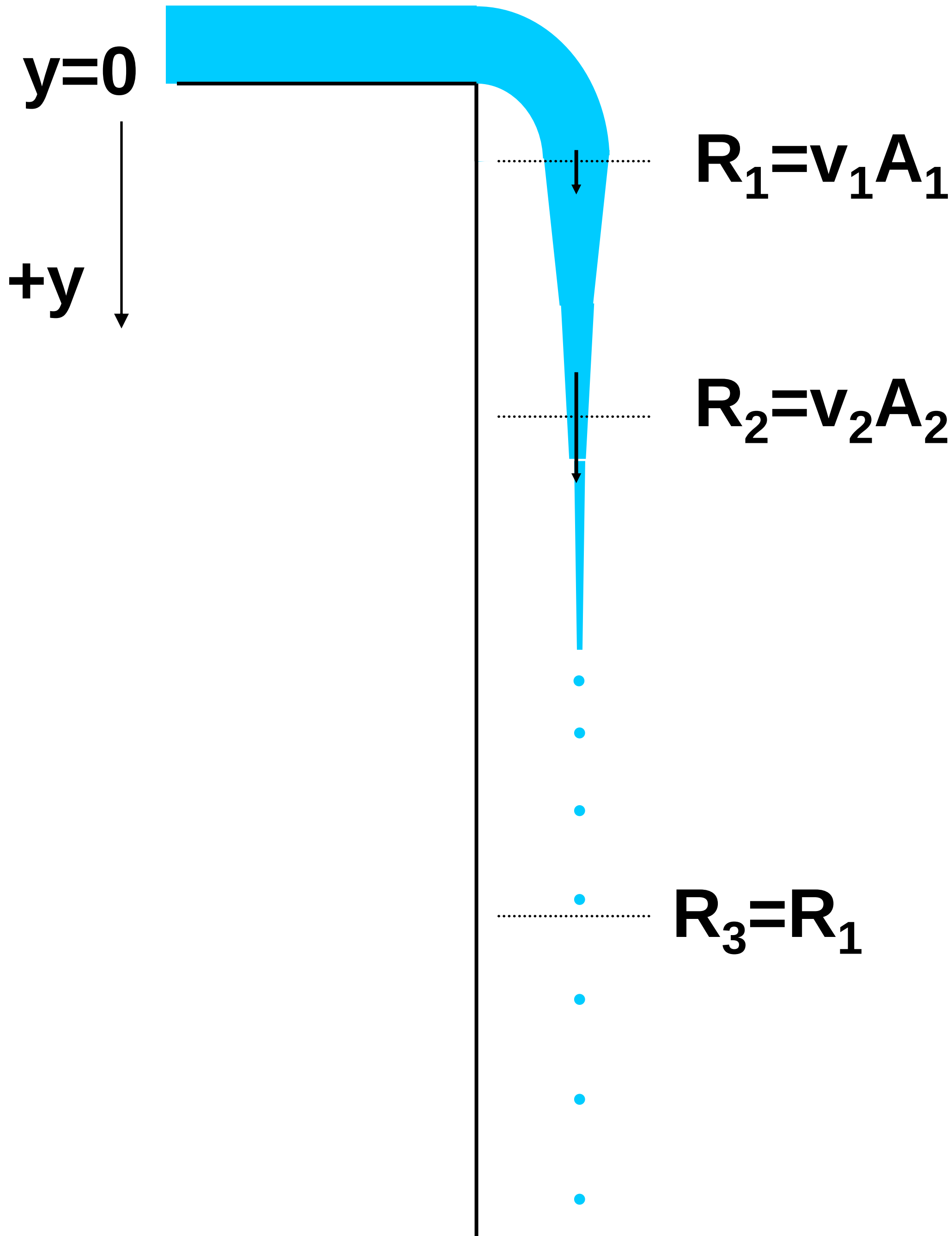
$R_1=v_1A_1$

$R_2=v_2A_2$



$R=R_1=R_2=\text{constant}$

Upper Enfield Glen:



$$R_1=R_2=R_3$$

$$v\sim(2gy)^{1/2}$$

Taughannock Falls