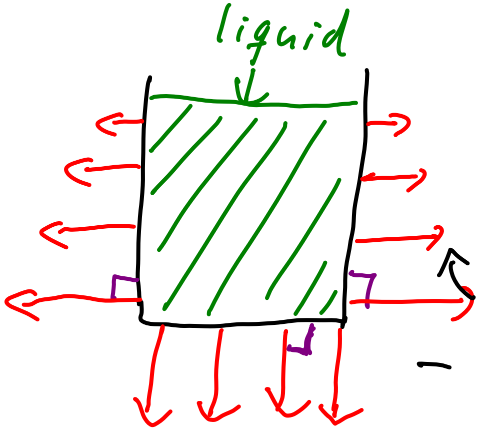
Recap: Fluids and Pressure

- **Pressure:**



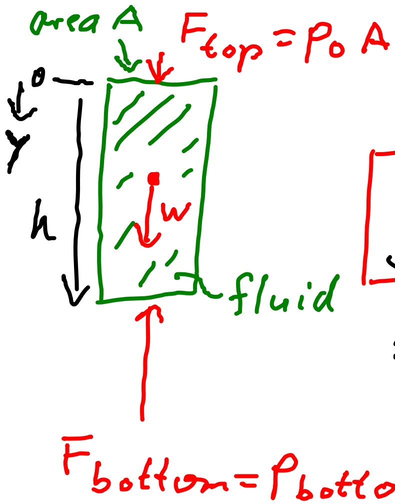
- p = pressure = $\frac{F}{A}$ ← uniform force on surface; A ← of a flat surface. $[p] = Pa = N/m^2$

Pressure is a scalar (no direction), but produces a force ⊥ to a surface!

force on surface from liquid

- measuring pressure: $p_{gauge} = p - p_{ref}$

- **Pressure variation with depth in a fluid:**



$$\boxed{\underbrace{p_{bottom}}_{=p(h)} = p_0 + \frac{W}{A}}$$

→ in a static fluid, p depends only on depth (same y-position ⇒ same p)

→ p at given h must support the weight/area of everything above it.

Today:

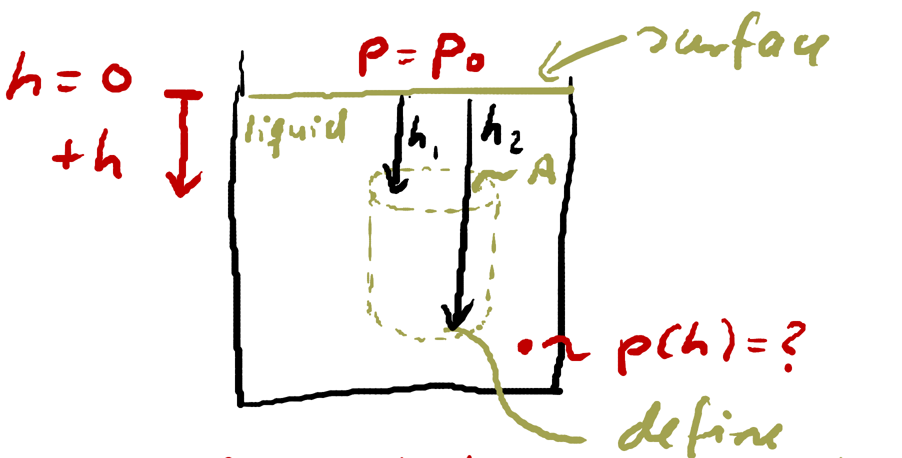
- Pressure variation with depth
- Pascal's principle
- Atmospheric pressure
- The giraffe
- Spiders
- Buoyancy

→ Pressure Variation with Depth in a Liquid:

⇒ for a static liquid (i.e. not for gas)

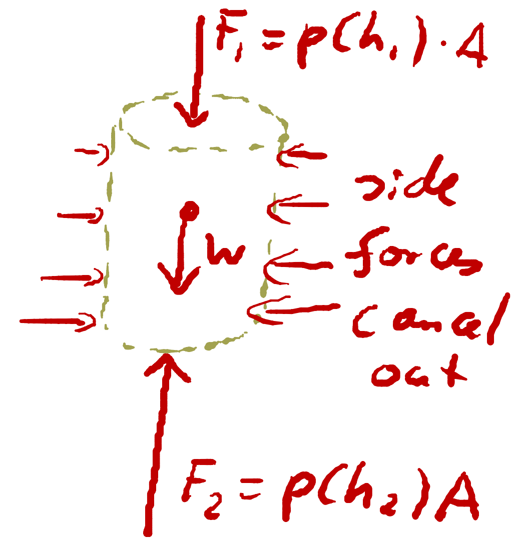
ρ_{Liquid} ~ indep. of pressure

$\Rightarrow \underline{\rho_L = \text{const}!}$



h = depth below surface

⇒ FBD of liquid cylinder:



$$N\text{II}: \sum F_y = 0 \Rightarrow |F_2| = |F_1| + |W|$$

$$W_{\text{cyl. of liquid}} = mg = \rho_L V g = \rho_L A(h_2 - h_1) g$$

$$\Rightarrow p(h_2)\underline{A} = p(h_1)\underline{A} + \rho_L \underline{A}(h_2 - h_1) g$$

$$\Rightarrow \boxed{p(h_2) = p(h_1) + \rho_L g (h_2 - h_1) = p(h_1) + \rho_L g \Delta h}$$

$\Rightarrow$ for $h_1 = 0$: $p(h_1 = 0) = p_0$, call $h_2 = h$

$$\Rightarrow \boxed{p(h) = p_0 + \rho_L g h}$$

h ← depth **below** surface

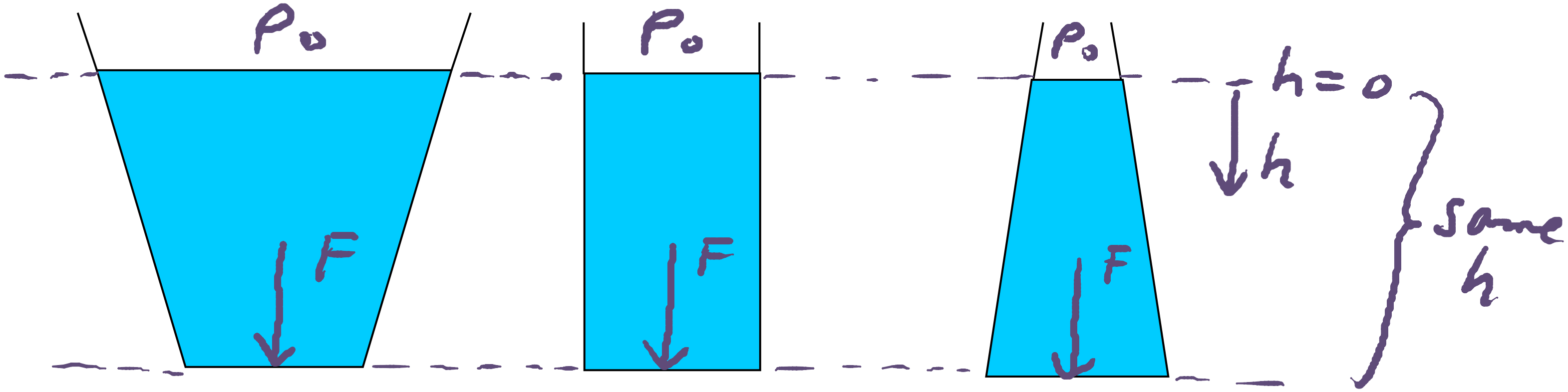
p_0: pressure at surface

ρ_L: density of liquid

$\Rightarrow$ only depends on depth h!
i.e. same y-position in a container $\Rightarrow$ same pressure

} for static liquids only! (no flow)

Three water-filled containers are shown below.



(A) **(B)** **(C)**

For which container is the **pressure** of the water **on the base** of the container **largest**?

same depth, same p_0
$\Rightarrow$ same pressure at base

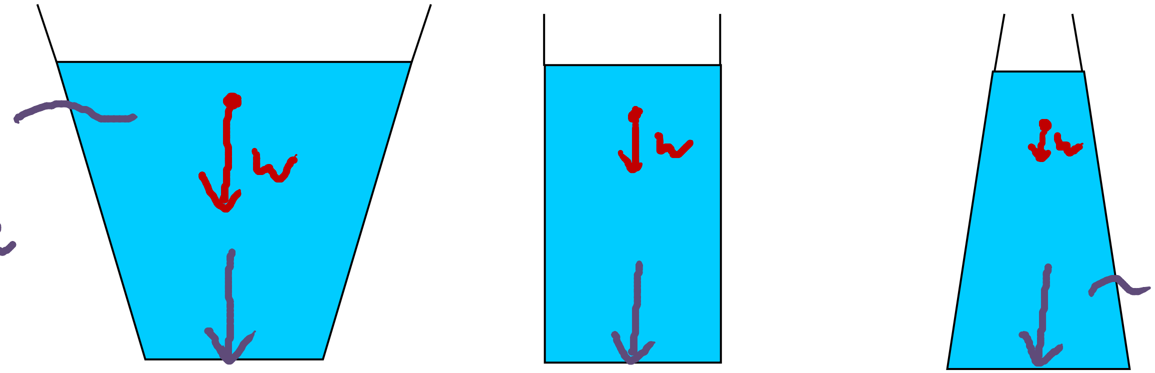
$p(h) = \underline{\underline{p_0}} + \rho g \underline{\underline{h}}$

$\Rightarrow$ same force from pressure on base (same p and A)

A. **(A)**
B. **(B)**
C. **(C)**
D. **All pressures are the same**
E. **Insufficient information**

Three water-filled containers are shown below.

more water ⇒ large |w|

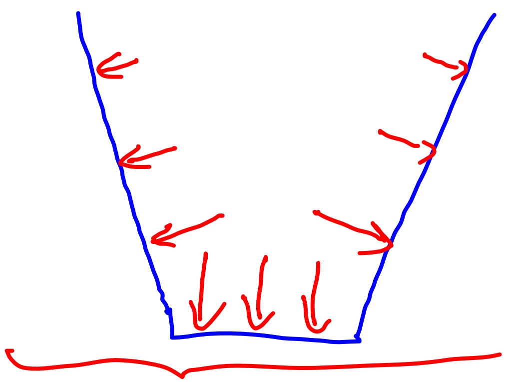


same force from water pressure on base surface

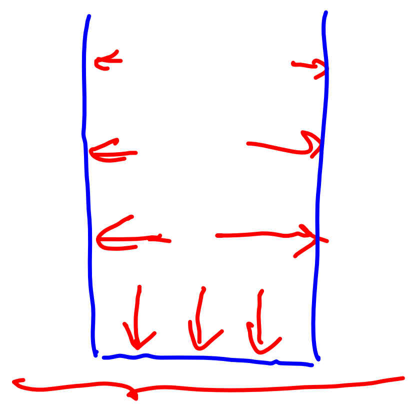
For which container is the **weight of the container + water the largest**?

A. **(A)**
B. **(B)**
C. **(C)**
D. **All weights are the same**
E. **Insufficient information**

- bottom surface: same area, same $p \Rightarrow$ same force on base
- But: Net force from water on container $= |W|$

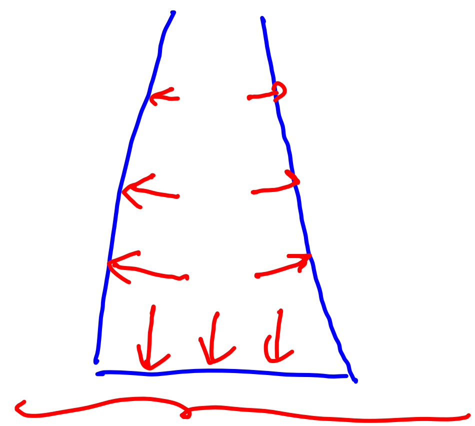


$|W| = |\text{net force on container}| > |F_{\text{on bottom}}|$



side wall forces cancel out

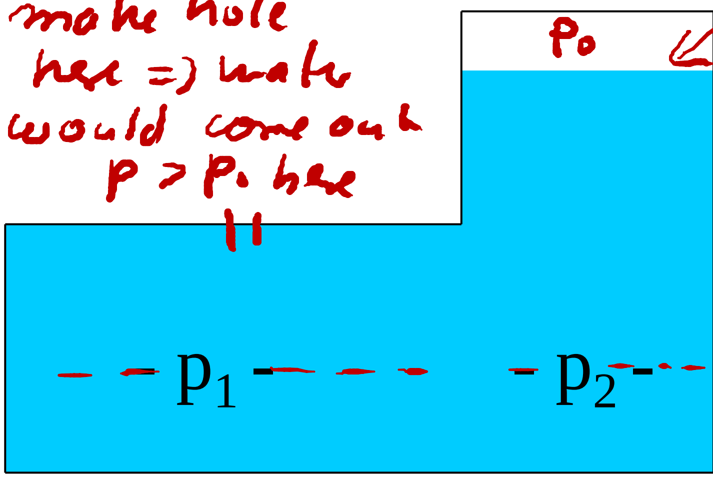
$|W| = |\text{net force on the container}| = |F_{\text{on bottom}}|$



$|W| = |\text{net force on the container}| < |F_{\text{on bottom}}|$

Consider the water-filled container shown.

make hole here ⇒ water would come out
$p > p_0$ here



surface

$h = 0$

h

$p = p_0 + \rho g h$

How do the pressures p_1 and p_2 (at equal height above the bottom of the container) compare?

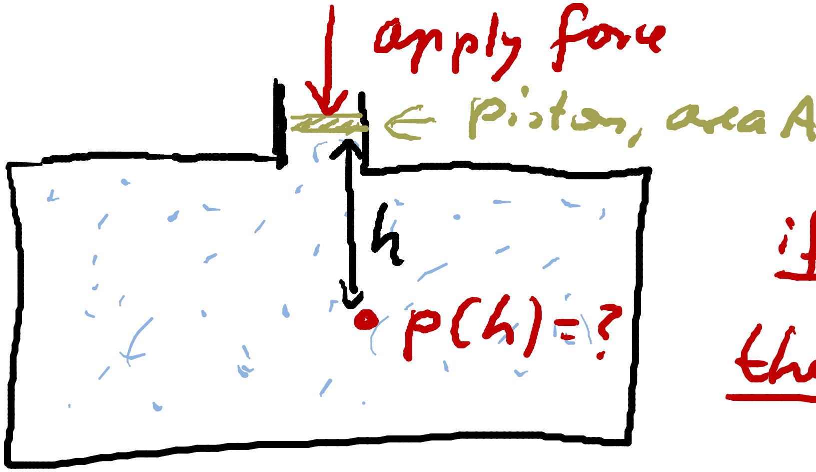
⇐ same y-position ⇒ same pressure if <u>no</u> flow!

A. $p_1 < p_2$

B. $p_1 = p_2$

C. $p_1 > p_2$

→ Pascal's Principle



container with incompressible liquid

$$\Rightarrow p(h) = \frac{F}{A} + \rho_L g h$$

force on piston (F)

area of piston (A)

if: $F \to F + \Delta F$

then: $p(h) \to p(h) + \dfrac{\Delta F}{A}$

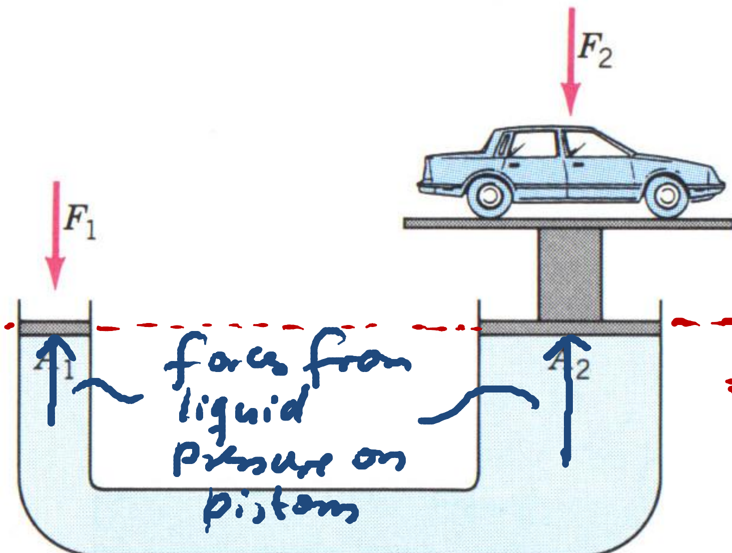
$\Rightarrow \Delta p = \dfrac{\Delta F}{A}$ } at every depth/point in liquid

→ A force produces Δp in an enclosed incompressible liquid, that is transmitted to every portion of the liquid (and to the walls of its container)

The hydraulic lever:

Consider a hydraulic lift for an automobile. The areas of the pistons are A_1 and A_2.

If the lift is stationary, what must be the **ratio of the forces** exerted on the pistons?



same magnit.

force from liquid pressure on piston

same y-position

=) same pressure

F_2/F_1 = ?

A. 1

B. A_2/A_1

C. A_1/A_2

p(h) is equal at given h if no flow: $P_1 = P_2$

$$\Rightarrow \frac{|F_1|}{A_1} = \frac{|F_2|}{A_2} \Rightarrow \left|\frac{F_2}{F_1}\right| = \frac{A_2}{A_1} > 1 \text{ here}$$

Atmospheric Pressure at the Earth's Surface:

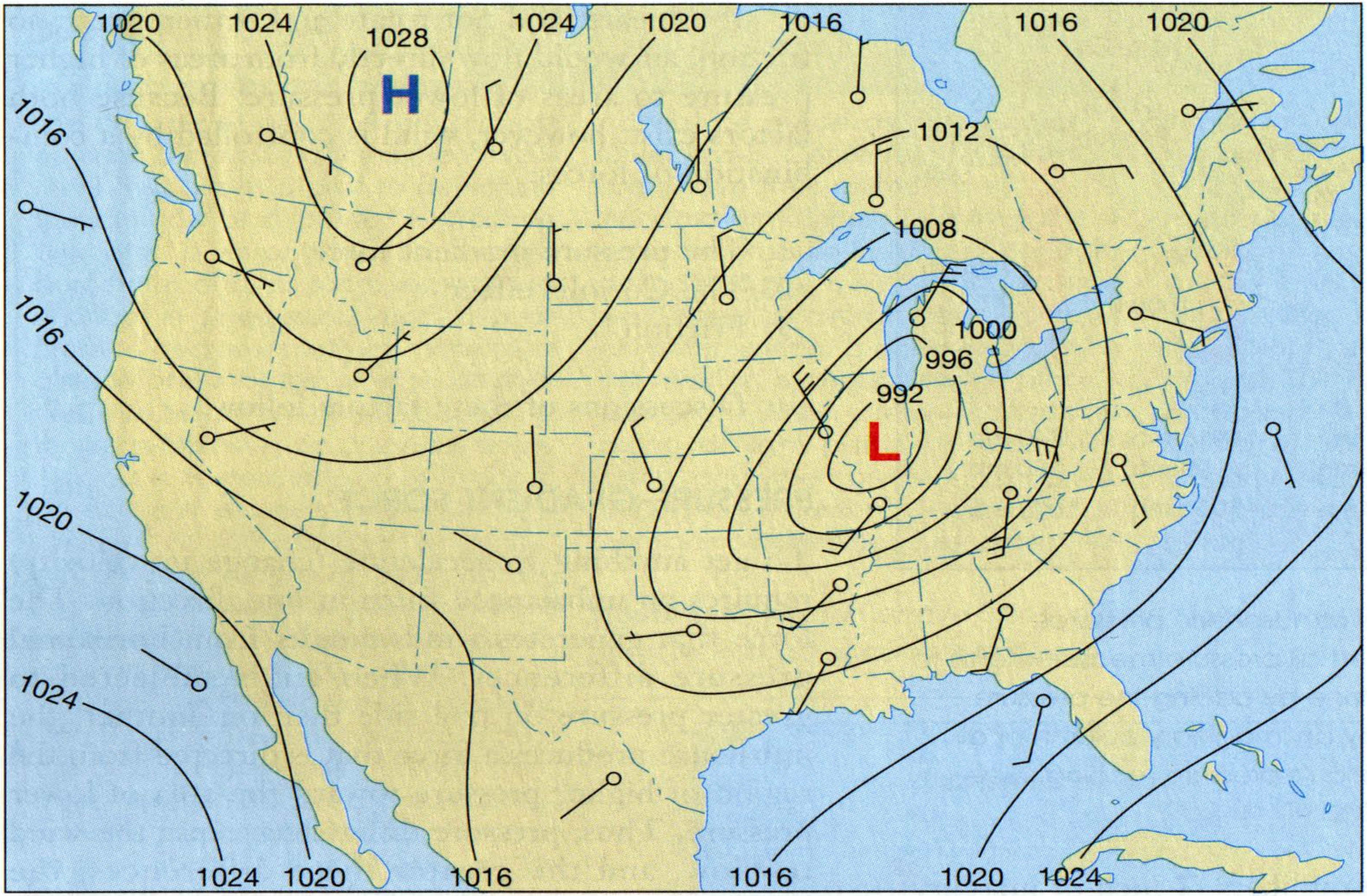
Lines indicate "**Isobars**":

-points of equal pressure (normalized to sea level)

Pressures indicated in **millibars**

1 bar =100 kPa

Average atmospheric pressure at sea level = 101.3 kPa ~ 1 bar


1020
1024
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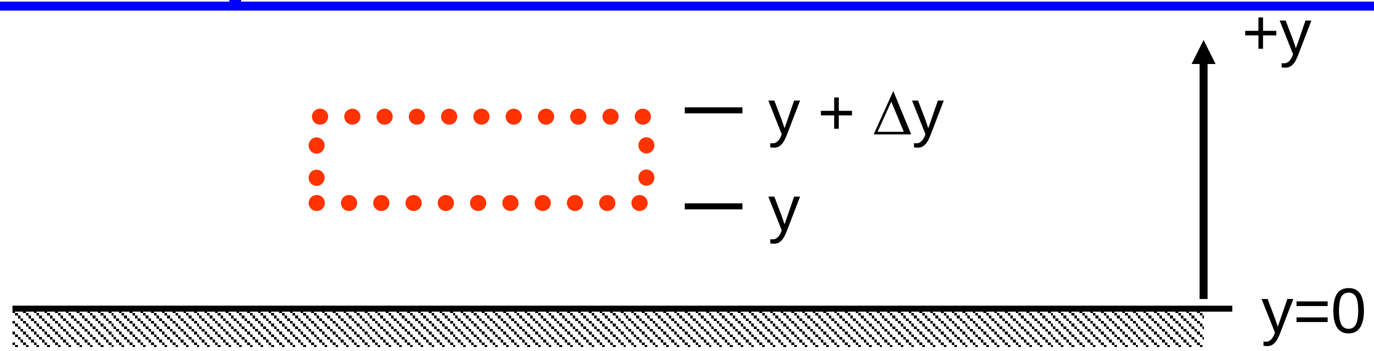
Atmospheric Pressure versus Altitude



$$\Delta p = p(y + \Delta y) - p(y)$$

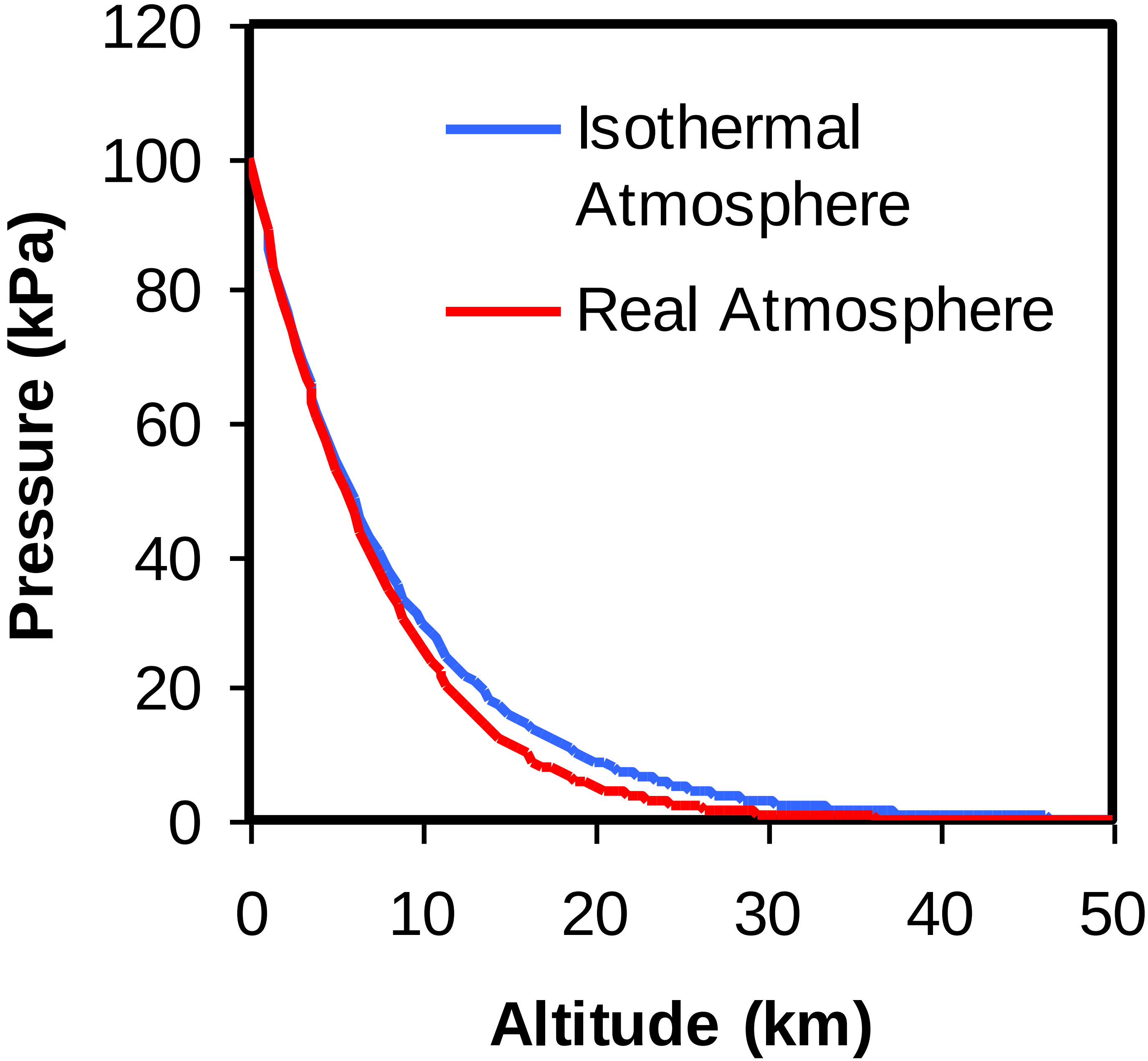
$$= -\rho\, g\, \Delta y$$

$\mathbf{\Delta y \rightarrow dy}$: $dp = -\rho\, g\, dy$ or $\mathbf{dp/dy = -\,\rho\, g}$

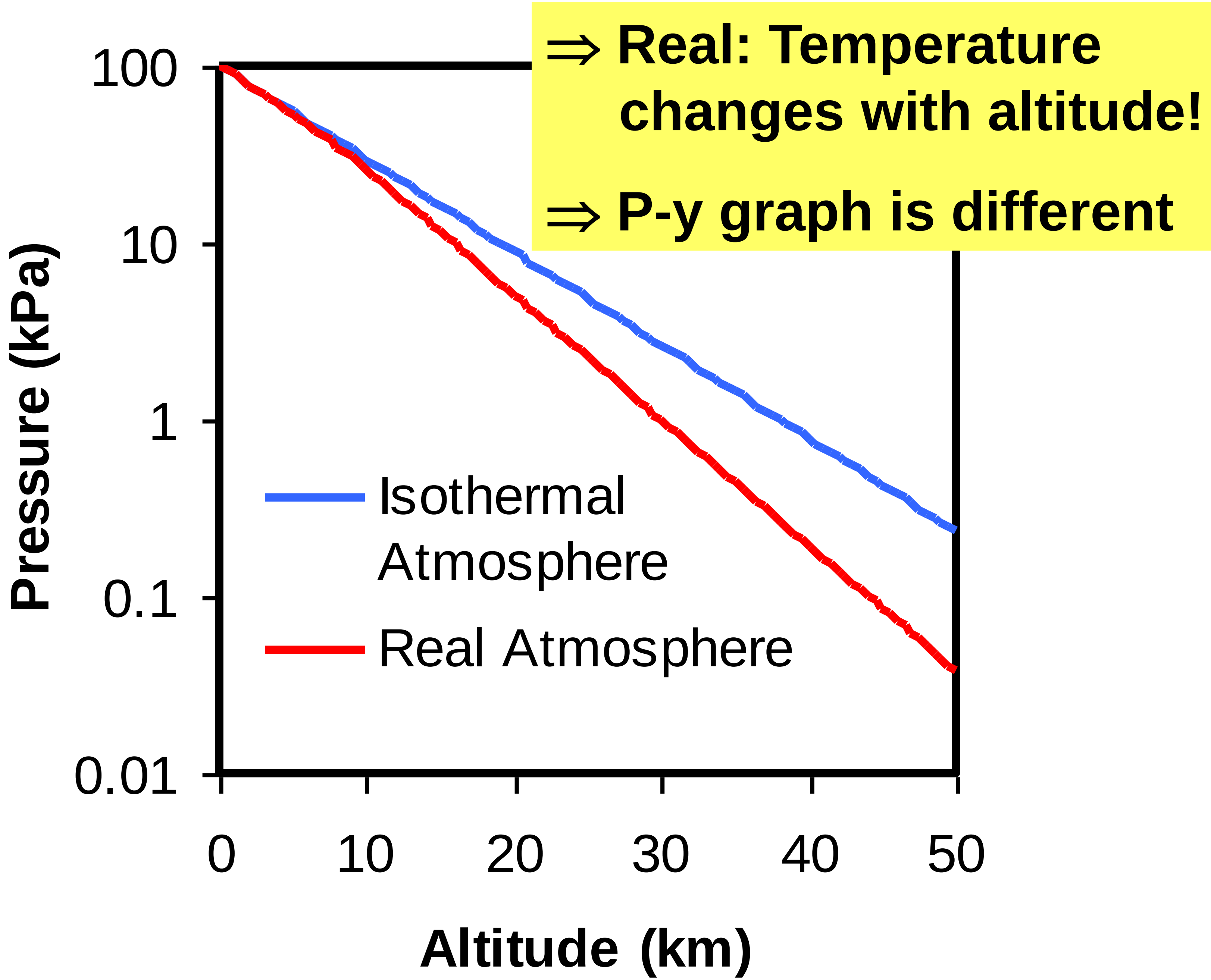
Ideal gas: $p \propto \rho\, T \;\Rightarrow\; \rho \propto p/T$

If $T(y)$= constant then $dp/dy \propto -\,p$

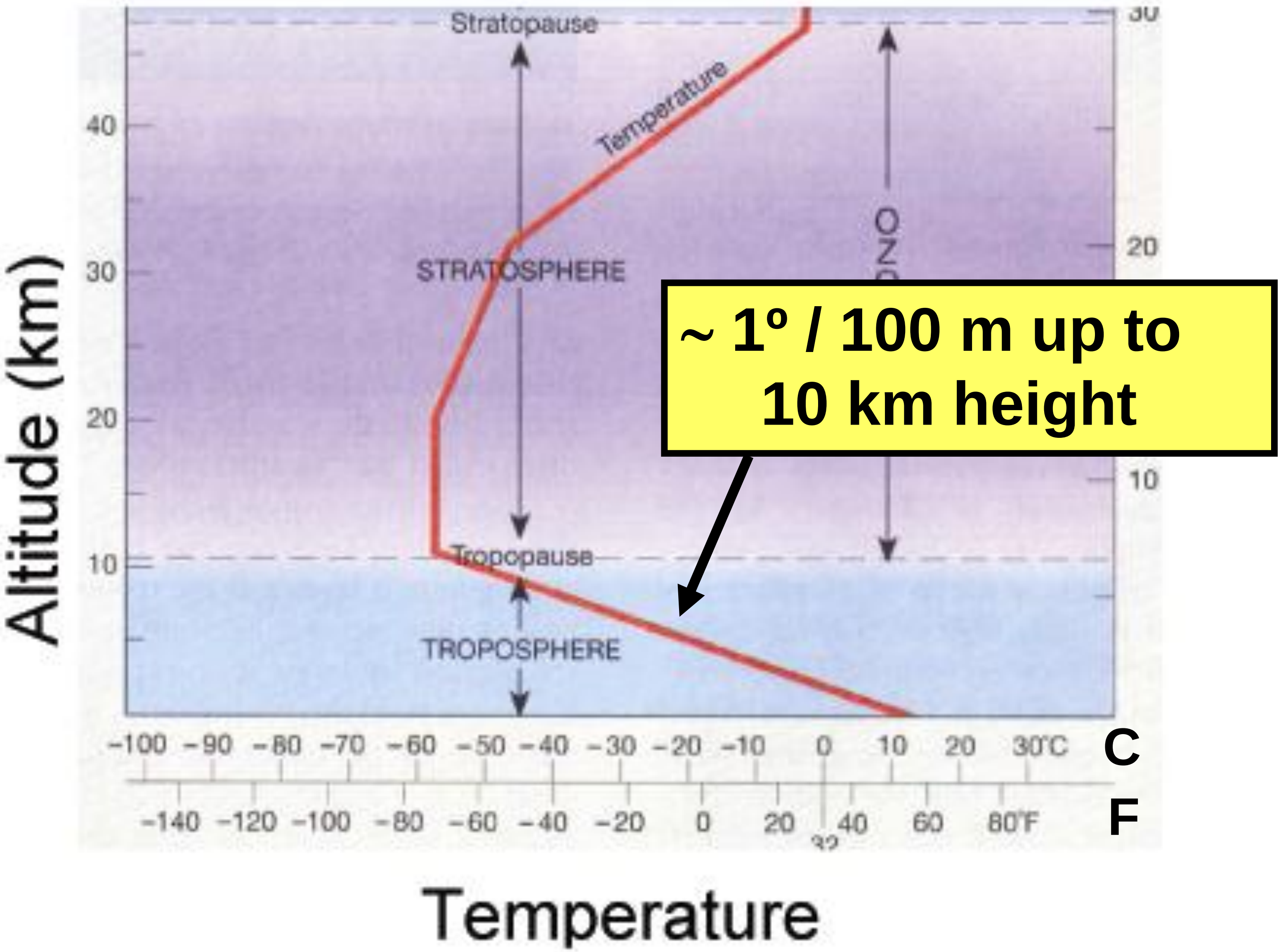
$\Rightarrow \boxed{p(y)=p_0 \exp(-y/y_s)}$, y_s= "scale height"



Pressure (kPa)
120
100
80
60
40
20
0
Isothermal
Atmosphere
Real Atmosphere
0
10
20
30
40
50
Altitude (km)


⇒ Real: Temperature changes with altitude!
⇒ P-y graph is different
100
10
1
0.1
0.01
Pressure (kPa)
Isothermal Atmosphere
Real Atmosphere
0
10
20
30
40
50
Altitude (km)

Temperature versus Altitude:



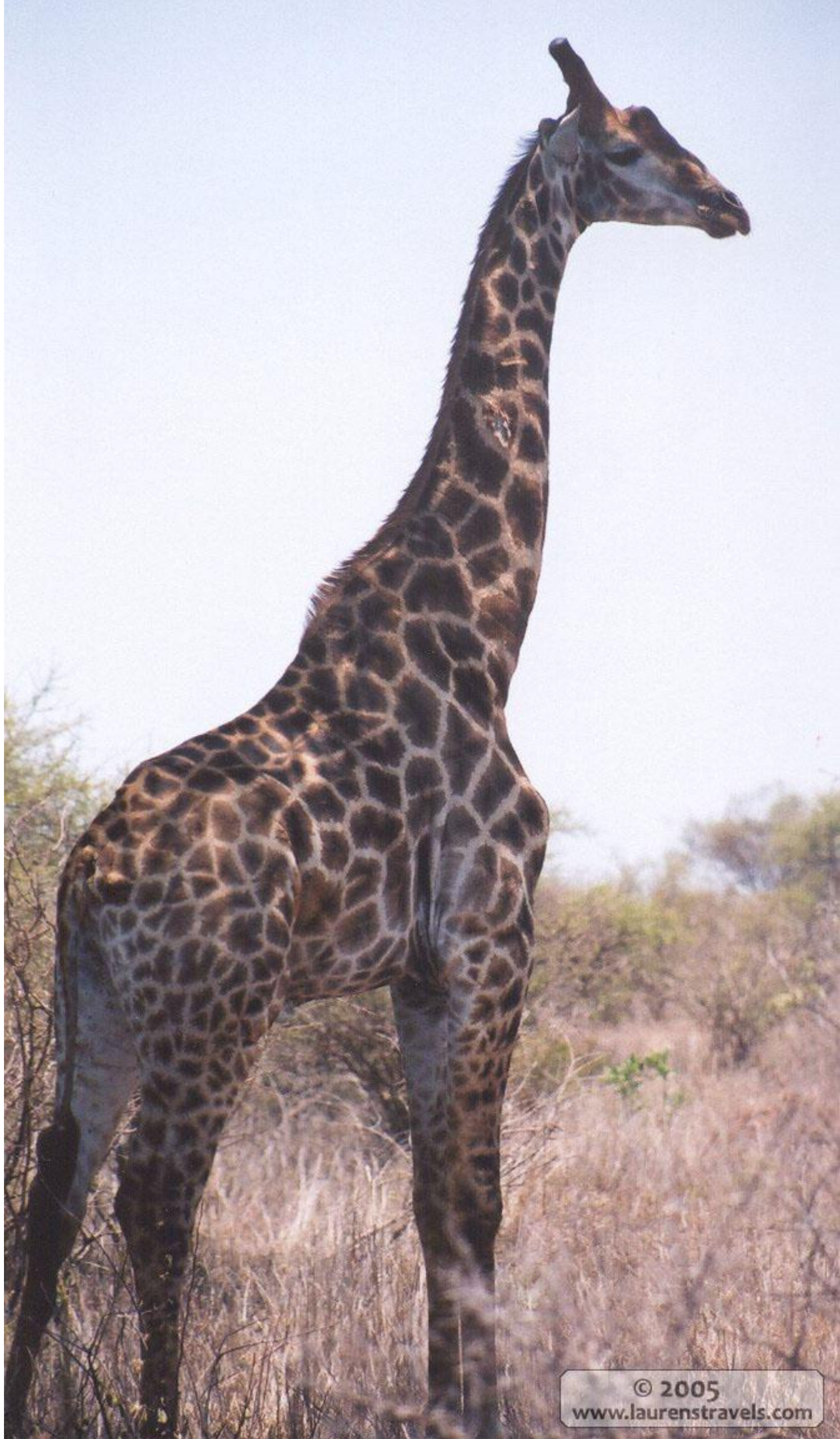
Why are the horizontal bands on this grain silo more closely spaced near the bottom?

Why doesn’t a giraffe’s head explode when it lowers its head for a drink?

Why don’t its legs bulge out from **hydrostatic pressure**?

$$\Delta p = \rho g h$$

16 – 18 feet !!

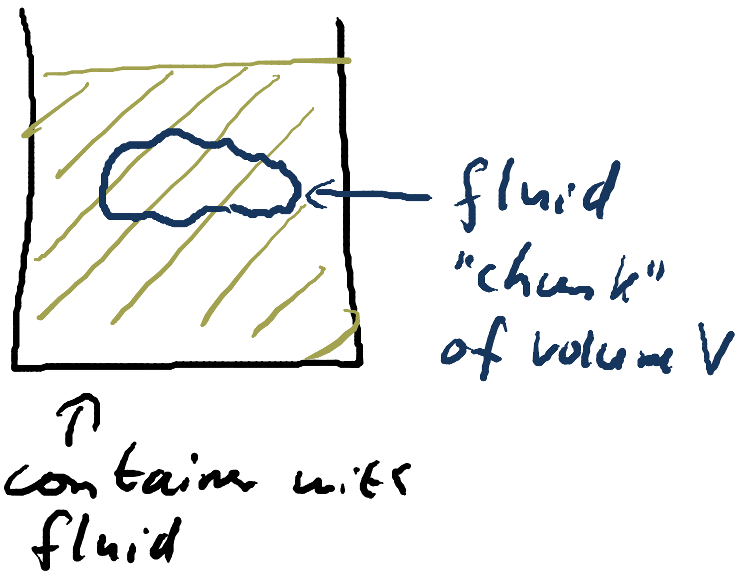


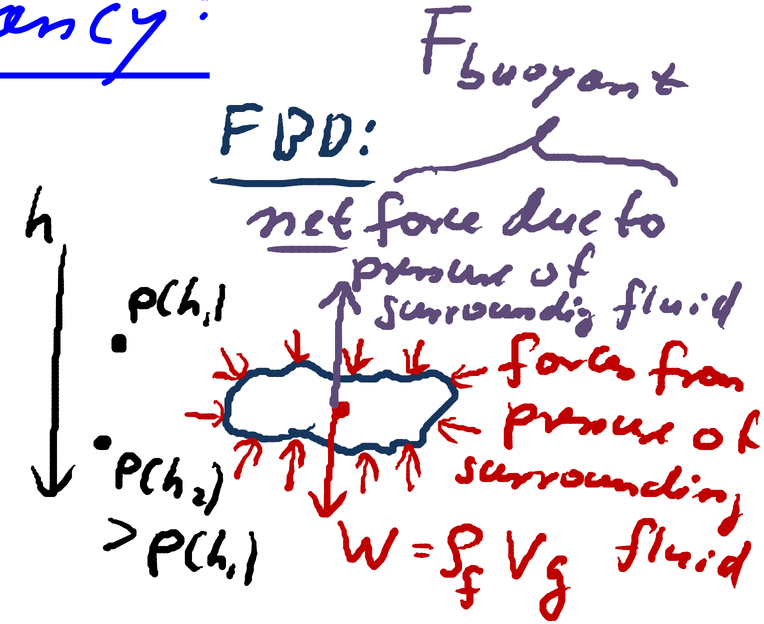
- A **giraffe's heart** (can weigh up to 10 kg and measure about 2 feet long), has to generate around **double** the normal blood **pressure** for an average large mammal to **maintain blood flow to the brain against gravity**.
- A complex **pressure-regulation system** prevents excess blood flow to the **head** when the giraffe **lowers** its head to drink.
- Giraffes have a very **tight, thick skin over their lower limbs** which prevents fluids from accumulating in the legs.

Pascal's Principle and Spiders:

Pascal's Principle and Spiders:

→ Buoyancy:





⇒ $F_{buoyant}$ is a consequence of pressure variation with depth in a fluid (not a new force)

⇒ need $\sum \vec{F} = 0$ ⇒ $|F_{\text{net from pressure of surrounding fluid on surface of fluid "chunk"}}| = |F_{buoy}| = |W_{\text{fluid chunk}}|$

$\Rightarrow$ If we replace the chunk of fluid by an object of same shape and volume, it must have the same F_{buoy} due to the pressure of the surrounding fluid!

$$\Rightarrow \boxed{|F_{\text{buoy on object in a fluid}}| = |W_{\text{of fluid displaced by the object}}| = \rho_{\text{fluid}} V_{\text{fluid displaced by object}}\, g}$$

density of fluid <u>not</u> object!