Recap: Motion in 1-D

Lecture 3

$$
\begin{tikzcd}
x(t) \arrow[r, shift left, "\text{slope}"] \arrow[rr, bend left, "\text{curvature}"] & v(t) \arrow[l, shift left, "\text{area} = \underline{\underline{\Delta}} x"] \arrow[r, shift left, "\text{slope}"] & a(t) \arrow[l, shift left, "\text{area} = \underline{\underline{\Delta}} V"]
\end{tikzcd}
$$

velocity $= v = \dfrac{dx}{dt} =$ slope of $x-t$ graph

acceleration $= a = \dfrac{dv}{dt} =$ slope of $v-t$ graph

$a = \dfrac{dx^2}{dt^2} =$ curvature of $x-t$ graph

$\Delta V =$ <u>change</u> of velocity $= V(t) - V(0) = \int_0^t a(t)\,dt$

$=$ area "under" $a-t$ graph

initial position at $t=0$

$\Delta x = x(t) - x(0) = \int_0^t v(t)\,dt =$ area "under" $v-t$ graph

change!

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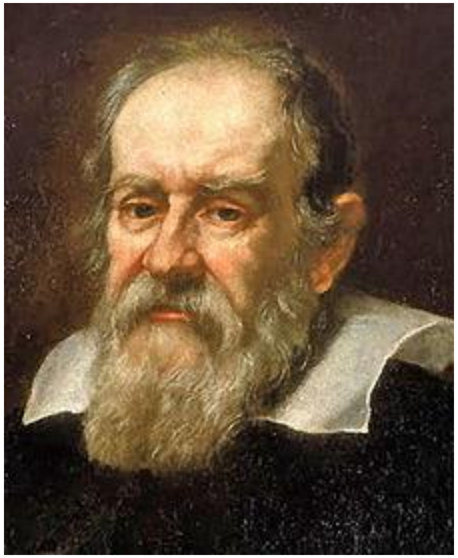
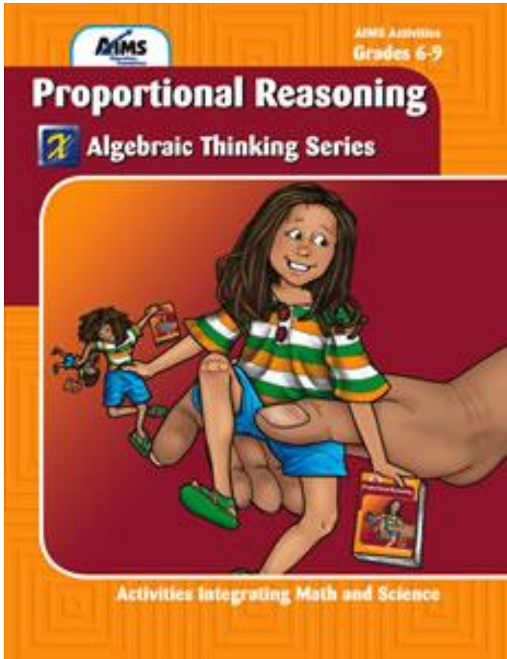
Today:

- **1-D motion with constant acceleration (a=const)**
  - Free fall
  - How high is the Suspension Bridge above Fall Creek Gorge?
  - Galileo
  - Kitchen faucets
- **Proportional reasoning**





Cornell Suspension Bridge spanning Fall Creek

How **high** is the bridge above the gorge floor?

h = ?	
A.	15 m (50 feet)
B.	30 m (100 feet)
C.	45 m (150 feet)
D.	60 m (200 feet)
E.	65 m (250 feet)

Special case: 1-D Motion with constant acceleration

$$\boxed{a = \text{const}}$$

Example: free fall $\qquad |a| = g = 9.8\ m/s^2$

$\approx 10\ m/s^2$ in Phy 2207

Methode I: Integration:

a = const here

$$a(t) = a = \text{const} \Rightarrow \underline{\underline{\Delta V}} = V(t) - \underbrace{V(t=0)}_{\substack{\text{initial} \\ \text{at } t=0 \\ = V_0}} = \int_0^t a\,dt = a \int_0^t dt$$

$$= \underline{\underline{a\,t}}$$

$$\Rightarrow (1) \qquad \boxed{V(t) = V_0 + at}$$

initial velocity at $t=0$

$$\Delta x = x(t) - \underbrace{x_0}_{\substack{x(t=0)\\ \text{initial position}}} = \int_0^t v(t)\,dt = \int_0^t (\underbrace{v_0 + at}_{①})\,dt$$

$$= v_0 t + \frac{1}{2} a t^2$$

$$\Rightarrow ② \quad \boxed{x(t) = x_0 + v_0 t + \frac{1}{2} a t^2}$$

x_0: initial position; v_0: initial velocity

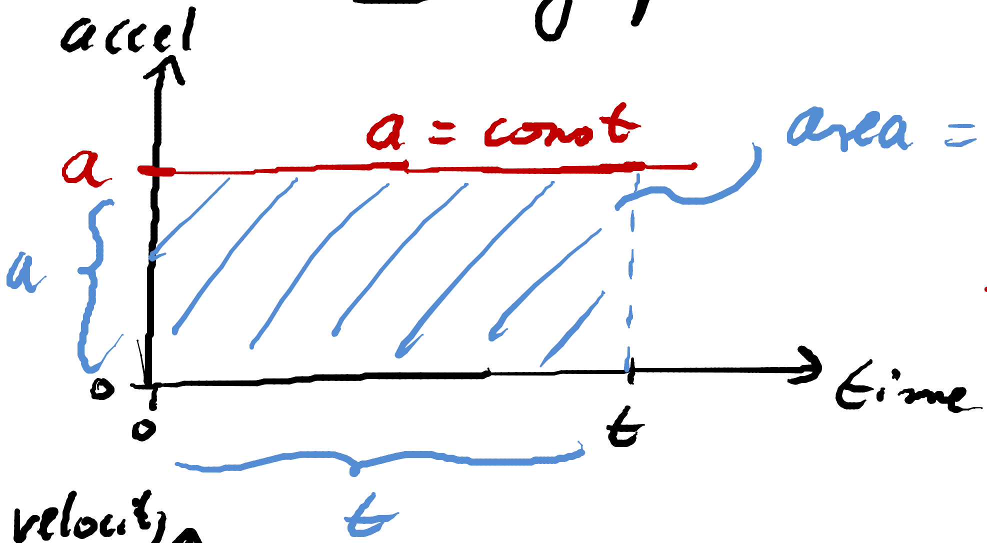
$\Rightarrow$ solve ① for t: $\quad t = \dfrac{v(t) - v_0}{a}$ and insert into ②

$$\Rightarrow 2a\,(x(t) - x_0) = 2a\,\Delta x = v^2(t) - v_0^2$$

$$\Rightarrow ③ \quad \boxed{\underbrace{v^2(t) - v_0^2}_{\text{note } v^2 - v_0^2 \neq (v - v_0)^2} = 2a\,\Delta x}$$

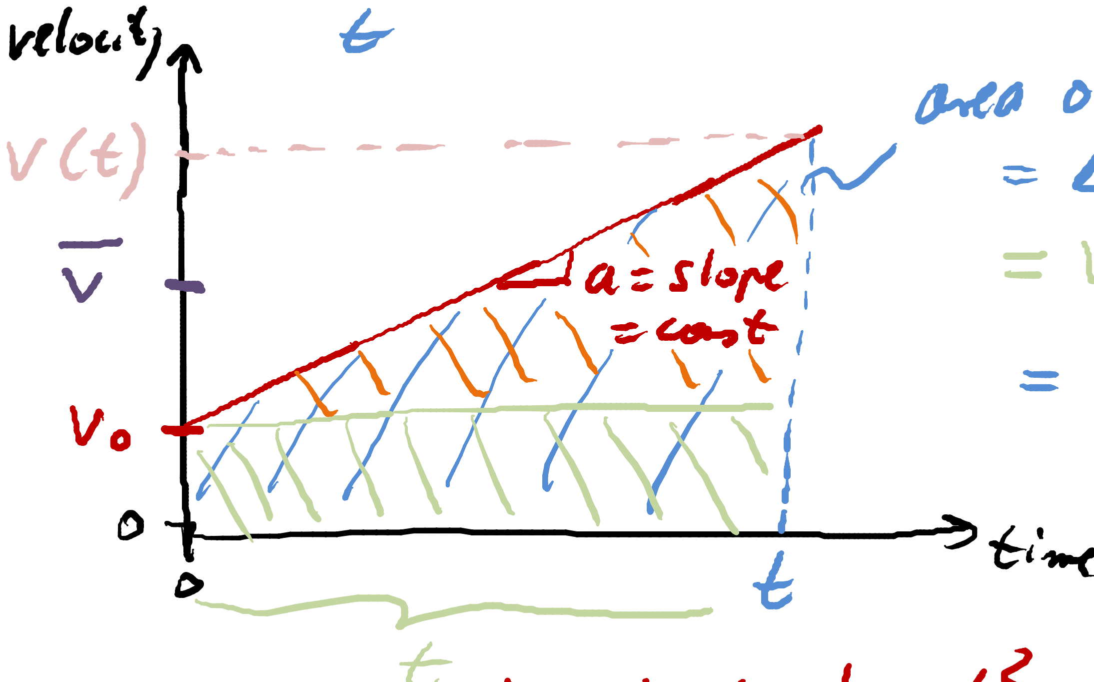
$\Rightarrow$ Note: Equ. ①, ②, ③ only for $a = \text{const}$! (incl. $a = 0$)

Methode II: Graphical



$$\text{area} = \Delta V = V(t) - V_0 = at$$

$$\Rightarrow \boxed{v(t) = v_0 + at} \quad (1)$$



area of trapezoid

$$= \Delta x = x(t) - x_0$$

$$= v_0 \cdot t + \frac{1}{2}(v(t) - v_0) \cdot t$$

$$= \underbrace{\frac{1}{2}(v_0 + v(t))}_{\bar{v}} \cdot t$$

$$= \frac{1}{2}(v_0 + v_0 + at) \cdot t$$

$$= v_0 t + \frac{1}{2} a t^2$$

$$\Rightarrow x(t) = x_0 + v_0 t + \frac{1}{2} a t^2 \quad (2)$$

Cornell Suspension Bridge spanning Fall Creek

$\boxed{\text{use } |a| = g = 10\ m/s^2 \text{ for free fall}}$

The time for a rock to drop from the Suspension Bridge to the floor of Fall Creek Gorge is measured to be ~3 s.

What is the rock's **speed** when it hits the ground?

+y

initial speed = 0

$v(t) = \underbrace{\cancel{v_0}}_{=0 \text{ here}} + at = -gt$

$= -10\ m/s^2 \cdot 3s = \underline{\underline{-30\ m/s}}$

Define sign!

speed = ?	
A.	10 m/s
B.	20 m/s
C.	30 m/s
D.	40 m/s
E.	50 m/s

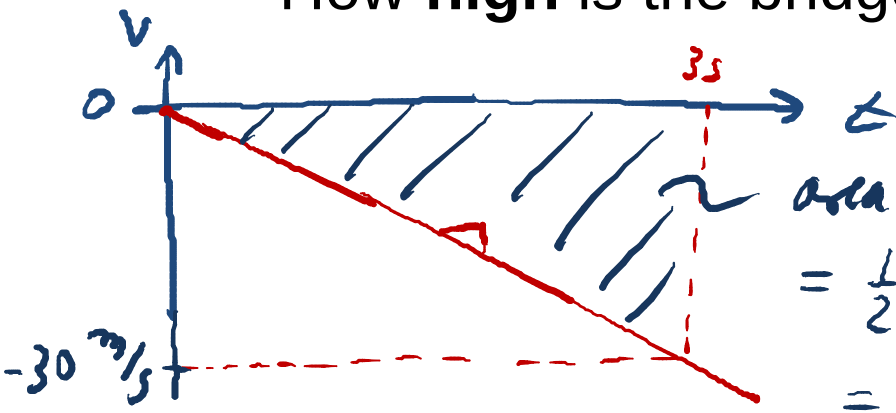
$\Delta v = -10\ m/s^2 \cdot 3s = -30\ m/s$

$= -65\ mi/h$

$1\ mi/h \approx 0.45\ m/s$

The time for a rock to drop from the Suspension Bridge to the floor of Fall Creek Gorge is measured to be ~3 s.

How **high** is the bridge above the gorge floor?



+y up

area = Δy

$= \frac{1}{2}(-30\ \mathrm{m/s}) \cdot 3\mathrm{s}$

$= -45\ \mathrm{m}$

or: $\Delta y = \cancel{v_0} t + \frac{1}{2} a t^2 = \frac{1}{2}(-10\ \mathrm{m/s^2})(3\mathrm{s})^2$

= 0 here

$= -45\ \mathrm{m}$

$\approx 150'$

h = ?

A. 30 m

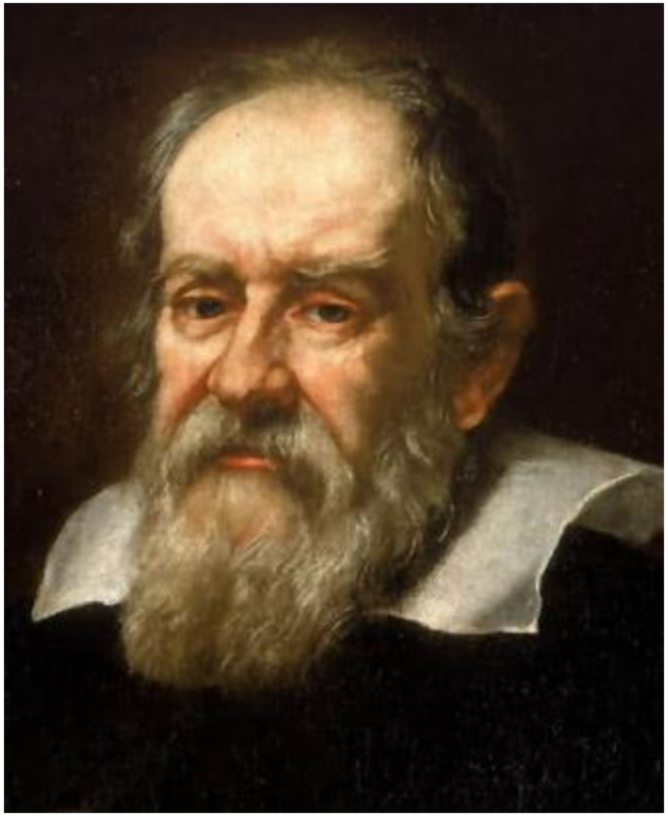
B. 45 m

C. 60 m

D. 75 m

E. 90 m

Galileo is said to have dropped balls of different masses from the Leaning Tower of Pisa to demonstrate that their time of descent was independent of their mass (excluding the effect of air resistance).



Kitchen Faucets:

- h = height of faucet outlet above sink

  Assume v_{water} at faucet outlet is ~0, and ~same for all faucets ($\Rightarrow$ same flow rate).

- Then $v_{water} = \sqrt{2gh}$ when it reaches the sink, so that

  $v_{water} \propto \sqrt{h}$ at sink.

$V(t)^2 - \cancel{V_0^2} = 2a\Delta y$

$v_{water} \propto \sqrt{h}$

Standard faucet:

h≈15 cm (~6 inches)

Gooseneck faucet:

h≈30 cm (~12 inches)

$\therefore$ At level of sink, $\dfrac{v_{water,goose}}{v_{water,std}} = \sqrt{\dfrac{h_{goose}}{h_{std}}} \approx \sqrt{2}$

$\Rightarrow$ **more splashing!**

Suppose that the height of the Suspension Bridge were doubled.

By what **multiplicative factor** would the **time** for the rock to fall change?

t(2h) / t(h) = ?

A. $1/\sqrt{2}$

B. 1

C. $\sqrt{2}$

~~D.~~ 2

E. 4

(2) $\Delta y = \underbrace{\cancel{v_0 t}}_{=0 \text{ here}} + \frac{1}{2} a t^2$

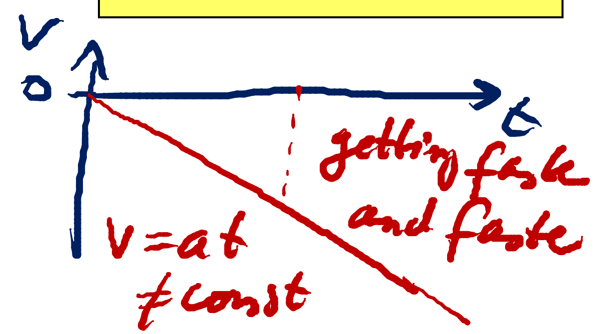
$\Delta y = \underbrace{\tfrac{1}{2} a}_{\text{const}} t^2$ here

$\Delta y \propto t^2 \Rightarrow \sqrt{\Delta y} \propto t$

"proportional to"

$\Rightarrow t \propto \sqrt{\Delta y}$

$\Rightarrow \dfrac{t(2h)}{t(h)} = \sqrt{\dfrac{2h}{h}} = \sqrt{2}$



Proportional Reasoning:

examples: $y = \overset{\swarrow \text{const}}{a}\, x \Rightarrow y \propto x$

$y = a\sqrt{x} \Rightarrow y \propto \sqrt{x}$

$y = a x^2 \Rightarrow y \propto x^2$

what happens to y if you double x?

General case: suppose $y = a x^\beta \Rightarrow y \propto x^\beta$

so $y_1 = a x_1^\beta$ $\quad$ $y_2 = a x_2^\beta$

$$\Rightarrow \frac{y_2}{y_1} = \frac{a x_2^\beta}{a x_1^\beta} = \left(\frac{x_2}{x_1}\right)^\beta$$

$\Rightarrow \frac{x_2}{x_1}$ = multiplicative factor by which x changes

$\frac{y_2}{y_1}$ = multiplicative factor by which y changes

$\Rightarrow$ If $y \propto x^{\beta}$, then if x changes by a factor c, then y change by a factor c^{β}.

Note: (1) if $y = a x^{\beta} + b$

$\Rightarrow$ can't use proportions!

(2) if $z = a x^{\alpha} y^{\beta}$

$$\Rightarrow \frac{z_2}{z_1} = \left(\frac{x_2}{x_1}\right)^{\alpha} \left(\frac{y_2}{y_1}\right)^{\beta}$$

Suppose that the Suspension Bridge and gorge were transported to the Moon, where the acceleration due to **gravity is 1/6 that on Earth**.

By what **multiplicative factor** would the **time** for a rock to fall to the bottom of the gorge change?

$$\Delta y = \underbrace{\cancel{v_0 t}}_{=0 \text{ here}} + \frac{1}{2} a t^2$$

$$\Delta y = \frac{1}{2} a t^2 \Rightarrow t^2 = \frac{2\,\overset{\text{const here}}{\Delta y}}{a}$$

$$\Rightarrow t^2 \propto \frac{1}{a}$$

$$\Rightarrow t \propto \sqrt{\frac{1}{a}}$$

$$\Rightarrow \frac{t_{moon}}{t_{earth}} = \sqrt{\frac{a_{earth}}{a_{moon}}} = \sqrt{\frac{g}{g/6}} = \sqrt{6}$$

t (Moon) / *t* (Earth) =?

A. 1/6

B. 1/√6

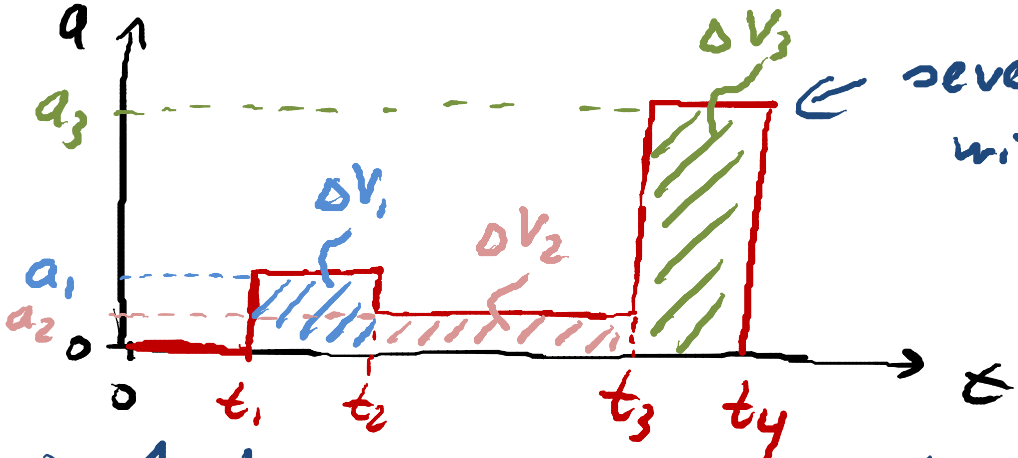
C. 1

D. √6

E. 6

Complicated Motions:

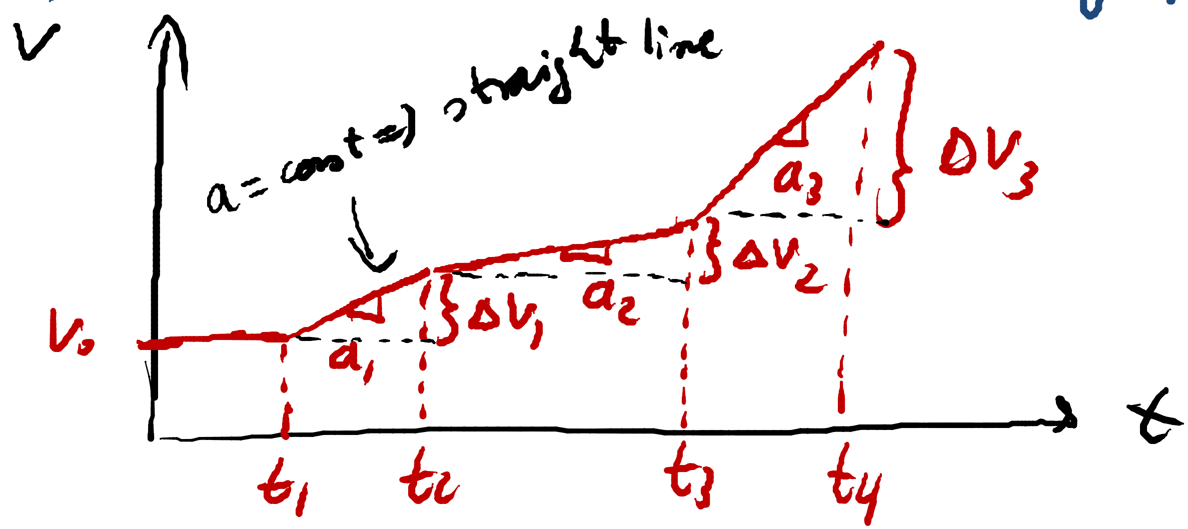
① what if:



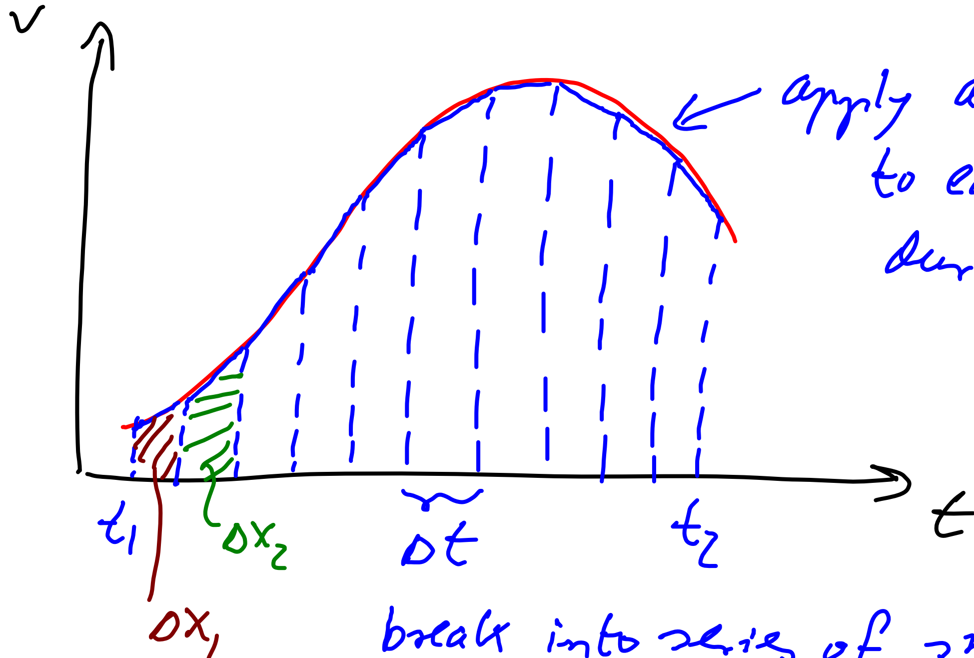
⟸ several time intervals with $a = $ const each

$\Rightarrow$ Apply $a=$ const analysis to each interval with $a=$ const independently

$\Rightarrow$ use $\Delta v = $ area "under" $a-t$ graph to get $v-t$ graph



② General case:



apply $a = \text{const}$ approximation to each interval of duration Δt

break into series of small time intervals Δt

$\Rightarrow$ apply $a = \text{const}$ analysis to each interval Δt

$\Rightarrow$ then get sum