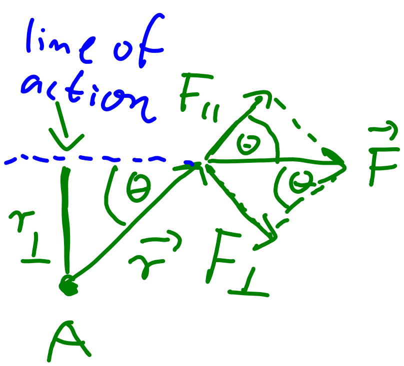
Recap: Torque and Equilibrium

Translational Motion		Rotation
x	$\rightarrow$	θ
$v = \frac{dx}{dt}$	$\rightarrow$	$\omega = \frac{d\theta}{dt} = \frac{v_t}{r}$
$a = \frac{dv}{dt}$	$\rightarrow$	$\alpha = \frac{d\omega}{dt}$
$\sum \vec{F}_{ext} = m\vec{a}_{cm}$	$\rightarrow$	$\sum \tau_{about\,A} = I_A \alpha_A$
$K = \frac{1}{2} m v^2$	$\rightarrow$	$K_R = \frac{1}{2} I_A \omega^2$
$p = m v$	$\rightarrow$	$L = I\omega$ } conserved if $\sum \tau_{ext} = 0$

$$\text{Torque} = \tau = F r \sin\phi = F_\perp r = F r_\perp \;\} \text{ has sign!}$$



Conditions for Equilibrium:

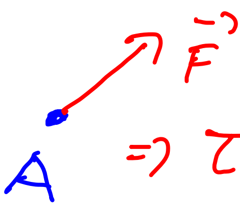
$$\boxed{\sum \vec{F}_{ext} = 0 \qquad \sum \tau_{about\ \underline{any}\ axis} = 0}$$

static: $\vec{v}_{cm} = 0$, $\omega = 0$

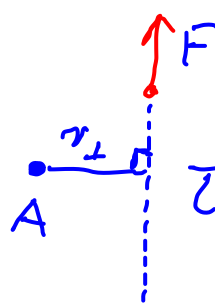
Solving for Equilibrium

$$\sum \vec{F}_{ext} = 0 \qquad \sum \tau_{\text{about any axis}} = 0$$

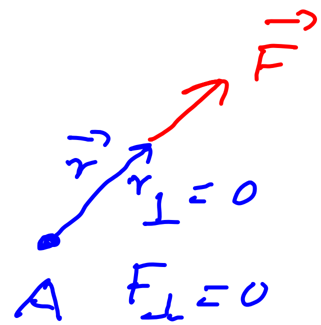
- Forces act at specific points! Must show on FBD!
- Weight of object acts through its center of gravity = center of mass (if $\vec{g}$ same for all parts of object)
- Can calculate $\sum \tau$ about any axis
  $\Rightarrow$ take $\sum \tau$ about a point/axis where unknown forces act $\Rightarrow r = 0 \Rightarrow \tau = 0$ for these forces
  $\Rightarrow$ eliminate these forces from $\sum \tau$



$\Rightarrow \tau_{F \text{ about } A} = 0$



$\tau = F\, r_\perp$



$\Rightarrow \tau_{\text{about } A} = 0$

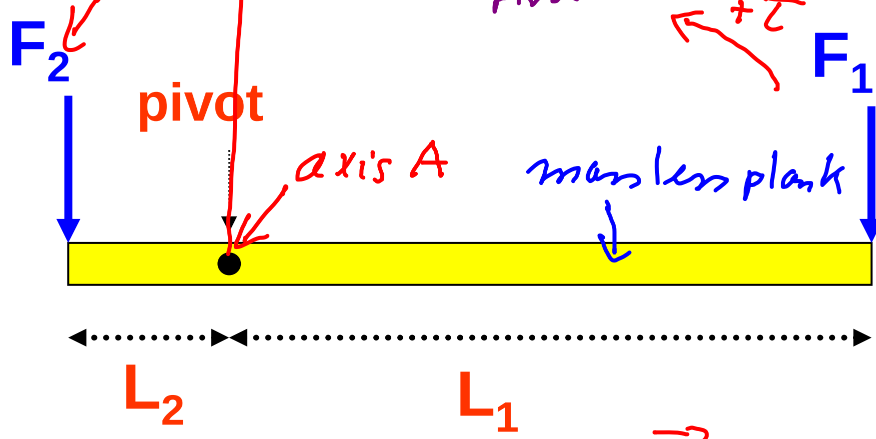
$F_{pivot} \Leftrightarrow \sum \vec{F} = 0$

but $\tau_{F_{pivot}} = 0 \quad (r = 0)$

large force

$+\tau$

small force



For rotational equilibrium,

$F_2/F_1 = ?$

A.	**1**
B.	**-1**
C.	**L_1/L_2**
D.	**L_2/L_1**

for equilibrium: $\sum \vec{F} = 0 \qquad \sum \tau = 0$

$$\sum \tau_{\text{about axis } A} = -F_1 L_1 + F_2 L_2 = 0$$

$$\Rightarrow \frac{F_2}{F_1} = \frac{L_1}{L_2}$$

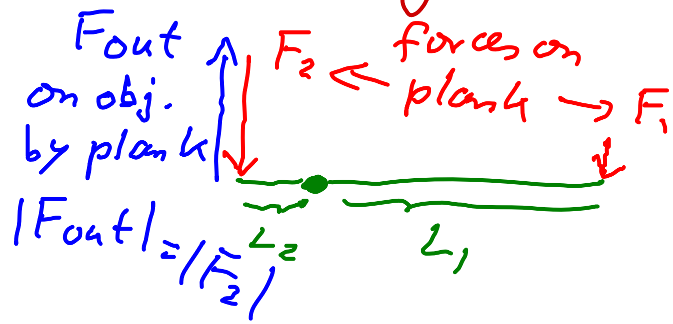
example of simple machine: levers

small input force $|F_1|$

$\Rightarrow$ large output force $|F_2|$

Levers: Mechanical Advantage MA:

$$MA = \frac{\text{"output force"}}{\text{" input force"}}$$



$\Rightarrow$ for previous example:

$|F_1|$ = input

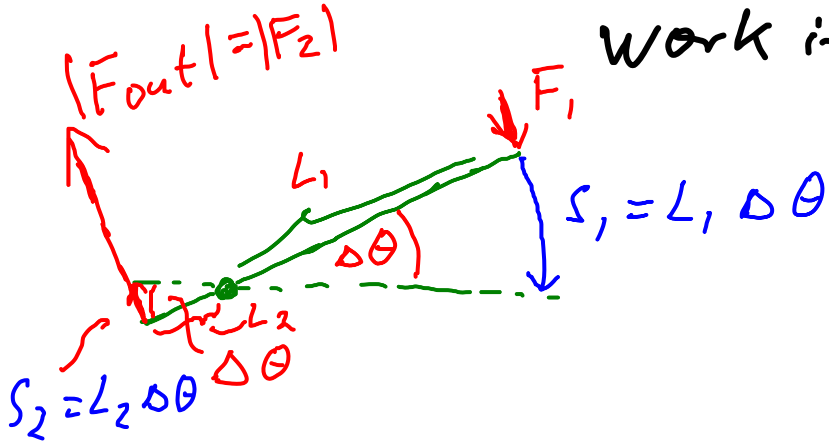
$|F_2|$ = output

$$\Rightarrow MA = \frac{F_2}{F_1} = \frac{L_1}{L_2} > 1$$

But: Need work in = work out

even if $F_{in} < F_{out}$?!

work in $\stackrel{?}{=}$ work out



$$\text{work} = \int_{\text{along path}} \vec{F}\cdot d\vec{s}$$

$$\Rightarrow |W_{\text{by } F_1}| = F_1\, S_1 = F_1\, L_1 \cdot \Delta\theta$$

$$|W_{\text{by } F_2}| = F_2\, S_2 = F_2\, L_2\, \Delta\theta$$

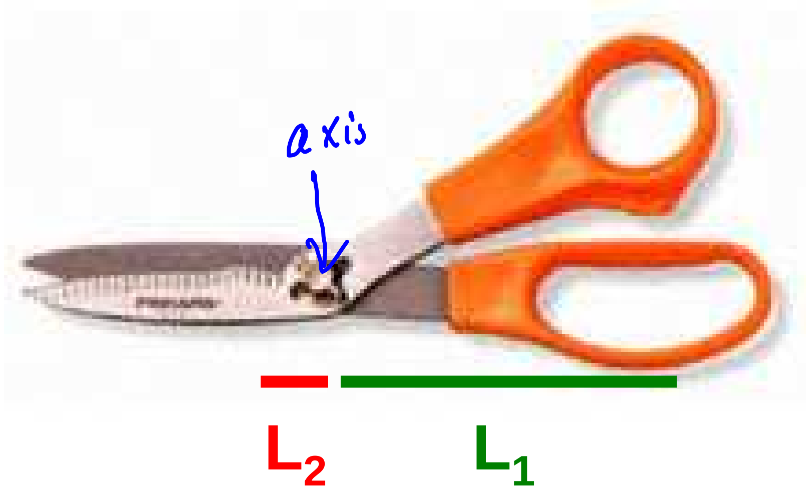
$$\Rightarrow \frac{|W_{\text{by } F_1}|}{|W_{\text{by } F_2}|} = \frac{F_1\, L_1\, \Delta\theta}{F_2\, L_2\, \Delta\theta} = \underbrace{\frac{F_1}{F_2}}_{} \cdot \frac{L_1}{L_2} = \underline{\underline{1}} \quad ✓$$

$F_1/F_2 = L_2/L_1$ } for equilibr.

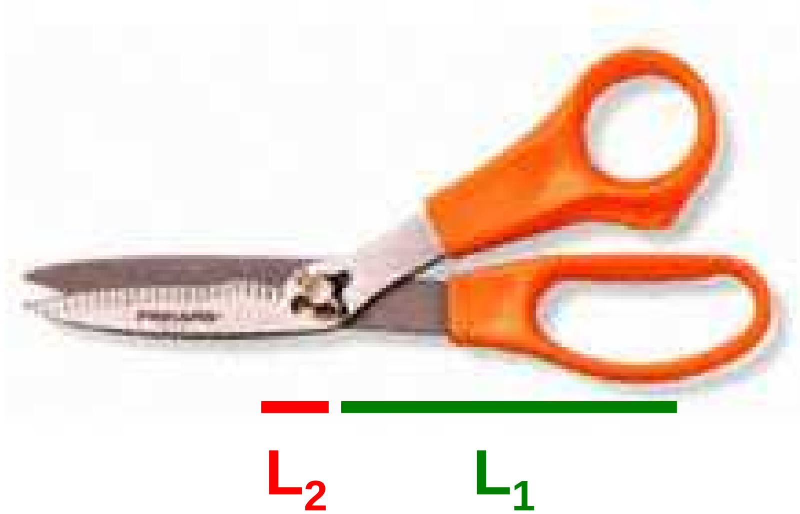
$\Rightarrow$ work by small force F_1 acting through big distance S_1

= work by large force F_2 acting through small distance S_2.

Levers in Kitchen Utensils:



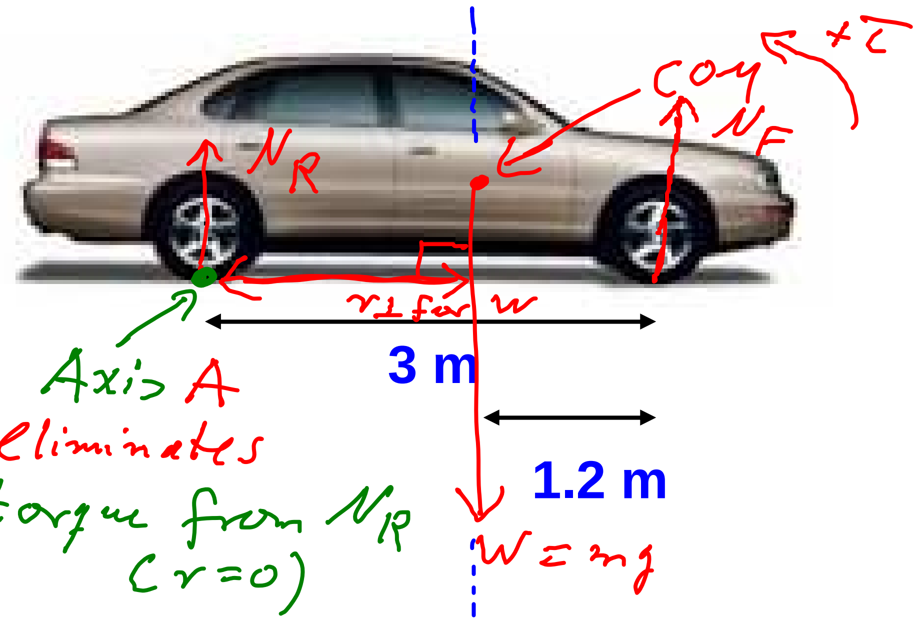
Levers in Kitchen Utensils:



Typical **mechanical advantage**:

$F_2/F_1=L_1/L_2$:	**~ 5**
Maximum hand grip force:	~ 10 - 200 lb
Maximum utensil force:	~ **50 - 1000 lb**

A car with weight W has a **distance of 3 m between its front and rear wheels**. Its **center of gravity** is **1.2 m behind the front wheels**.



What is **the total force exerted on the two front wheels** by the ground?

A.	**0.4 W**
B.	**0.5 W**
C.	**0.6 W**
D.	**1.0 W**

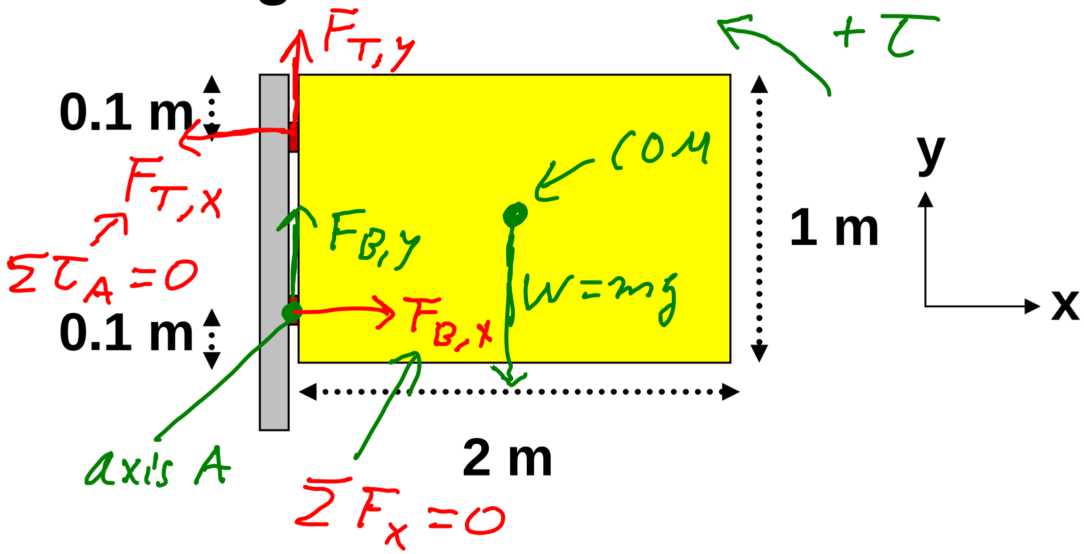
Equilibrium: $\sum \vec{F} = 0 \Rightarrow W = N_F + N_R$

$\Rightarrow$ can't find N_F just by this...

$$\sum \tau_{\text{about axis A}} = 0 = -W \cdot \underbrace{1.8m}_{r_\perp} + N_F \cdot 3m + N_R \cdot 0 \leftarrow r=0$$

$$\Rightarrow N_F = \frac{1.8m}{3m} W = 0.6\,W$$

A gate of uniform mass distribution and **weight W** is supported by two hinges as shown. **The y component of the hinge force is the same for both hinges.**



$\Sigma \vec{F}_{on\ gate} = 0$

$\Sigma \tau_{about\ \underline{any}\ axis} = 0$

need horizontal forces from hinges, otherwise gate would start to rotate!

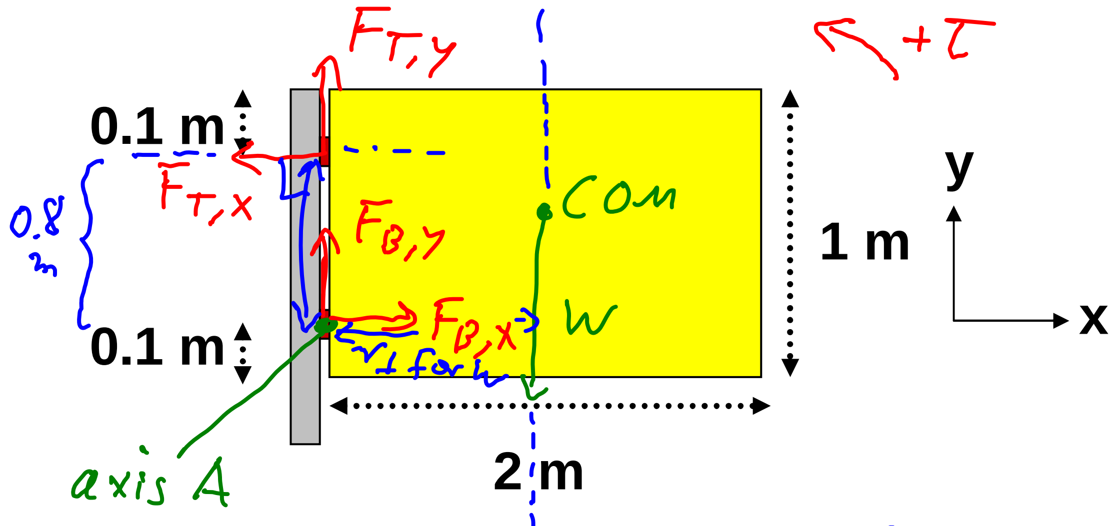
In what **direction** is the **force** exerted **on the gate by the top hinge**?

bottom hinge

F_{on}

A. **B.** **C.** **D.**

A gate of uniform mass distribution and **weight W** is supported by two hinges as shown. **The y component of the hinge force is the same for both hinges.** $F_{T,x} = ?$



What is the **horizontal force** exerted **on the gate by the top hinge**?

A. W / 2

B. W

C. 1.25 W

D. 2.5 W

E. None of the above

$$\Sigma \tau_{about\ A} = 0 = -W \cdot 1\,m + F_{T,x} \cdot 0.8\,m$$

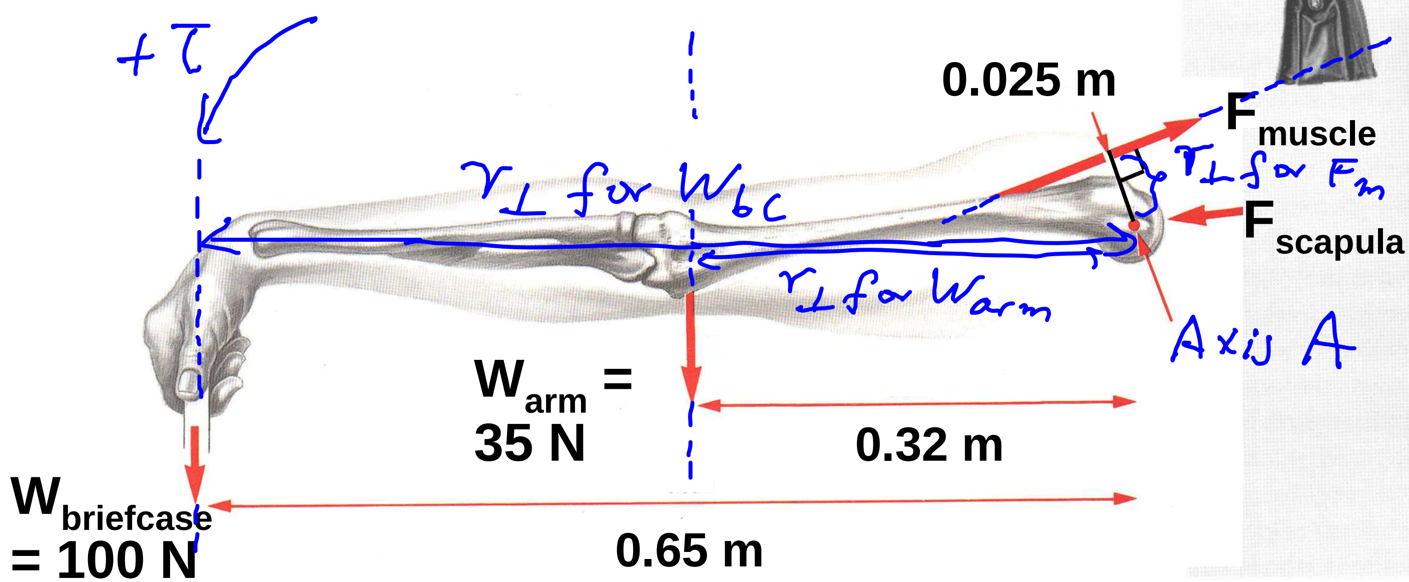
$$\Rightarrow F_{T,x} = \frac{W}{0.8} = 1.25\,W > 0$$

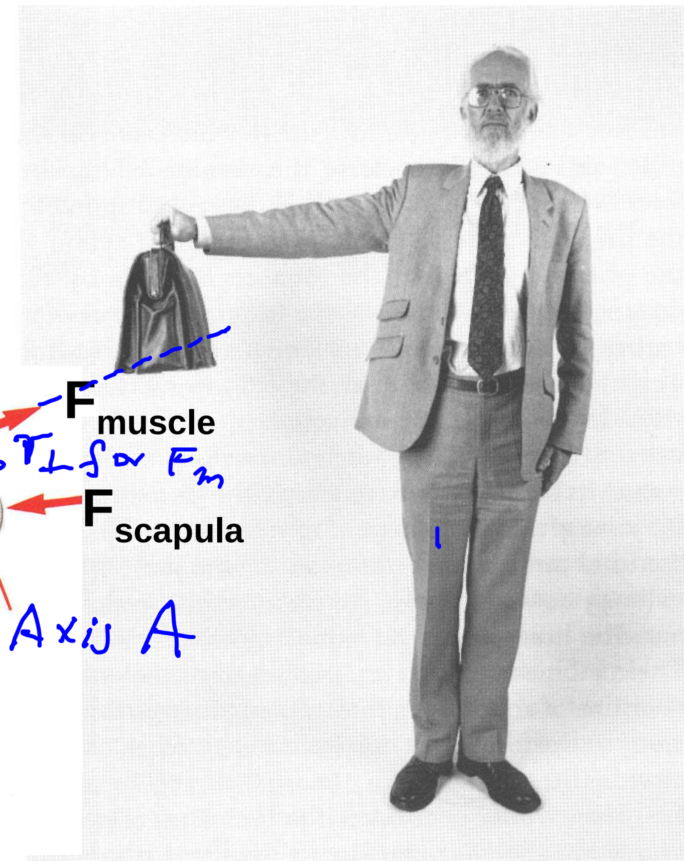
Note: $\tau_{about\ A}$ of $\vec{F}_b = 0$ $(r = 0)$

$\tau_{about\ A}$ of $F_{T,y} = 0$ $(r_\perp = 0)$

Forces in lifting at 10 kg briefcase:

What is the force from the muscle?





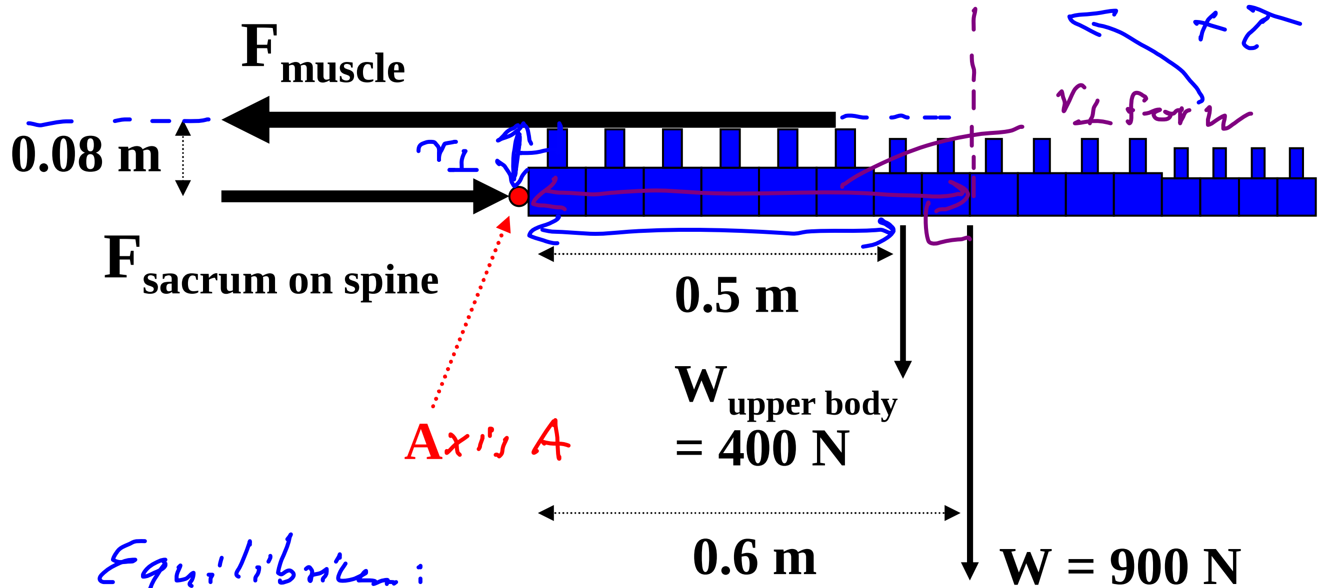
Equilibrium: $\sum \tau_{about\,A} = 0 = W_{bc} \cdot \underbrace{0.65m}_{r_\perp} + W_{arm} \cdot 0.32m - \underbrace{F_{muscle}} \cdot \underbrace{0.025m}_{r_\perp}$

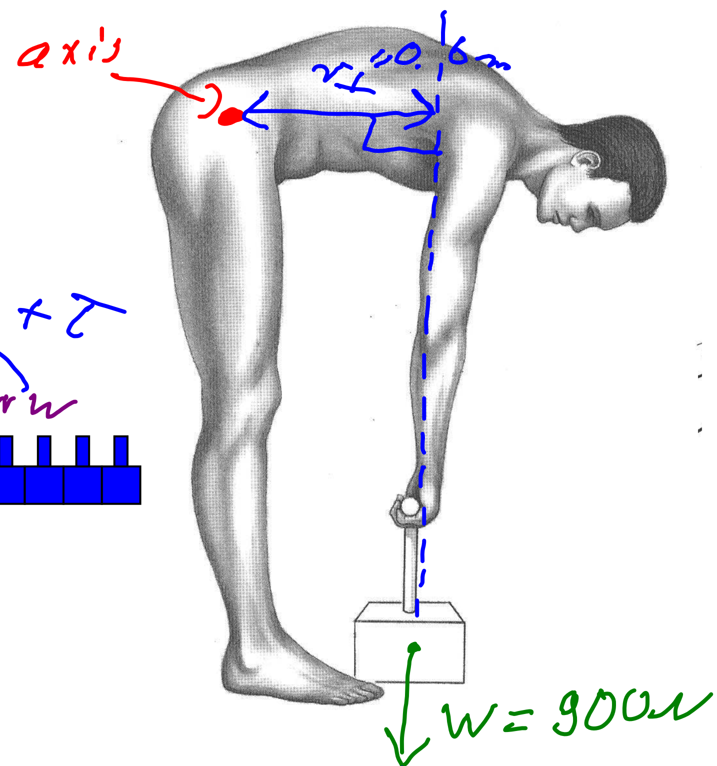
$\Rightarrow F_{muscle} = \dfrac{65\,Nm + 11.2\,mN}{0.025m} = \underline{3000N} = 0.3 \text{ tons}$ ~ weight of 650 lbs

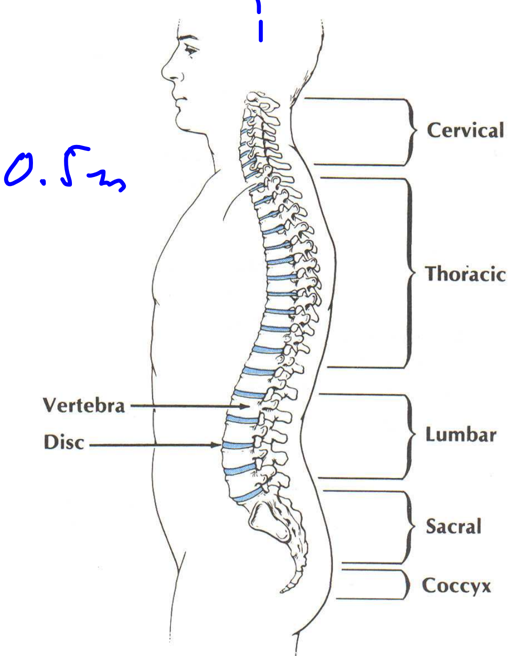
$\Rightarrow MA = \dfrac{\text{output force}}{\text{input force}} = \dfrac{100N + 35N}{3000N} \approx 0.04 \ll 1$ "mechanical <u>disadvantage</u>"

Forces when lifting a 90 kg weight:

Simplified model for forces on spine:







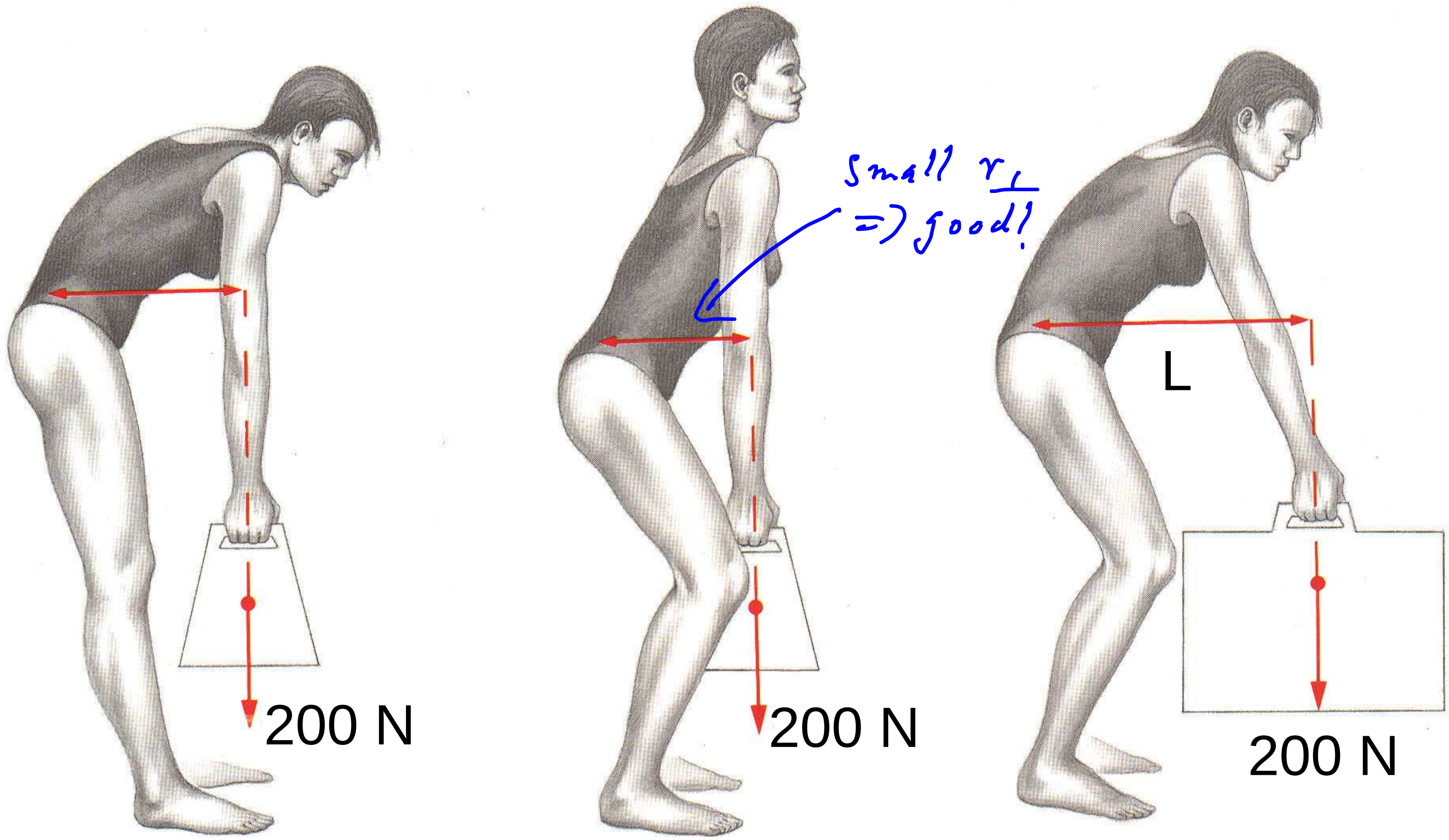
Equilibrium:

$$\sum \tau_{\text{about } A} = 0 = -900\,N \cdot 0.6\,m - 400\,N \cdot 0.5\,m + \underline{F_{muscle}} \cdot 0.08\,m$$

$$\Rightarrow F_{muscle} = 9000\,N \approx \text{weight of 1 ton}$$

$$\Rightarrow MA = 0.14 \ll 1$$

Torques exerted by muscles in your lower back depend on how you lift:

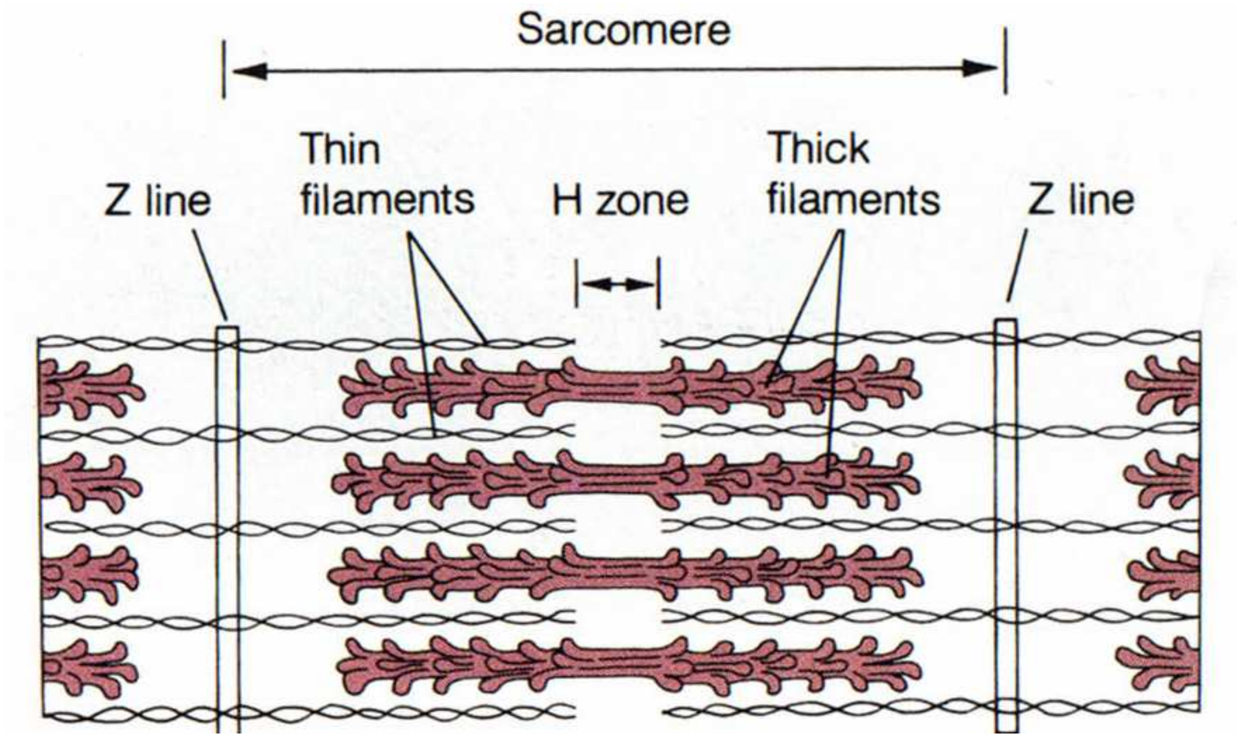



UESAKA-TOKYO
25kg

How to throw a ball at 50 m/s using molecules that move at 10^{-5} m/s:

1. Start with **basic unit**: sarcomeres made of opposed actin filaments and myosin "motors" that pull filaments together.

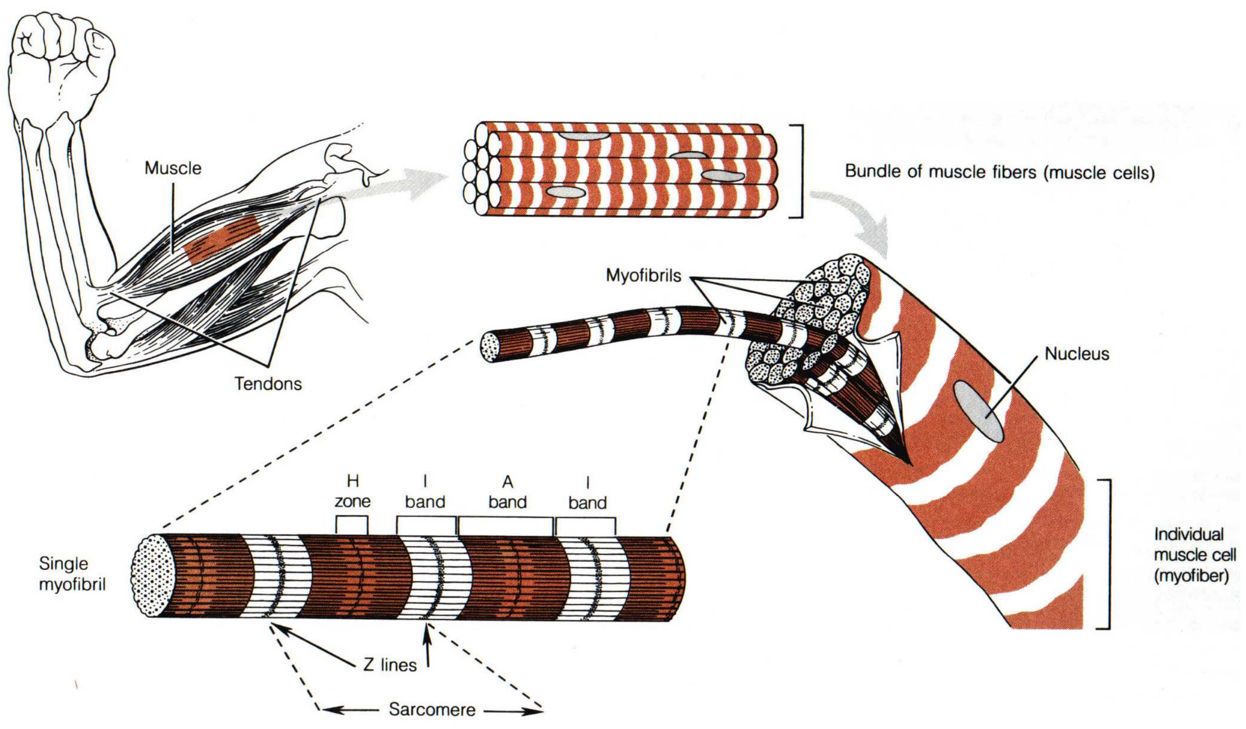
L~2.5 μm, **shortens by ΔL/L ~ 40% in ~0.1 s**



How to throw a ball at 50 m/s using molecules that move at 10^{-5} m/s:

2. Connect sarcomeres in **parallel** to get big force F (up to 10,000 N!)

   Connect sarcomeres in **series** to get big L, ΔL (up to 0.1 m) $\Rightarrow$ **v_{max} (muscle) < 1 m/s**

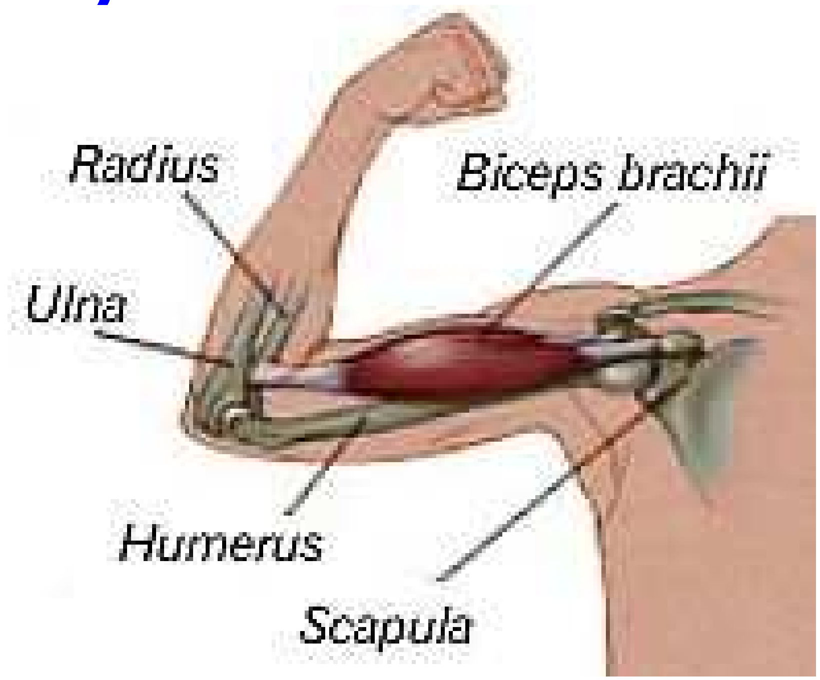


How to throw a ball at 50 m/s using molecules that move at 10^{-5} m/s:

3. Use tendons (with $d_{tendon} << d_{muscle}$) to **connect muscles close to pivot points of long bones**.

   Mechanical *disadvange* then produces **large limb displacements for given muscle ΔL**.

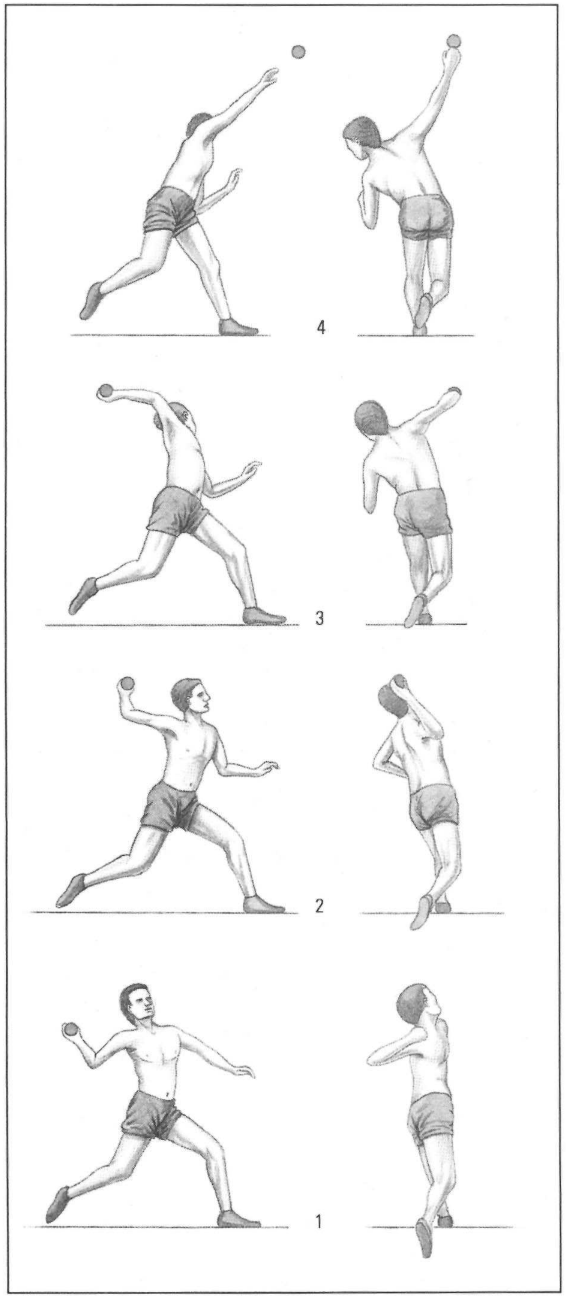
$\Rightarrow$ **$v_{rel,max}$ (limb) ~ 5-10 m/s**



How to throw a ball at 50 m/s using molecules that move at 10^{-5} m/s:

4. **Use several mechanical "stages" that can rotate or move relative to each other**. (E.g., legs, hips, torso, arms, wrists, fingers).

5. Execute relative motion of each stage so that **relative velocities of each stage add**. "Whip-like" motion taking advange of elastic energy storage and release by tendons and ligaments maximizes impulse delivered to the ball.



Speed of body parts relative to ground during a shot-put: