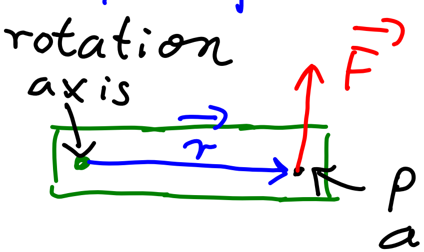
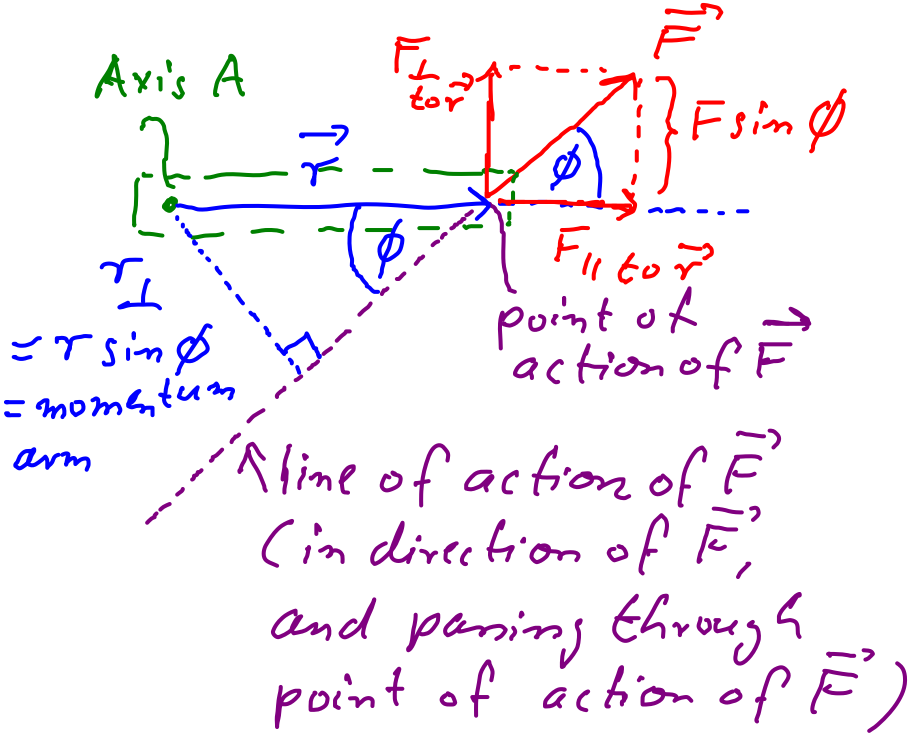
Recap: Rotational Motion

Translation (motion of COM)		Rotation
Position	x	θ
Displacement	Δx	$\Delta\theta$
velocity	$v = \dfrac{dx}{dt}$	$\omega = \dfrac{d\theta}{dt} = \dfrac{v}{r}$ = rate of change of θ w.r.t. time
acceleration	$a = \dfrac{dv}{dt}$	$\alpha = \dfrac{d\omega}{dt}$ = rate of change of ω w.r.t. time
Forces cause acceleration of center of mass of object.		Torque causes angular acceleration, i.e rotation.



$$\text{Torque} = \tau = F \cdot r_{\perp} = F_{\perp} \cdot r$$

perpendicular comp.



ϕ: angle between $\vec{F}$ and $\vec{r}$

$F_\parallel$: produces no torque and no rotation about axis A

$$\boxed{\tau_{\text{of } \vec{F} \text{ about axis A}} = F_\perp \, r = F \, r \, \sin\phi = F \underbrace{r_\perp}}$$

"<u>momentum arm</u>" of $\vec{F}$ about axis A

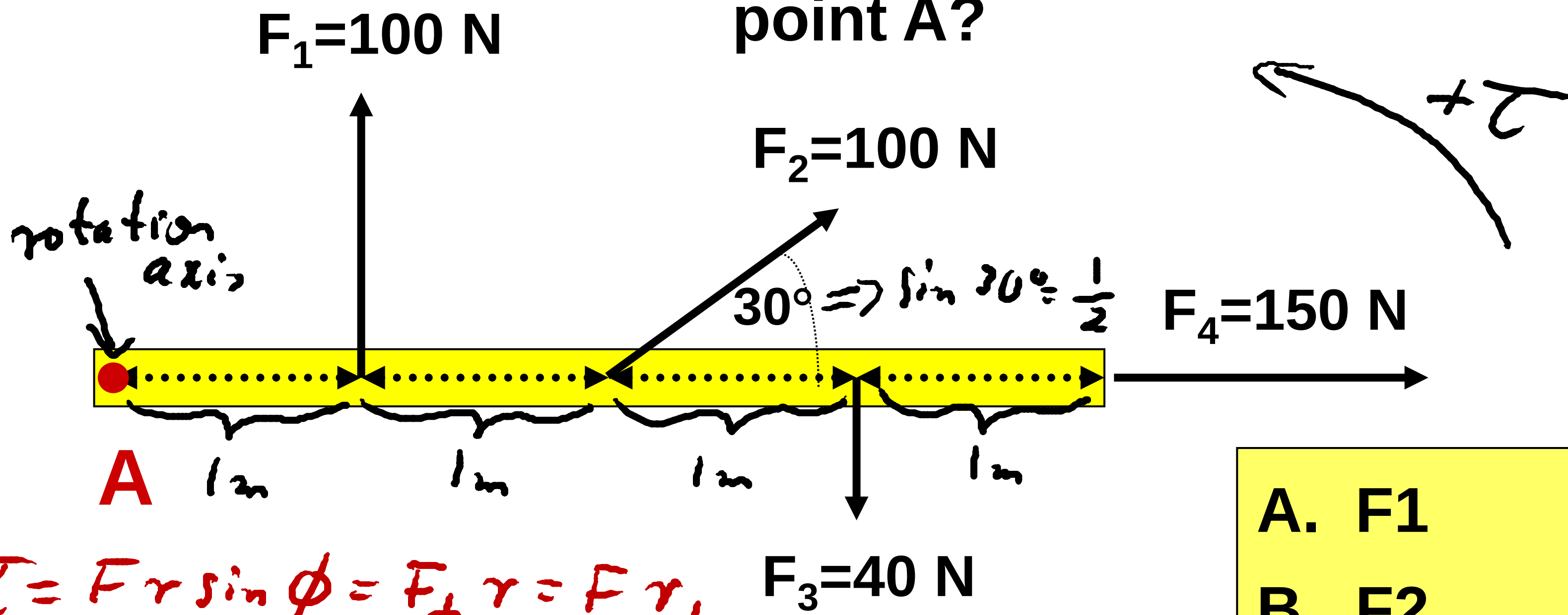
= $\perp$ distance from axis A to the line of action of force $\vec{F}$

Note:

- τ of given force $\vec{F}$ depends on position of axis **and** the point of action of $\vec{F}$
- Torque is a **vector**! For 2-D problems: $+\tau$

  τ has a sign! $\Rightarrow$ Forces that would produce **counter-clockwise** rotations about choosen axis of rotation produce **positive** torque.
  (same convention as for $+\theta$ direction)
- $[\tau] = Nm$ (same as energy, but don't use $J = Nm$ for torque, only for energy/work)

Which force exerts the largest magnitude torque about the point A?



A. F1

B. F2

C. F3

D. F4

E. F1 and F2

$\tau = F r \sin\phi = F_\perp r = F r_\perp$

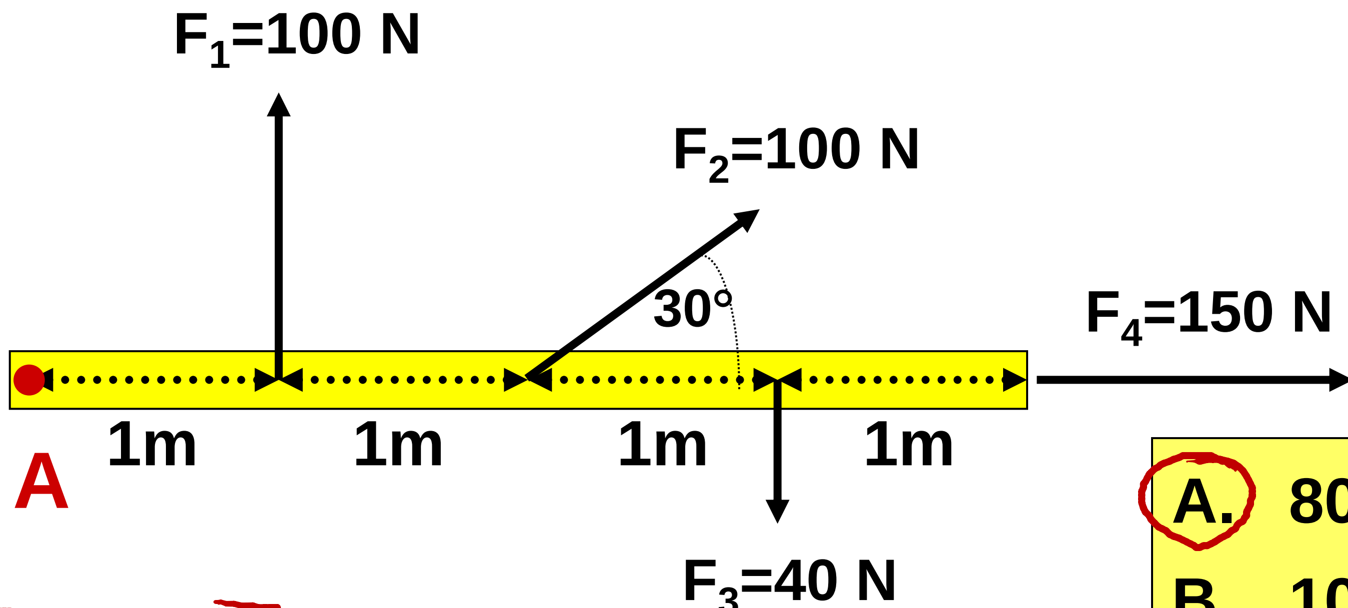
$\Rightarrow \tau_{1\,\text{about}\,A} = F_1 \cdot 1\text{m} = 100\ \text{Nm}$

$\tau_{2\,\text{about}\,A} = F_2 \cdot 2\text{m} \cdot \sin 30^\circ = 100\text{N} \cdot 2\text{m} \cdot \frac{1}{2} = 100\ \frac{\text{N}}{\text{m}}$

$\tau_{3\,\text{about}\,A} = -40\text{N} \cdot 3\text{m} = -120\ \text{Nm}$

$\tau_{4\,\text{about}\,A} = F_4 \cdot 4\text{m} \cdot \sin 0^\circ = 0 \quad (r_\perp = 0)$

What is the **net torque** exerted by the 4 forces about the point A? Assume **counterclockwise** torques are **positive**.



A. 80 N m

B. 100 N m

C. 180 N m

D. 200 N m

E. None of the above

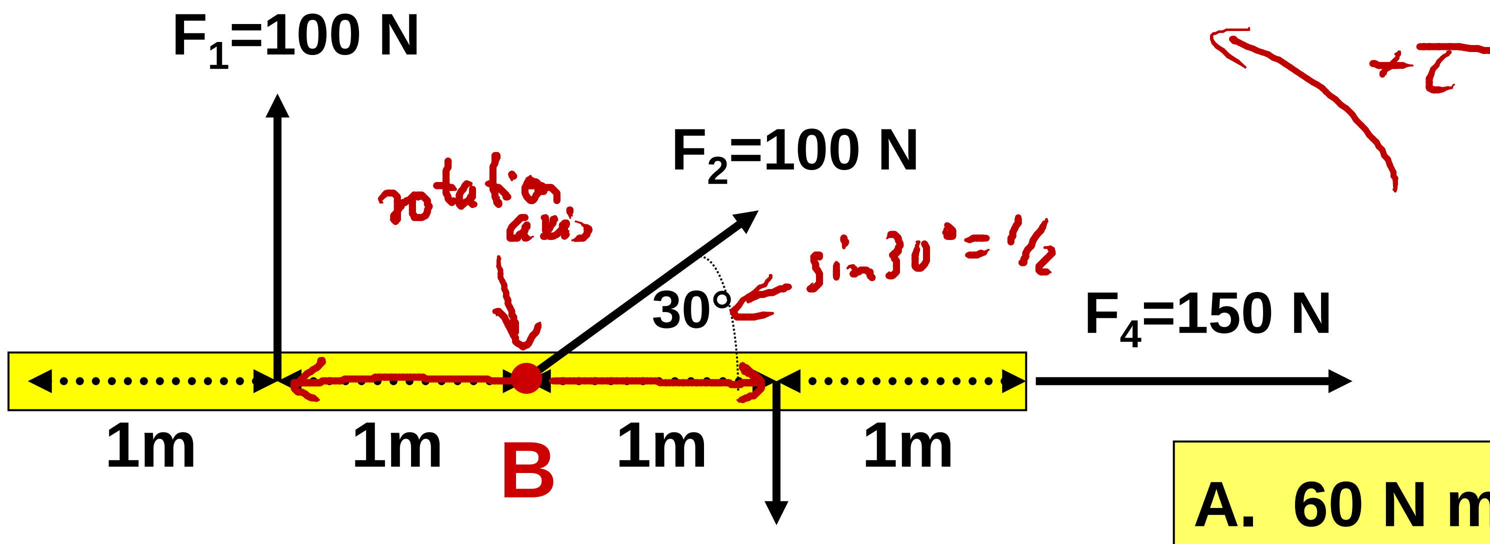
$$\tau_{net\ about\ A} = \sum \tau_{about\ A} = \tau_1 + \tau_2 + \tau_3 + \tau_4$$

$$= 100\ Nm + 100\ Nm - 120\ Nm + 0$$

↑ sign!

$$= +80\ Nm$$

What is the **net torque** exerted by the 4 forces about the point **B**? Assume **counterclockwise** torques are **positive**.



A. 60 N m

B. - 60 N m

C. 140 N m

D. - 140 N m

E. None of the above

τ_1 about B $= -100N \cdot 1m = -100\ Nm$

τ_2 about B $= (100N \cdot 0m) = \underline{0}$ $(r=0)$

τ_3 about B $= -40N \cdot 1m = -40\ Nm$

τ_4 about B $= 0$ $(r=0)$

$\Rightarrow \Sigma \tau_{\text{about B}} = -140\ Nm$

- N II for translational motion of COM point:

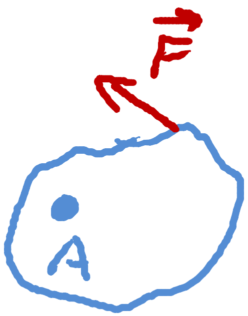
$$\boxed{\vec{F}_{net,ext} = \sum \vec{F}_{ext} = m\,\vec{a}_{com}}$$

$F \leftrightarrow \tau$

$a \leftrightarrow \alpha$

$m \leftrightarrow I$

- N II for rotational motion:



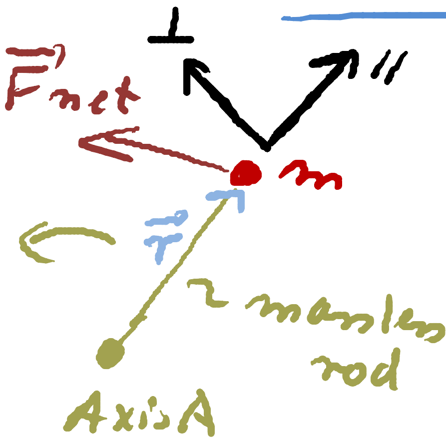
$$\boxed{\tau_{\substack{net\\ \text{about}\\ \text{axis } A}} = \sum \tau_{\text{about axis } A} = I\,\alpha}$$

angular acceleration about axis A

I: "moment of inertia" of the object about axis A (depends on position and orientation of axis A)

$$[I] = \frac{[\tau]}{[\alpha]} = \frac{Nm}{1/s^2} = \frac{kg\ m^2/s^2}{1/s^2} = kg\ m^2$$

"Proof" of $\tau_{net} = I\alpha$ for point mass:



$$\tau_{net} = F_{net,\perp} \cdot r = m a_\perp r$$

use NII: $\vec{F}_{net} = m\vec{a} \Rightarrow F_\perp = m a_\perp$

$$a_{\perp \text{ to } \vec{r}} = \frac{dV_\perp}{dt} = \frac{d}{dt}(\omega r) = r\frac{d\omega}{dt} = r\alpha$$

$$\omega = \frac{V_\perp}{r} \Rightarrow V_\perp = \omega r$$

Note:
$|a_{\parallel}| = \frac{V_\perp^2}{r}$

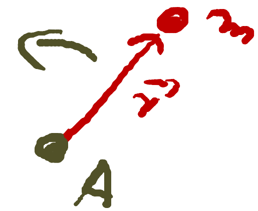
for circ. motion

$$\Rightarrow \tau_{net} = m a_\perp r$$
$$= m(r\alpha) r$$
$$= m r^2 \alpha$$
$$= I\alpha$$

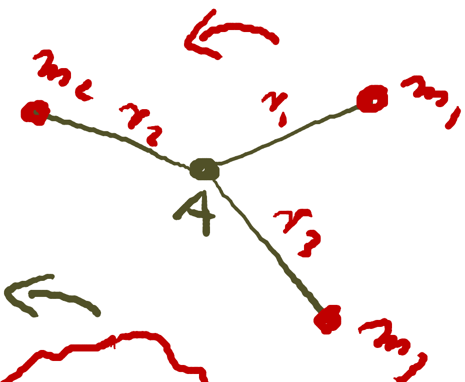
with $\boxed{I = m r^2}$ for point mass

Moment of Inertia I:

$$I_{\substack{\text{about}\\\text{axis A}}} = m r^2 \quad \Big\} \text{ point mass}$$

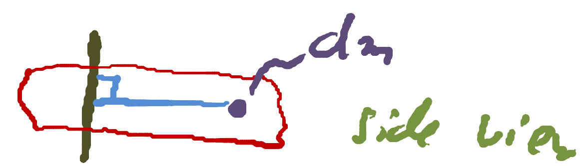


$$I_{\substack{\text{about}\\\text{axis A}}} = \sum_{i=1}^{N} m_i r_i^2 \quad \Big\} \text{ for } N \text{ point masses rotating about } \underline{\text{same}} \text{ axis A}$$



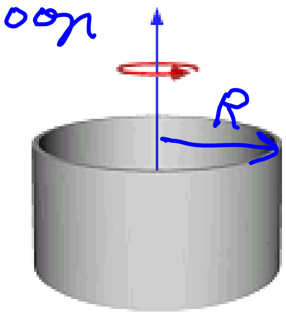
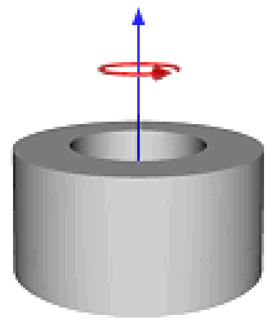
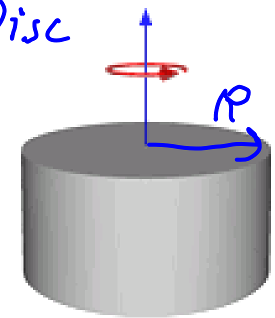
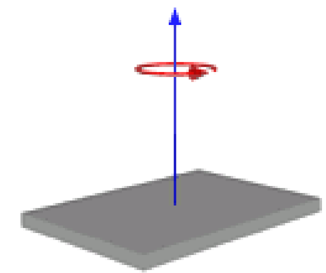
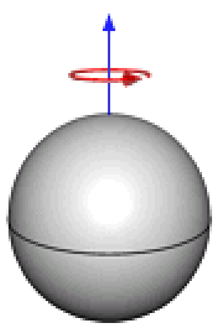
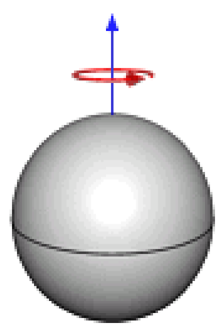
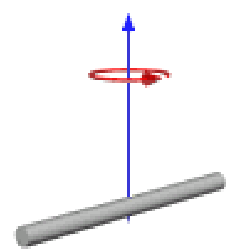
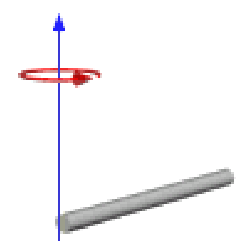
$$I_{\substack{\text{about}\\\text{axis A}}} = \int r^2 \, dm \quad \Big\} \text{ for rigid object}$$

r: perpendicular distance from rotation axis



__Note:__ I depends on
- mass and how it is distributed
- axis about which we are considering rotation!

$I = m_{hoop} \cdot R^2$ $\qquad$ $I_{about\ center} = \frac{1}{2} m_{disc} \cdot R^2$

Hoop 		Disc 	
thin hoop or ring of radius R & mass M:	thick ring of inner radius R1, outer radius R2, and mass M:	solid cylinder or disc of radius R and mass M:	flat plate with sides of length A and B and mass M:
M * R^2	**M * (R1^2 + R2^2) / 2**	**(M * R^2) / 2**	**M * (A^2 + B^2) / 12**
			 GVG/PD/2.0
solid sphere of radius R and mass M:	thin-walled hollow sphere of radius R & mass M:	slender rod of length L and mass M, spinning around center:	slender rod of length L and mass M, spinning around end:
(2 / 5) * M * R^2	**(2 / 3) * M * R^2**	**(M * L ^ 2) / 12**	**(M * L ^ 2) / 3**

MOMENTS OF INERTIA

Kinetic Energy and Momentum

1-D lin. motion	rotation of rigid object about fixed axis A

1-D lin. motion

- Kinetic energy:

$$\mathcal{K} = \frac{1}{2} m v^2$$

- Linear momentum

$$\underbrace{\vec{p} = m\vec{v}}$$

conserved if

$$\vec{F}_{net,ext} = \sum \vec{F}_{ext} = 0$$

rotation of rigid object about fixed axis A

- Kinetic energy from rotation:

$$\boxed{\mathcal{K} = \frac{1}{2} I_{about\,A}\, \omega^2}$$

- Angular momentum L

$$\underbrace{L = I\omega}$$

angular momentum is conserved if

$$\tau_{net} = \sum \tau_{ext} = 0$$

$$m \leftrightarrow I$$

$$v \leftrightarrow \omega$$

$$F \leftrightarrow \tau$$

Requirements of Equilibrium:

① $\vec{a}_{com} = 0 \iff \boxed{\sum \vec{F}_{ext} = m\,\vec{a}_{com} = 0}$

for translational equilibrium

for static equilibrium: $\boxed{\vec{v}_{i,com} = 0}$

② $\alpha_{\text{about any axis}} = 0 \iff \boxed{\sum \tau_{\text{ext, about axis A}} = I\alpha = 0}$

for rotational equilibrium

for static equilibrium: $\boxed{\omega_i = 0}$

for any axis you choose!

Solving for Equilibrium

$$\sum \vec{F}_{ext} = 0 \qquad \sum \tau_{\text{about any axis}} = 0$$