Recap: Lecture 23

- **Center of mass and its motion:**

Where is the CoM? $\Rightarrow$ $X_{com} = \dfrac{\sum_i m_i x_i}{\sum m_i}$ $\qquad Y_{com} = \dfrac{\sum m_i Y_i}{\sum m_i}$

How does the CoM of a system of particles/objects or a composite object move?

$$\Rightarrow \sum_{i} \vec{F}_{ext,i \text{ on system}} = \vec{F}_{net,ext} = m_{total}\,\vec{a}_{com}$$

} path of CoM point determined by net external force on system!

$\Rightarrow$ if $\vec{F}_{net,ext \text{ on system}} = 0 \Rightarrow \vec{V}_{com} = \text{const}$ } NI for system of particles

- **Linear Momentum: $\vec{p}$**

for one object $\left\{ \vec{P}_{obj} = m_{obj}\,\vec{V}_{obj} \right.$

vector!

$\Rightarrow \pm$ sign for 1-D motion

$$\frac{d\vec{p}_{obj}}{dt} = \underbrace{\sum \vec{F}_{on\,obj}}_{=\vec{F}_{net,obj}} = \left(\text{rate of change of momentum}\right)$$

$$= m\,\vec{a}_{obj}$$

A **superball** and a **putty ball** are dropped onto a tabletop from the **same height**. They both have the **same mass *m***.

Which changes its momentum the most in the collision with the tabletop?

$|\vec{\Delta p}|$

A. The superball (bounce)
B. The putty ball (does not bounce)
C. Both change momentum by the same amount
D. Not enough information

$\uparrow +y$

before | after

putty ball: $\downarrow v_i < 0$ | $\bullet\ v_f = 0$, $p_f = 0$ $\Big\}\ \Delta p = p_f - p_i = -p_i$

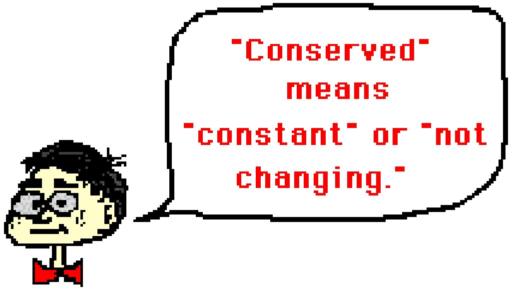
same $\Rightarrow$ same $\vec{p}_i$

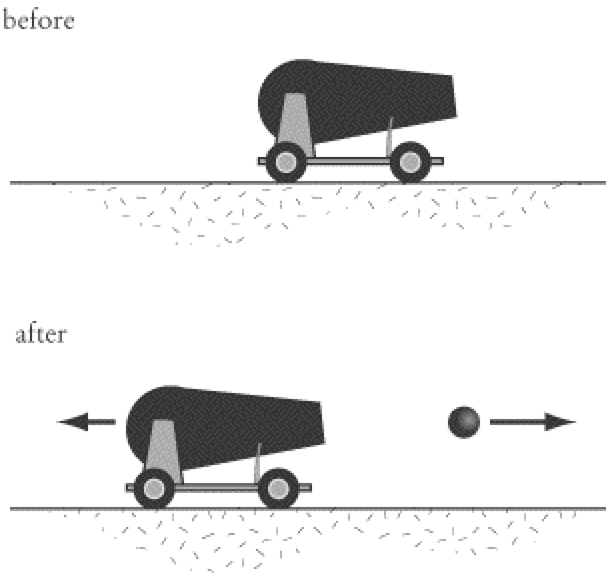
superball: $\downarrow v_i < 0$ | $\uparrow v_f \simeq -v_i$ $\Big\}\ \Delta p = p_f - p_i$
$= m v_f - m v_i$
$= -m v_i - m v_i$
$= -2 m v_i = \underline{-2} p_i$

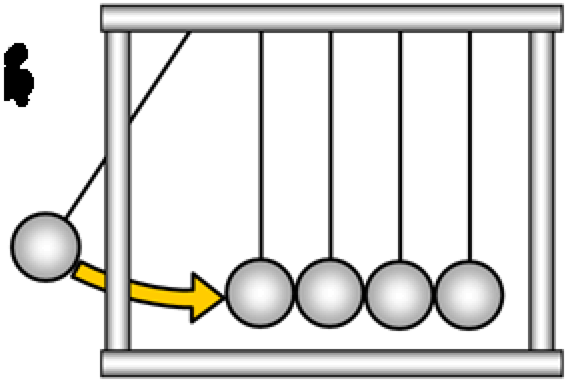
$\Rightarrow$ Momentum is a vector!

Today:

- Conservation of momentum
- Impulse
- Collisions







Now: consider a system of interacting objects:

$$\boxed{\vec{P}_{\text{total of system}} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots = m_{\text{total}} \cdot \vec{v}_{\text{com}}}$$

total momentum of system of objects

$\Rightarrow$ rate of change of total momentum:

$$\frac{d\vec{P}_{\text{total of system}}}{dt} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} + \ldots = \sum \vec{F}_{\text{on }1} + \sum \vec{F}_{\text{on }2} + \ldots$$

$\Rightarrow$ all internal forces (interaction pairs) in this total sum will cancel, leaving only external forces!

Example: assume $\vec{F}_{ext,net} = 0$

$\vec{F}_{2on1} = -\vec{F}_{1on2} \Rightarrow \frac{d\vec{p}_1}{dt} = -\frac{d\vec{p}_2}{dt} \Rightarrow \frac{d\vec{p}_{total}}{dt} = 0$

N III

$\vec{F}_{1on2} = -\vec{F}_{2on1} \Longleftrightarrow$ conservation of momentum $\Delta\vec{P}_1 = -\Delta\vec{P}_2$

Same thing!

integrate over time interval t_1 to t_2

$$\int_{t_1}^{t_2} \frac{d\vec{p}_1}{dt}\,dt = -\int_{t_1}^{t_2} \frac{d\vec{p}_2}{dt}\,dt$$

$$\Rightarrow \Delta\vec{P}_1 = -\Delta\vec{P}_2$$

$$\Rightarrow \Delta\vec{p}_1 + \Delta\vec{p}_2 = 0 = \Delta\vec{P}_{total}$$

$\Rightarrow$ Internal forces can't change the total momentum of a system of objects, but they re-distribute it among objects in the system!

$$\Rightarrow \boxed{\frac{d\vec{p}_{total}}{dt} = \sum \vec{F}_{\substack{ext\ on\\ system}} = \vec{F}_{net,ext} = m_{total}\,\vec{a}_{com}}$$

$\Rightarrow$ <u>If</u> $\sum \vec{F}_{ext,\ on\ system} = 0$ (net external force is zero)

then: $\vec{a}_{com} = 0$

$$\frac{d\vec{p}_{total}}{dt} = 0 \Rightarrow \vec{p}_{\substack{total\ of\\ system}} = \text{const}\ !$$

$\Rightarrow$ <u>Momentum is</u> <u>conserved</u> !

$\Rightarrow \vec{p}_{total,initial} = \vec{p}_{total,final}$ if $\vec{F}_{net,ext} = 0$

$\Rightarrow \vec{p}_{1,i} + \vec{p}_{2,i} + \vec{p}_{3,i} + \ldots = \vec{p}_{1,f} + \vec{p}_{2,f} + \vec{p}_{3,f} + \ldots = \text{const}$

for system of objects

An **ice skater of mass M=100 kg** is **initially at rest**, and throws a **snowball of mass m=1 kg with a speed of 10 m/s** relative to the ground. horizontally

What is the **skater's speed after throwing the snowball**? (Assume friction can be ignored.)

$|v_{skater}| = ?$

A. 0 m/s

B. 0.100 m/s

C. 1 m/s

D. 10 m/s

E. not enough information

$\sum \vec{F}_{ext} = 0$ here $\Rightarrow \vec{p}_{total} =$ const

$$\vec{p}_{total,\ initial} = 0 \overset{!}{=} \vec{p}_{total,\ final} = \vec{p}_{sb} + \vec{p}_{sk}$$

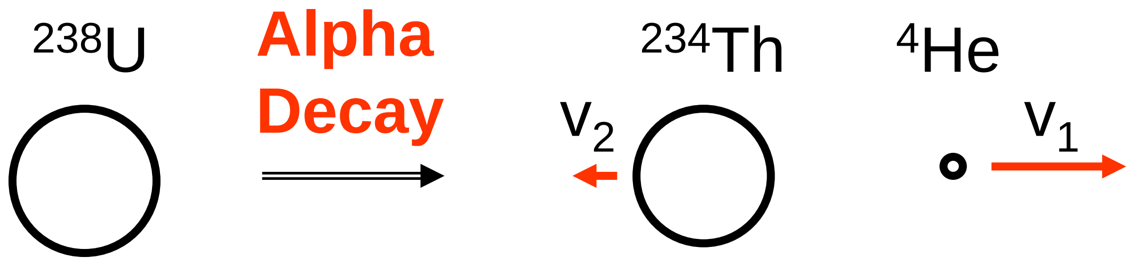
$\vec{p}_{sb}$ p_{sk}

$$p_{total} = 0 = \underbrace{m_{sb} v_{sb}}_{>0} + \underbrace{m_{sk} v_{sk}}_{<0} \Rightarrow v_{sk} = -\frac{m_{sb}}{m_{sk}} v_{sb} = -\frac{1}{100}\, 10 \frac{m}{s}$$

$$= -0.1\ m/s$$

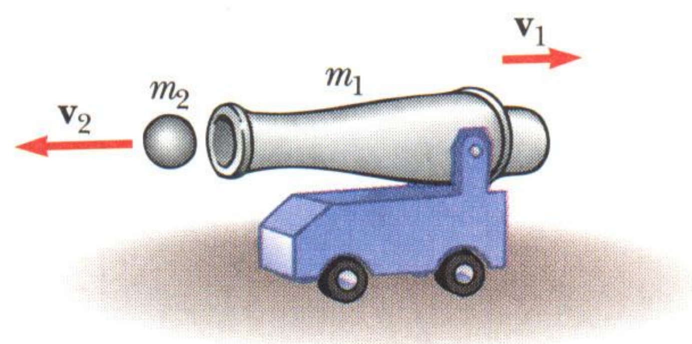
Applications of Momentum Conservation in Propulsion

Radioactive decay:



Guns, Cannons, etc.:

$\Delta \vec{p}_A = -\Delta \vec{p}_B \Rightarrow \vec{p}_{total} = const$



Rockets:

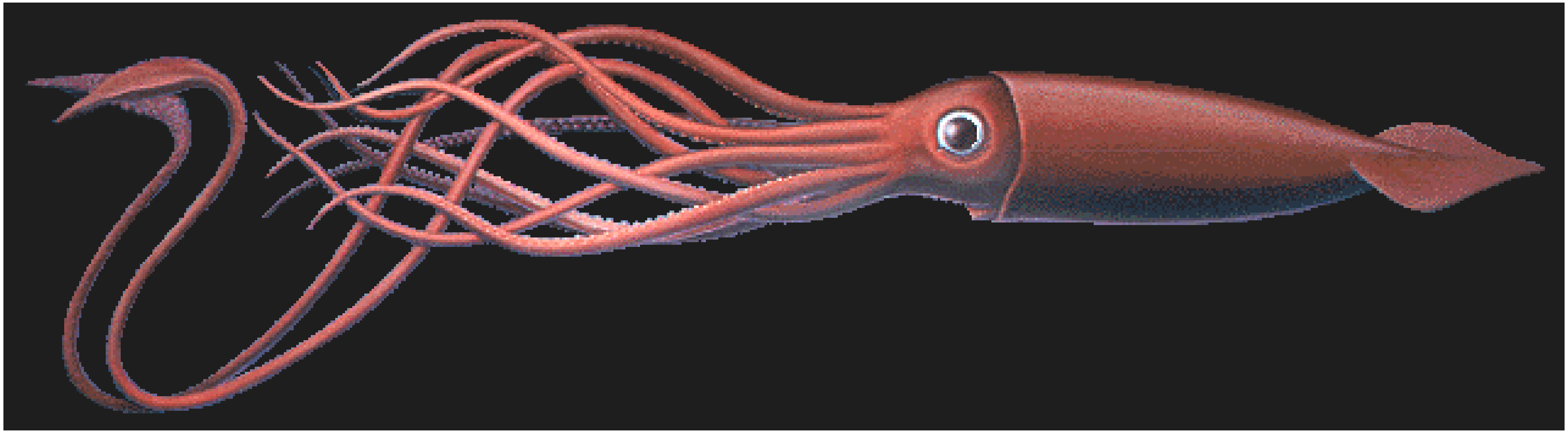
"Professor Goddard does not know the relation between action and reaction and the need to have **something better than a vacuum against which to react**. He seems to lack the basic knowledge ladled out daily in high schools."

*New York Times editorial, **1921**, about Robert Goddard's revolutionary rocket work*.

"Correction: It is now definitely established that a rocket can function in a vaccuum. The 'Times' regrets the error."

*New York Times editorial, July **1969**.*

Cephalopods (Squid, Ocotopi, Cuttlefish):



Collisions:

- brief, intense interaction between 2 (or more) object

"brief": time of interaction $\Delta t_{coll} = t_f - t_i$ short compared with Δt motion

"intense": other, external forces on objects can usually be neglected during collision ($\sum \vec{F}_{ext} \approx 0$)

$\Rightarrow$ $\vec{P}_{total}$ = const during the collision!

- Impulse $\vec{J}$:

$$\vec{J}_{\text{on object during a collision}} = \int_{t_i}^{t_f} \vec{F}_{coll}(t)\,dt = \int_{\Delta t_{coll}} \vec{F}_{coll}(t)\,dt = \left(\begin{array}{c}\text{area "under"}\\ F-t \text{ graph}\end{array}\right)$$

vector!

force on object during collision by other object

$= \vec{F}_{avg} \cdot \Delta t_{coll} = \vec{F}_{avg}(t_f - t_i)$

define average force during collision: $\vec{F}_{avg} = \vec{J}/\Delta t_{coll}$

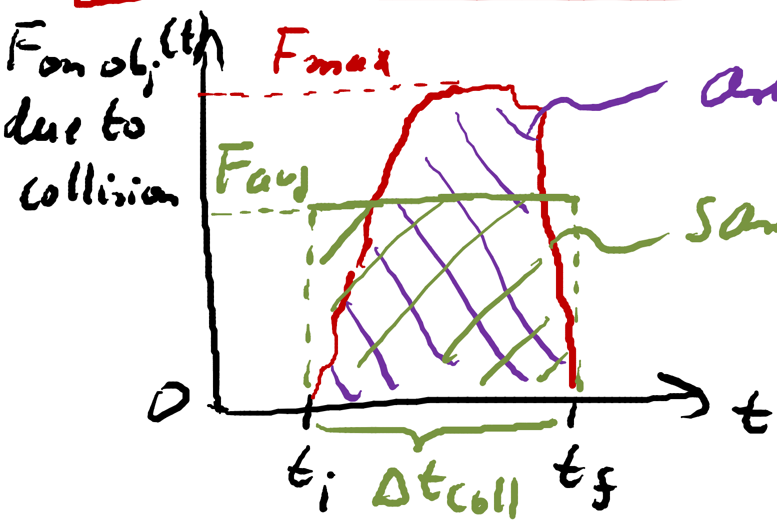
$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

– why important:

$$\underline{\vec{J}_{\text{on object}}} = \int_{t_i}^{t_f} \vec{F}_{\substack{coll \\ \text{on obj}}}\, dt = \int_{t_i}^{t_f} \frac{d\vec{p}_{obj}}{dt}\, dt = \vec{p}_f - \vec{p}_i = \underline{\Delta \vec{p}_{obj}}$$

⟹ Impulse – Momentum Relation:

$$\boxed{\vec{J}_{\substack{\text{on object} \\ \text{during collision}}} = \Delta \vec{p}_{obj} = \vec{p}_f - \vec{p}_i = \left(\begin{array}{c}\text{area "under"} \\ F\text{–}t \text{ graph}\end{array}\right)}$$



area $= |\vec{J}| = |\Delta \vec{p}_{obj}|$

same area $= |\vec{J}| = |\vec{F}_{avg}| \cdot \Delta t_{coll}$

A **Neon of mass m=1000 kg** and moving at **v_i=20 m/s** is rear-ended by a **Wegman's truck of mass M=20,000 kg**.

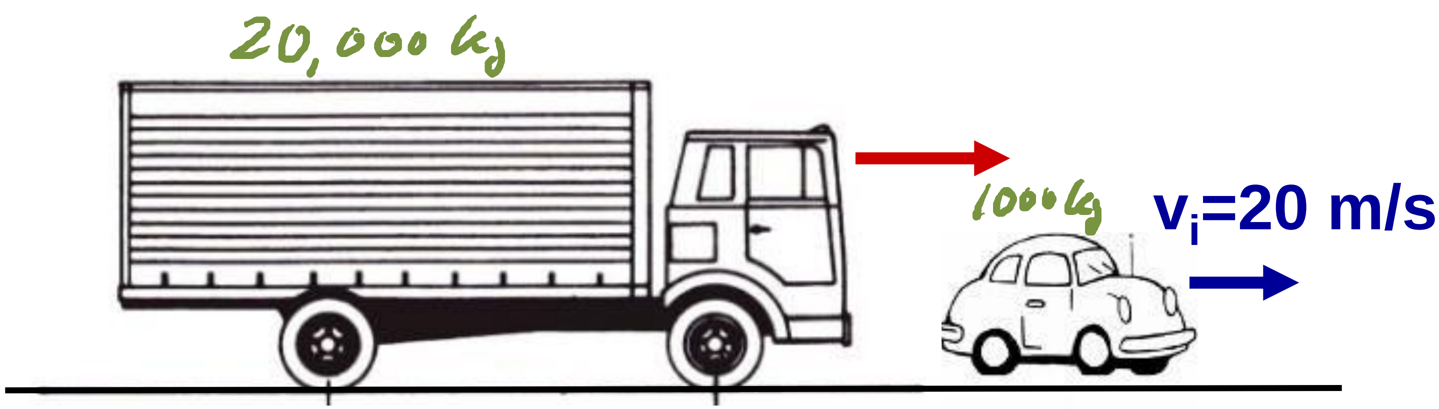
Which vehicle exerts the **larger |force|** on the other vehicle during the collision?

N III

$$\vec{F}_{N\,on\,T} = -\vec{F}_{T\,on\,N}$$

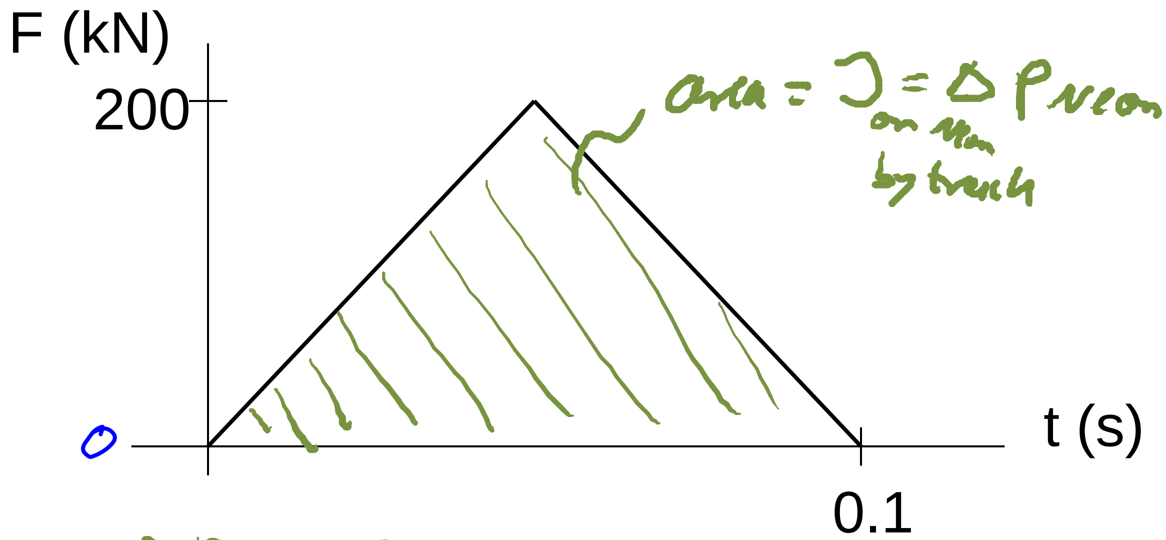
⇒ same force magnitude!

A.	The Neon
B.	The truck
C.	Both exert equal forces
D.	Not enough information



The **force** exerted **by the truck on the Neon** during the collision is shown below. $v_i = 20\ m/s$ $m_N = 1000\ kg$

What is the **final velocity** of the Neon?



v_f = ?

A. 10 m/s

B. 20 m/s

C. 30 m/s

D. 40 m/s

E. 50 m/s

$$\Rightarrow \Delta p_N = m_N v_f - m_N v_i = J = \text{area} = \frac{1}{2}\, 200\cdot 10^3 N \cdot 0.1 s = 10{,}000\ Ns$$

$$\Rightarrow v_f = \frac{J}{m_N} + v_i = 10\ \frac{m}{s} + 20\ \frac{m}{s} = 30\ m/s$$

A **Neon of mass m=1000 kg** and moving at **v_i=20 m/s** is rear-ended by a **Wegman's truck of mass M=20,000 kg**.

Which one experiences the **larger change in momentum**?

A. The Neon

B. The truck

C. Both change momentum by the same amount

D. Not enough information

$$\vec{F}_{N\,on\,T} = -\vec{F}_{T\,on\,N}$$

$$\Rightarrow \int_{\Delta t_{coll}} \vec{F}_{N\,on\,T}\,dt = -\int_{\Delta t_{coll}} \vec{F}_{T\,on\,N}\,dt$$

$$\Rightarrow J_{on\,T} = -J_{on\,N}$$

$$\Rightarrow \Delta\vec{p}_T = -\Delta\vec{p}_N \quad \Rightarrow \Delta\vec{p}_{total} = 0$$

same magnitude of force, impulse, and change in momentum!

A **Neon of mass m=1000 kg** and moving at **v_i=20 m/s** is rear-ended by a **Wegman's truck of mass M=20,000 kg**.

Which vehicle experiences the **larger acceleration** during the collision?

- A. The Neon
- B. The truck
- C. They have the same magnitude of acceleration
- D. Not enough information

$$|F_{N \text{ on } T}| = |F_{T \text{ on } N}|$$

$$\Rightarrow |m_T a_T| = |m_N a_N|$$

$$\Rightarrow \left|\frac{a_N}{a_T}\right| = \frac{m_T}{m_N} = \frac{20}{1}$$

$\Rightarrow$ That's why you want to be in the truck and not the neon during the collision...

A **Neon of mass m=1000 kg** and moving at **v_i=20 m/s** is rear-ended by a **Wegman's truck of mass M=20,000 kg**.

Which ***driver*** experiences the **larger impulse** during the collision?

A. The driver of the Neon

B. The driver of the truck

C. Both experience the same impulse

$$\left|\frac{F_{\text{on driver of N}}}{F_{\text{on driver of T}}}\right| = \left|\frac{m_{\text{driver}}\, a_N}{m_{\text{driver}}\, a_T}\right| = \frac{20}{1} \text{ from before}$$

$$\Rightarrow |J_{\text{on driver of neon}}| \gg |J_{\text{on driver of T}}|$$

$$\left(\vec{J} = \int_{\Delta t_{coll}} \vec{F}\, dt\right)$$