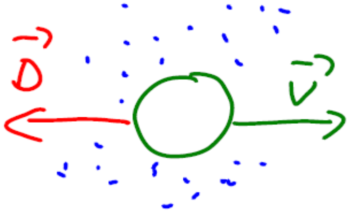
Recap: Fluid Friction - Drag Force



- fluid: gas or liquid
- Drag force: force of liquid on object; opposes relative motion

$$D_{\substack{\text{turbulent}\\ \text{on object}}} = \frac{1}{2} C_{obj}\, \rho_{fluid}\, A_{object}\, v^2_{\text{object relative to fluid}}$$

- A_{obj}: effective cross-sectional area of object perpendicular to $\vec{v}$

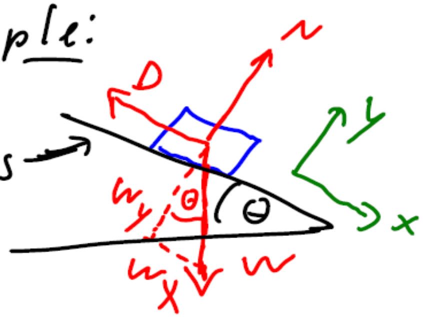
Example:

- D o $\Rightarrow$ object reaches terminal speed:

at $v = v_t$: $\vec{D}$ such that $\vec{a}_{obj} = 0$

Example:

frictionless



$$\text{at } v = v_t:\ \Sigma F_x = 0 = W_x - \overbrace{D(v_t)}^{\text{drag force at } v = v_t}$$

$$\Rightarrow D(v_t) = W_x \quad \text{here}$$

$$\Rightarrow \frac{1}{2} C \rho A v_t^2 = mg \sin\theta$$

Which force in a Gravitron makes you go on a circle?

A. Gravity

B. Friction

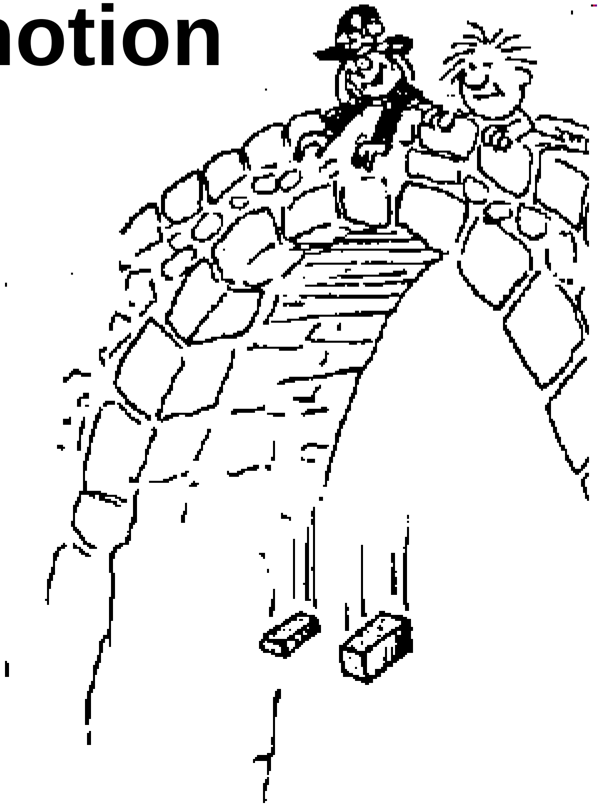
C. Normal force

D. Spring force

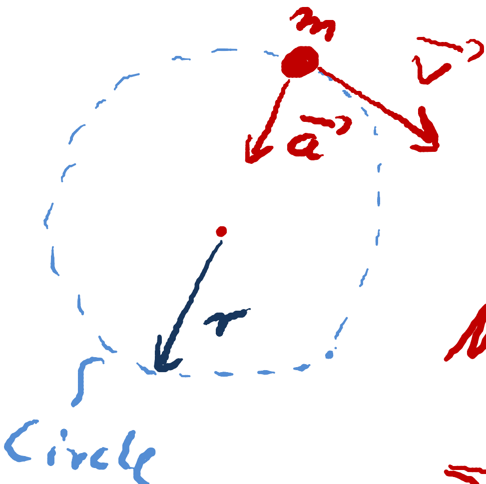
E. Tension

Today:

- Forces in uniform circular motion
  - Gravitrons
  - A spinning water bucket
- Work and Energy:
  - Kinetic energy
  - Work done by a single force



Forces in uniform circular motion



$a = \frac{v^2}{r}$, points toward center of circle; $|\vec{v}| = \text{const}$

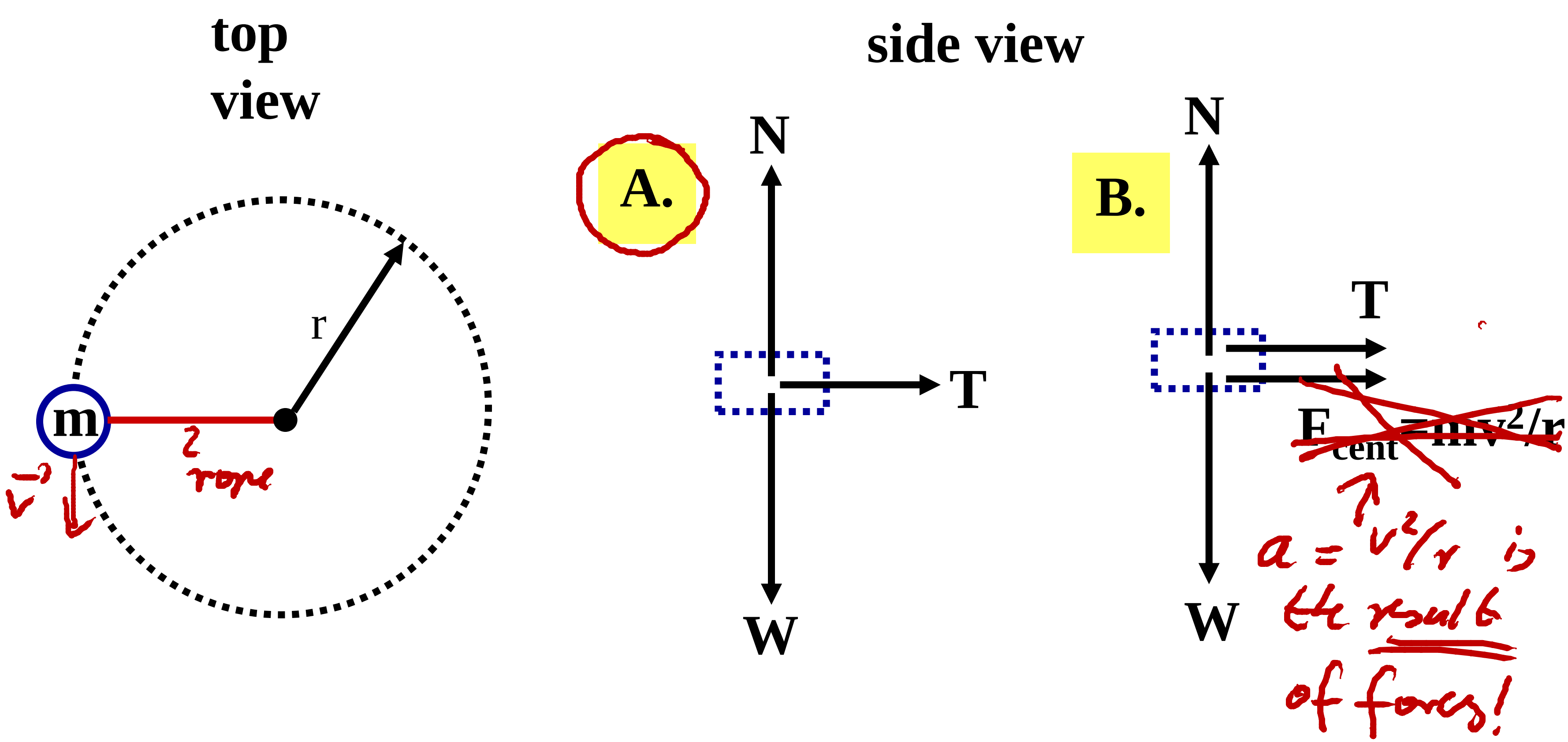
Now: If $a = \frac{v^2}{r}$, then according N II:

$$\underbrace{\sum \vec{F}_{\text{on object}}}_{\text{cause of motion}} = m\vec{a} \text{ such that } \underbrace{|\vec{a}| = \frac{v^2}{r}}_{\text{effect of force on object}}$$

and $\sum \vec{F}_{\text{on obj.}}$ points to the center of the circle, $\perp$ to path at each point along the path.

A mass *m* rotates with constant speed at the end of a rope in a circle of radius *r* on a horizontal frictionless surface.

Which of the following is the correct **free-body diagram** for the **forces acting on the mass**?



- $\sum \vec{F}_{on\ object}$ is <u>cause</u> of $\vec{a}$, of circular motion!

  $a = \frac{v^2}{r}$ is effect of forces

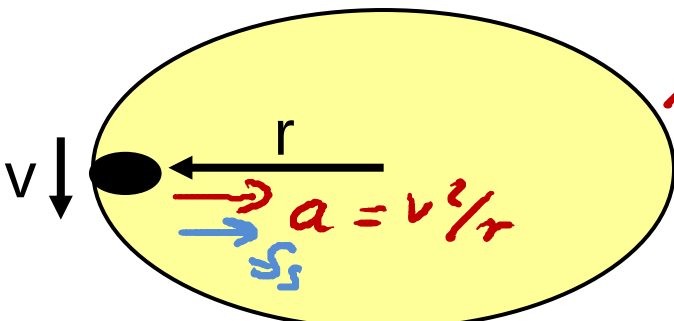
  $\Rightarrow |\sum \vec{F}_{on\ obj}| = m|\vec{a}| = m\frac{v^2}{r}$ if moving in a circle at speed v

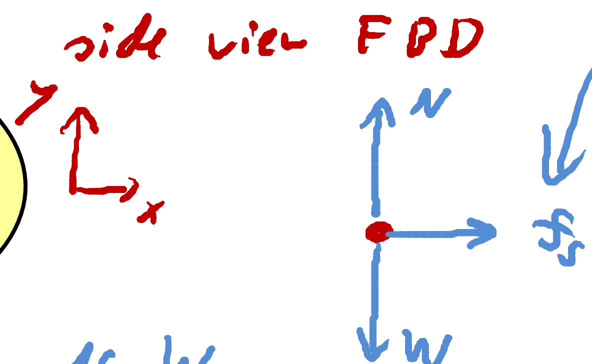
- no mysterious forces! (only gravity, forces direct physical contact)
- Never show $\sum \vec{F}_{on\ obj} = \vec{F}_{net}$ on a FBD!

  $\Rightarrow$ never show $m\frac{v^2}{r}$ as a force on a FBD for circular motion!

A coin of mass m rests at the edge of a horizontal platter of radius r.

If the coefficient of static friction between coin and platter is μ_s, what is the **maximum speed** v of the coin so that the **coin does not slip**?





cause of circ. motion

v_{max} = ?

A. m g r

B. $\sqrt{m\,g\,r}$

C. μ_s g r

D. $\sqrt{\mu_s\,g\,r}$

E. $(\mu_s\,g\,r)^2$

check units!

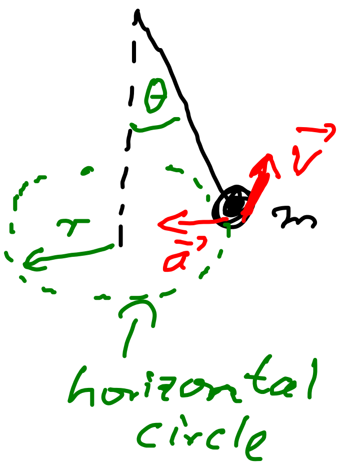
$\Rightarrow \sum F_y = m a_y = 0 = N - W \Rightarrow N = W = mg$

$\Rightarrow \sum F_x = m a_x \overset{!}{=} m \frac{v^2}{r} = f_s \le (f_s)_{max} = \mu_s N$

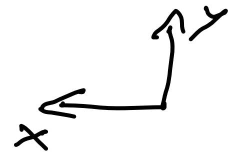
$\Rightarrow \frac{m v_{max}^2}{r} = \mu_s \underbrace{mg}_{= N \text{ here}} \Rightarrow v_{max} = \sqrt{\mu_s g r}$

More Examples:

① Conical Pendulum:



side view of m:



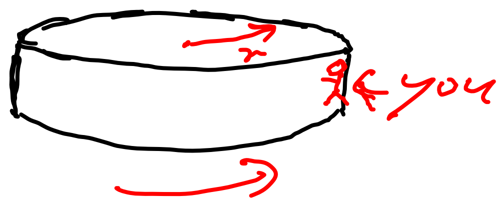
x ← along direction of acceleration!

$$\sum F_y = m\,a_y = 0 = T_y - W \Rightarrow T_y = W = mg$$

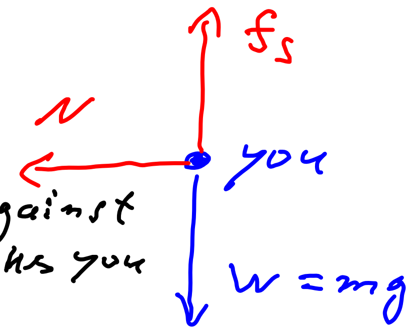
$$\sum F_x = m\,a_x \overset{!}{=} m\frac{v^2}{r} \quad \Big\}\text{ for circ. motion!}$$

$$= T_x \Rightarrow T_x = \frac{mv^2}{r}$$

② Gravitron:



side-view FBD:



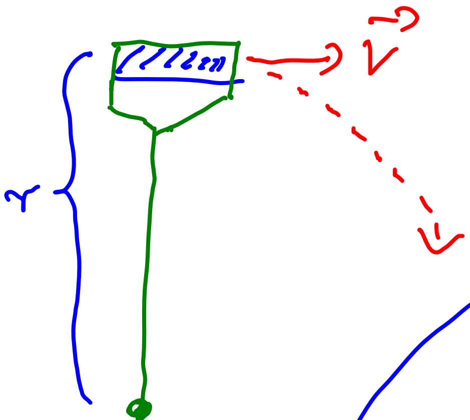
N force of wall against your back makes you go on circle

⇒ If you don't fall: $f_s = W \le (f_s)_{max} = \mu_s N$

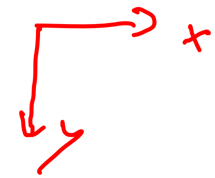
If going on circle: $N = m\frac{v^2}{r}$

Needs to go fast enough, or you fall!

③ Water in bucket:



FBD at top:



$$\sum F_y = W + N = m a_y \overset{!}{=} m\frac{v^2}{r} = mg + \underbrace{N}_{\geq 0}$$

$\Rightarrow \quad v \geq \sqrt{rg}$, or you get wet...

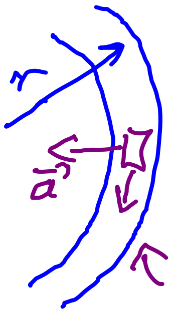
Note: normal force points down here!

Note!!!!!!

4 Car on banked curve:

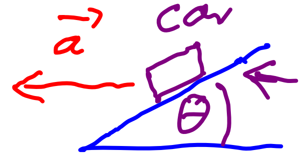
along $\vec{a}$!

top view:



car travels in horizontal circle of radius r at speed v

side view



assume $\mu_s, \mu_k = 0$ here

FBD of car, side view

$$\Rightarrow \sum F_y = m_{car}\, a_y \doteq 0 = N_y - w$$

$$\Rightarrow N_y = w = mg$$

$$\Rightarrow \sum F_x = N_x = m_{car}\, a_x \overset{!}{=} m\frac{v^2}{r} \Big\}\text{ to go on circle}$$

$$\Rightarrow N_x = \frac{mv^2}{r} = N_y \tan\theta$$

$$= mg \tan\theta$$

works only at one speed v for given θ

Until now:

- how things move: $\vec{r} \rightleftarrows \vec{v}(t) \rightleftarrows \vec{a}(t)$ (Kinematics)
- why things move (I): Forces, Newton's laws

$$\sum \vec{F} = m\vec{a}$$

Next: why things move (II):

Energy and Work

Types of energy:

Energy due to position or motion of object } potential energy, kinetic energy

thermal, chemical, nuclear, ... } "Internal energy"

What is energy?

- Energy: Scalar associated with the state of an object
- state (condition): position, velocity, temperature, chemical bonding state, ...
- Energy can be transformed from one type to another type, and transferred from one object to another.
- Total amount of energy is always the same! (Energy is conserved!)

Kinetic Energy K:

- Energy associated with the state of <u>motion</u> of an object

Equation? $\boxed{K = \frac{1}{2} m_{obj} v^2}$

$v = 0 \rightarrow K = 0$

$v \uparrow \rightarrow K \uparrow$

Units? $[K] = [\text{energy}] = kg \frac{m^2}{s^2} = N\,m = \text{Joule} = \underline{\underline{J}}$

$N = kg \frac{m}{s^2}$

Work: $\underline{\underline{W}}$ What is work?

work = energy transferred to or from an object by a force acting on the object

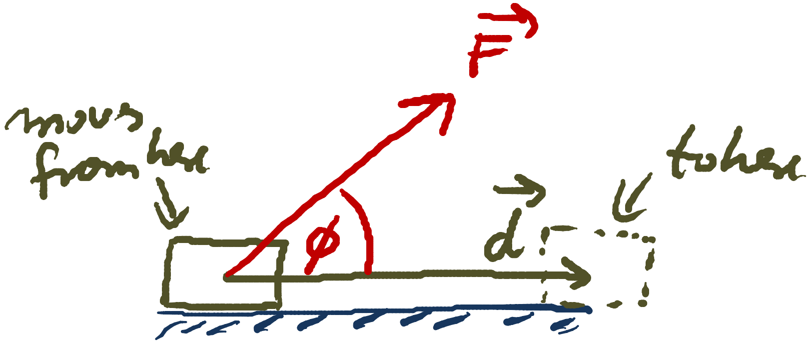
- If $W_{\text{done on object}} > 0 \iff$ energy is transferred to object
- If $W_{\text{done on object}} < 0 \iff$ energy is transferred from object

Equation? look at units:

$$[\text{work}] = [\text{energy}] = J = N\,m$$

$$\Rightarrow W \propto F d$$

↑ displacement of object



ϕ : angle between $\vec{F}$ and $\vec{d}$

$\vec{d}$: displacement vector $= \vec{r}_2 - \vec{r}_1$

$$\text{Work} = W_{\substack{\text{by } F \\ \text{on object}}} = F d \cos\phi$$

note: if $\phi = 90^\circ$

$\Rightarrow W_{\text{done}} = 0$