Recap: Lecture 11,

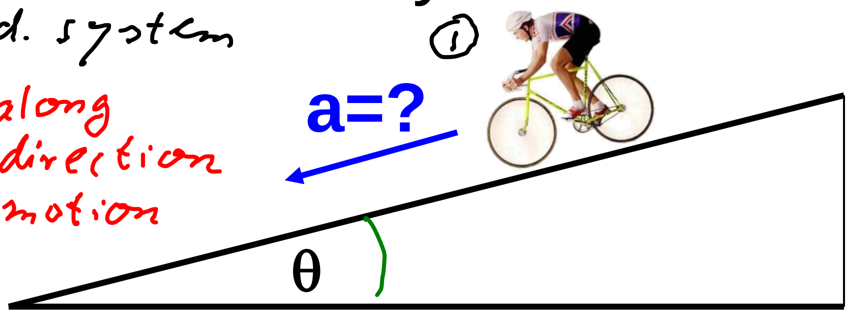
What is the acceleration of a cyclist if he coasts without pedaling down Buffalo St.? (Neglect air resistance and any other forces from friction)

② choose coord. system

y, x

along ← direction of motion

① **a=?**



Eddy St.

Aurora St.

③ FBD of cyclist:

$\vec{N}$ ← always ⊥ to surface!

w_x, w_y, θ, $W = mg$

④ resolve into components:

$W_x = W \sin\theta$ } these are the magnitudes of the components

$W_y = W \cos\theta$

check for θ=0!

⑤ Use N II:

$\Sigma F_x = m a_x = + W_x$

$= W \sin\theta$

$= mg \sin\theta$

$\Rightarrow \boxed{a_x = g \sin\theta}$

$\vec{N}$ self-adjusts, so that $a_\perp = 0$

$\Sigma F_y = m a_y = 0$

"–" sign for direction

$= N - W_y$

$\Rightarrow N = W_y = mg\cos\theta$

Note: $N \neq W$ here!

a=?

A. g cos (θ)

B. g sin (θ)

C. g tan (θ)

D. g / cos (θ)

E. g / sin (θ)

T=?

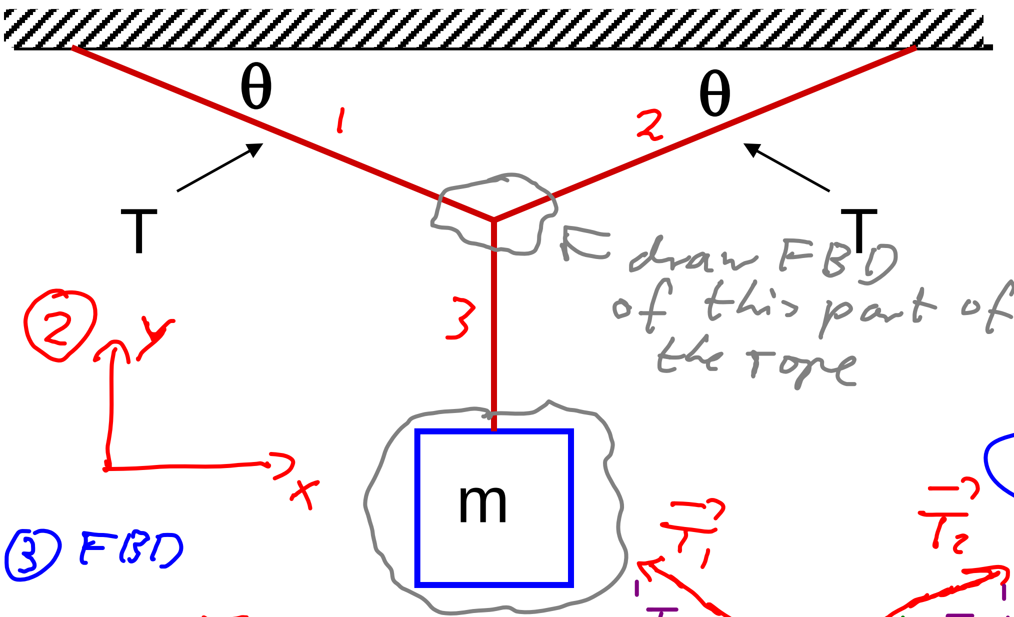
A. mg / 2

B. mg /sin(θ)

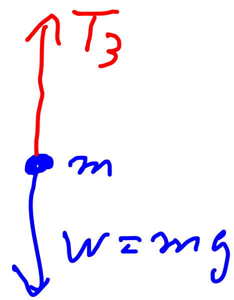
C. mg / cos(θ)

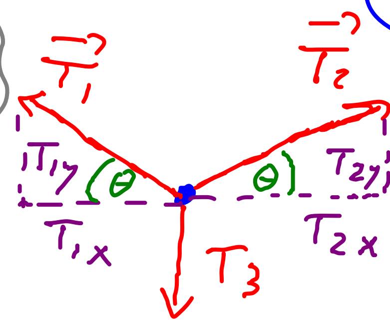
D. mg / 2 sin(θ)

E. mg / 2 cos(θ)



③ FBD





$\Sigma F_y = 0$

$\Rightarrow \underline{T_3 = mg}$

$\Sigma F_x = m a_x = 0 = T_{2x} - T_{1x} \Rightarrow T_{1x} = T_{2x}$

$\Sigma F_y = m a_y = 0 = T_{1y} + T_{2y} - T_3 = T_{1y} + T_{2y} - mg$

symmetry: $T_{1y} = T_{2y} = T_y = T \sin\theta$

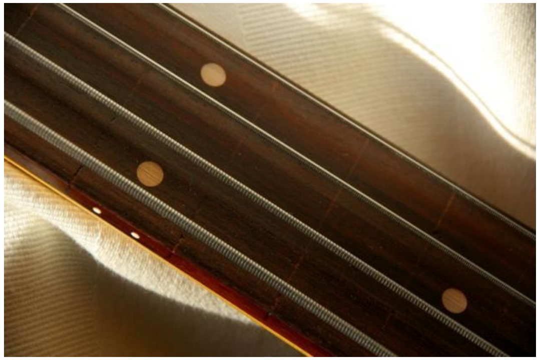
$\Rightarrow 2T_y - mg = 0 \Rightarrow T_y = mg/2 = \underline{T} \sin\theta$

$\Rightarrow T = mg/2\sin\theta$

Other Applications

- sag of power lines

- power line and pole snapping by trees, ice



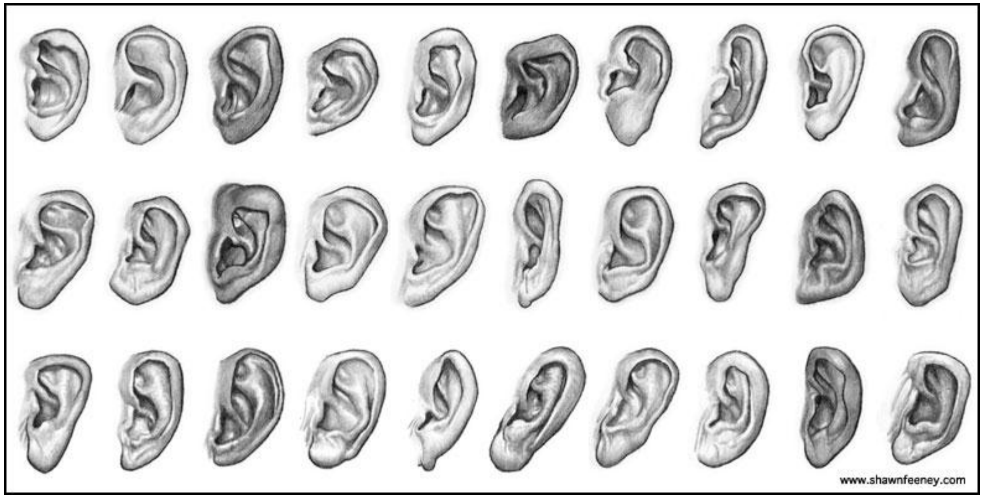
- plucking of guitar strings
- retrieving your car from a ditch

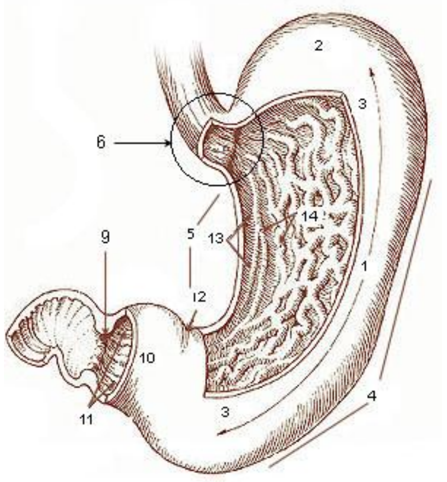
How does your body detect acceleration?

A. With the eyes

B. With the ears

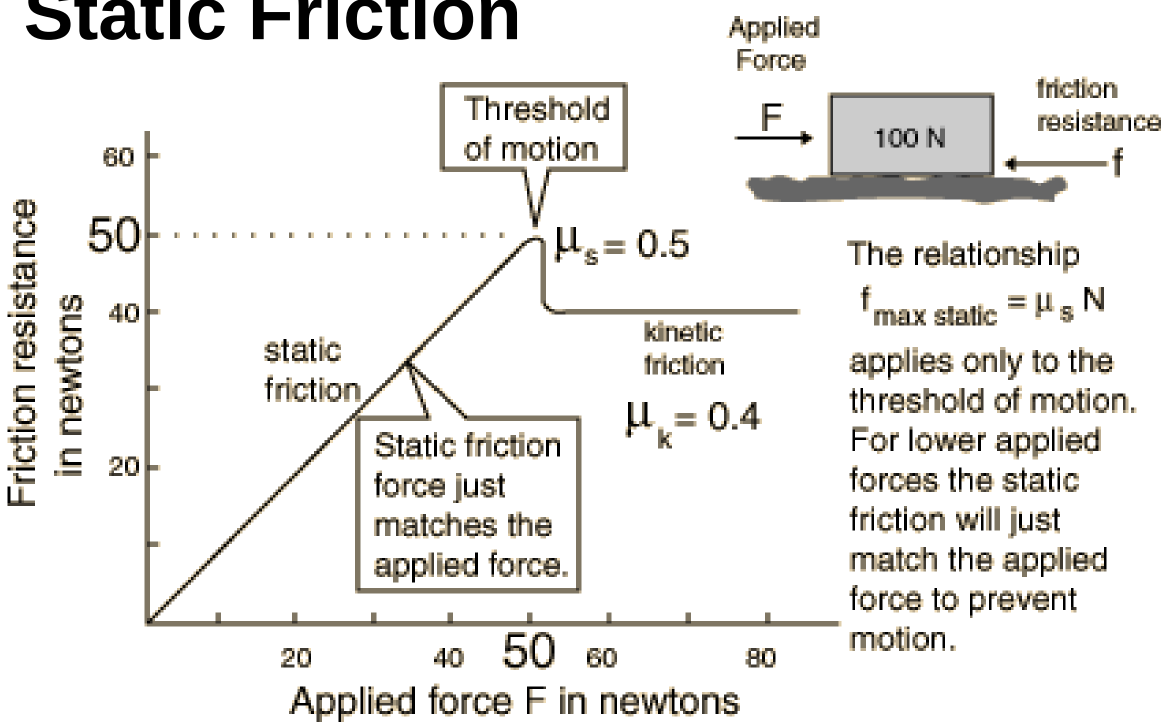
C. With the stomach



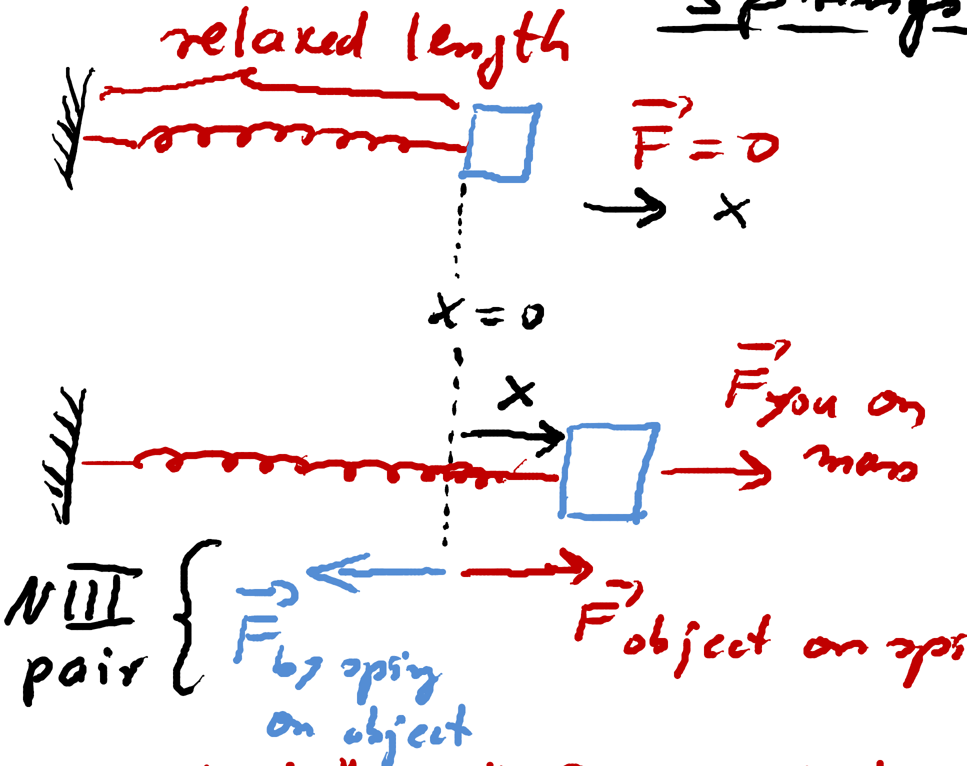


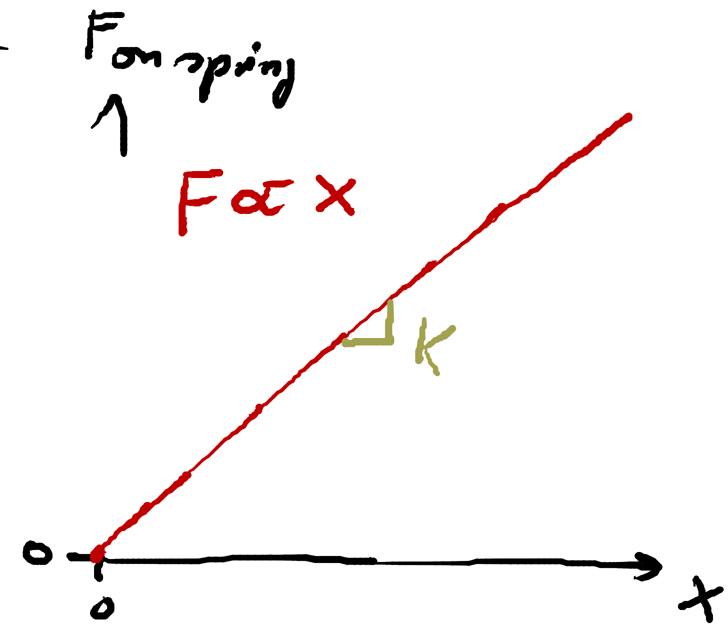
Today:

- **Forces**
  - Spring forces
  - Springs in our bodies
  - Solid on solid friction
    - Static Friction



Springs:





K: spring constant

$[k] = N/m$

Hooke's "Law" for an ideal spring

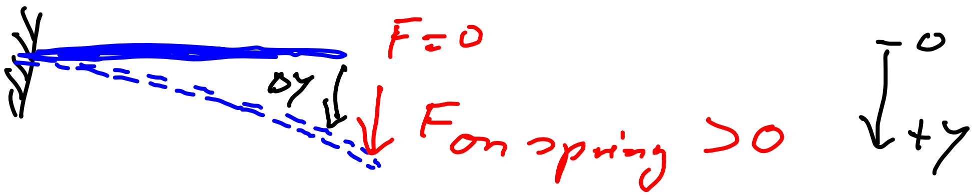
$F_{by\ spring\ on\ object} = -Kx$

restoring force: acts in direction to try to restore spring back to its equilibrium length

$F_{by\ object\ on\ spring} = +Kx$

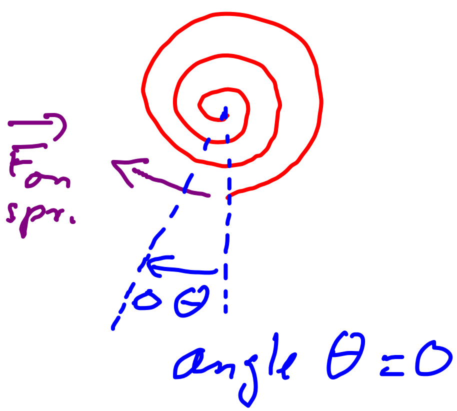
Other types of springs:

- **Cantilever spring:**



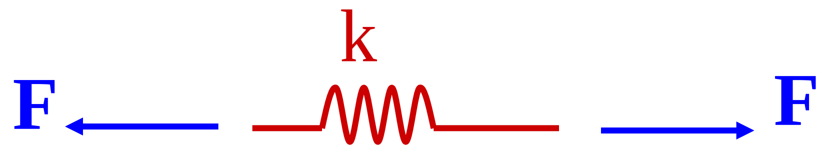
$$\boxed{F_{\text{on spring}} = k\,\Delta y}$$

- **Coil spring:** (watch...)

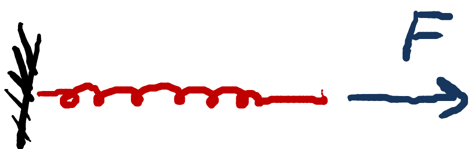


$$F \propto \Delta\theta$$

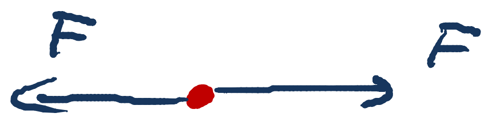
By how much does the spring stretch from its relaxed length?

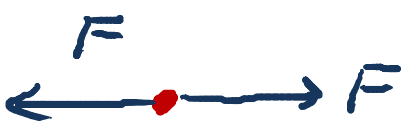


↓ FBD



↓ FBD





same!

$F = kx$

$x = F/k$

$F = kx$

$x = F/k$

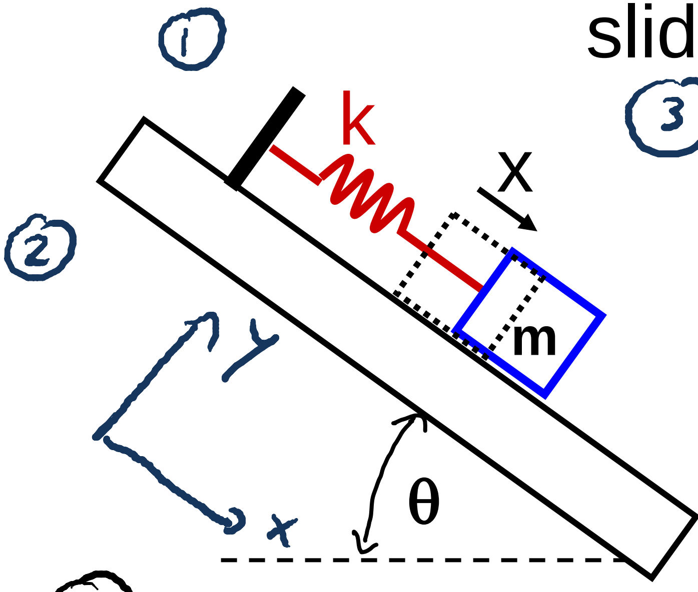
x=?

A. x=F/2k

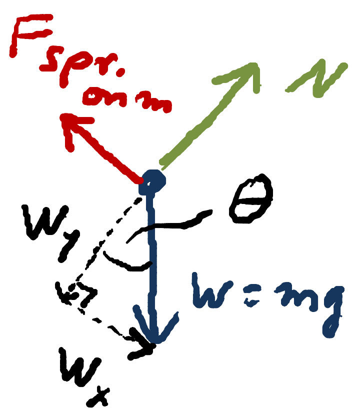
B. x=F/k

C. x=2F/k

The spring has stretched an amount x. What is the angle θ? (Assume the surface on which the mass m slides is frictionless.)



3 FBD of m



θ=?

A. $\sin^{-1}$ (kx/mg)

B. sin (kx/mg)

~~C.~~ $\cos^{-1}$ (kx/mg)

~~D.~~ cos (kx/mg)

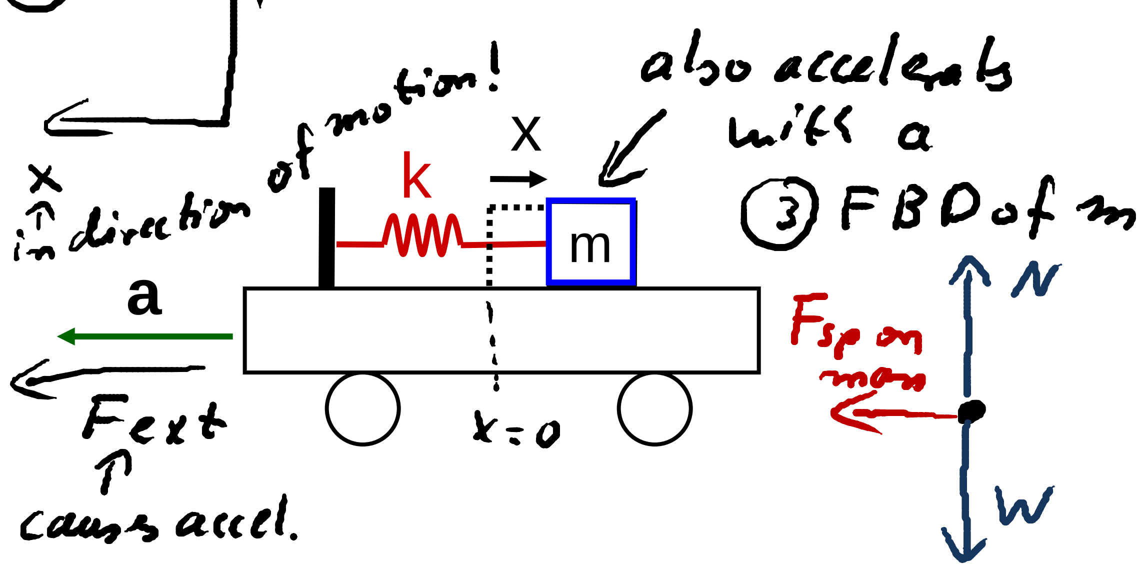
4 $W_x = mg\sin\theta \qquad |W_y| = mg\cos\theta$

5 $\Sigma F_x = m\underbrace{a_x}_{\vec{a}=0\text{ here}} = 0 = W_x - |F_{sp\ on\ mass}| = mg\sin\theta - kx = 0$

$\Rightarrow \sin\theta = kx/mg$

$\Rightarrow \theta = \sin^{-1}\left(kx/mg\right)$

The spring has stretched an amount x. What is the acceleration a? (Assume the surface on which the mass m slides is frictionless.)



a = ?

A. a=-kx/m

B. a=-k x m

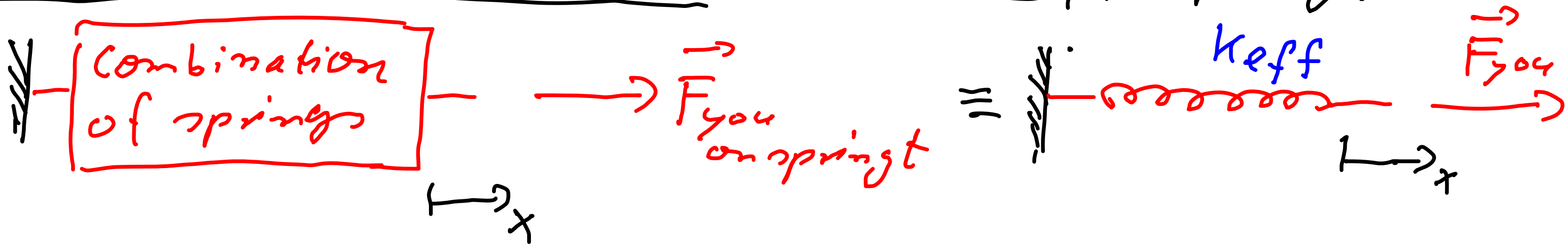
C. a=-kx

D. Insufficient information

$$N\,II: \quad \sum F_x = m a_x = F_{\text{by spring on mass}} = -k\underset{x<0}{x}$$

$$\Rightarrow a_x = -\frac{kx}{m}$$

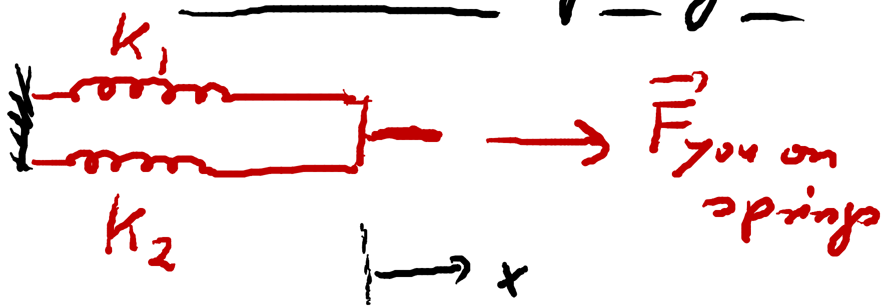
Series and parallel combinations of springs:



- For a given F on springs, k_{eff} gives the same stretch x as the combination of springs.

- Ask: which quantity – F or x – is the same for the individual springs and for the "effective" spring, and which quantities add to give the quantity for the "effective" spring?

Parallel Springs:



Same: $x_1 = x_2 = x$

add: forces

$$F_{\text{you on spring}} = F_1 + F_2$$

$$\Rightarrow \quad F_{\text{you on springs}} = F_1 + F_2 = k_1 x_1 + k_2 x_2 = k_1 x + k_2 x$$

$$= (k_1 + k_2)\, x$$

$$F_{\text{on springs}} = k_{eff} \cdot x$$

with $\boxed{k_{eff} = k_1 + k_2}$

Note: $k_{eff} > \max\{k_1, k_2\}$

Series of Springs:

Same: forces

add: Δx

$\vec{a} = 0 \Rightarrow \Sigma \vec{F} = 0$

$F = k_1 x_1$

$F = k_2 x_2$

$$x = x_1 + x_2 \Rightarrow \frac{F}{k_{eff}} = \frac{F}{k_1} + \frac{F}{k_2}$$

$F_{\text{on spring}} = k_{eff} \cdot x$

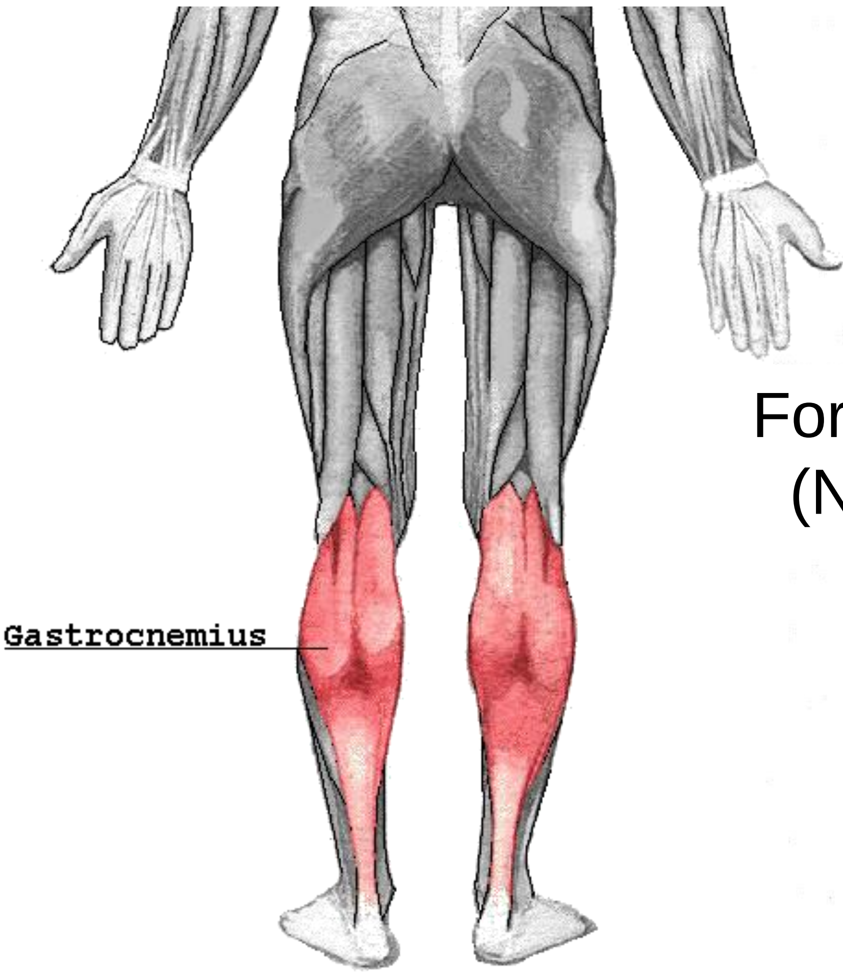
$$= F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

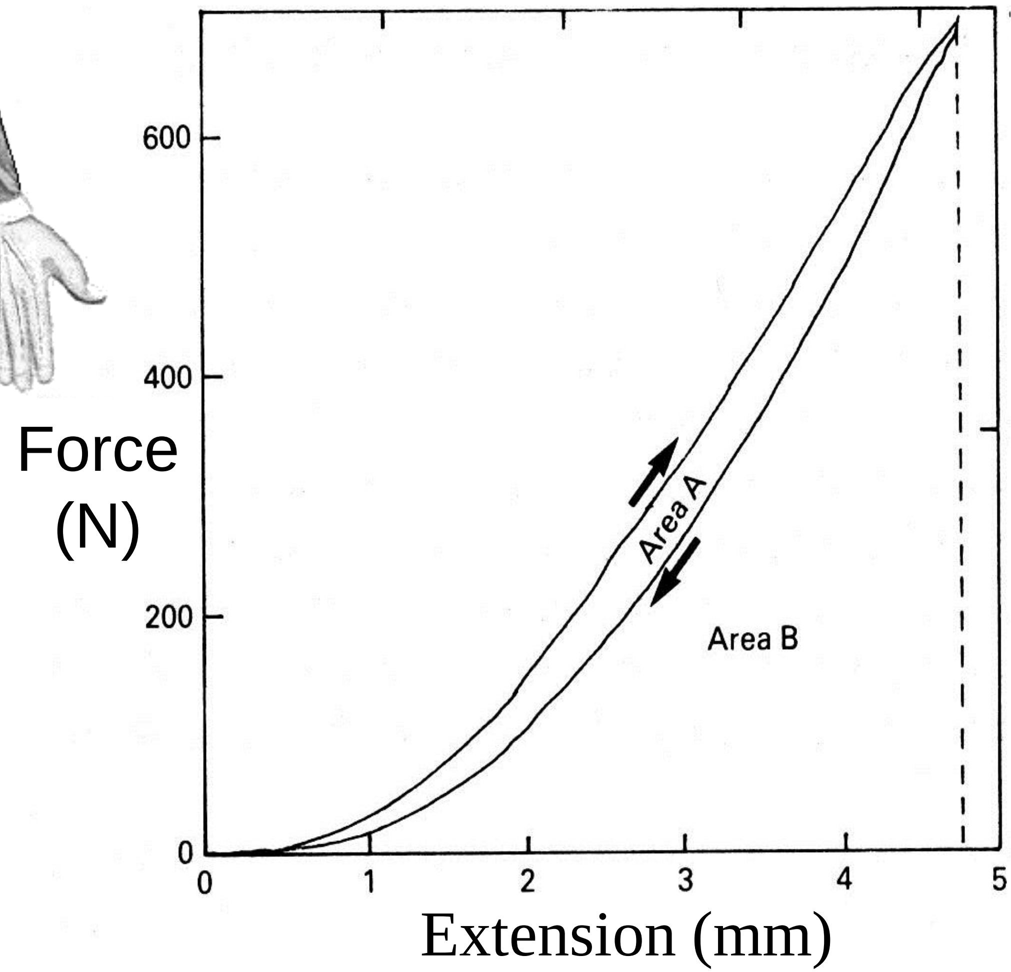
$$\Rightarrow \boxed{\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}} \quad \text{or} \quad k_{eff} = \frac{k_1 \cdot k_2}{k_1 + k_2}$$

Note: $k_{eff} < \min\{k_1, k_2\}$

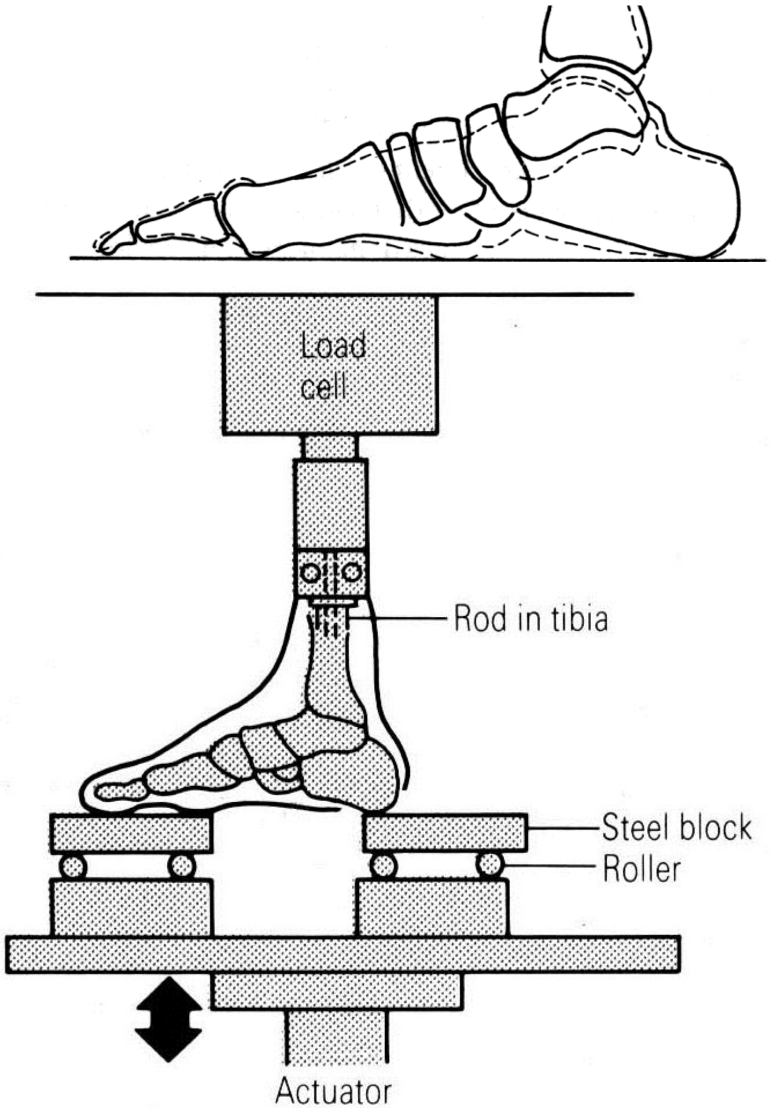
Springs in our Bodies

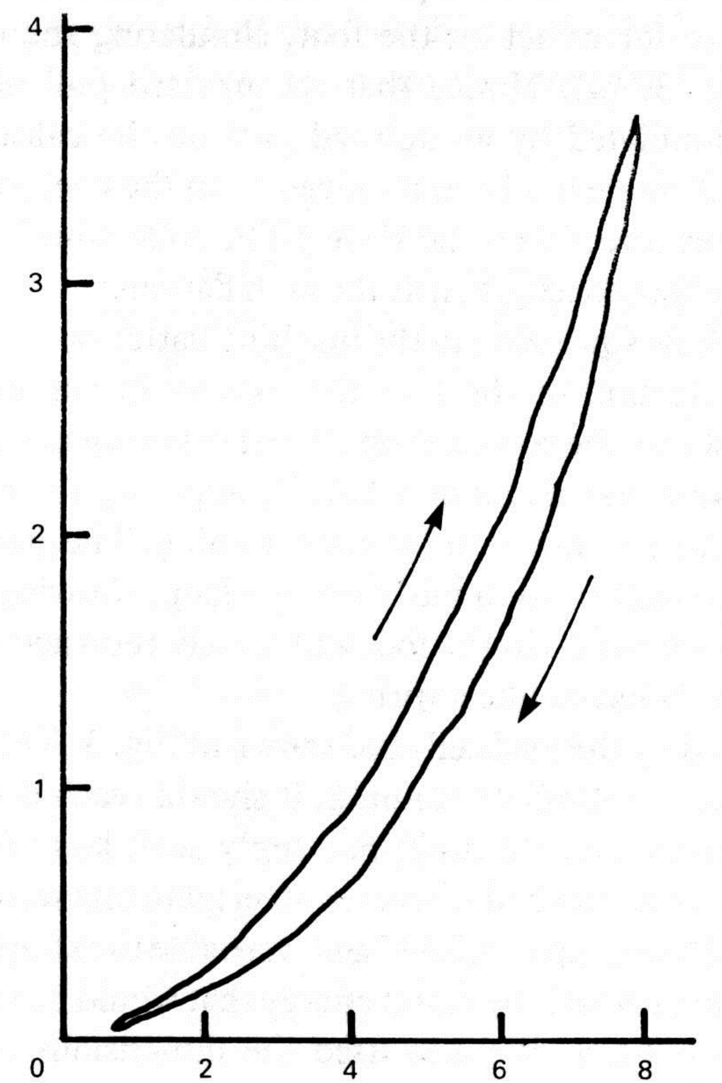
F-x relation for a gastrocnemius tendon:



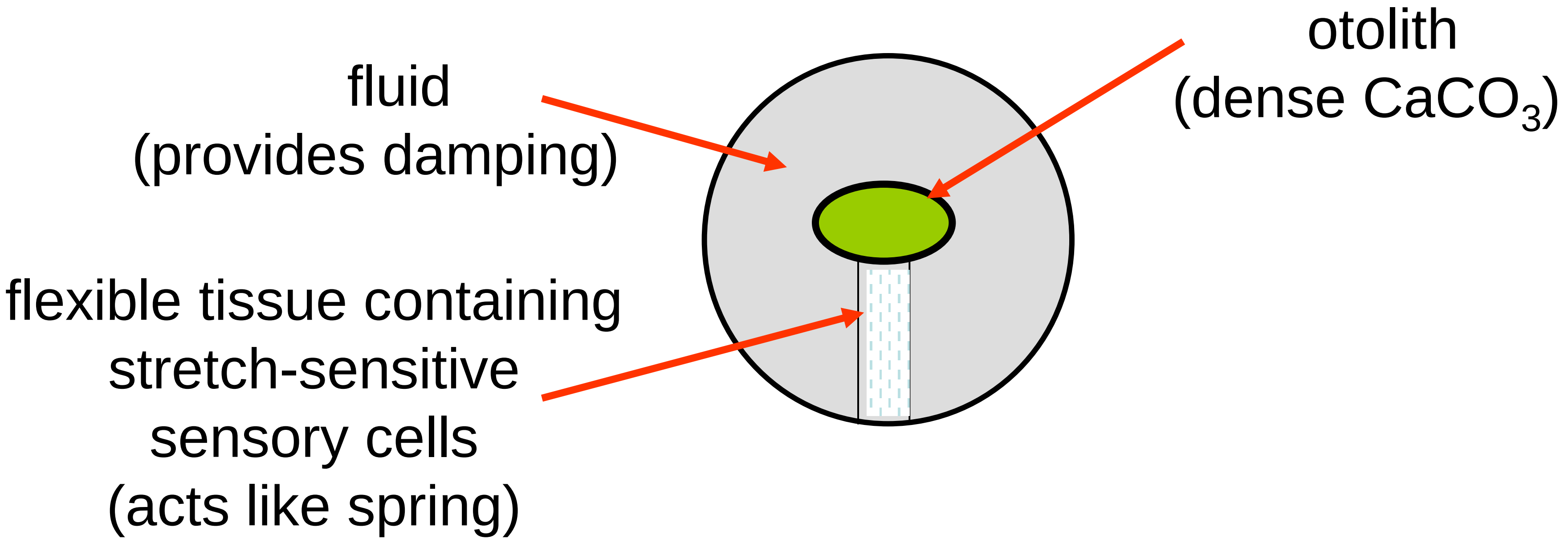


F-x relation for a foot arch:

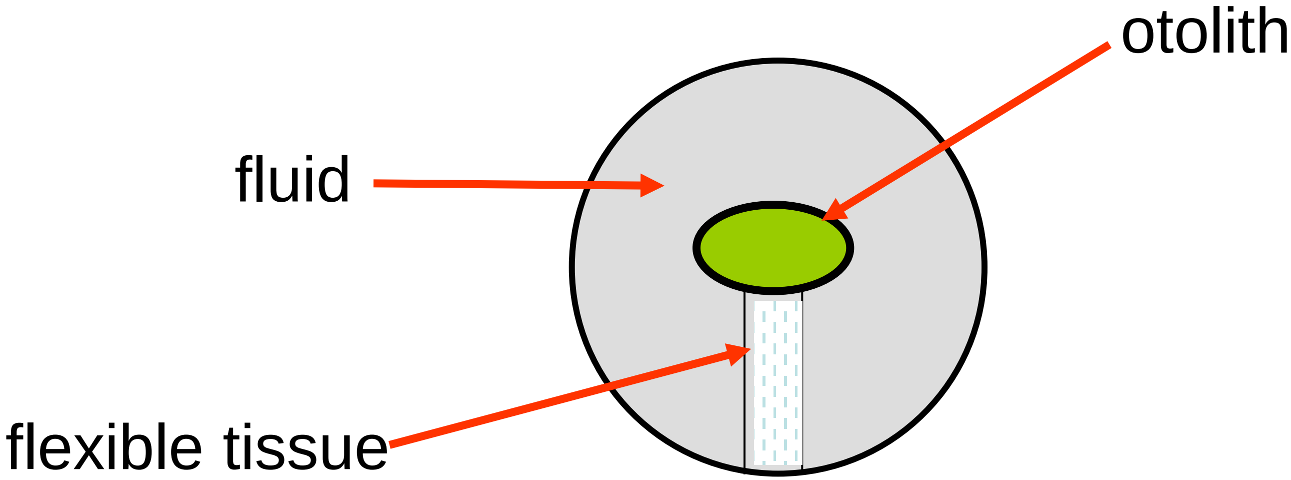




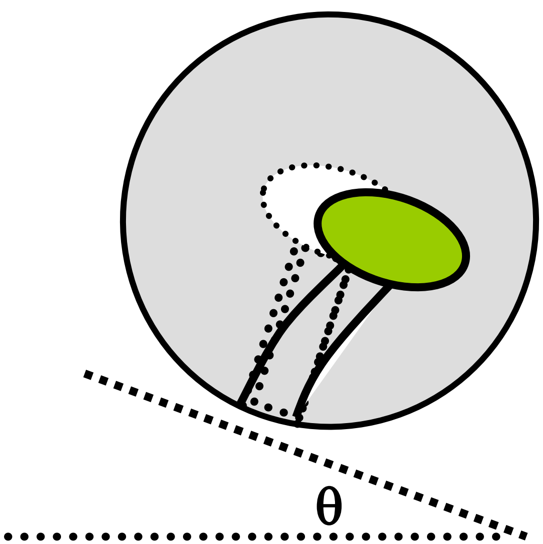
Otolith Organ



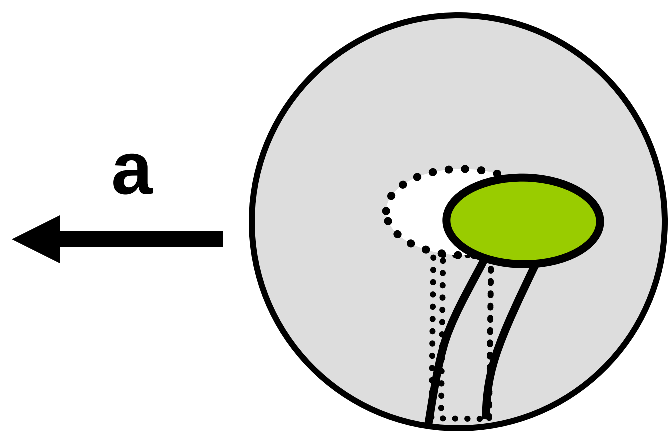
- All vertebrates have at least 2 or 3 in each ear
- Measures orientation and acceleration.



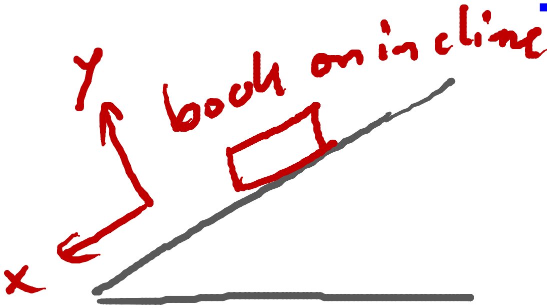
Detecting orientation:

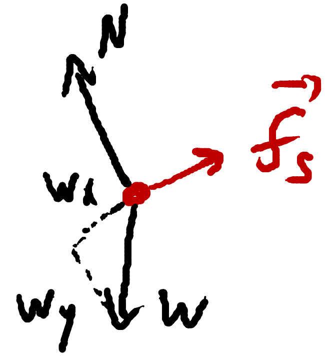


Detecting acceleration:



Solid on solid friction:



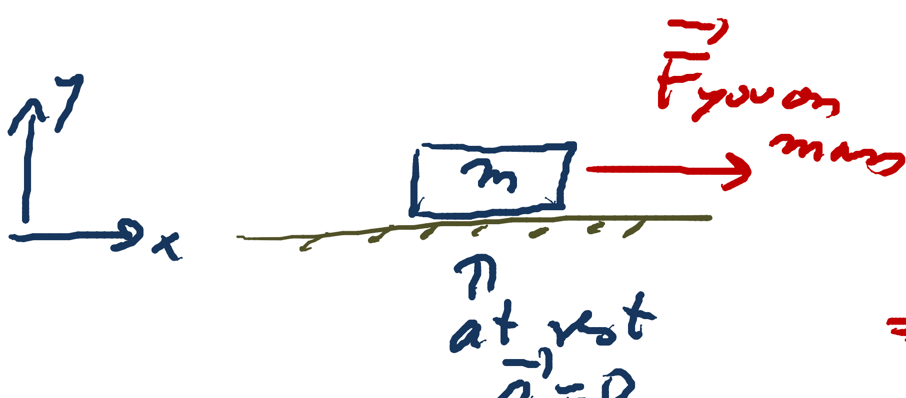


book does not slide!

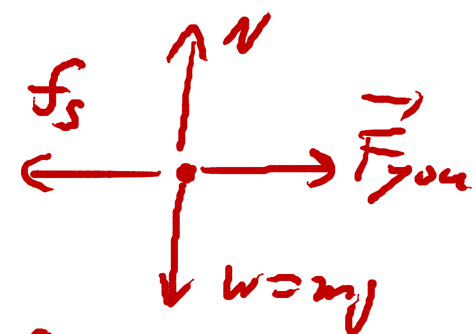
$\Rightarrow \vec{a} = 0 \Rightarrow \Sigma \vec{F} = 0$

$\Rightarrow$ friction force

- case 1: static friction force $\vec{f}_s$
  - prevents relative motion of surfaces in contact
  - Example:



FBD $\Rightarrow$



$\Rightarrow \Sigma F_x = m a_x = 0 \Rightarrow F_{you} = f_s$!

$\vec{f}_s$ is $\parallel$ to surface

coefficient of static friction (surface property)

$$\boxed{f_s \leq (f_s)_{max} = \mu_s N}$$

normal force on object

static friction force $\vec{f}_s$ self-adjusts to cancel $\vec{F}$ to prevent relative motion, but only up to maximum value $(f_s)_{max}$