Lecture 2

1.4 RF field accelerators

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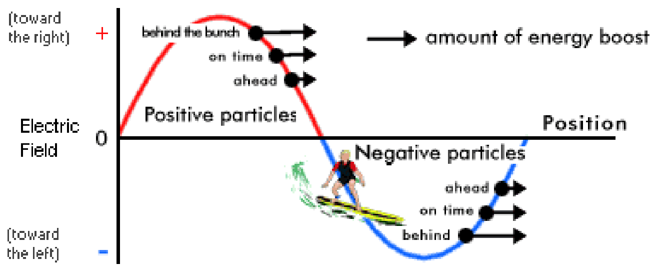
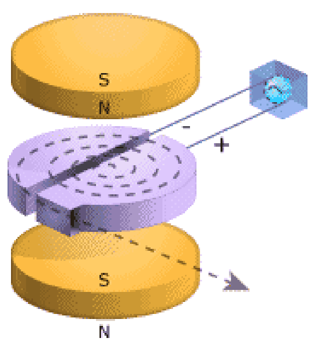
Cyclotron
Microtron
Wideroe linear accelerator
Accelerating cavities
Alvarez linear accelerator
Phase focusing
RF quadrupole
Synchrotrons

RF field accelerators

- Key idea: using rapidly changing high frequency voltages instead of electrostatic voltages avoids corona formation and discharge
  - -> much higher accelerating voltages possible
- But: Particles must have the correct phase relation to the accelerating voltage.
- But: need high power RF sources!

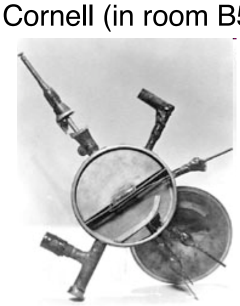


The Cyclotron



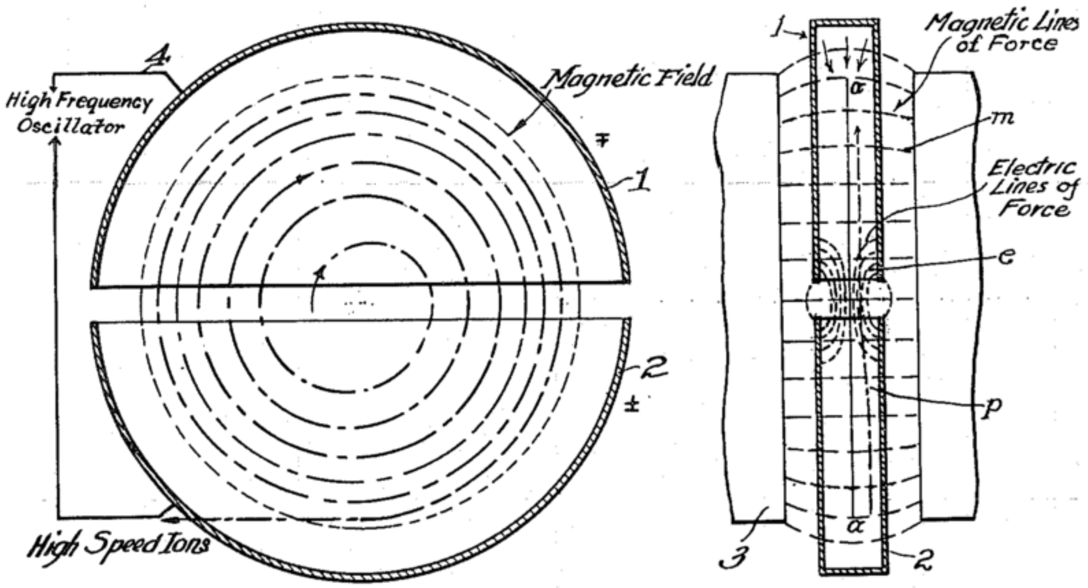
- 1930: Lawrence proposes the Cyclotron
  (before he develops a workable color TV screen)
- 1931: Lawrence and Livingston built first cyclotron (80 keV)
- 1932: Lawrence and Livingston use a cyclotron for
  1.25MeV protons and mention longitudinal
  (phase) focusing
- 1934: Livingston builds the first Cyclotron
  away from Berkeley (2MeV protons)
  at Cornell (in room B54)

NP 1939
Ernest O Lawrence
USA 1901-1958



M Stanley Livingston
USA 1905-1986

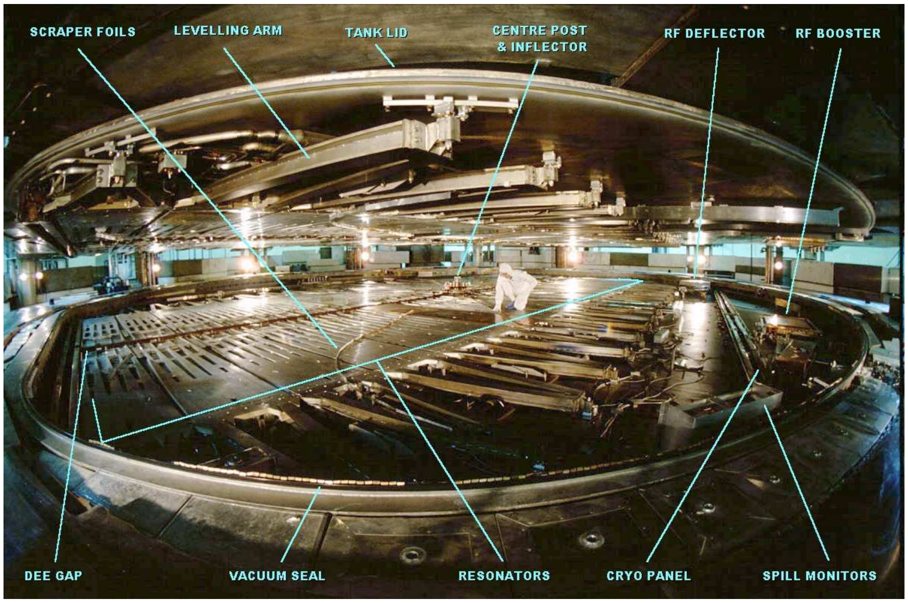
Diagram of cyclotron operation from Lawrence's 1934 patent.



The Cornell Cyclotron

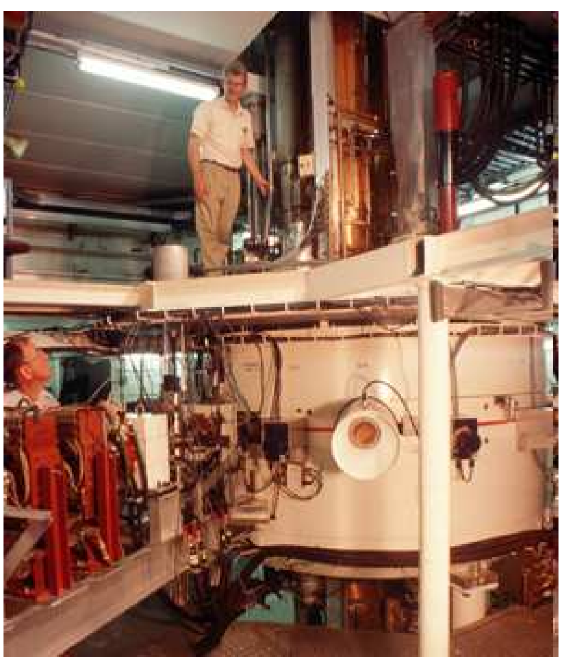
The Cornell cyclotron (2 MeV protrons) was built about 1935 and decommissioned in 1956. It was sent to the Hebrew Univeristy in Jerusalem. This photo with Assistant Professor Boyce D. McDaniel was taken in 1955.

The 500 MeV Proton Cyclotron at Triumf, CA



- ~ 100 μA
- 59' magnetic pole diameter
- Magnet: 4400 tons
- 23 MHz RF

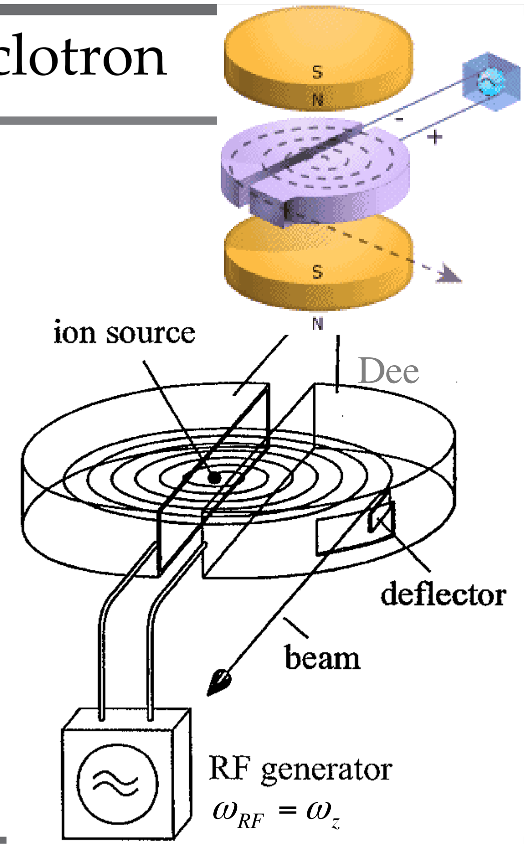
Superconducting Cyclotrons



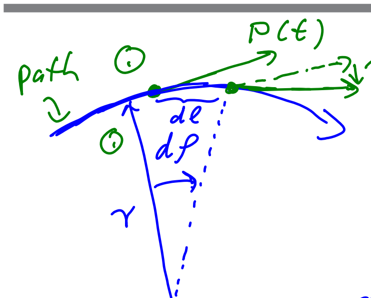
- First superconducting cyclotron at the National Superconducting Cyclotron Laboratory
- Large magnetic fields of several T reduce size of cyclotron

Principle of the cyclotron

- Key idea: circular structure to use same accelerating structure ("Dees") many times
- Iron magnet produces constant magnetic field (B≈2T) between its two round poles to keep beam on spiral path
- Cyclotron:
  $R_{path} = R_{path}(t)$, B=const
- Vacuum chamber between poles with D-shaped electrodes (DEE) with RF voltage from a generator applied between the two halves to accelerate beam



The cyclotron frequency



$\Delta p = p\, d\rho \Rightarrow d\rho = \frac{dp}{p}$

$$\frac{dp}{dt} = F = q v B_z$$

$$r = \frac{dl}{d\rho} = \frac{v\, dt}{dp/p} = \frac{p}{q B_z}$$

⇒ revolution time:

$$\tau = \frac{2\pi r}{v} = 2\pi \frac{p}{v q B_z} = 2\pi \frac{\gamma m_0}{q B_z}$$

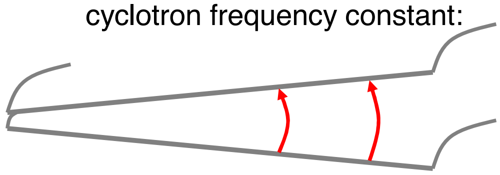
⇒ revolution frequency: cyclotron frequency

$$\omega_z = \frac{2\pi}{\tau} = \frac{q}{\gamma m_0} B_z = \text{const}$$ as long as $\gamma \approx 1$

i.e. indep. of particle velocity and orbit radius!

Cyclotron condition

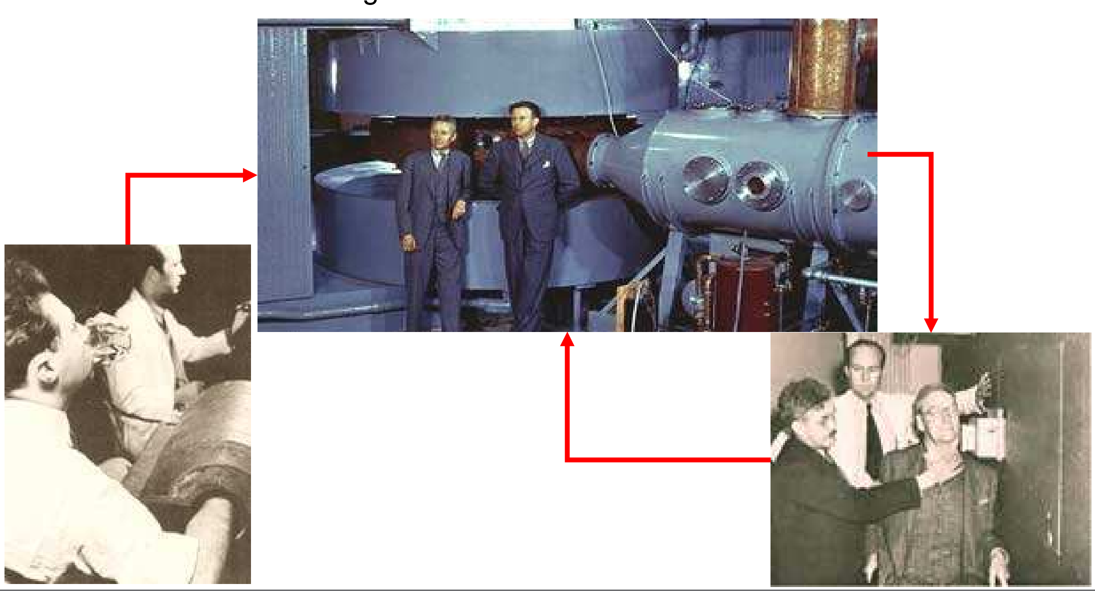
- Condition for continuous acceleration: need $\omega_{RF} = \omega_z$ so that particles always encounter accelerating field in gap between Dees
- But: at higher energies (v>0.15c), cyclotron frequency decreases significantly: $$\omega_z = \frac{q}{m_0\gamma(E)} B_z$$
  -> Only for non-relativistic particles. Therefore not for electrons.
- To reach higher energies:
  - The synchrocyclotron: Acceleration of bunches with decreasing RF frequency (with short pulses of beam): $\omega_z(E) = \frac{q}{m_0\gamma(E)} B_z$
  - Or isocyclotron with magnetic field increasing with radius to keep cyclotron frequency constant: $\omega_z = \frac{q}{m_0\gamma(E)} B_z(r(E))$



Up to 600MeV but changing magnetic field vertically defocuses the beam.

First Medical Applications

- 1939: Lawrence uses 60” cyclotron for 9MeV protons, 19MeV deuterons, and 35MeV 4He. First tests of tumor therapy with neutrons via d + t $\mapsto$ n + α With 200-800keV d to get 10MeV neutrons.

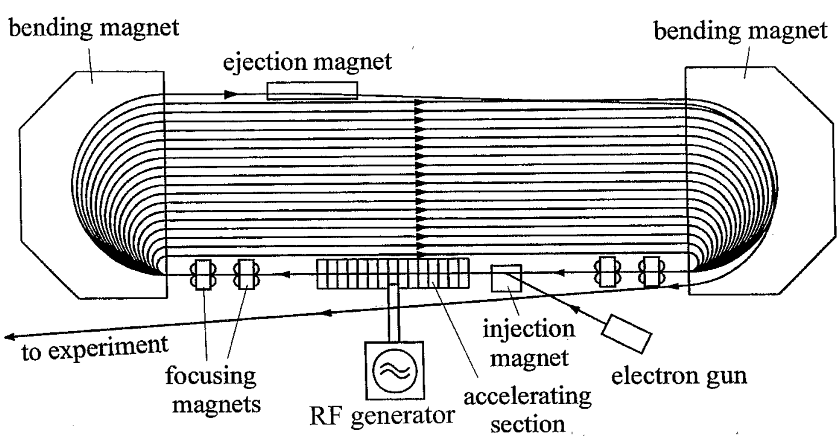


Modern Medical Cyclotrons

- Used for radio isotope production for medical applications (carbon-11, Gallium-67, Thallium-201, Iodine-123, Iodine-123…) for medical imaging
- Ion beams from cyclotrons are used, as in proton therapy, to penetrate the body and kill tumors by radiation damage, while minimizing damage to healthy tissue along their path.

The microtron

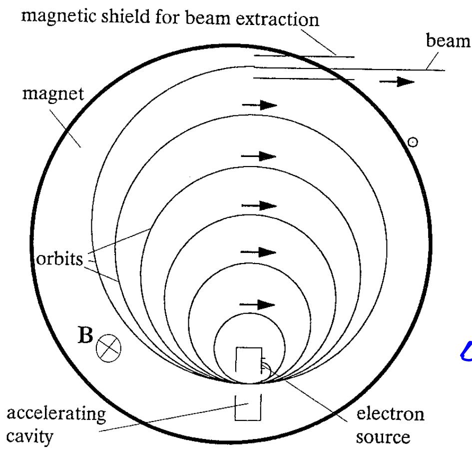
- Electrons are quickly relativistic and cannot be accelerated in a cyclotron.
- In a microtron the revolution frequency changes, but each electron misses an integer number of RF waves.



- Today: Used for medical applications with one magnet and 20MeV.
- Nuclear physics: MAMI (Univ. of Mainz) designed for 820MeV as race track microtron.

The microtron condition

- The extra time that each turn takes must be a multiple of the RF period!



$$r_{path} = \frac{p}{q B_z} = \frac{\gamma m_0 v}{q B_z} \qquad B_z = \text{const ind. of } r$$

revolution time of nth revolution

$$T_n = 2\pi \frac{r_n}{v_n} = 2\pi \frac{\gamma_n m_0}{q B_z}$$

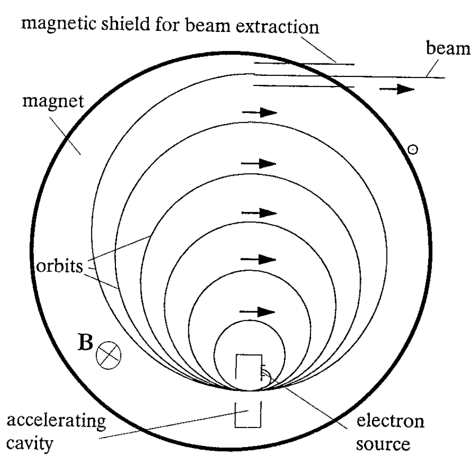
$\Rightarrow \Delta t$: time difference in the periods of the nth and (n+1)th revolution

$$\Delta t = T_{n+1} - T_n = \frac{2\pi}{q B_z}(\gamma_{n+1} m_0 - \gamma_n m_0)$$

$$= \frac{2\pi}{q B_z c^2} \Delta E \overset{!}{=} i\, T_{RF} = i \frac{2\pi}{\omega_{RF}}$$

i = integer

The microtron: Energy gain



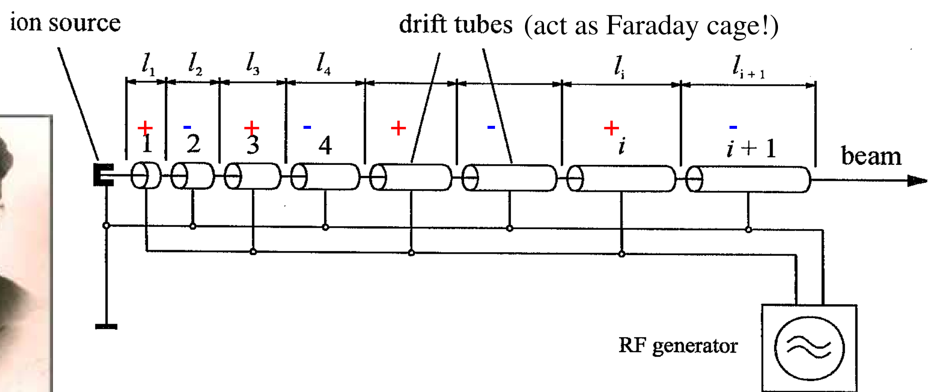
$\Rightarrow$ required energy gain per pass through accelerating structure:

$$\Delta E = i \cdot \frac{q B_z c^2}{\omega_{RF}}$$

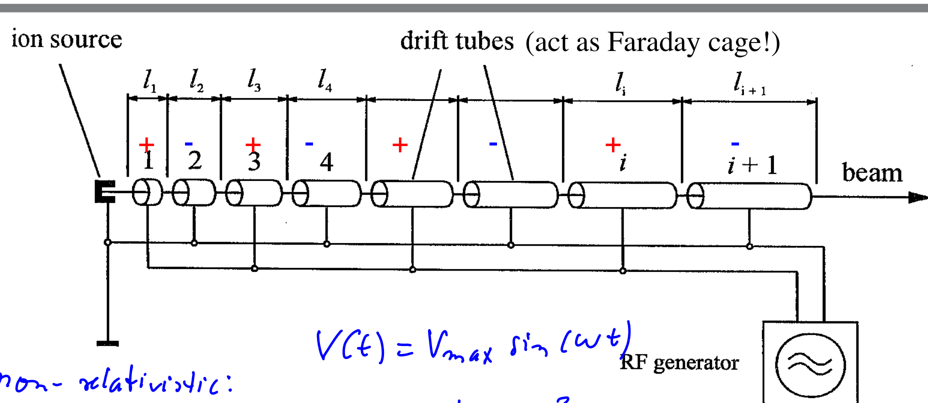
Example: B=1T, ~~n~~ i=1, and f_{RF}=3GHz leads to 4.78MeV. This requires a small linear accelerator.

Wideroe linear RF accelerator

- 1928: Series of metal drift tubes, connected to radio frequency (RF) supply
- Oscillating electric fields in gaps between drift tubes accelerates beam
- Only for bunched beam with bunch length $\sigma_t \ll 1/f_{RF}$ and bunch spacing $\Delta t = 1/f_{RF}$!



Wideroe linear RF accelerator: Gap spacing



$V(t) = V_{max} \sin(\omega t)$

non-relativistic:

$K_n = n\, q\, V_{max} \sin \psi_0 = \frac{1}{2} m v_n^2$

average phase of RF voltage seen by bunch as it passes the gaps

$\Rightarrow$ for constant ψ_0 in all gaps: need time to travel from gap to next gap $= T_{RF}/2$

$\Rightarrow l_n = v_n \frac{T_{RF}}{2} = \frac{1}{2} \beta_n \lambda_{RF} = \frac{\sqrt{n}}{f_{RF}} \sqrt{\frac{q V_{max} \sin \psi_0}{2m}} \propto \sqrt{n}$

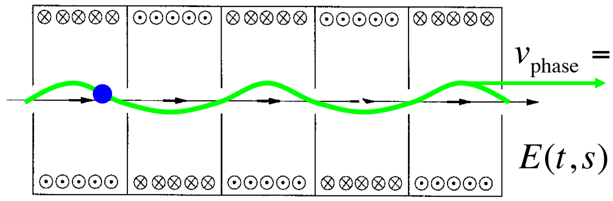
nth drift tube

called the π mode or $1/2\ \beta\lambda$ mode

Accelerating cavities

- 1933: J.W. Beams uses resonant cavities for acceleration

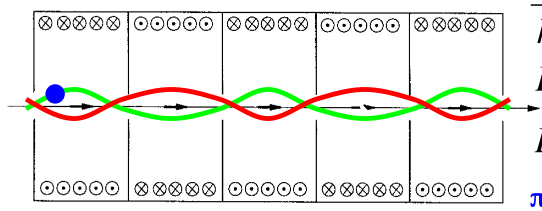
Traveling wave cavity:



$$v_{\text{phase}} = \frac{\omega}{k} = v_{\text{particle}}$$ Here v=c for electrons

$$E(t,s) \approx E_{\max} \sin(\omega t - ks)$$

Standing wave cavity:



$$\frac{\omega}{k} = v_{\text{particle}}$$

$$E(t,s) \approx E_{\max} \sin(\omega t) \sin(ks)$$

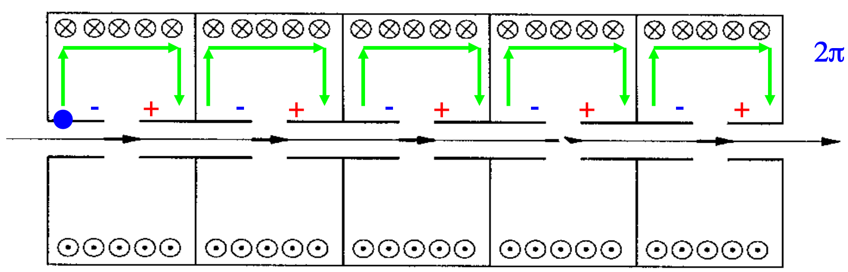
$$E\left(\frac{s}{v_{\text{particle}}}, s\right) \approx E_{\max} \sin^2(ks)$$

π or the $1/2\beta\lambda$ mode

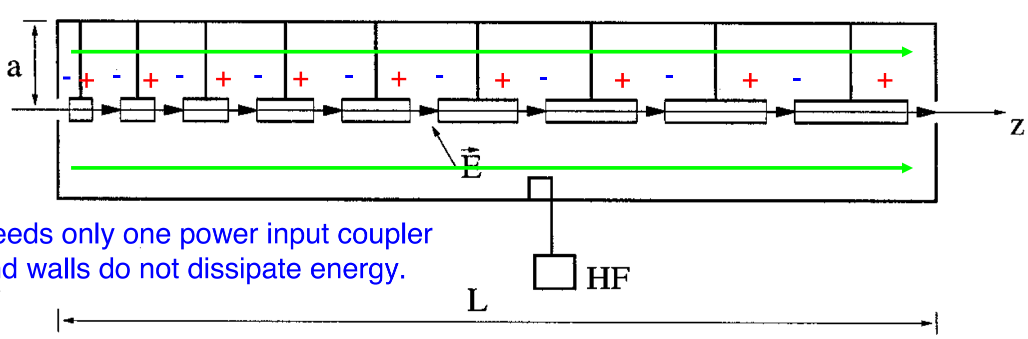
Transit factor (for this example): $\langle E \rangle = \frac{1}{\lambda_{RF}} \int_0^{\lambda_{RF}} E\left(\frac{s}{v_{\text{particle}}}, s\right) ds \approx \frac{1}{2} E_{\max}$

The Alvarez Linear Accelerator

- Gaps between the tubes are enclosed by metallic cavities. This avoids the significant losses to electromagnetic radiation in the Wideroe structure at higher RF frequencies.



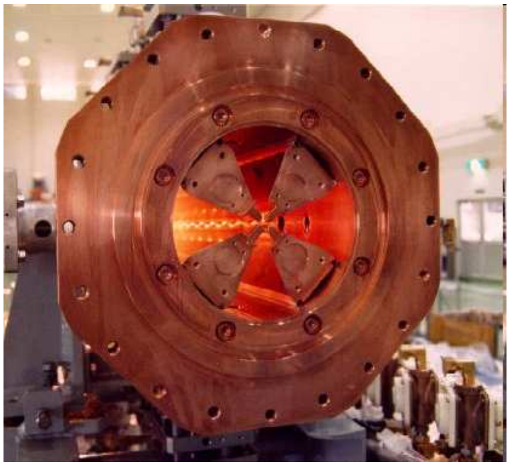
2π or the $\beta\lambda$ mode

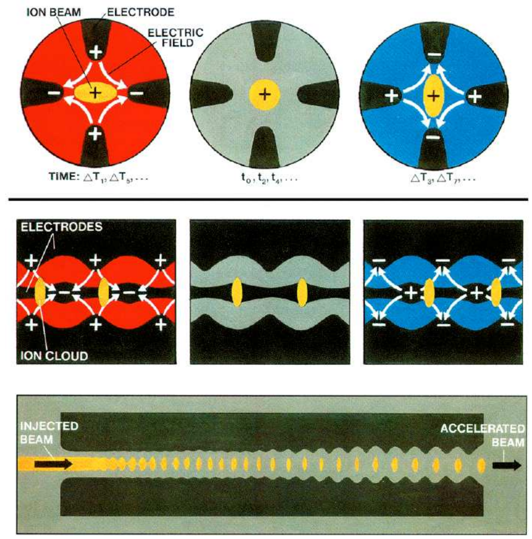


Needs only one power input coupler and walls do not dissipate energy.

The RF quadrupole (RFQ)

- 1970: Kapchinskii and Teplyakov invent the RFQ
- 4 vanes, excited in an electric quadrupole mode

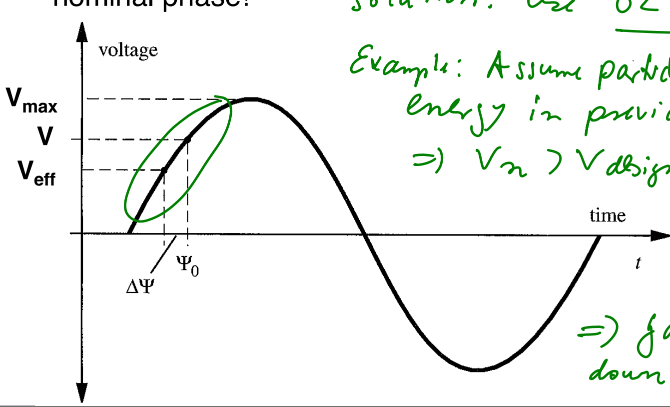




Phase focusing

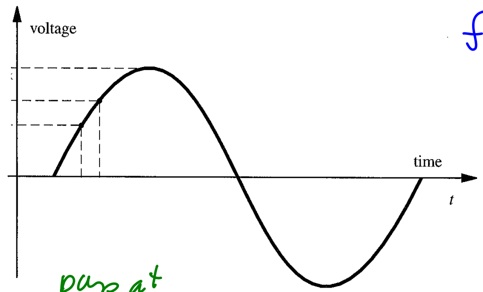
- 1945: Veksler (UDSSR) and McMillan (USA) realize the importance of phase focusing
- RF accelerator: energy gain depends critically on the voltage V_{max} and the nominal accelerating phase Ψ_0 : $\Delta E = q\, V_{max} \sin(\Psi_0)$
- Small error in V_{max} or Ψ_0 -> particle velocity no longer matches design velocity fixed by length of drift tube -> phase shift relative to design Ψ_0 phase in subsequent stages -> longitudinal instability / large energy error
- Phase focusing is required in any RF accelerator to bring particles back to nominal phase!

Solution: use $\underline{0 < \Psi_0 < \pi/2}$ => phase focusing!



Example: Assume particle has gained too much energy in previous gap: $E_n > E_{design,n}$
=> $V_n > V_{design,n}$ => $\Psi_{n+1} < \Psi_0$ in next stage/gap
=> $V_{eff,n+1} = V_{max} \sin(\Psi_0 - \Delta\Psi) < V_{max} \sin \Psi_0$
=> gains less energy and slows down => longitudinal focusing

Phase focusing: more formally



for given particle in bunch:

$\Delta E = q V(t) = q V_{max} \sin\{\omega(t-t_0)+\psi_0\}$

t_0: time bunch center passes accel. structure

ψ_0: average acc. phase in bunch

pass at $t>t_0$ pass at $t<t_0$ s σ s_0

longitudinal position in bunch:

$\sigma = s - s_0 = -v(t-t_0)$

$\Rightarrow \Delta E(\sigma) = q V_{max} \sin\{-\frac{\omega}{v}(s-s_0)+\psi_0\}$

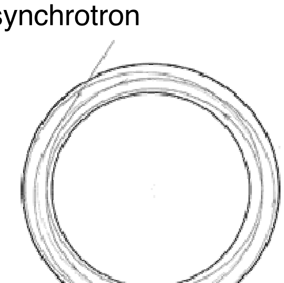
- for acceleration: need $\Delta E(0) > 0 \Rightarrow \psi_0 \in (0,\pi)$
- for phase focusing: need $\Delta E(\sigma) < \Delta E(0)$ for $\sigma > 0$

$\Rightarrow \frac{d}{d\sigma}\Delta E(\sigma) < 0 \Rightarrow \psi_0 \in (-\frac{\pi}{2}, \frac{\pi}{2})$ (same as $\left.\frac{dV(t)}{dt}\right|_{t=t_0} > 0$)

$\Rightarrow$ for acceleration and phase focusing: require $\psi_0 \in (0, \pi/2)$

The Synchrotron

- 1945: Veksler (UDSSR) and McMillan (USA) invent the synchrotron
- 1946: Goward and Barnes build the first synchrotron (using a betatron magnet)
- 1949: Wilson et al. at Cornell are first to store beam in a synchrotron (later 300MeV, magnet of 80 Tons)
- 1949: McMillan builds a 320MeV electron synchrotron

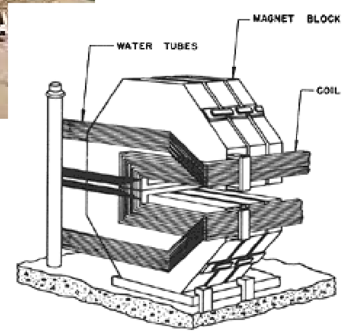


- Key idea: use individual, narrow banding magnets instead of one large magnet
  - -> fixed orbit radius $R = \frac{p(t)}{qB(R,t)} = \text{const.}$
  - -> magnetic bending field needs to increase synchronously with particle energy
  - -> **Synchrotron**
- Only one acceleration section is needed, with $\omega = 2\pi \frac{v_{\text{particle}}}{L} n$

for an integer *n* called the harmonic number

The Synchrotron (II)

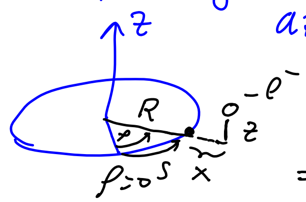
- 1952: Operation of the Cosmotron, 3.3 GeV proton synchrotron at Brookhaven: Beam pipe height: 15cm.



- Particles circulate many thousand times:
  -> transverse focusing needed!
  -> Prior to 1954: used **weak focusing**
  -> but: needed large beam pipe aperture (large transverse oscillation amplitudes)!

Weak focusing Synchrotrons (I)

weak focusing: use guide field that does not depend on azimuthal angle φ



$\Rightarrow$ equilibrium orbit is circle with radius: $R = \frac{mv}{eB_z}$

$\Rightarrow$ for stable motion: for small deviations from the design orbit, restoring forces must arise

$\Rightarrow$ oscillation around design orbit (betatron oscillations)

- for horizontal plane ($z=0$)

need $F_r = evB_z(r) \begin{cases} < \frac{mv^2}{r} & \text{for } r<R \\ > \frac{mv^2}{r} & \text{for } r>R \end{cases}$

for small horizontal deviations x from the equilibrium orbit

$r = R+x = R(1+\frac{x}{R})$ $\qquad \frac{mv^2}{r} \approx \frac{mv^2}{R}(1-\frac{x}{R})$

Weak focusing Synchrotrons (II)

$evB_z(r) \approx evB_0\left(1 - n\frac{x}{R}\right)$ with $B_0 = B_z(R)$

and field index $n = -\frac{R}{B_0}\left(\frac{\partial B_z(r)}{\partial r}\right)_{r=R}$

$$\Rightarrow evB_0\left(1-n\frac{x}{R}\right) \begin{cases} < \frac{mv^2}{R}\left(1-\frac{x}{R}\right) & \text{for } x<0 \\ > \frac{mv^2}{R}\left(1-\frac{x}{R}\right) & \text{for } x>0 \end{cases}$$

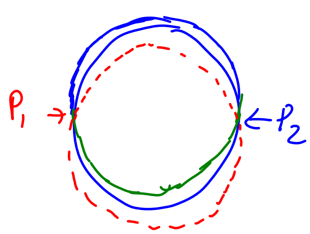
since $R = \frac{mv}{qB_0}$

$$\Rightarrow \text{need: } \quad -\frac{nx}{R} \begin{cases} < -\frac{x}{R} & \text{for } x<0 \\ > -\frac{x}{R} & \text{for } x>0 \end{cases}$$

$\Rightarrow$ need $\underline{\underline{n<1}}$ for horizontal focusing!

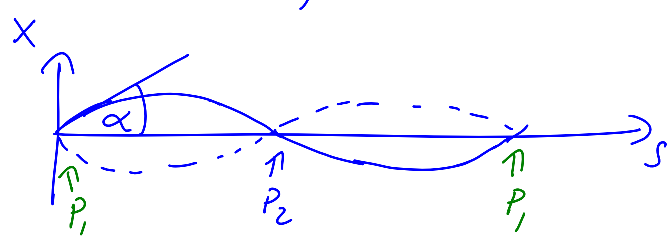
Weak focusing Synchrotrons (III)

Note: also satisfied for $n=0$, i.e. for homogeneous field: natural ring focusing (for $n=0$, all orbits are circles)



sample particle trajectory

Particles diverging from point P_1 meet again after 180° at point P_2



$X_{max} \simeq \alpha R \Rightarrow$ for $\alpha = 1$ mrad

$X_{max} = 1\,mm$ for $R = 1\,m$

$X_{max} = 1\,m$ for $R = 1000\,m$