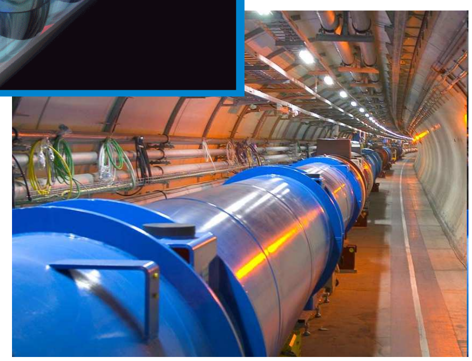
Introduction to Accelerator Physics
Phys 4456/7656, Spring 2010



Tuesdays, Thursdays:

10:10 - 11:25

Newman Lab 311

Starting Jan. 26

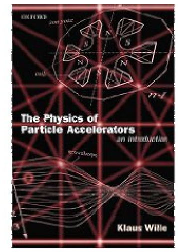
http://www.lns.cornell.edu/~liepe/webpage/P4456.html

Logistics

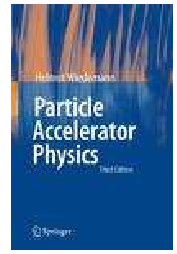
- **Lectures**: 10:00 11:15
  - Tu. and Th. ~~10:10am~~ - 11:25am, Newman 311
- **Homework**:
  - New weekly assignment each Thursday. Suggested due date for turning homework in (just leave it in my mail box): one week later, but there will be no penalty for turning it in after that.
- **Final exam**:
  - Take home exam.
- **Grading**:
  - S/U only, based on homework (50%) and final exam (50%). You will need >55% to pass.
- **Office hours**:
  - Anytime I'm in my office (just call to check if I'm in)
  - Office: 110 Newman Lab, 4-8937
- **Course Website**:
  - http://www.lns.cornell.edu/~liepe/webpage/P4456.html
  - Lecture notes, homework assignments and solutions, …

Literature

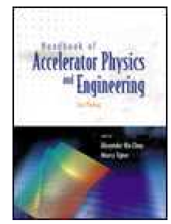
- Required:
  - The Physics of Particle Accelerators, Klaus Wille, Oxford University Press, 2001, ISBN: 0198505493



- Optional:
  - Particle Accelerator Physics, Helmut Wiedemann, Springer, 3nd edition, 2007, ISBN 3540490434



- Related material:
  - Handbook of Accelerator Physics and Engineering, Alexander Wu Chao and Maury Tigner, 2nd edition, World Scientific, ISBN: 9810238584



Schedule

- First lecture: January 26
- No class on: February 18
- Spring break: March 21 – 28
- TTC meeting: April?
- Last lecture: May 6
- Final exam: sometime between May 12 and 21
- IPAC: May 23 to 28

Syllabus (I)

1. **A short history and principles of particle accelerators**
   - 1.0 Forces on charged particles
   - 1.1 First steps
   - 1.2 Direct voltage accelerators
   - 1.3 Transformer accelerators – the Betatron
   - 1.4 RF field accelerators
   - 1.5 Particle sources
   - 1.6 Accelerators for particle physics and energy reach
   - 1.7 Accelerator based light sources
   - 1.8 Accelerators around the world

2. **Charged particles in electromagnetic fields**
   - 2.1 Multipole expansion
   - 2.2 Magnets (dipole, quadrupole, sextupole)
   - 2.3 Building blocks for beam transport lines

3. **Linear beam optics**
   - 3.1 Equation of motion; trajectory
   - 3.2 Transfer matrices
   - 3.3 Beam emittance, Liouvill's theorem
   - 3.4 Betatron function and oscillations
   - 3.5 Dispersion and momentum compaction

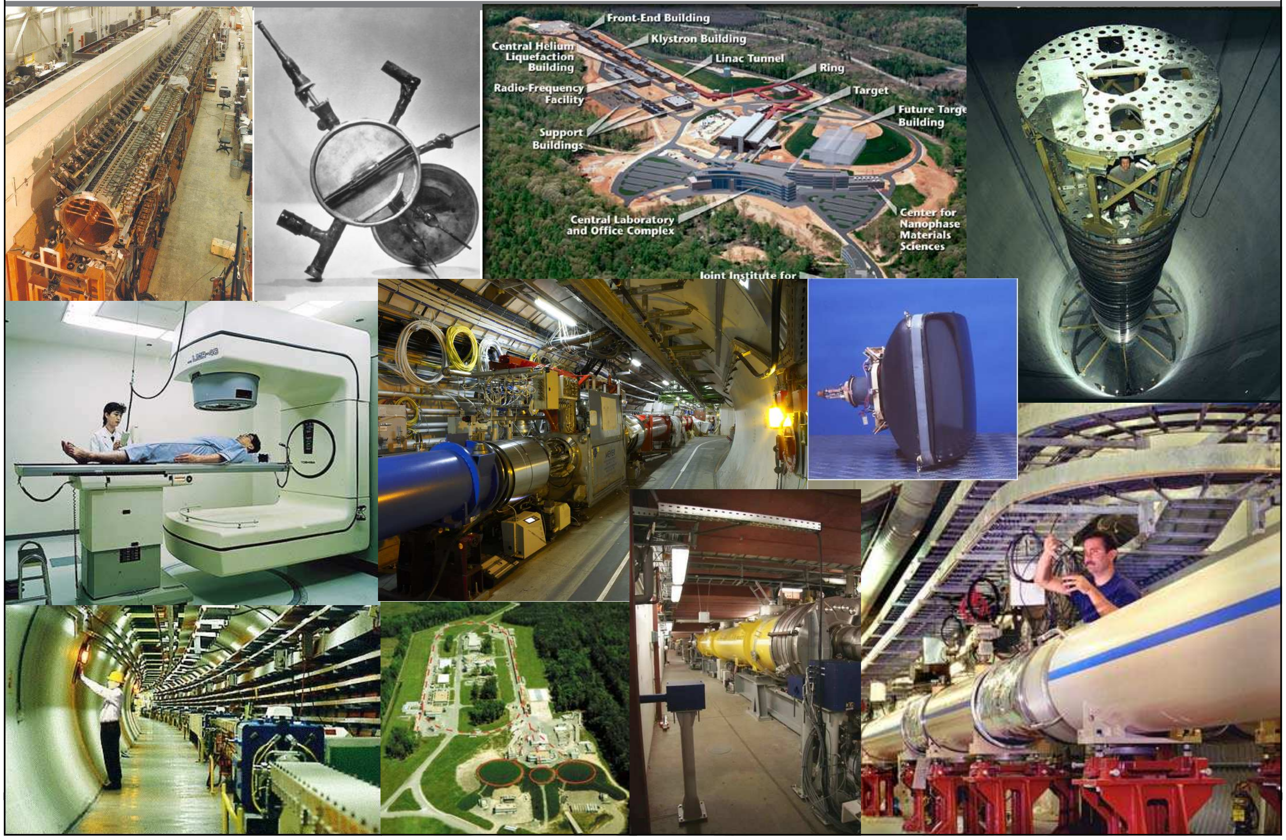
Syllabus (II)

4. **Linear beam optics in circular accelerators**
   - 4.1 Periodicity conditions
   - 4.2 FODO structure
   - 4.3 Hill's equation, …

5. **RF systems for particle acceleration**
   - 5.1 Waveguides
   - 5.2 RF cavities (nc, sc, low beta, …)
     - 5.2.1 RF superconductivity
     - 5.2.2 HOMs and excitation
   - 5.3 RF power sources: klystrons, IOTs, …
   - 5.4 Longitudinal beam oscillations and stability

6. **Synchrotron radiation and radiative damping effects**
   - 6.1 Bends
   - 6.2 Damping of oscillations
   - 6.3 Wigglers and undulators
   - 6.4 FELs

Particle Accelerators



Lectures 1 - 3

1. A short history and principles of particle accelerators

1.0 Forces on charged particles
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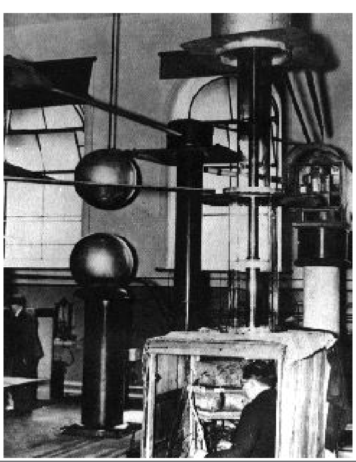


From Cockroft and Walton's electrostatic accelerator (1932) to LHC at CERN (2009)

1.0 Forces on charged particles

Forces on charged particles (I)

- velocity of particles generally close to speed of light c in particle accelerators
  $\Rightarrow$ use relativistic equations

  energy: $E = \sqrt{m_0^2 c^4 + p^2 c^2} = \gamma m_0 c^2$

  with m_0 = rest mass of particle

  $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ with $\beta = \frac{v}{c}$

  momentum: $\vec{p} = \gamma m_0 \vec{v}$

- Momentum changes by force $\vec{F}$: $\frac{d\vec{p}}{dt} = \vec{F}$

- particle accelerators: use electromagnetic force:

  Lorentz force on charged particle of velocity v: $\vec{F} = q(\vec{v} \times \vec{B} + \vec{E})$

Forces on charged particles (II)

⇒ energy change of particle moving $\vec{r}_1$ to $\vec{r}_2$

$$\Delta E = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = q\int_{\vec{r}_1}^{\vec{r}_2} (\vec{v}\times\vec{B}+\vec{E})\cdot d\vec{r}$$

↑ $\parallel$ to $\vec{v}$

so $(\vec{v}\times\vec{B})\cdot d\vec{r} = 0$

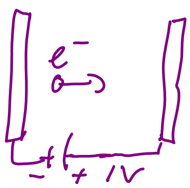
⇒ magnetic field does not change energy of particle, since force acts always ⊥ to direction of motion!

⇒ use to steer, bend, focus beam

⇒ acceleration / increase in energy by electric field only!

$$\Delta E = q\int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\cdot d\vec{r} = q\,\Delta V$$

↖ voltage crossed by particle



Units: J / electron volt (eV)

$\Delta E = 1\,eV = 1.602\cdot 10^{-19}\,C \cdot 1\,V = 1.602\cdot 10^{-19}\,J$

Rest energies $E_0=m_0c^2$ of particles

electron	e^- :	$E_0 = 511\ keV = 0.511\ MeV$
proton	p :	$E_0 = 938\ MeV$
b quark	b :	$E_0 = 4725\ MeV = 4.7\ GeV$
vector boson	Z_0 :	$E_0 = 91\,190\ MeV = 91.9\ GeV$
t quark	t :	$E_0 = 174\,000\ MeV = 0.174\ TeV$

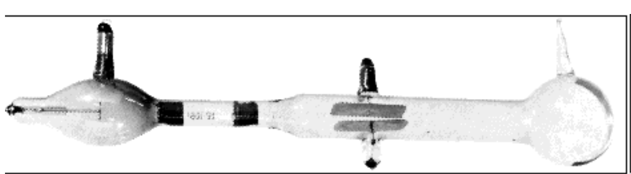
⇒ high energy physics accelerators: GeV range

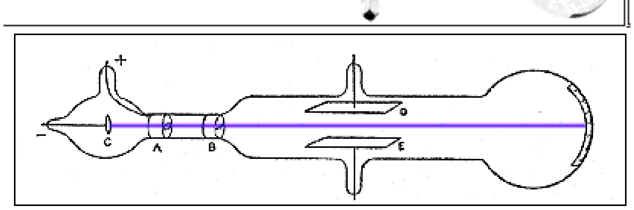
1.1 First steps (prior to ~1930)

First steps…

- **1862: Maxwell theory of electromagnetism**
- **1887: Hertz discovery of the electromagnetic wave**
- **1886: Goldstein discovers positively charged rays (ion beams)**
- **1894: Lenard extracts cathode rays (with a 2.65um Al Lenard window)**
- **1897: JJ Thomson shows that cathode rays are particles since they followed the classical Lorentz force** $m\vec{a} = e(\vec{E} + \vec{v} \times \vec{B})$ **in an electromagnetic field**
- **1926: GP Thomson shows that electrons have also wave properties (1929-1930 in Cornell, NP in 1937)**

NP 1905, Philipp E.A. von Lenard, Germany 1862-1947

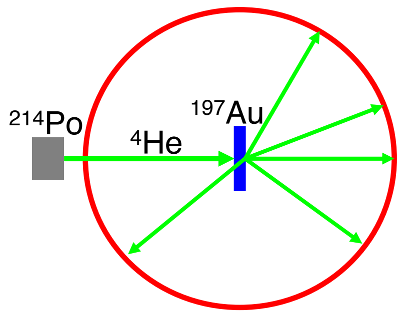




NP 1906, Joseph J. Thomson, UK 1856-1940

First steps…

- 1911: Rutherford discovers the nucleus with 7.7MeV ^4He from ^{214}Po alpha decay measuring the elastic cross-section of $^{197}Au + ^4He \mapsto ^{197}Au + ^4He$.



$$E = \frac{Z_1 e Z_2 e}{4\pi\varepsilon_0 d}$$

d = smallest approach for back scattering

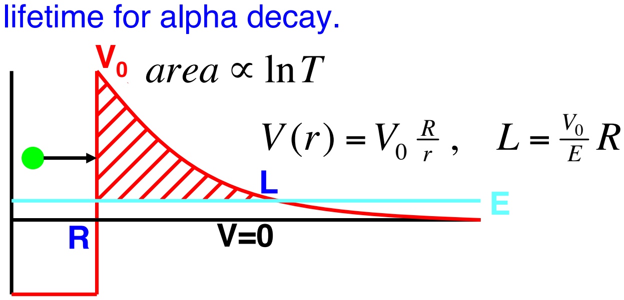
- 1914: Greinacher proposes the cascade (voltage multiplier) circuit for several 100 keV
- 1919: Rutherford produces first nuclear reactions with natural ^4He
  $^{14}N + ^4He \mapsto ^{17}O + p$
- Rutherford is convinced that several 10 MeV are in general needed for nuclear reactions. He therefore gave up the thought of accelerating particles.

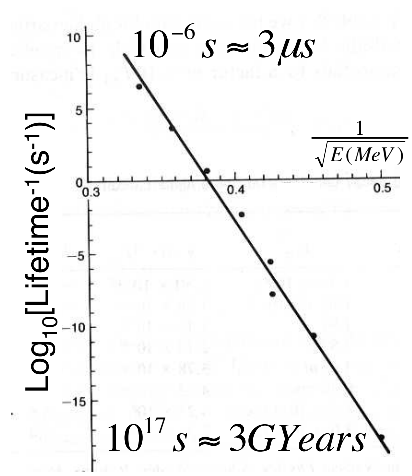
Tunneling allows low energies

- 1928: Explanation of alpha decay by Gamov as tunneling showed that several 100keV protons might suffice for nuclear reactions

Schroedinger equation: $\frac{\partial^2}{\partial r^2} u(r) = \frac{2m}{\hbar^2}[V(r) - E]u(r)\ ,\quad T = \left|\frac{u(L)}{u(0)}\right|^2$

The transmission probability T for an alpha particle traveling from the inside towards the potential well that keeps the nucleus together determines the lifetime for alpha decay.





$$T \approx \exp\left[-2\int_R^L \frac{\sqrt{2m[V(r)-E]}}{\hbar} dr\right]$$

$$\ln T \approx A - \frac{C}{\sqrt{E}}$$

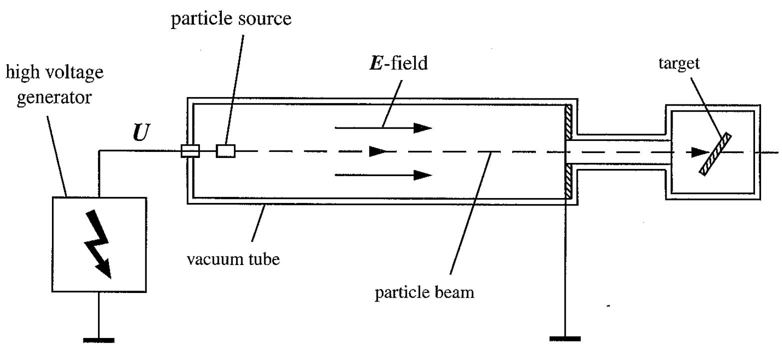
1.2 Direct voltage accelerators

Van de Graaff Accelerator
Tandem Accelerator
Cockcroft-Walton Cascade Generator
Marx Generator

Direct Voltage / Electrostatic Accelerators

General Principle and Limitations:

- Constant electric field between two electrodes
- One of the electrodes contains the particle source
- Example: Cornell ERL DC Electron Gun (~500 kV)



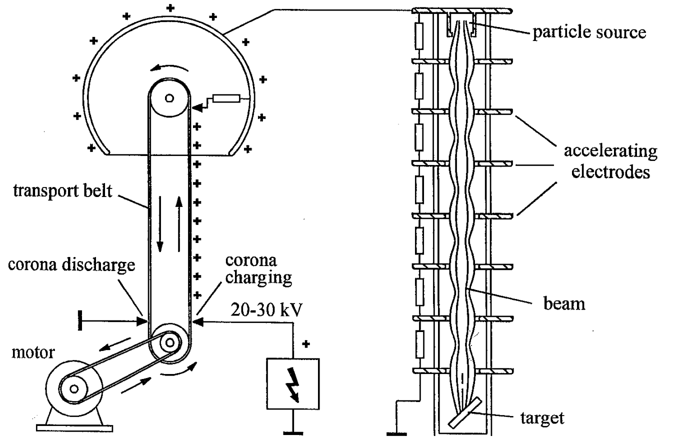
$\Delta E = qV$

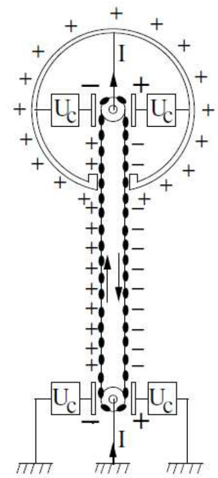
Voltage 1MV
Charge Ze
Energy Z MeV

- The energy limit is given by the maximum possible voltage. At the limiting voltage, electrons and ions are accelerated to such large energies that they hit the surface and produce new ions. An avalanche of charge carries (corona formation) causes a large current and therefore a breakdown of the voltage.
- Maximum DC voltage: **a few MV** are technically possible

The Van de Graaff Accelerator

- 1930: van de Graaff builds the first 1.5MV high voltage generator



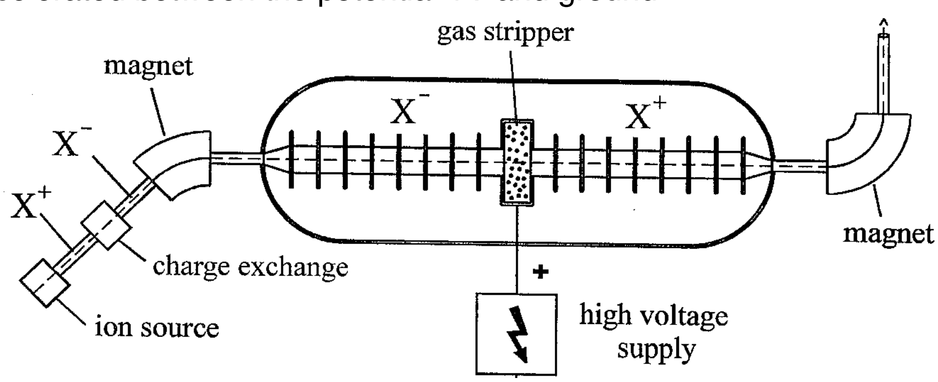


Van de Graaff

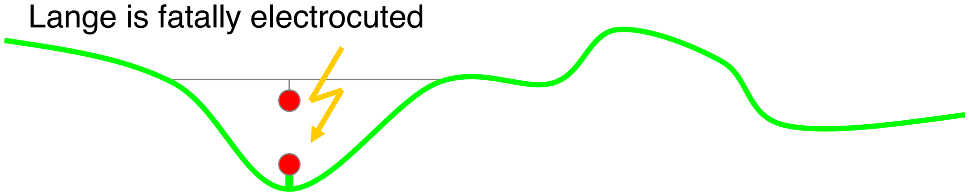
- Peletrons (chains with metal pellets) or Laddertron (with metal plates) that are charged by influence are commercially available.
- Series of resistors connected to electrodes give uniform distribution of electric field and act as electrostatic lens -> focus beam
- Used as injectors, for electron cooling, for medical and technical n-source via $d + t \mapsto n + \alpha$
- Up to ~20 MV with insulating gas (1MPa SF_6), ~1.5 MV in air

The Tandem Accelerator

- 1936, for ions: give twice the energy $\Delta E = e2V$ for p, d; even higher for other ions
- *Negative* ions cross potential difference from ground to +V once, then collide with gas molecules and lose their added electrons, then *positive* ions are further accelerated between the potential +V and ground.

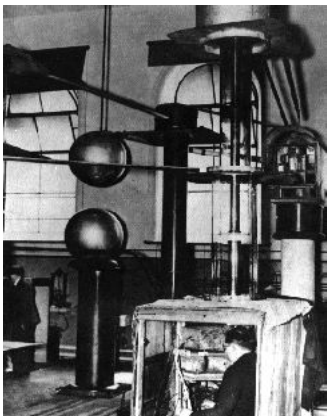


- 1932: Brasch and Lange use potential from lightening, in the Swiss Alps, Lange is fatally electrocuted

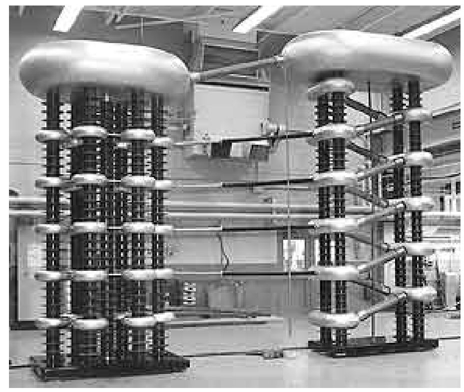


The Cockcroft-Walton Accelerator

- 1932: Cockcroft and Walton: 700keV *cascade* generator (planed for 800keV); used initially to produce 400keV protons for $^7Li + p \mapsto {}^4He + {}^4He$ and $^7Li + p \mapsto {}^7Be + n$
- First "high energy" accelerator: first atomic reaction initiated by accelerated protons
- Used voltage multiplier / cascade circuit proposed by Greinacher (see next slide)
- Voltages up to 4 MV can be reached; pulsed (few μs) beam up to several 100 mA



Nobel price 1951, Sir John D Cockcrof and Ernest T S Walton



Greinacher voltage multiplier circuit

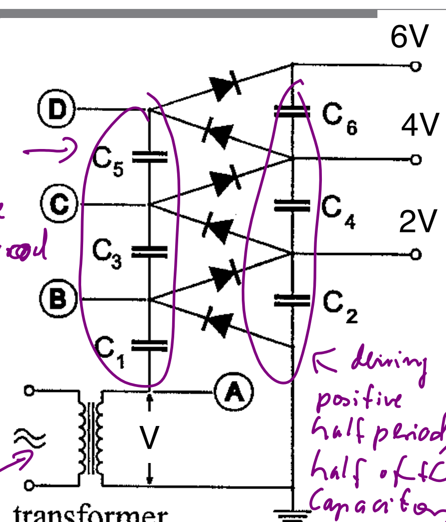
⇒ 2N capacitors: multiply voltage by factor of ~N

⇒ max. voltage with n stages:

- without loading/extracting current: $V_n = 2nV_0$ (↑ max AC voltage ampl.)
- with loading: → causes voltage drop

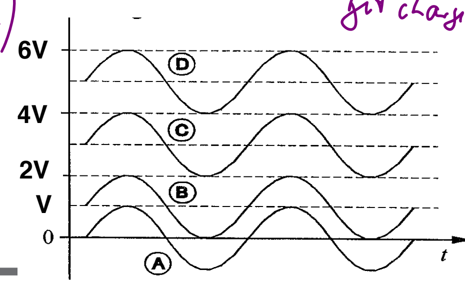
$$V_n = 2nV_0 - \frac{2\pi I}{\omega C}\left(\frac{2}{3}n^3 + \frac{1}{4}n^2 + \frac{1}{12}n\right)$$

typical: $f = 0.5 - 10$ kHz, $C = 1 - 10$ nF, $n = 3 \dots 5$



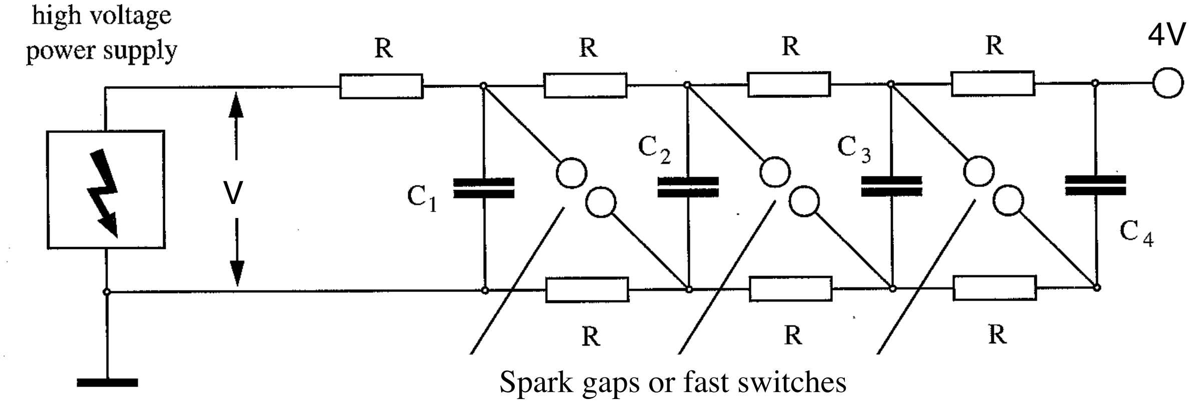
charged during negative half period

during positive half period half of the capacitors get charged



The Marx Generator

- 1932: Marx Generator achieves 6 MV at General Electrics



- Cascade design with network of resistors and capacitors
- Initially: capacitors are charged in parallel
- At firing voltage of the spark gaps: capacitors become connected in series
  $\Rightarrow V_{total} = n\,V$
- Example: 100 capacitors of around 2μF; filled to about 20kV; the spark gaps or switches close as fast as 40ns, allowing up to 500kA for short pulses << 1 μs.

1.3 Transformer accelerators

The Betatron

The Betatron

- 1924: Wideroe invents the betatron
- 1940: Kerst and Serber build a betatron for 2.3MeV electrons and understand betatron (transverse) focusing (in 1942: 20MeV)
- Today: Betatrons with typically about 20MeV for medical applications
- Betatron = transformer with secondary winding replaced by a circulating electron (“beta rays”) beam

Betatron:

R=const, B=B(t)

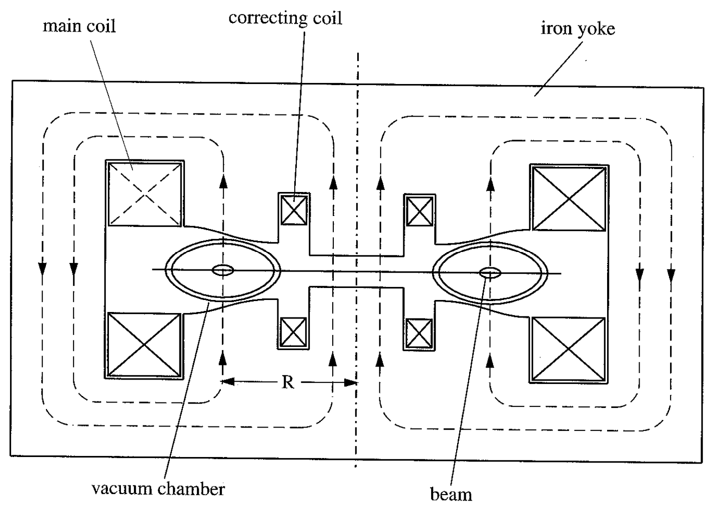
Whereas for a cyclotron:

R(t), B=const

No acceleration section is needed since: law of induction

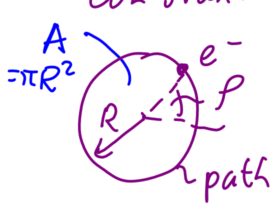
$$\oint_{\partial A} \vec{E}\cdot d\vec{s} = -\iint_A \frac{d}{dt}\vec{B}\cdot d\vec{a}$$

Accelerating electric field arises from time dependent magnetic field!



The Betatron Condition (1)

1) Magnetic field needs to keep particles on circle with constant radius R



R = const

rotational symmetry: $\frac{d\vec{B}}{d\rho} = 0$

for circular motion:

$$|F_R| = \underbrace{|q v B_z|}_{\text{component of Lorentz force along } R} \overset{!}{=} \gamma m_0 \frac{v^2}{R}$$

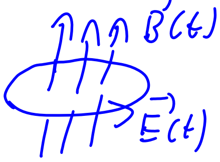
$$\Rightarrow |q B_z| = \gamma m_0 \frac{v}{R} = \frac{p}{R}$$

⇒ particle momentum: $p(t) = |q R B_z(R,t)|$ (1)

⇒ need $|B_z(R,t)| = \left|\frac{p(t)}{qR}\right|$ as p increases, B_z must increase too to keep R = const!

The Betatron Condition (2)

2) Time dependent magnetic field generates accelerating electric field via induction:



$$\oint \vec{E} \cdot d\vec{s} = -\iint_A \frac{d\vec{B}}{dt} \cdot d\vec{a} = -\frac{d\Phi}{dt} \quad \Big\}\ \text{law of induction}$$

A = surface enclosed by path

note: $\frac{d\vec{B}}{d\varphi} = 0$ (symmetry), but $\frac{d\vec{B}}{dR} \Rightarrow B = B(R,t)$

⇒ average magnetic field inside path ⊥ to surface

$$\langle |B_z(t)| \rangle = \frac{1}{\pi R^2} \iint_A B_z \, da$$

also: $|\oint \vec{E} \cdot d\vec{s}| = 2\pi R \; E_\varphi(R,t)$ since $\frac{d\vec{E}}{d\varphi} = 0$ (symmetry)

$$\Rightarrow 2\pi R \, E_\varphi(R,t) = \pi R^2 \frac{d}{dt} \langle |B_z(t)| \rangle$$

$$\Rightarrow E_\varphi(R,t) = \frac{R}{2} \frac{d}{dt} \langle |B_z(t)| \rangle \quad (2)$$

The Betatron Condition (3)

3) combine statement (1) and (2)

NII $F_\varphi = \frac{dp(t)}{dt} = |q E_\varphi(R,t)| = |q \frac{R}{2} \frac{d}{dt} \langle |B_z(t)| \rangle|$ with (2)

$$\Rightarrow p(t) = p(0) + |q \frac{R}{2} [\langle |B_z(t)| \rangle - \langle |B_z(0)| \rangle]|$$

$$= |q R B_z(R,t)| \quad \text{according to (1)}$$

$$\Rightarrow B_z(R,t) - B_z(R,0) = \frac{1}{2} [\langle |B_z(t)| \rangle - \langle |B_z(0)| \rangle]$$

with $B_z(R,0) = \left| \frac{p(0)}{qR} \right|$

Wideröe's betatron condition for stable particle motion during acceleration

"field on orbit" = $\frac{1}{2}$ "average field inside orbit circle"

The Betatron: Max. momentum

- small deviations from this condition lead to transverse beam oscillation about design orbit called betatron oscillations (for all accelerators)

4) Maximum particle momentum

from before: $p(t) = |q R B_z(R,t)|$ (1)

$\Rightarrow p_{max} = |q R B_{z,max}(R,t)|$

i.e. depends on orbit radius and max. magnetic field on orbit, but not on rate of change of field

- typical: $\vec{B} = \vec{B}_0 \cos(2\pi f t)$ with $f = 50\,Hz$ or $60\,Hz$
- largest betatron: $R = 1.23\,m$, $B_{max} = 8.1\,kG$, $p_{max} = 300\,\frac{MeV}{c}$