Lecture 9: 02/06/09

- Wave Packets and Group Velocity
- λ: Order of Magnitude
- Evidence for wave Behavior of Particle
- The "old Quantum Theory"

Clinton Davisson and Lester Germer

I_3 Particle Waves

Recap:

$I_{3,1}$ De Broglie Hypothesis:

Particle: Energy E, Momentum p $\longleftrightarrow$ associated wave

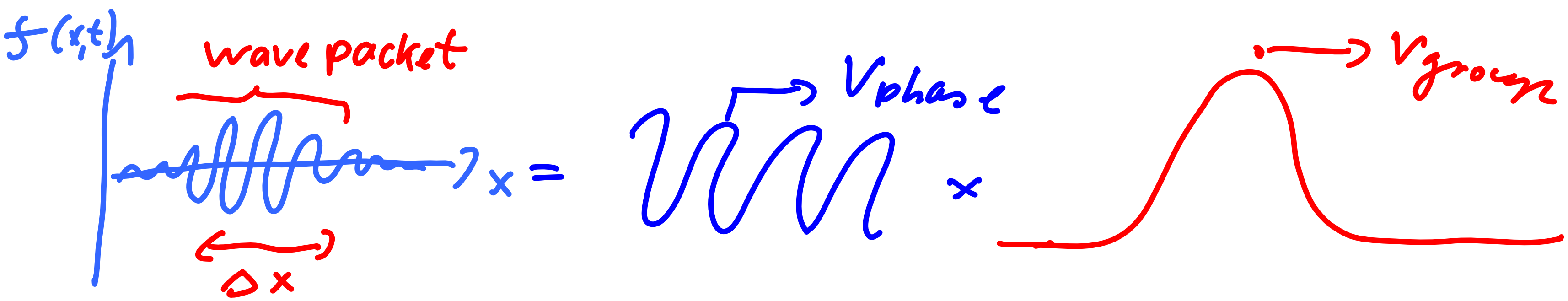
$$\lambda = \frac{h}{p} \qquad \nu = \frac{E}{h}$$

$I_{3,3}$ Superposition of Particle Waves

$$f(x,t) = \mathrm{Re}\left\{ e^{i[k_0 x - \omega(k_0)t]} \int \phi(s)\, e^{is\left(x - \left.\frac{d\omega}{dk}\right|_{k_0} t\right)} ds \right\}$$

infinite plane wave: crests move at $v_{phase} = \frac{\omega}{k} = \frac{c^2}{v}$

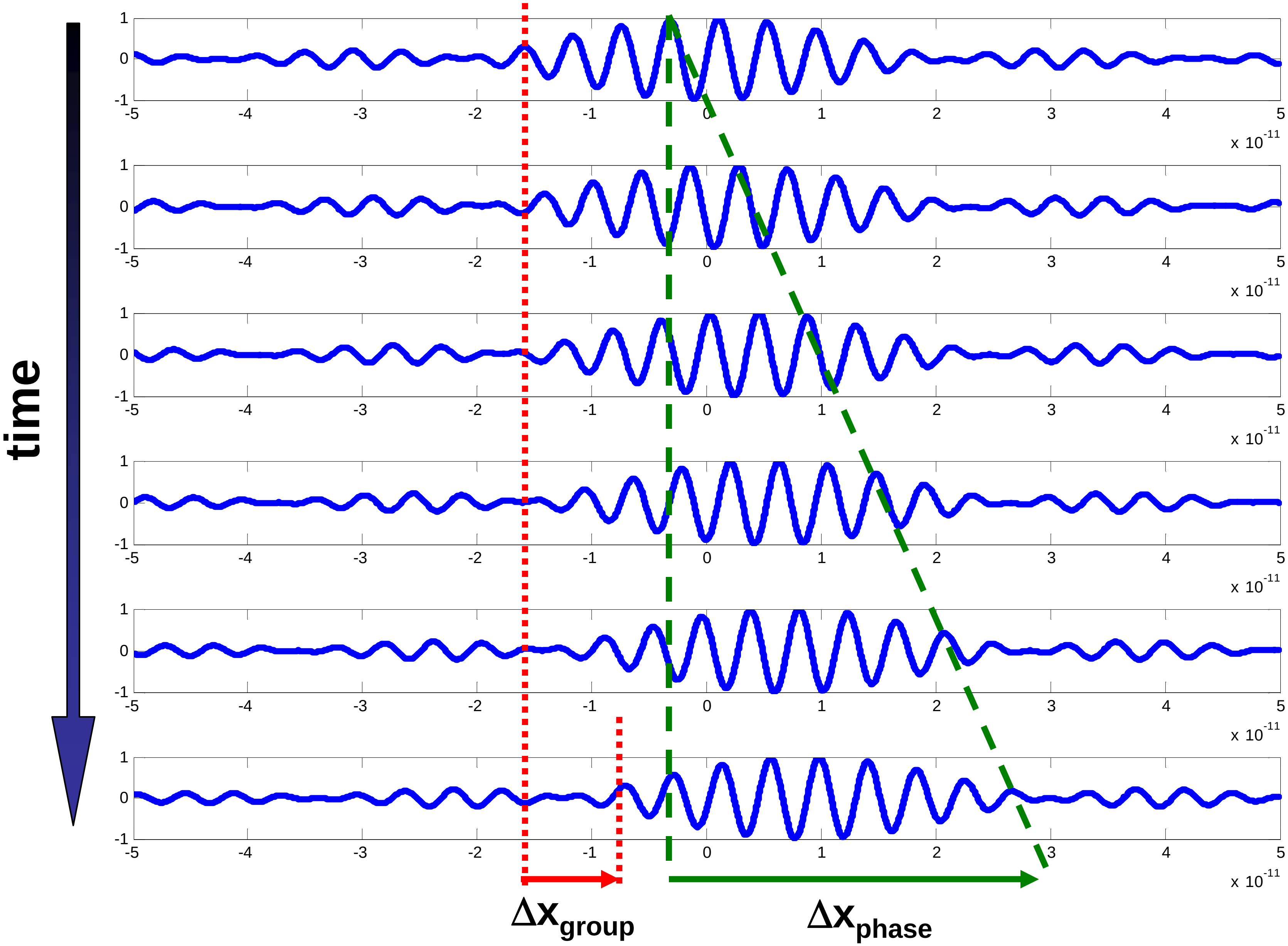
envelope function travels at group velocity $v_{group} = \frac{d\omega(k)}{dk} \overset{!}{=} v_{particle}$



Conclusion:

- wave packet associated with "localized" particle:
  - ⇒ position of envelope function (max. wave amplitude) matters
  - ⇒ group velocity matters, not phase velocity
  - ⇒ localized wave packet ⇔ momentum of particle is not well defined (uncertainty) (small $\Delta x \rightarrow$ large Δp_x)
  - ⇒ see free particle in Schrödinger's QM

Example: $v_{group} = v_{particle} = c/2$ $v_{phase} = c^2/v = 2c$



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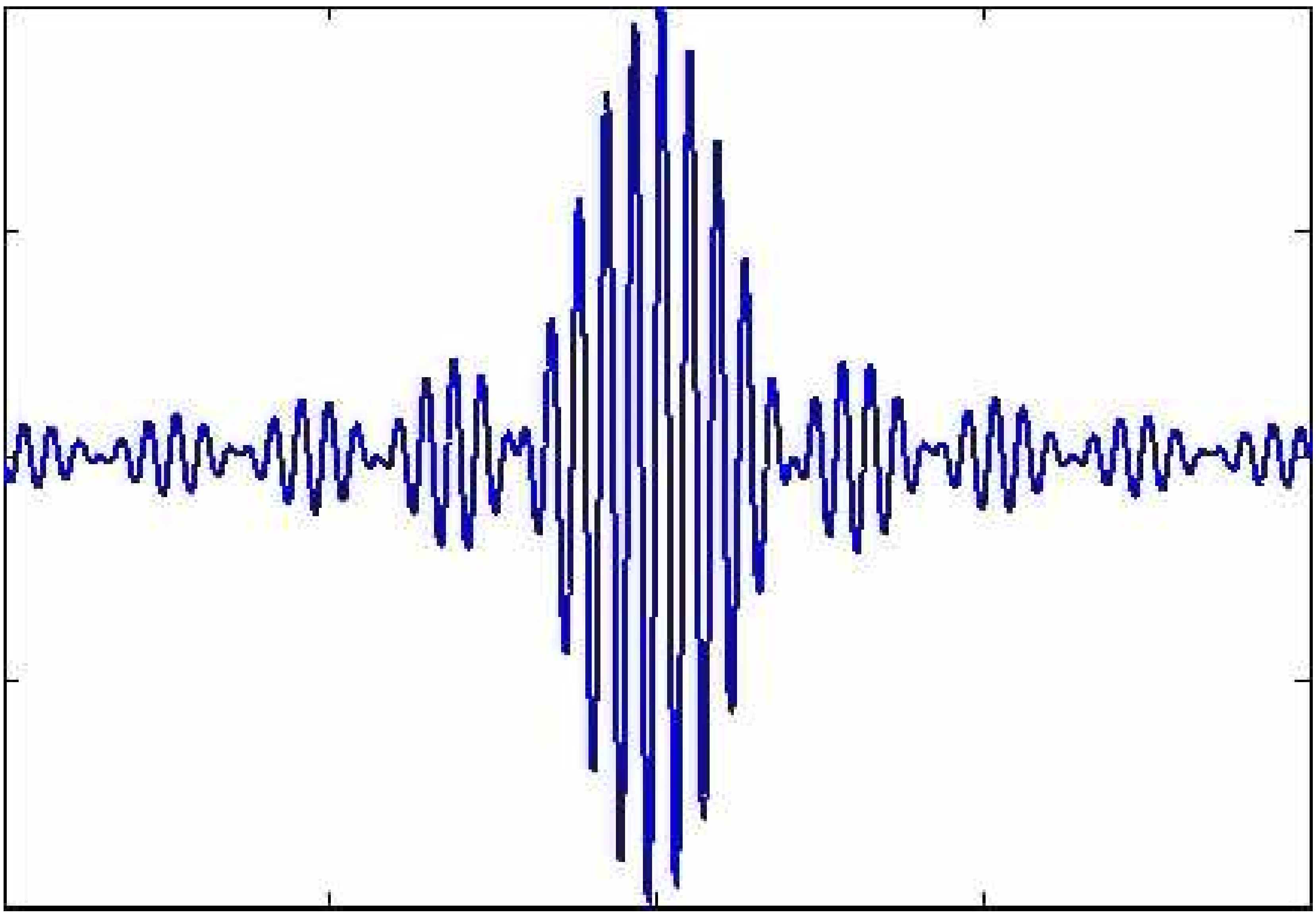


The envelope function of the wave packet associated with a localized particle should be related to…

A. The size of the particle (smaller size -> shorter envelope function)

B. The range of space in which the particle might be found if its position would be measured

C. Something else

$I_{3,4}$ Group and Phase Velocity for de Broglie's Particle Waves:

phase velocity: $V_{ph} = \frac{\omega}{k} = \lambda\nu = \frac{c^2}{V}$ ← speed of particle

group velocity: $V_{group} = \frac{d\omega(k)}{dk}$ ⇒ need $\omega(k)$

$$\omega = 2\pi\nu = 2\pi\frac{E}{h} = \frac{2\pi}{h}\sqrt{p^2c^2 + m_0^2c^4} = \frac{2\pi}{h}\sqrt{\frac{h^2}{\lambda^2}c^2 + m_0^2c^4}$$

↑ $p = h/\lambda$

$k = 2\pi/\lambda$

$$= \frac{2\pi}{h}\sqrt{\frac{h^2}{4\pi^2}k^2c^2 + m_0^2c^4} = \omega(k)$$

$$\Rightarrow \frac{d\omega}{dk} = \frac{2\pi}{h}\,\frac{1}{2}\,\frac{1}{\sqrt{\frac{h^2}{4\pi^2}k^2c^2 + m_0^2c^4}}\,\frac{h^2}{4\pi^2}c^2 2k = \frac{1}{E}\frac{h}{2\pi}c^2k$$

$$= \frac{1}{E}\frac{h}{2\pi}c^2\frac{2\pi}{\lambda} = \frac{c^2}{E}p = \frac{c^2\gamma m_0 V}{\gamma m_0 c^2} = V$$ = particle speed ☺ = group velocity ☺

⇒ group velocity = speed of envelope function of particle wave packet = particle speed

good!

$I_{3,5}$ λ=h/p: Order of Magnitude Estimate

Or: Why wasn't this noticed before?

thermal neutrons (300K) $\Rightarrow \lambda = 1.5\,\text{Å}$ } $\approx$ atom size

electrons at 100 eV $\Rightarrow \lambda = 1.2\,\text{Å}$

neutrons at 10 MeV $\Rightarrow \lambda = 9 \cdot 10^{-15}\,m$ } size of nucleus

$m = 1g$ at $1\,m/s$ $\Rightarrow \lambda = 7 \cdot 10^{-31}\,m$

compare to visible light $\Rightarrow \lambda = 400 - 700\,nm$

$= 4\ \text{to}\ 7 \cdot 10^{-7}\,m$

$\rightarrow$ recall 2-slit exp.: maxima for $\sin\theta = \frac{n\lambda}{d} < 1$

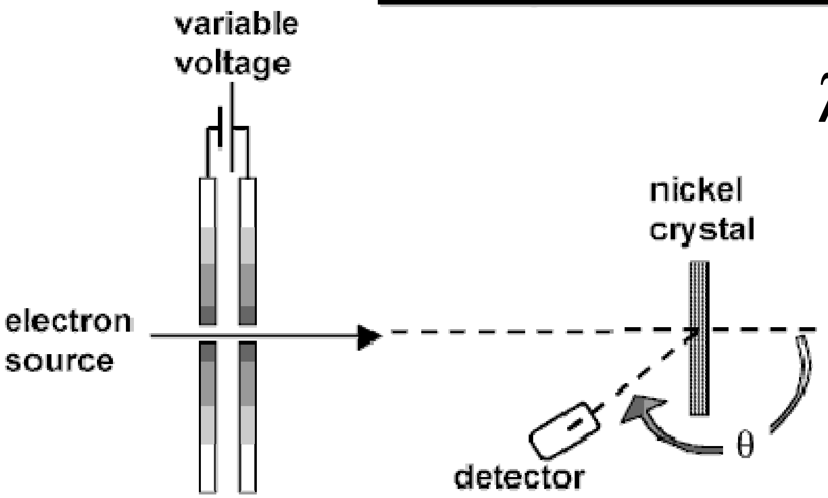
need $\lambda \simeq d$

$\Rightarrow$ for particle: need "slit" spacing / diffraction grid on Å scale (or less)

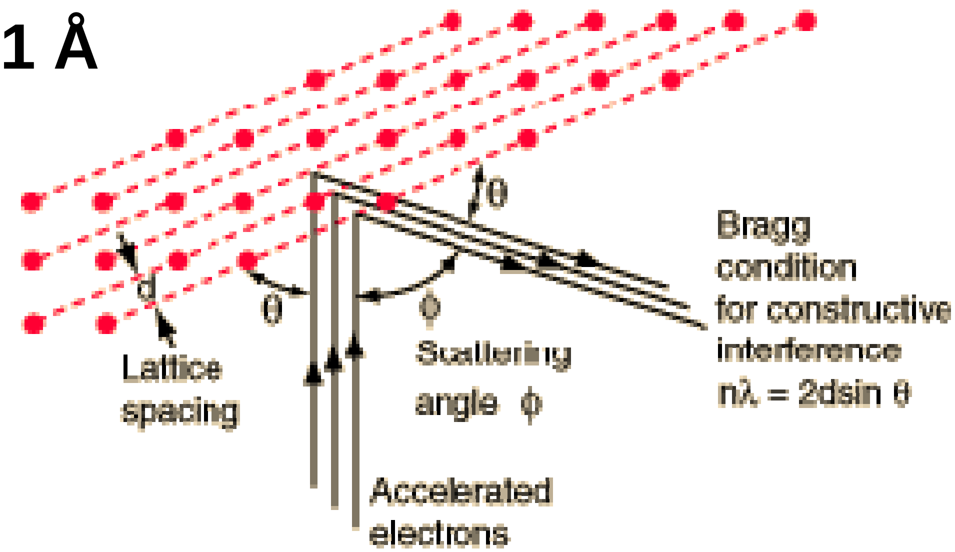
$\Rightarrow$ use crystals!

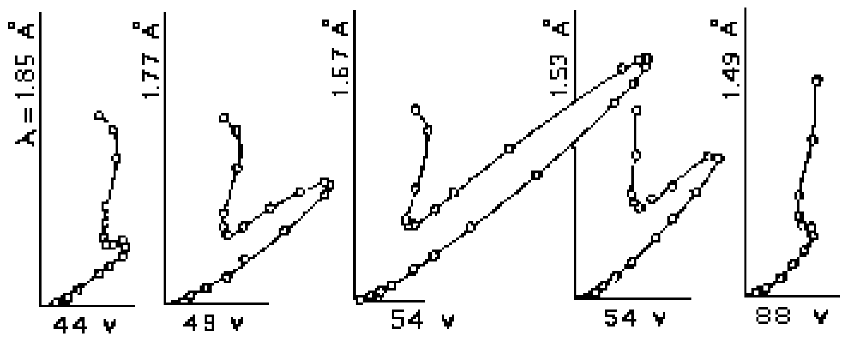
$I_{3,6}$ Evidence for de Broglie's Particle Waves:

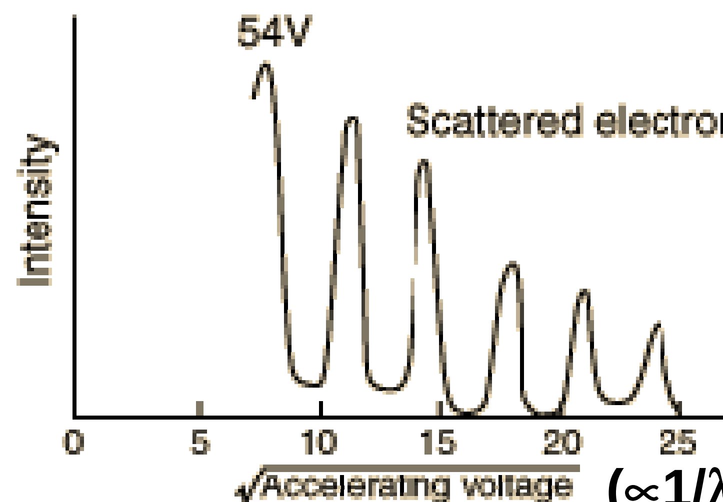
Davisson-Germer Experiment (1925): Scattering of low energy electrons by a crystal surface



$\lambda \approx 1$ Å



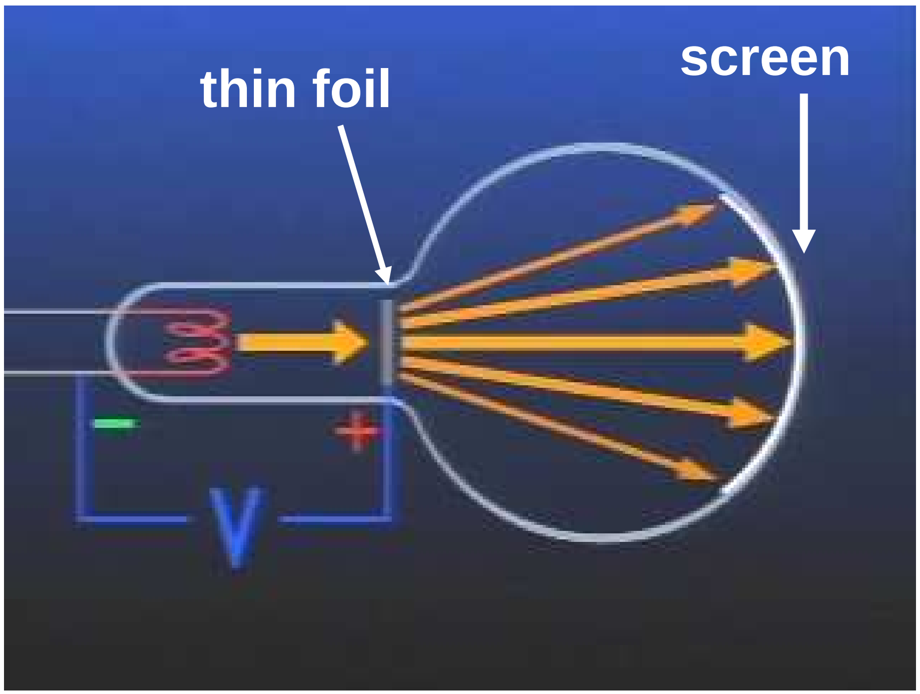


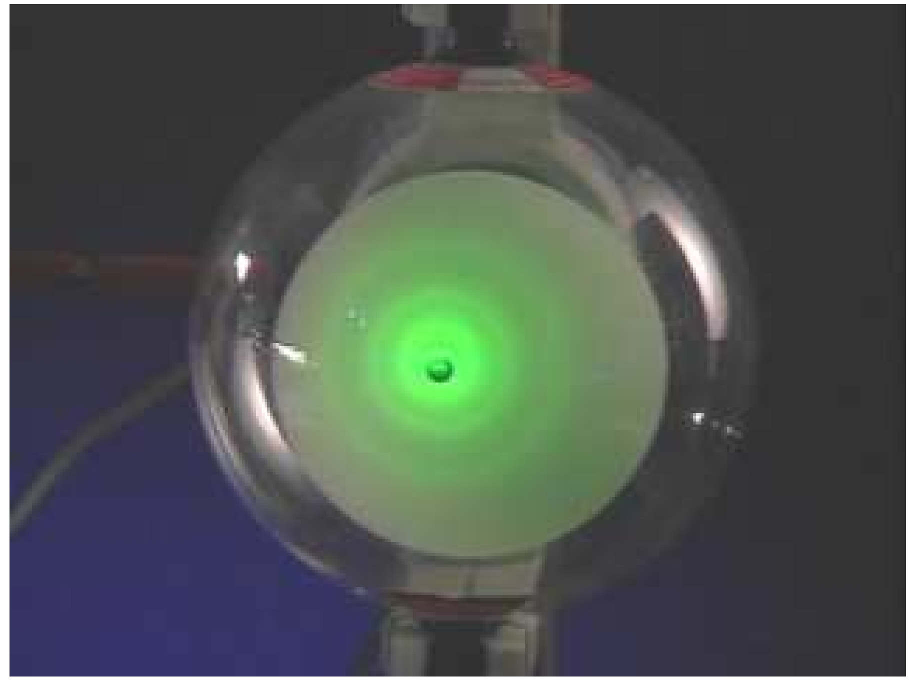


$(\propto 1/\lambda)$

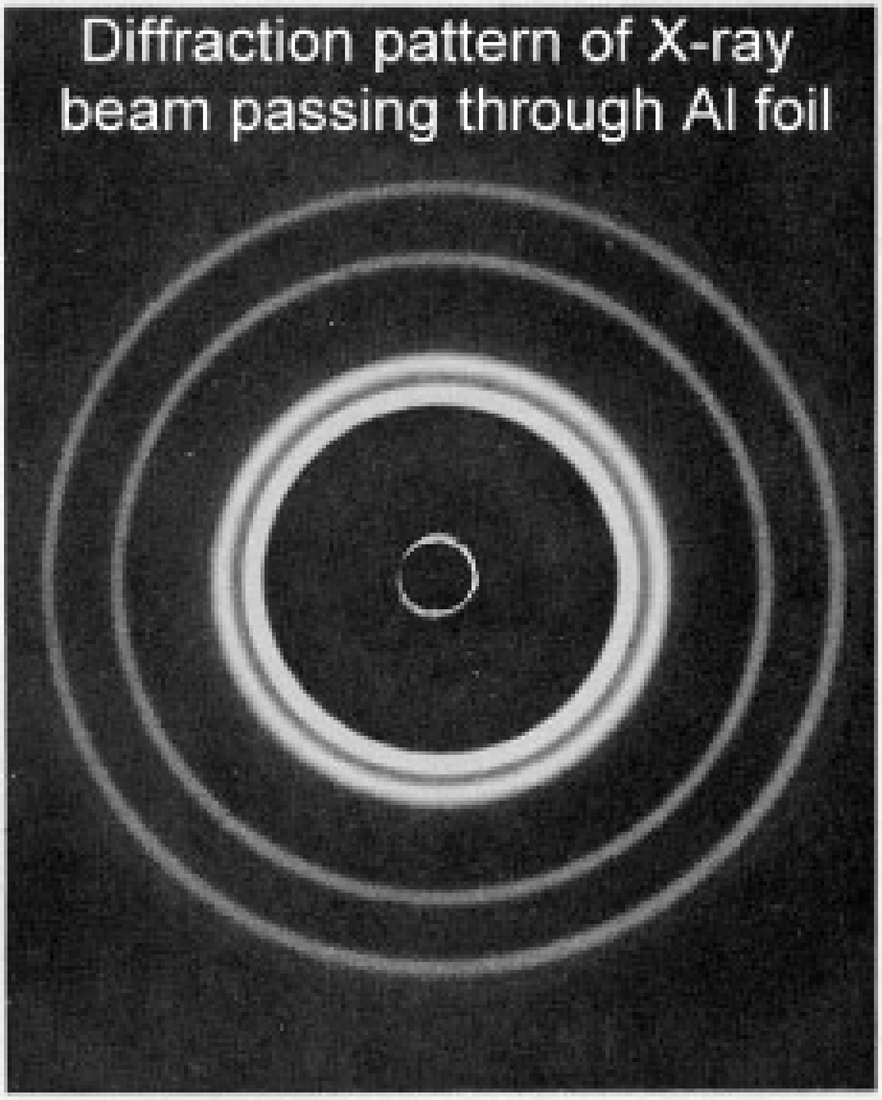
G. P. Thompson’s Experiment: Diffraction of 10 – 40 keV electrons by a thin polycrystalline foil

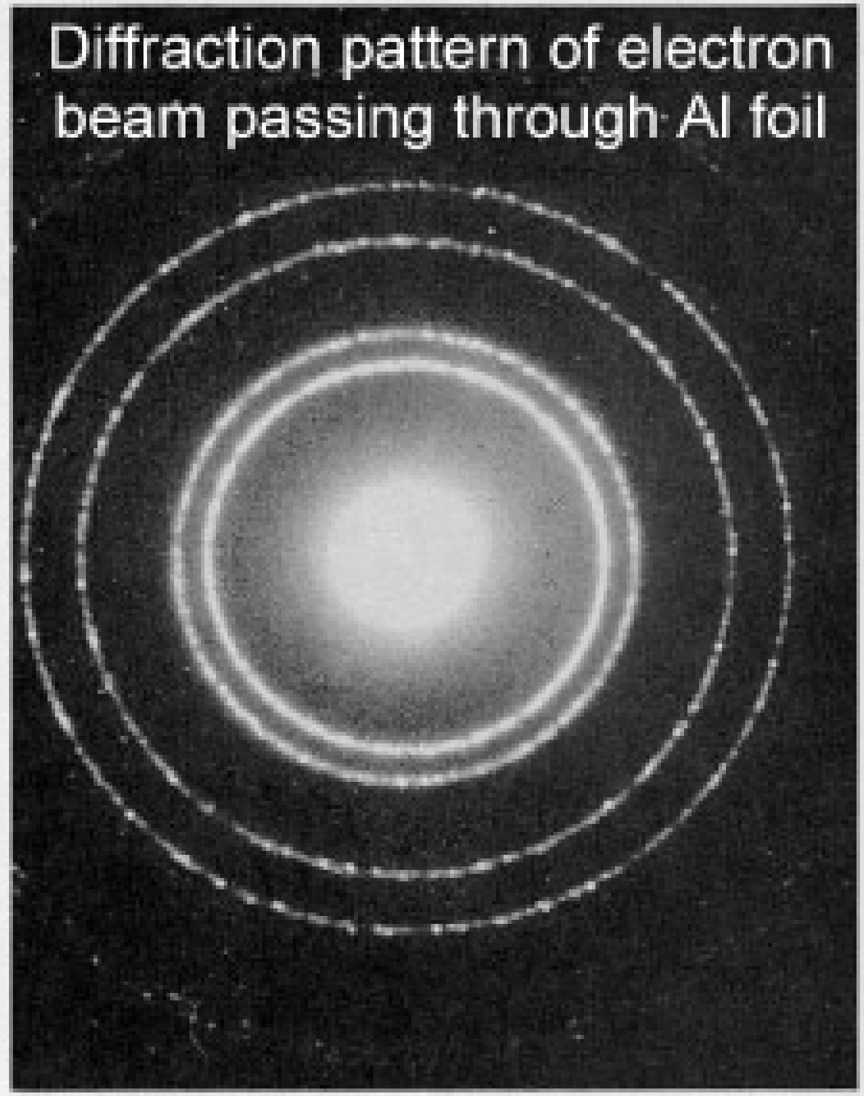
$\lambda \approx$ **0.1 Å**



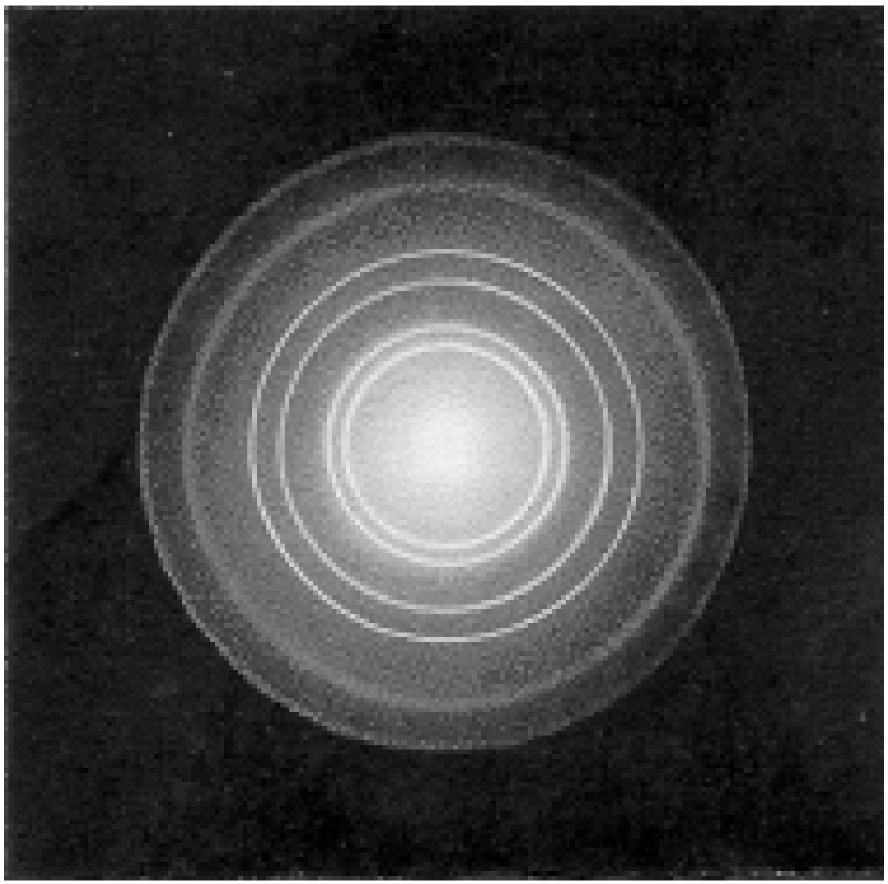


polycrystalline film ⇒ Bragg condition satisfied for any given reflecting plane ⇒ concentric circles

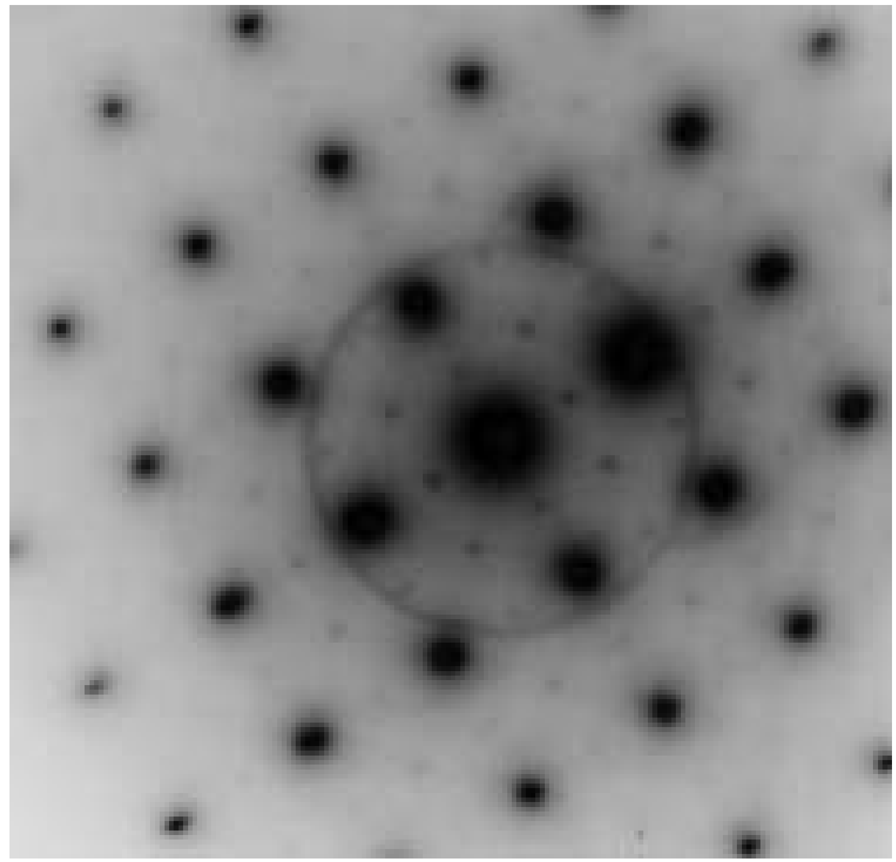

Diffraction pattern of X-ray
beam passing through Al foil


Diffraction pattern of electron
beam passing through Al foil

Electron diffraction by polycrystalline aluminum

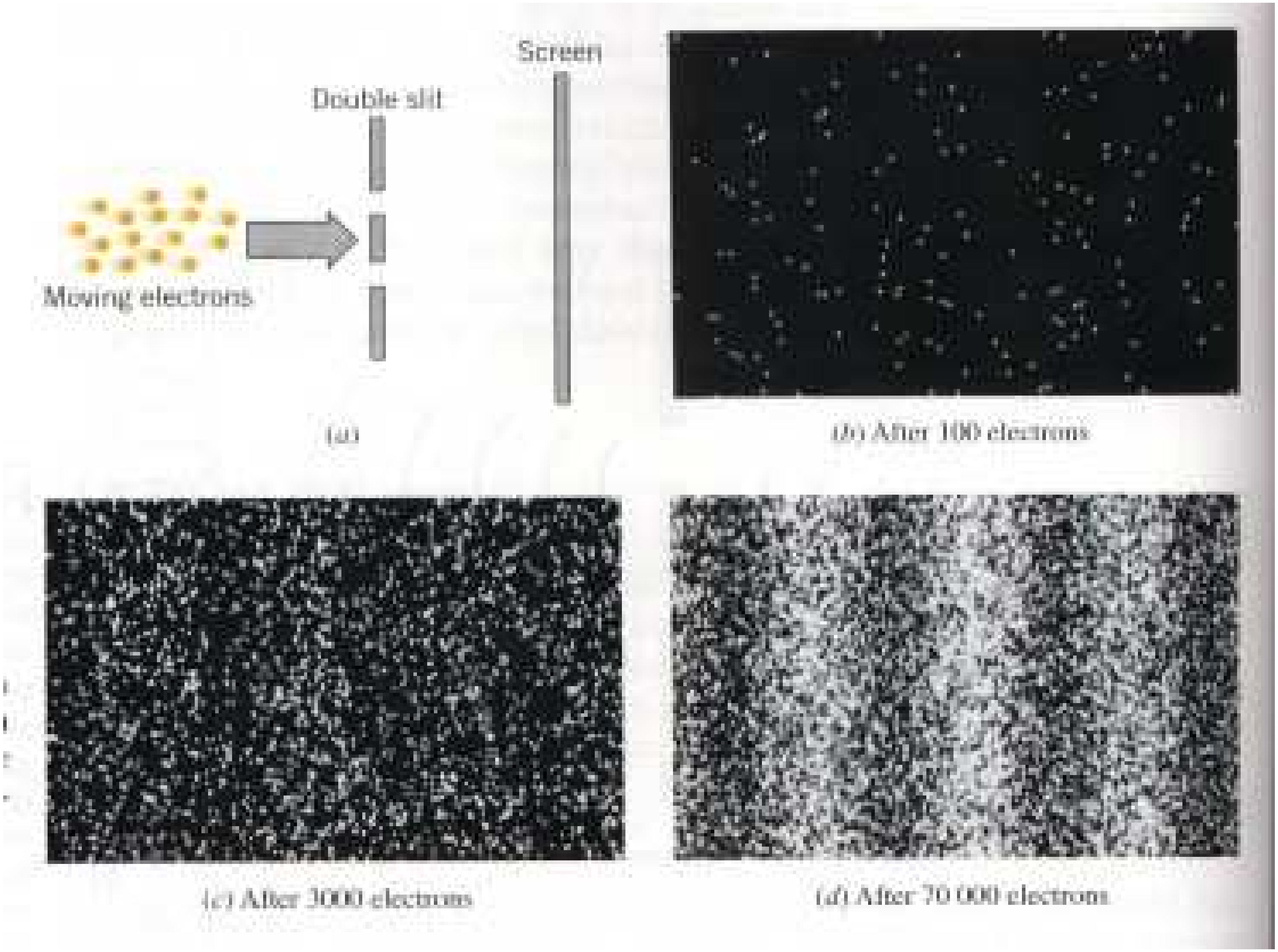


Laue pattern of electron diffraction by a single crystal



(Courtesy of Prof. Y. Soejima, Dept. of Physics, Kyushu Univ.)

2-slit Interference of Electrons



Diffraction of Neutrons

λ = several Å down to $<10^{-14}$ m

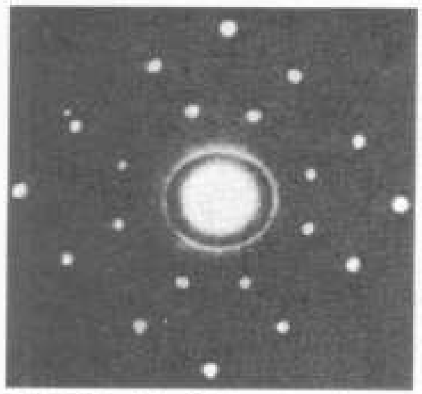
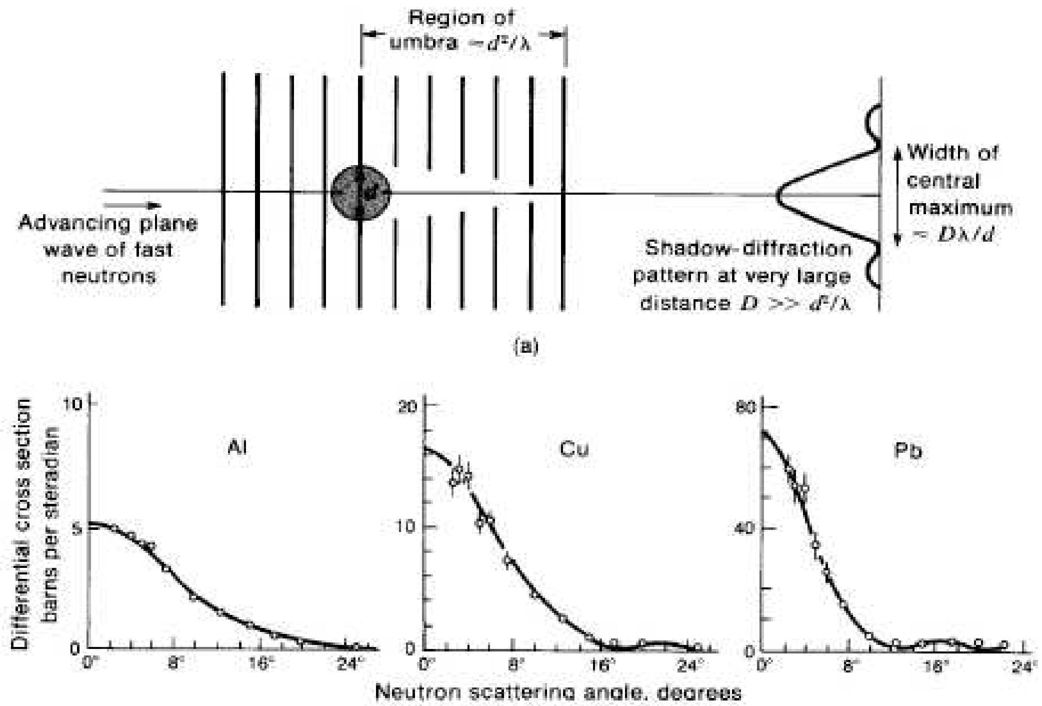
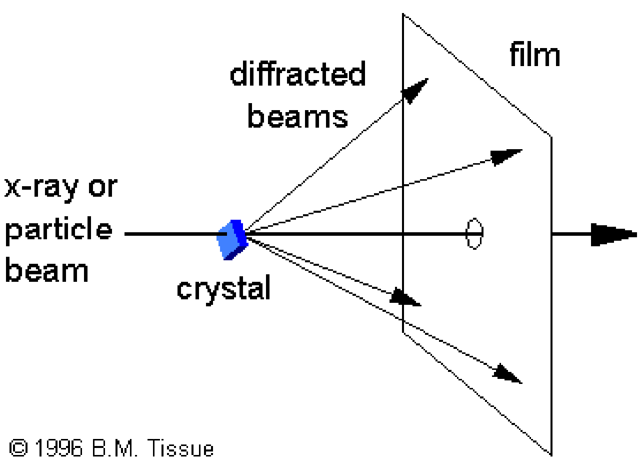


FIGURE 4.7 Diffraction of neutrons by a sodium chloride crystal.

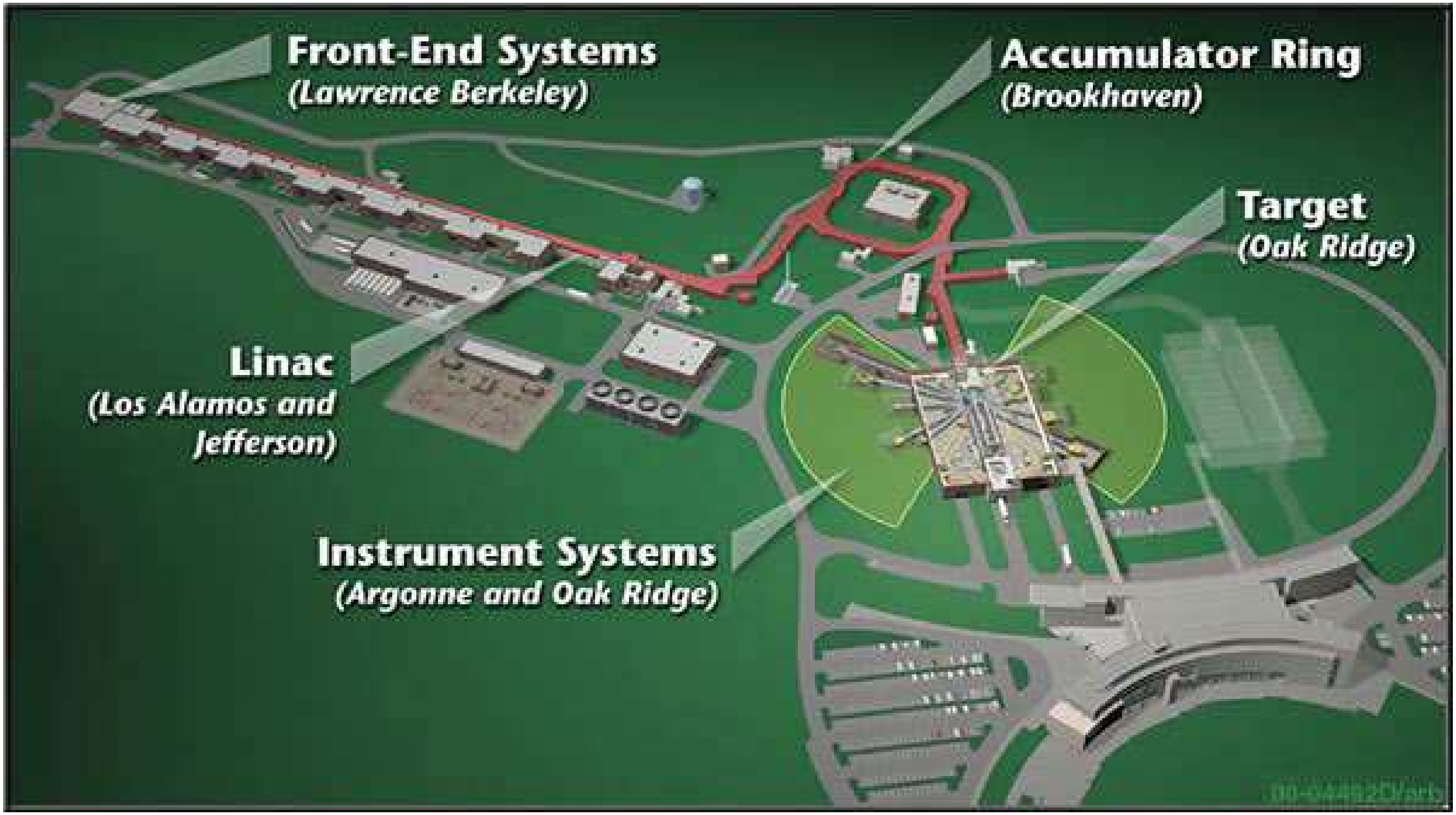
from Krane



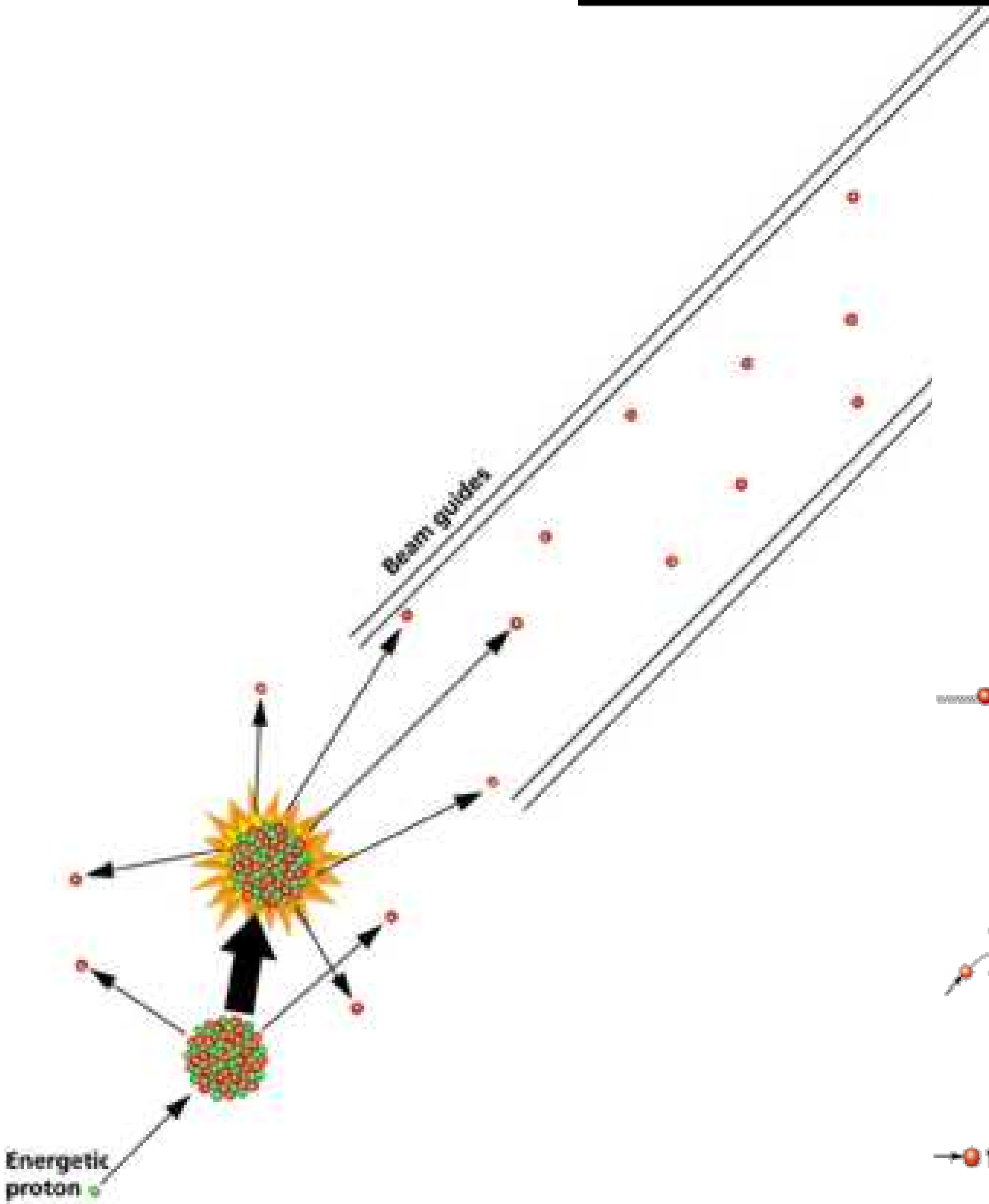


Diffraction of fast neutrons from Al, Cu, and Pb nuclei. [from French, after A Bratenahl, Phys Rev 77, 597 (1950)]

The Spallation Neutron Source (SNS) in Oak Ridge, TN



Why Neutrons?



Neutrons are NEUTRAL particles. They

- are highly penetrating,
- can be used as nondestructive probes, and
- can be used to study samples in severe environments.

Neutrons have a MAGNETIC moment. They can be used to

- study microscopic magnetic structure,
- study magnetic fluctuations, and
- develop magnetic materials.

Neutrons have SPIN. They can be

- formed into polarized neutron beams,
- used to study nuclear (atomic) orientation, and
- used for coherent and incoherent scattering.

The ENERGIES of thermal neutrons are similar to the energies of elementary excitations in solids. Both have similar

- molecular vibrations,
- lattice modes, and
- dynamics of atomic motion.

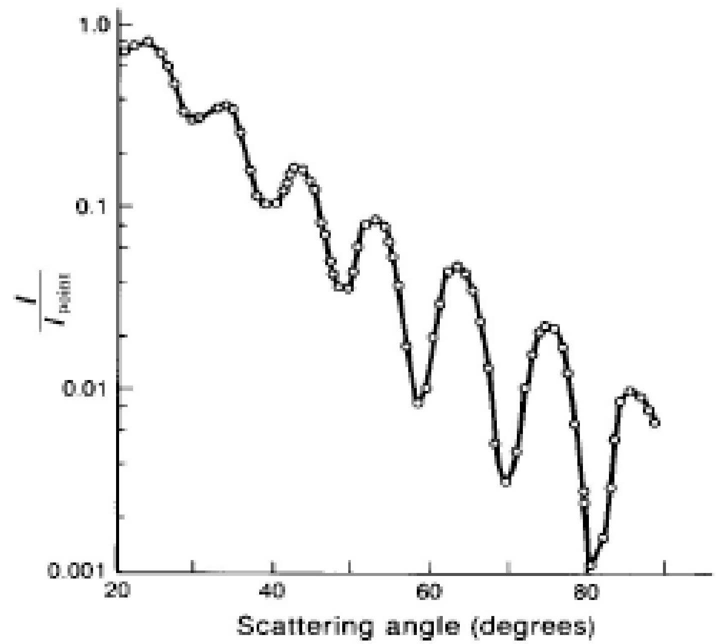
The WAVELENGTHS of neutrons are similar to atomic spacings. They can determine

- structural sensitivity,
- structural information from 10^{-13} to 10^{-4} cm, and
- crystal structures and atomic spacings.

Neutrons "see" NUCLEI. They

- are sensitive to light atoms,
- can exploit isotopic substitution, and
- can use contrast variation to differentiate complex molecular structures.

Scattering of Alpha Particles



Angular distribution of 40 MeV alpha particles scattered from niobium nuclei.
[from French after G. Igo et al., Phys Rev 101, 1508 (1956)]

Crystal Diffraction of Neutral Helium (1930)

$\lambda \approx$ **1 Å**

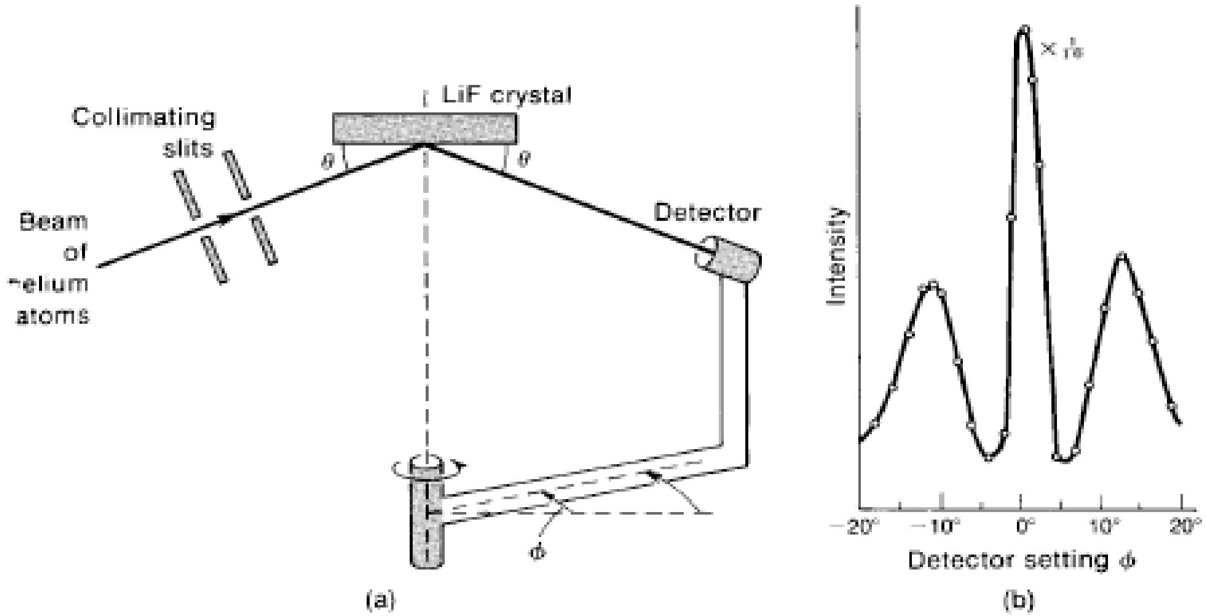
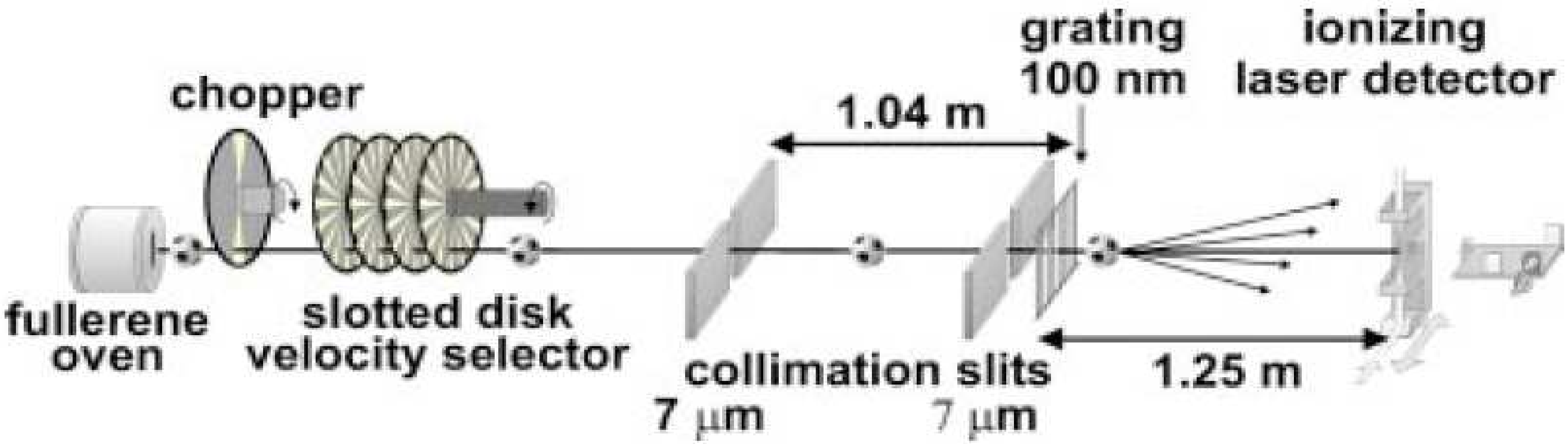


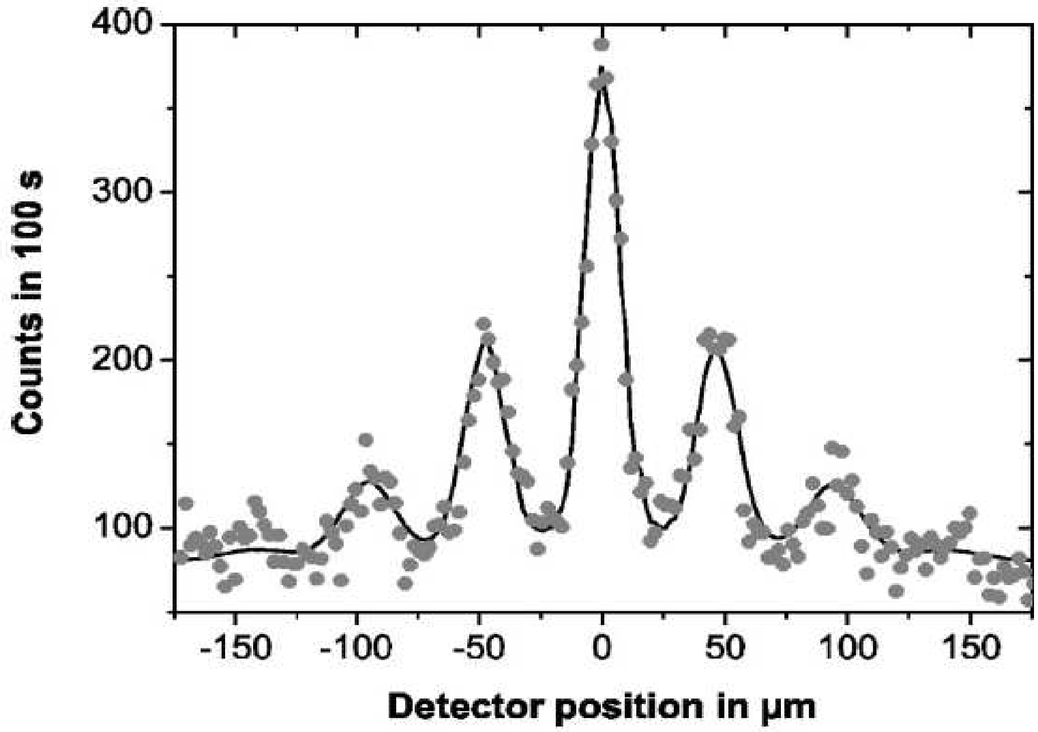
Fig. 2-16 (a) Experimental arrangement used by Stern et al. *to investigate crystal diffraction of neutral helium atoms. (b) Experimental results showing central reflection peak* ($\phi = 0°$)*, plus first-order diffraction peaks* ($\phi = 11°$)*. In the experiment,* $\theta = 18.5°$.

from French after Estermann and Stern, Z Phys 61, 95 (1930)

Interference of Molecules



Fullerene molecule C60 , consisting of 60 carbon atoms



I_4 The "Old Quantum Theory"

$I_{4,1}$ Key Ideas / Concepts / Postulates:

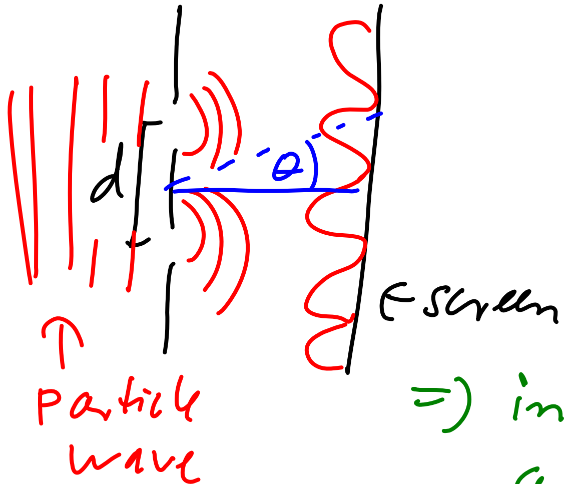
1) Photons, all particles have both particle-like and wave-like properties
2) Precisely-defined trajectories do not exist at the quantum level
3) The exact behavior of a given particle cannot be predicted → only its probable behavior

$\Rightarrow$ statistical interpretation

4) The probability that a single particle is observed in a given region is proportional to intensity of its associated wave field: $I \propto |A|^2$

$$\Rightarrow P \propto |A|^2$$

⇒ recall 2-slit experiment.



wave amplitude on screen:

- one slit open: A_0
- both slits open: interference

$$A_{total} = 2 A_0 \cos\left(\frac{\pi d}{\lambda} \sin\theta\right)$$

⇒ intensity on screen ∝ probability for a particle to arrive at a given region along the screen ∝ statistical distribution of large number of particles on the screen

$$I(\theta) \propto P(\theta) \propto |A_{total}|^2 = 4|A_0|^2 \cos^2\left(\frac{\pi d}{\lambda} \sin\theta\right)$$

↑ probability ($P(\theta)$)

⇒ $\sqrt{P} \propto |A|$ also called probability amplitude (quantum amplitude)

later: wavefunctions ψ (complex) ⇒ $P \propto |\psi|^2$

2-slit experiment with particles: Assume that only one slit is open, and that the probability of a particle to arrive at a small section Δx of the screen is **F**. What is the maximum probability of finding a particle in that section Δx of the screen if both slits are open simultaneously?

A. $\sqrt{2}$ **F**

B. **2 F**

C. **4 F**

D. **2** $\sqrt{F}$

E. F^2

– one slit open: probability F ⇒ prob. amplitude = $\sqrt{F}$ · complex phase factor

– both slits open:
⇒ add probability amplitudes!
⇒ max. probability amplitude = $2\sqrt{F}$
⇒ max. probability = $(2\sqrt{F})^2 = 4F$

5) If a particle is confined into a small volume, its energy is quantized ⇒ "energy levels"
"energy states"

6) de Broglie - Einstein postulates:

$$\lambda = h/p \qquad (p = \hbar k) \qquad k = \text{wave number}$$

$$\nu = E/h \qquad (E = \hbar \omega) \qquad \omega = \text{angular frequ.}$$

7) Superposition principle