Lecture 4: 01/26/09

M. Planck (1858 –1947) won Nobel price in Physics for his work on blackbody radiation (1918)

Blackbody Spectrum:

1) Classical Theory

⇒ Rayleigh-Jeans formula

⇒ "ultraviolet catastrophe"

2) Planck's Theory

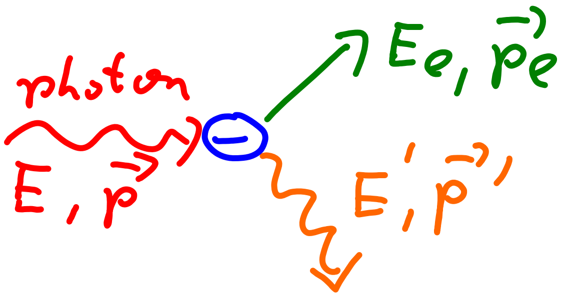
⇒ Quantized energy

Recap:

$I_{1,3}$ The Compton Effect

$\Rightarrow$ **Demonstrates particle like nature of radiation**

Elastic collision between photon and free e^-



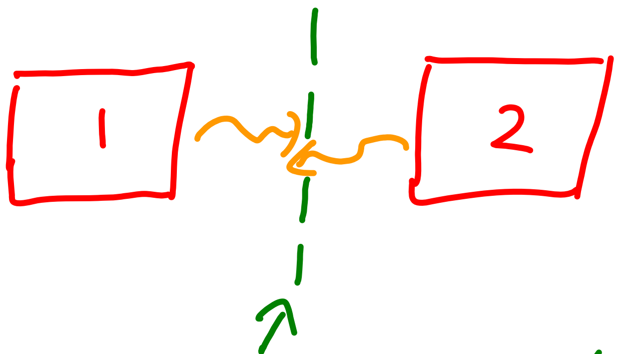
$\boxed{\text{Photon: Energy } E = h\nu \quad \text{Momentum } p = h/\lambda}$

Compton shift: $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$

$I_{1,4}$ Blackbody Radiation

$\Rightarrow$ Demonstrates quantization

- Blackbody: 100% Absorbs at all wavelengths – no reflection
  $\Rightarrow$ emit a universal spectrum that only depends on temperature T
- why?



Two black bodies at same T. Radiation emitted by 1 is absorbed by 2 and vice versa.

notch filter: transmission in one narrow frequency range

$\Rightarrow$ Energy emitted at given frequency must be same, or one would heat up and other cool down
– violates thermodynamics

$\Rightarrow$ whole spectrum must be same! $\Rightarrow$ universal

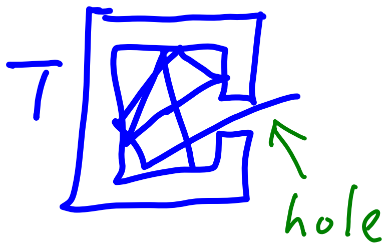
Does a blackbody always appear black?

A. Yes

B. No

C. Maybe

- How to make a black body with real materials?



⇒ cavity with small hole, walls at T

⇒ absorbs everything (hole)

⇒ radiation from hole in cavity has black body spectrum

- Why interesting?

⇒ calculate spectrum from classical physics and thermodynamics ⇒ Result can not be correct! (predicts infinite energy...)

- Solution:

Planck (1900) ⇒ quantized chunks of light energy (photons)

allowed energies: $\underline{E_n = n h \nu}$ n: integer

- Blackbody spectrum:

$\rho_T(\nu)\,d\nu =$ energy of radiation per unit volume in a cavity at temperature T in the frequency intervall ν to $\nu + d\nu$

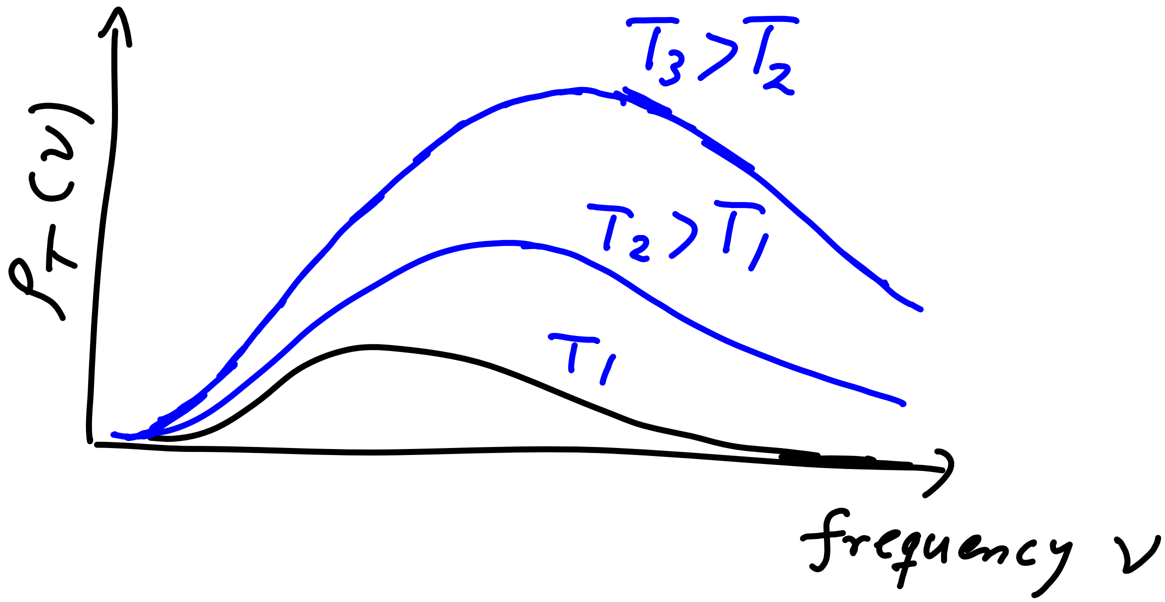
$[\rho_T(\nu)] = J/m^3\,Hz$

$R_T(\nu)\,d\nu =$ spectral radiancy = energy emitted by hole in cavity at temperature T per unit time per unit area of the hole in the frequency intervall ν to $\nu + d\nu$

$[R_T(\nu)] = W/m^2\,Hz$

$\rho_T(\nu) \propto R_T(\nu)$

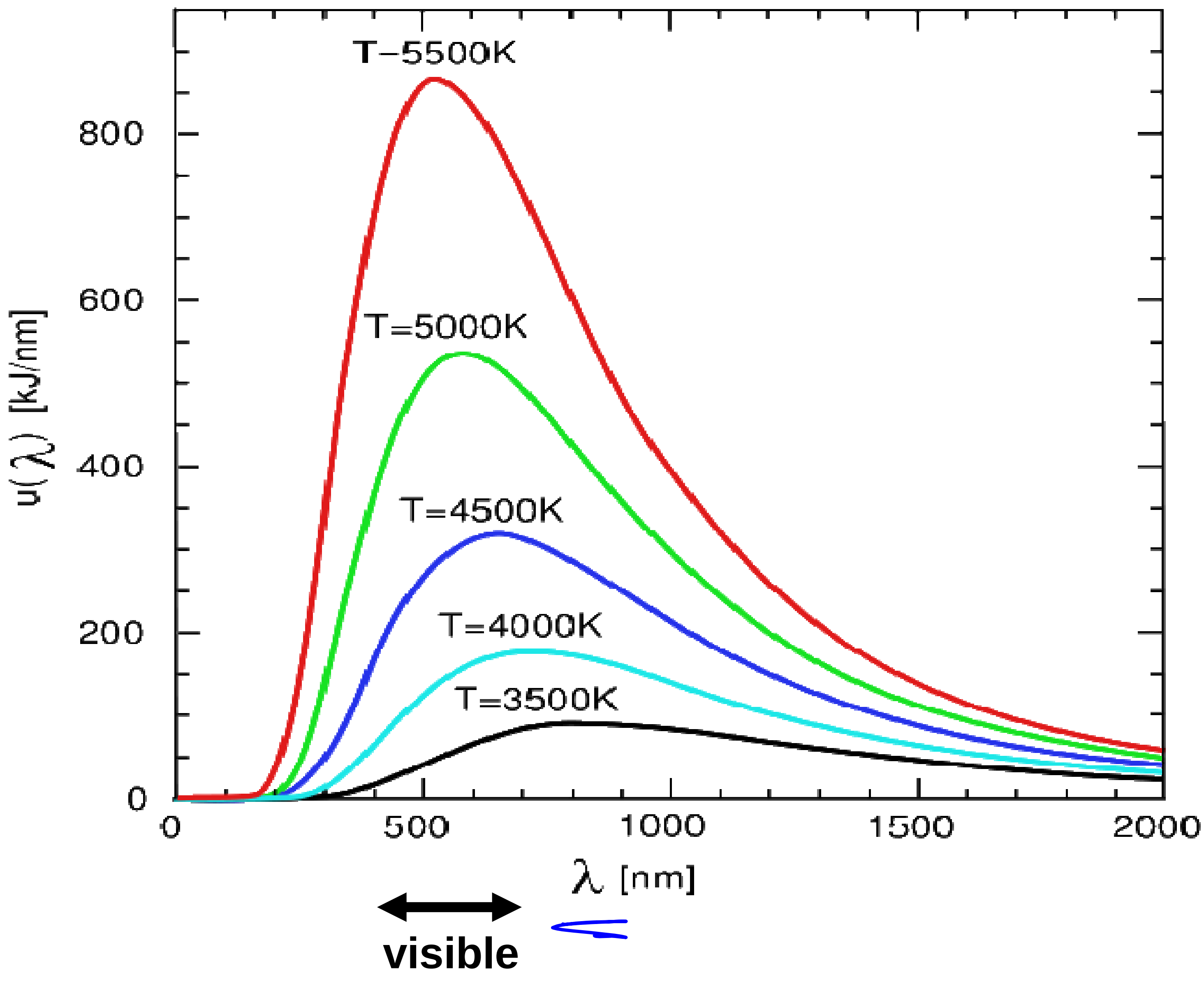
- ## Experimental Results:



$\Rightarrow$ frequency of maximum radiancy $\propto T$

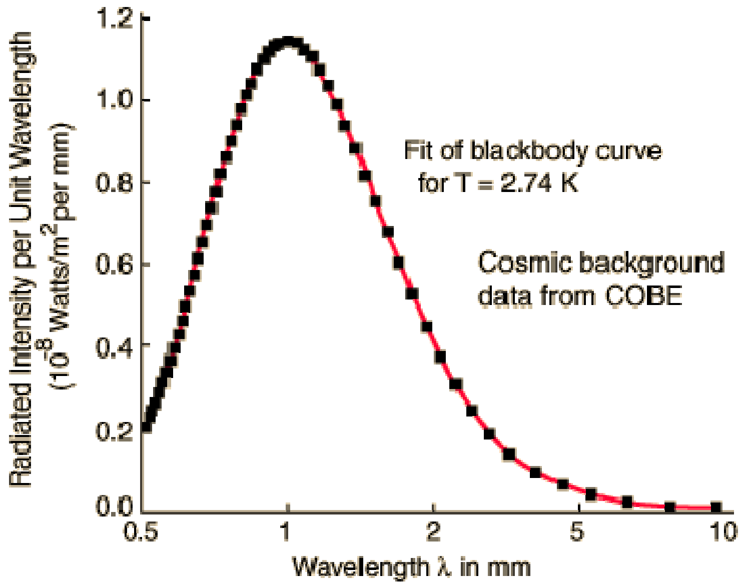
$\Rightarrow$ total power emitted per area $\propto T^4$

Blackbody Radiation Spectrum

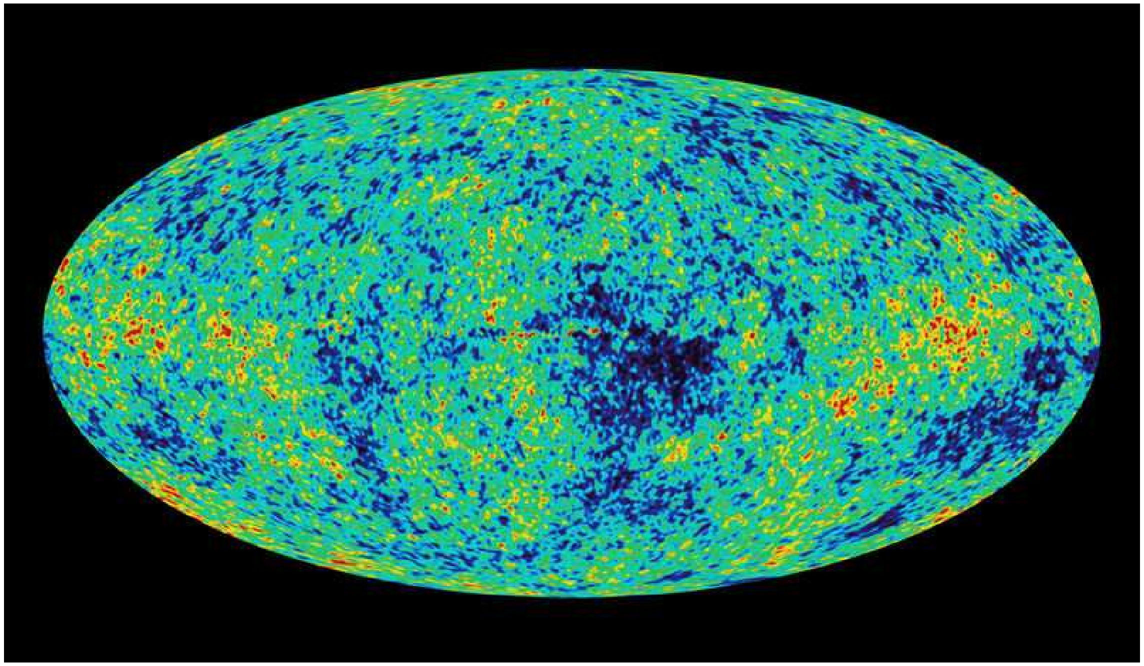


The Cosmic Microwave Background

The cosmic microwave background is a perfect blackbody emission and corresponds to a temperature of 2.725 Kelvin with an emission peak of 160.4 GHz.



WMAP image of the cosmic microwave background radiation anisotropy.



– towards $\rho(\nu)$:

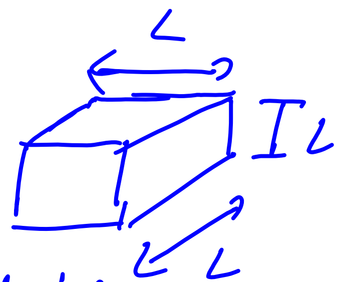
1. Count # of modes for standing waves of electromagnetic radiation inside a metal box in the frequency range ν to $\nu + d\nu$

2. Use statistical mechanic to compute the average energy per mode
   - 1st classically with continuous energies
   - 2nd with quantized energy

3. Calculate:

$$\rho_T(\nu) = \left(\frac{\#\text{ of modes between } \nu \text{ and } \nu + d\nu}{\text{volume of cavity}} \right) \left(\begin{array}{c}\text{average } E \\ \text{per mode}\end{array} \right)$$

step ①:

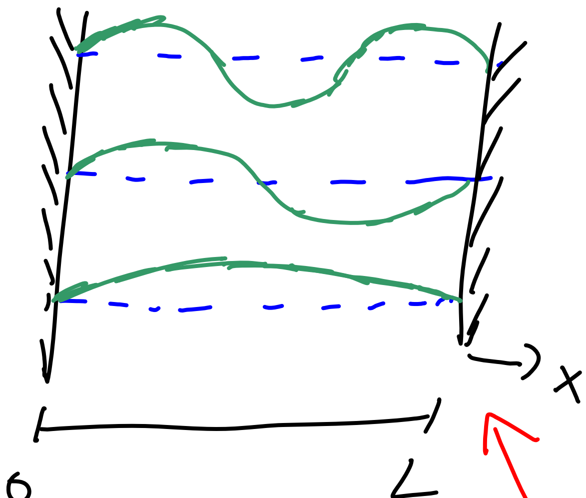


metal box
$V = L^3$

- $\vec{E}_{trans} = 0$ at metal walls
- E&M wave are transverse

$\Rightarrow$ standing waves have electric field nodes at walls

• start with 1-D:



- standing waves have integer number of half wavelengths

$$L = n_x\left(\frac{\lambda}{2}\right) \Rightarrow \lambda_x = \frac{2L}{n_x} \qquad n_x = 1, 2, 3, \ldots$$

$$\vec{E}(x,t) = \vec{E}_T \sin\left(\frac{2\pi}{\lambda_x}x\right)\sin(2\pi\nu t)$$

∞ number of modes!

now in 3D: standing waves in x, y, and z directions

$$\vec{E}(x,y,z,t) = \vec{E}_0 \sin\left(\frac{2\pi}{\lambda_x}x\right)\sin\left(\frac{2\pi}{\lambda_y}y\right)\sin\left(\frac{2\pi}{\lambda_z}z\right)\sin(2\pi\nu t)$$

with $\lambda_x = \frac{2L}{n_x}$, $\lambda_y = \frac{2L}{n_y}$, $\lambda_z = \frac{2L}{n_z}$ $n_x, n_y, n_z > 0$

Note: 1) ∞ number of modes

2) Two polarizations $\Rightarrow$ 2 standing waves for every set of integers (n_x, n_y, n_z)

- # of standing wave modes within interval ν to $\nu + d\nu$?

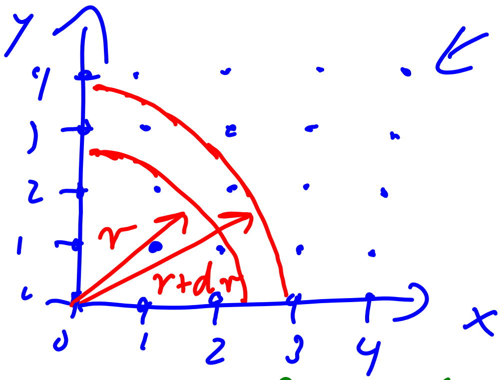
wave equation: $\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$ (free space) $\Rightarrow \frac{\partial^2}{\partial x^2}\vec{E} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$

$$\Rightarrow -\left(\frac{2\pi}{\lambda_x}\right)^2 - \left(\frac{2\pi}{\lambda_y}\right)^2 - \left(\frac{2\pi}{\lambda_z}\right)^2 = -(2\pi\nu)^2/c^2$$

$$\Rightarrow \nu = c\sqrt{1/\lambda_x^2 + 1/\lambda_y^2 + 1/\lambda_z^2} = \frac{c}{2L}\sqrt{n_x^2 + n_y^2 + n_z^2}$$

$\Rightarrow$ # of modes in $[\nu, \nu + d\nu] = 2 \cdot$ (combinations of n_x, n_y, n_z that give ν in this range) have $\frac{\nu}{c}2L = \sqrt{n_x^2 + n_y^2 + n_z^2} = r$ = radius of sphere

in 2-D:



← point $(x,y) = (n_x, n_y)$

– (n_x, n_y, n_z) define a grid of points in octant $(n>0)$

– density of points: 1 point per unit volume

2 polarizations!

$\Rightarrow \begin{pmatrix} \text{\# of modes between} \\ \nu \text{ and } \nu + d\nu \end{pmatrix} = \begin{pmatrix} 2 * \text{\# of points with radii} \\ r = \frac{\nu}{c}2L \text{ to } r + dr = \frac{\nu + d\nu}{c}2L \end{pmatrix}$

in 3D for large n:

$\begin{pmatrix} \text{\# of points} \\ \text{between } r \text{ and } r+dr \end{pmatrix} \simeq \begin{pmatrix} \text{volume of shell} \\ r \to r+dr \end{pmatrix} = \frac{\overbrace{4\pi r^2 dr}^{A}}{8}$ ← octant only

$\Rightarrow \begin{pmatrix} \text{\# of modes between} \\ \nu \text{ and } \nu + d\nu \text{ in cavity} \end{pmatrix} = 2 * \frac{\pi}{8} r^2 dr = 2\frac{\pi}{2}\left(\frac{\nu}{c}2L\right)^2 \frac{2L}{c} d\nu$

$$= 8\pi \frac{L^3}{c^3} \nu^2 d\nu$$

$$\frac{\#\text{ of standing wave modes between } \nu \text{ and } \nu + d\nu}{\text{volume of cavity}} = \underbrace{8\pi \frac{\nu^2}{c^3}}_{= 2.4 \cdot 10^{5} / m^3\,Hz} d\nu$$

Note: 1) for $\nu = 5.1 \cdot 10^{14}$ Hz (yellow): $= 2.4 \cdot 10^{5} / m^3\,Hz$

2) # of modes $\propto \nu^2 \rightarrow \infty$ number of modes

(power of ν = # of space dimensions -1)

step 2: average energy per standing wave at wall temp T

1st classically:

Boltzmann Principle: for a large number of physical entities of the same kind in thermal equilibrium, the relative probability that a system at temp. T will be found in a given state with energy E is

$$P(E) \propto e^{-E/kT}$$

k: Boltzmann's constant

Here: $P(E)$ = probability that a standing wave in cavity will have energy E