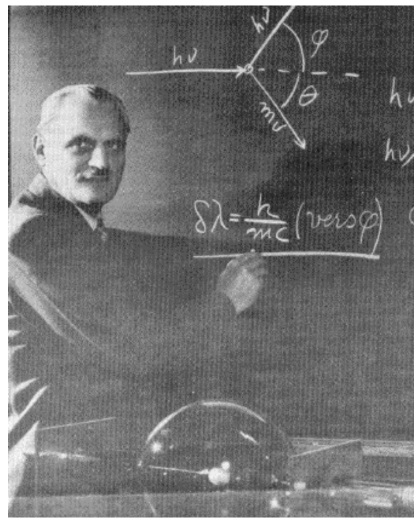
Lecture 3: 01/23/09

- Compton Effect



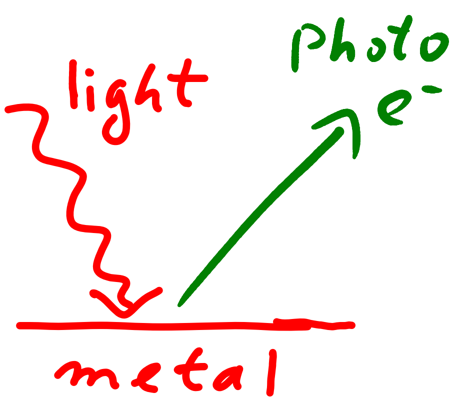
Arthur Holly Compton (1892 – 1962) won the Nobel Prize in Physics (1927)

Recap

• 2-slit interference: Wave-particle duality

≈ Light is composed of photons whose motion must be described by an analysis that closely parallels the classical wave description in terms of interfering amplitudes from both slits ⇒ "wave equation"

• Photoelectric effect:



~~classical picture~~: waves "shake" e^-

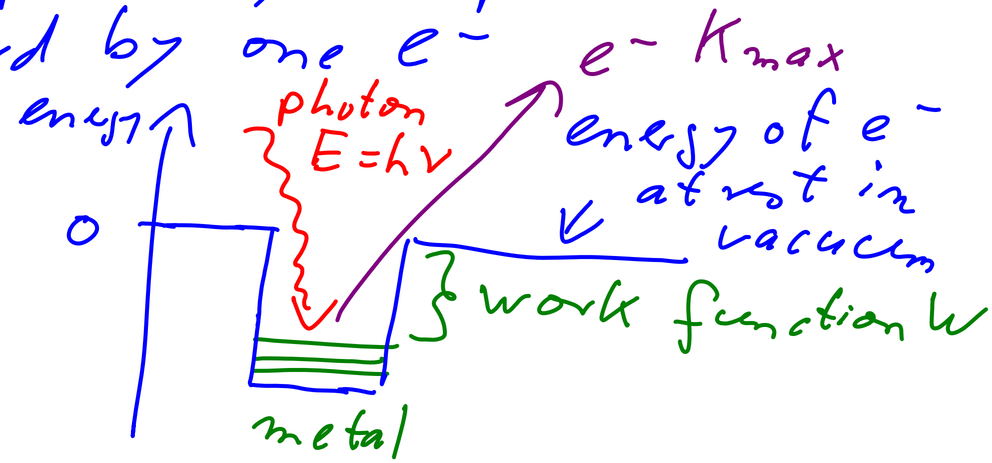
Quantum picture: Photons

Energy of photon: $E = h\nu$

⇒ Radiation energy is quantized

3. In the photoelectric process, one photon is completely absorbed by one e^-

- Some of the energy of the photon will go to pay for the e^- to escape from the metal



→ energy conservation: $K_{max} = h\nu - W$

allows to measure of h

material dep.

K_{max} : highest kinetic energy possible (less possible too)

↝ K_{max} does <u>not</u> depend on intensity (result 1)

↝ K_{max} increases linearly with ν (result 3)

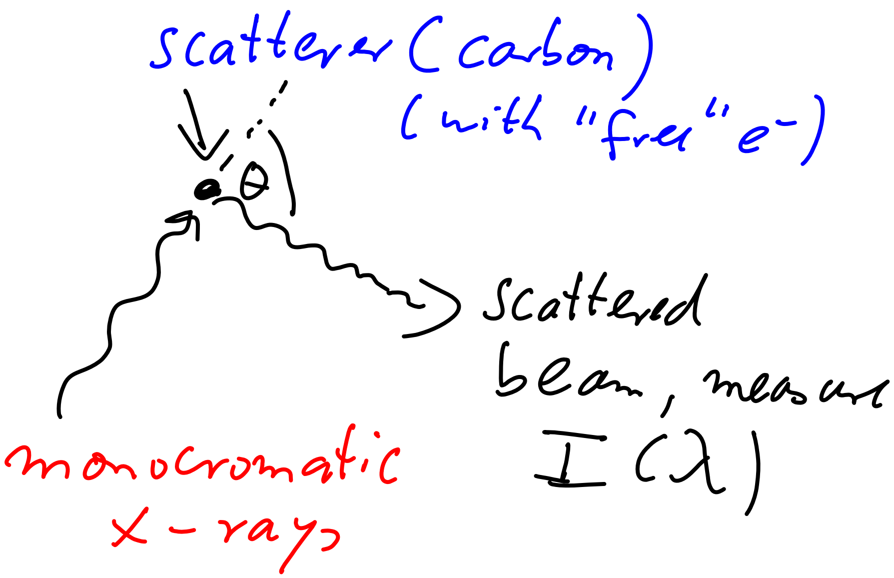
↝ No photoelectrons if $h\nu < W$ (result 2)

↝ No time lag (result 4)

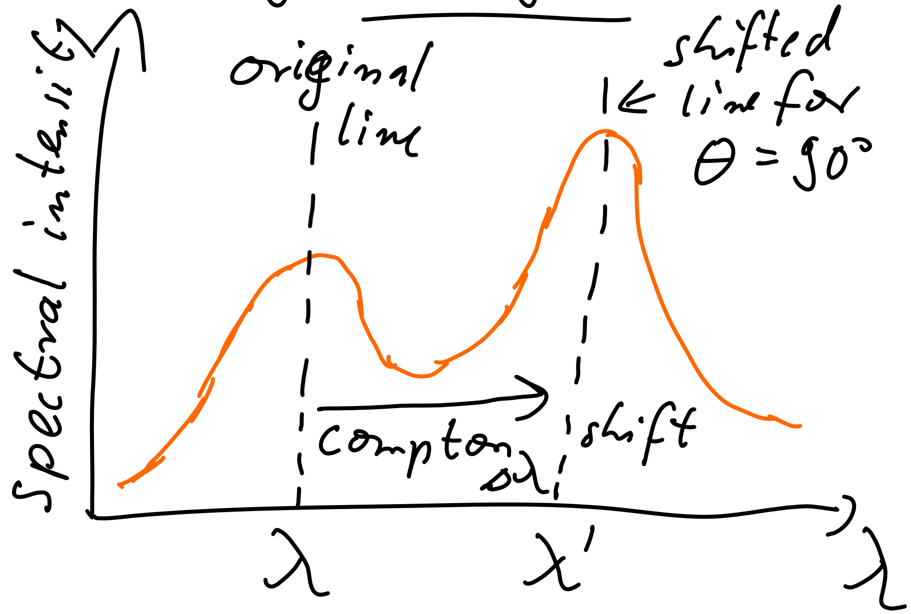
$I_{1,3}$ The Compton Effect

- Dynamics of individual x-ray photon in collision with <u>free</u> electrons
- <u>key idea:</u> photons carry momentum $\vec{p}$ and obey momentum and energy conservation laws

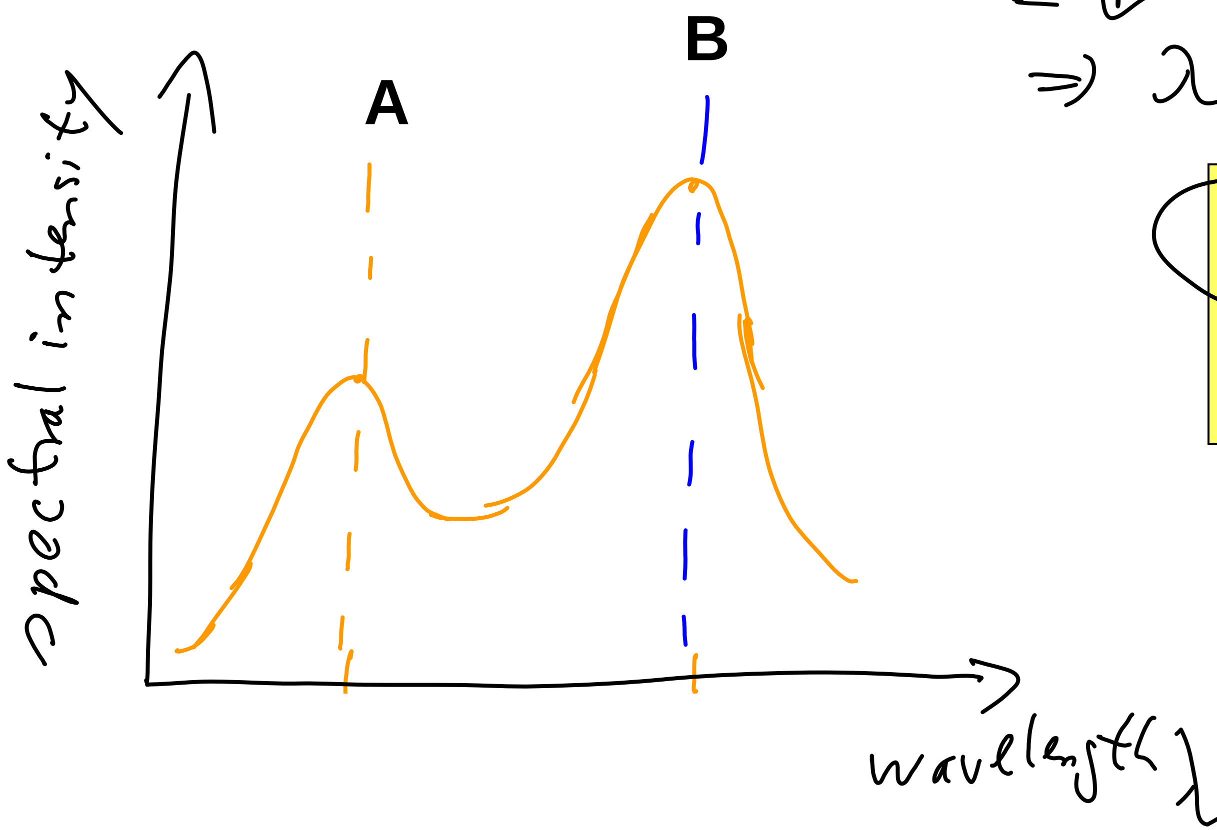
<u>Experiment.</u>



for <u>$\theta = 90°$</u>:



Which spectral line is at the original wavelength of the incident x-rays?



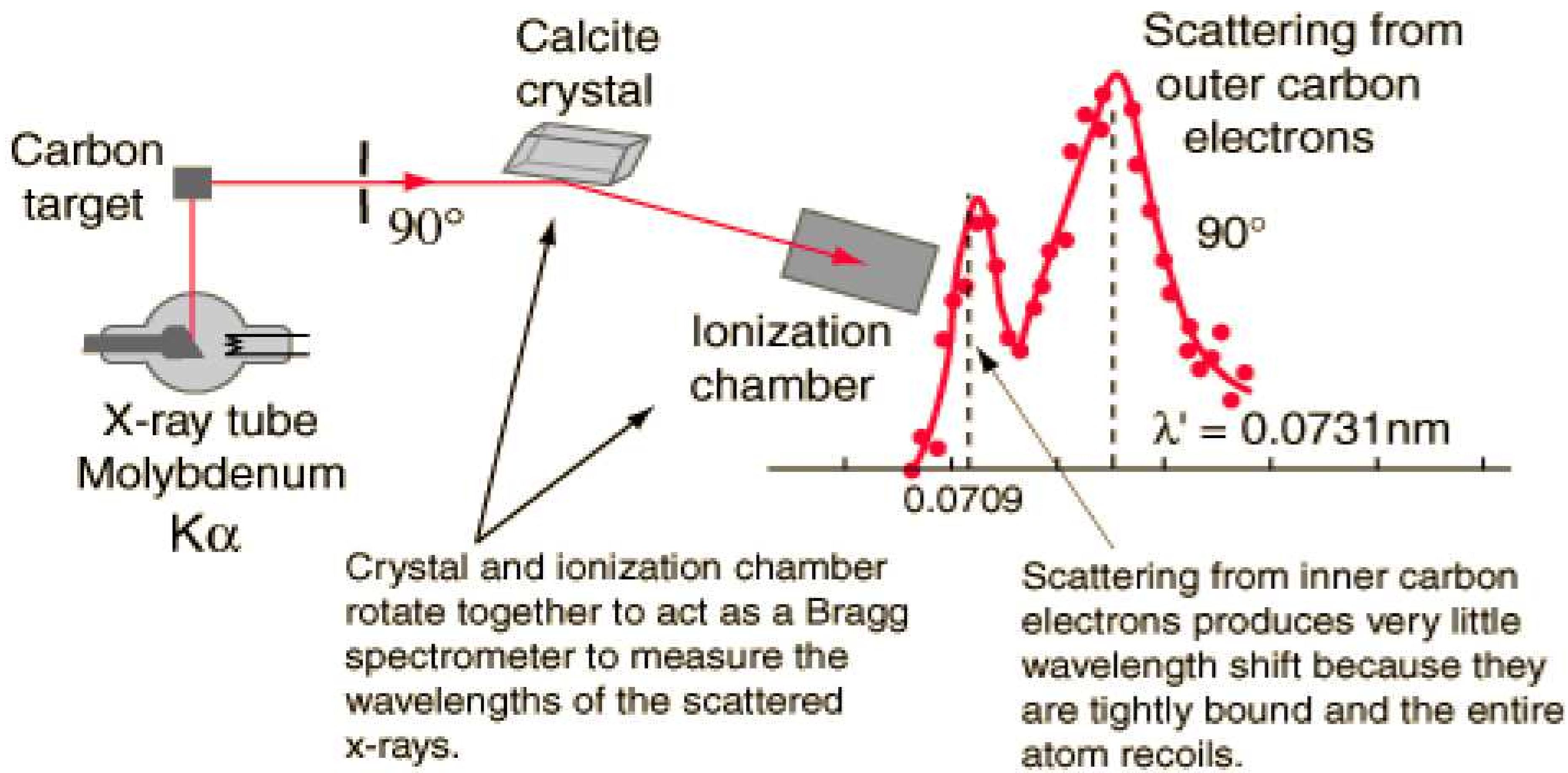
$E \downarrow \Rightarrow \nu \downarrow$

$\Rightarrow \lambda \uparrow \quad (\nu\lambda = c)$

A.

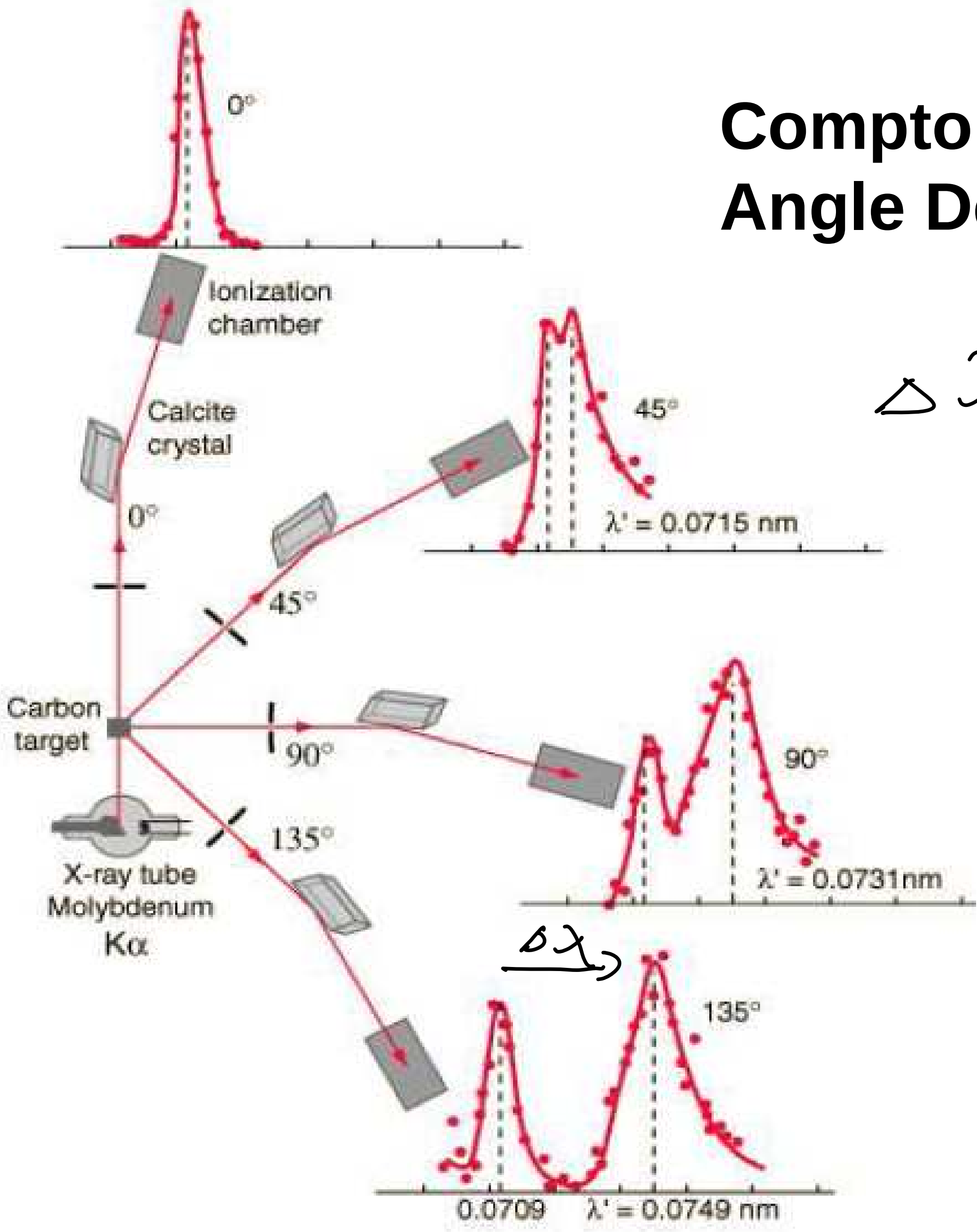
B.

Compton Effect: Setup and Result



Compton Effect: Angle Dependence

$$\Delta\lambda = \Delta\lambda(\theta)$$



$\Delta\lambda$

- **Classical picture:**

→ oscillating electric field

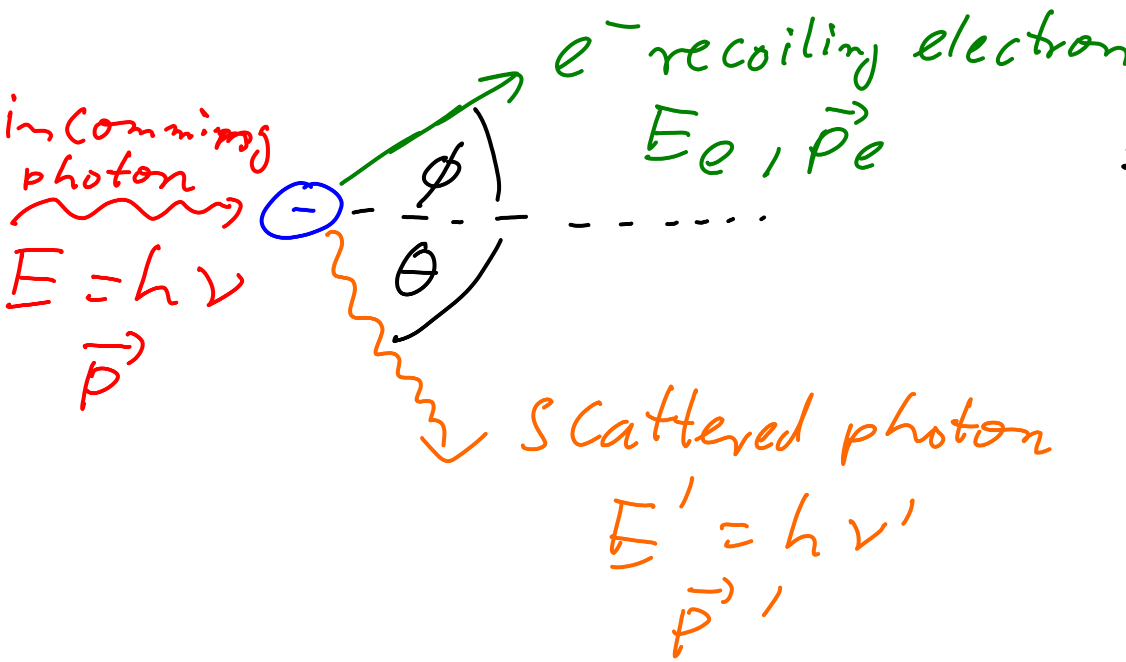
⇒ oscillating electrons (at same frequ.)

⇒ electrons produce radiation

<u>but</u>: incident and scattered radiation should have <u>same</u> frequency

⇒ no shift ($\Delta\lambda = 0$) ↯

- **Quantum Picture:**



⇒ collision (elastic) between photon and <u>free</u> electron

Qualitatively: incomming photon gives some of its energy to electron KE

$\Rightarrow$ scattered photon has less energy

$\Rightarrow$ " " " larger wavel. λ

$$E' = h\nu' = h\,c/\lambda'$$

Notes: 1) Electron is relativistic:

total relativistic energy of e^- of mass m_e $\quad E^2 = p_e^2 c^2 + (m_e c^2)^2$

2) Momentum of photon:

$$\Rightarrow \quad E = pc \quad \Rightarrow \underline{\underline{p}} = \frac{E}{c} = \frac{h\nu}{c} = \underline{\underline{\frac{h}{\lambda}}} \neq 0 \,!$$

momentum conservation | energy conservation

$$\vec{p} = \vec{p}\,' + \vec{p}_e$$

$$\vec{p}_e = \vec{p} - \vec{p}\,'$$

$$\vec{p}_e \cdot \vec{p}_e = (\vec{p} - \vec{p}\,') \cdot (\vec{p} - \vec{p}\,')$$

$$p_e^2 = p^2 + p'^2 - 2\underbrace{pp'\cos\theta}_{\vec{p}\cdot\vec{p}\,'}$$

$$p_e^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - 2\left(\frac{h\nu}{c}\right)\left(\frac{h\nu'}{c}\right)\cos\theta$$

$$h\nu + m_e c^2 = h\nu' + \sqrt{p_e^2 c^2 + m_e^2 c^4}$$

$$\sqrt{p_e^2 c^2 + m_e^2 c^4} = h\nu - h\nu' + m_e c^2$$

$$\cancel{p_e^2 c^2 + m_e^2 c^4} = (h\nu)^2 + (h\nu')^2 + 2(h\nu - h\nu')m_e c^2 + \cancel{m_e^2 c^4} - 2(h\nu)(h\nu')$$

$$\Rightarrow p_e^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 + 2(h\nu - h\nu')\cdot m_e - 2\left(\frac{h\nu}{c}\right)\left(\frac{h\nu'}{c}\right)$$

Combine:

$$-\left(\frac{h\nu}{c}\right)\left(\frac{h\nu'}{c}\right)\cos\theta = (h\nu - h\nu')m_e - \left(\frac{h\nu}{c}\right)\left(\frac{h\nu'}{c}\right)$$

$$\Rightarrow (h\nu - h\nu')m_e c^2 = (1 - \cos\theta)(h\nu)(h\nu')$$

$$\Rightarrow \left(\frac{1}{h\nu'} - \frac{1}{h\nu}\right)m_e c^2 = (1 - \cos\theta) = \left(\frac{c}{\nu'} - \frac{c}{\nu}\right)\frac{m_e c}{h}$$

$$\Rightarrow \left(\frac{c}{\nu'} - \frac{c}{\nu}\right) = \underbrace{\lambda' - \lambda}_{\Delta\lambda} = \frac{h}{m_e c}(1 - \cos\theta)$$

Compton shift: $\lambda' - \lambda = \Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) \geq 0$

Note:
- $\Delta\lambda \geq 0$ as measured
- $\Delta\lambda$ depends on θ, but not λ
- $\Delta\lambda$ varies from 0 ($\theta = 0°$) to $\frac{2h}{m_e c} = 0.049\,\text{Å}$ ($\theta = 180°$, "head on")
- 2 peaks:
  - I) photons scattered from free (or weakly bound) e^-
  - II) photons scatter from e^- strongly bound to atom
    - $\Rightarrow$ large "m" $\rightarrow \Delta\lambda \rightarrow 0$
    - $\Rightarrow$ Rayleigh scattering

Does a photon of Energy E have a non-zero mass?

A. Photons have momentum, so they must have a mass

B. It travels with the speed of light, independent of frequency and energy, so no mass

special relativity: total energy of particle

$$E^2 = c^2 p^2 + (mc^2)^2$$

$$\Rightarrow E^2 = c^2 p^2 = (h\nu)^2 \text{ for photon}$$

If one doubles the wavelength of the incoming light, the Compton shift $\Delta\lambda$ …

A. … doubles

B. … stays the same

C. … decreases by a factor ½

D. Something else

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

Can one observe the Compton effect with visible light?

A. Yes

B. No

$\Delta\lambda_{max} = 0.005\ nm$

$\lambda_{visible} = 400 - 700\ nm$

Why, in Compton scattering, would you expect $\Delta\lambda$ to be independent of the scatter material?

A. Because the equation says so
B. Because the photon scatters of a free electron in the Compton effect
C. It is not independent of the scatter material