RESEARCH DESCRIPTION
December 2012, André LeClair

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1.1 Work completed on the current grant

Prof. LeClair has primarily worked on a broad range of applications of quantum field theory to condensed matter theory. Some of this work was motivated by the AdS/CFT correspondence, for example his calculation of the ratio of the viscosity to entropy density for the unitary fermi gas, and certain 2d field theories that arise in string theory.

Two dimensional quantum field theories with supergroup symmetry. Current algebras based on Lie super-algebras, especially $OSP(2N|2N)$ and $GL(N|N)$, have important applications to disordered systems, in particular Dirac fermions in random potentials. The most important example is the Quantum Hall transition, which is believed to be describable by a disordered Dirac fermion system with 3 types of random potentials. In [**?**], the simplest super-current algebra, $gl(1|1)$ was constructed in detail for any level k. A simple free field representation was found consisting only of two bosons and a symplectic fermion. The logarithmic fields were explicitly constructed, and it was shown how the twist fields are important for constructing all the primary fields. This simple construction has subsequently been used by other researchers on string theory motivated models. With student Robinson[1], a super-spin charge separation was proven for these kinds of theories, which can lead to interesting examples of non-trivial disordered critical points when the potentials are such that the spin or charge degrees of freedom are gapped out.

Classification of Topological Insulators. Topological insulators are new topological phases of matter where the bulk electronic wave-functions are characterized by topological invariants. They were classified in arbitrary spatial dimension by Kitaev[4] and Ryu. et. al.[5] using K-theory and homotopy groups of certain sigma-models. LeClair, with Bernard and Kim[**?**, **?**], took a different, holographic approach. The special bulk topological properties lead to zero modes on the boundary that are typically Dirac fermions. Thus, they classified Dirac theories with zero modes on the boundary protected by the usual discrete symmetries, and reproduced the known classifications, but in a simple manner that relied only on the properties of Clifford algebras. Perhaps more interestingly, it was also shown that in two dimensions, because there are two inequivalent ways of implementing time-reversal symmetry, 6 additional classes of topological insulators were found in addition to the previously known 5.

Strongly interacting Cold Atoms and their viscosity to entropy density ratio. LeClair and collaborators have been studying various aspects of bosonic and fermionic gases using a new approach to the quantum statistical mechanics of gases at finite temperature and density developed with student How[**?**]. This approach is based entirely on the S-matrix. The advantage of this method is that the 2-body S-matrix can be calculated exactly for non-relativistic models. There is great interest in the so-called unitary limit, where the scatttering length diverges, and the S-matrix becomes simply -1. With student How, the unitary limit, for both bosons and fermions, was studied in [**?**, **?**]. In [**?**], LeClair and collaborators extended the analysis to arbitrary negative scattering length. One outcome of the Ads/CFT correspondence is a conjectured lower bound on the viscosity to entropy density ratio of $\hbar/4\pi k_B$[?]. LeClair computed this ratio, and obtained about 4.7 times the conjectured bound, in excellent agreement with recent experiments.

1.2 Proposed future research

Topological Insulators. Over the last few years, there has been great excitement over the discovery, both theoretically and experimentally, of new topological phases of matter referred to as topological insulators/superconductors[2, 3]. These systems have promising applications to spintronics, and possibly even to topological quantum computing. They were classified in arbitrary spatial dimension by Kitaev[4] and Ryu. et. al.[5] using relatively sophisticated mathematics, i.e. K-theory and homotopy groups of certain sigma-models. In order to help clarify these classifications, LeClair, with Bernard and Kim[?, ?], took a different, holographic approach, wherein symmetry protected Dirac zero modes on the boundary were classified according to the symmetries of time-reversal, particle-hole symmetry, and chirality. This simpler approach, which relied mainly on properties of Clifford algebras, made this classification understandable to a larger audience. The 8-found periodicity of the classification was related to the 8-fold periodicity of the reality properties of spinor representations of $SO(N)$, something familiar to most particle physicists, and a form of Bott-periodicity in K-theory.

There are a number of issues to follow up on based on the work [?]. There, symmetry protected zero modes on the edge of bulk 2-dimensional systems were classified. Eleven different classes had protected zero modes. This was interesting because previous classifications based on bulk properties predicted only 5 of these 11 classes. The natural open question is whether there are physical realizations of these 6 additional classes. Finding such realizations could have important applications, since perhaps such classes arise in materials that easier to work with. One of the additional classes was suggested to occur in strained graphene. In discussions with Prof. Kim, we intend to study these issues. The work [?] also investigated whether there could exist fractionalized topological insulators when there are interactions present, in analogy with the fractional Quantum Hall effect. The approach taken was to check whether the quartic interactions on the boundary were exactly marginal. If they were, then this suggests that the free Dirac theory is deformed to a Luttinger liquid, in analogy with the fractional Quantum Hall effect. Surprisingly, it was found that all interactions that preserved the symmetries were exactly marginal. We propose to further investigate the effect of interactions and possible experimental signatures.

Cold Atoms. The multitude of new and precise experimental results on cold atoms continues to present challenges to many-body theorists. With the use of Feshbach resonances, interactions can be tuned through the whole range of infinitely strongly repulsive to attractive. There has been great interest in the so-called “unitary limit” where the scattering length diverges in 3 spatial dimensions, and the theory becomes scale invariant and universal. Although the energy scales are very different, the same states of matter exist in the cold atom laboratory and at the surface of neutron stars. This unitary limit, for both bosons and fermions, was studied by LeClair with student How[?, ?], using the new S-matrix based approach developed in [?]. Good results were obtained for measured quantities such as the ratio of viscosity to entropy density. In [?], LeClair and collaborators extended the analysis to arbitrary negative scattering length. They showed that the second virial coefficient agrees with other calculations, and also calculated the 3rd and 4th virial coefficients in the 2-body approximation.

We propose several projects continuing our investigation of the strongly correlated states of matter realized with cold atoms. (i) With student J. Stout, we are investigating 2-dimensional gases, which have been studied in experiments. At small coupling the bosonic gas does not experience BEC, but there is a phase transition in the Kosterlitz-Thouless class. (ii) In experiments researchers have discovered methods of preventing the formation of bound states. We propose then to study this system, referred to as the "upper branch". The interesting question here is to map out the temperatures where this state is metastable, for both bosons and fermions. (iii) Whereas fermionic systems at arbitrary scattering length have been studied extensively both theoretically and experimentally, the study of bosonic atoms at arbitrary scattering length is in its infancy. This purely bosonic system also has a unitary limit and universal physics, and we propose to study it with our methods. (v) We propose to incorporate 3-body physics into the formalism. The work[**?**] in principle prescribes how to do this. One check would be the 3rd virial coefficent at unitarity, since it has been measured and calculated using other methods.

Another perspective on the Cosmological Constant Problem. The Cosmological Constant Problem is now regarded as a major crisis of theoretical physics, especially since a small non-zero value is suggested by astrophysical measurements. In very recent work, LeClair, with D. Bernard, offer a different perspective on the problem. A rough sketch of the ideas are as follows. In quantum mechanics without gravity, there is no definition of the zero point of energy, i.e. all that can be measured is changes in energy. The Casimir effect is correctly cited as proof of the reality of vacuum energy, however what is actually measurable is how the vacuum energy *changes* as one varies a geometric modulus. By analogy, this leads us to propose that the physical vacuum energy in a Friedman-Robertson-Walker expanding universe only depends on the time variation of the scale factor $a(t)$. In other words, requiring that empty Minkowski space is stable is a principle that fixes the zero point of energy. We describe two different choices of vacuum, one of which is consistent with the current universe consisting only of matter and vacuum energy. The resulting vacuum energy density $\rho_{\rm vac} \propto k_c^2 H_0^2$, where k_c is a momentum cut-off and H_0 is the Hubble constant. For a cut-off close to the Planck scale, values of $\rho_{\rm vac}$ in agreement with astrophysical measurements are obtained. Another choice of vacuum is more relevant to the early universe consisting of only radiation and vacuum energy, and we propose it as a model of inflation.

References

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