SCRUTINIZING THE COSMOLOGICAL CONSTANT PROBLEM (AND A PROPOSAL)

André LeClair
Cornell university

Cornell LEPP Journal club
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Outline

- The classical Cosmological Constant and its measurement. (review FRW cosmology)
- What is the problem?
- Is it really a problem? I.e. is it properly defined? Something IS real: Casimir effect
- A principle that fixes the ambiguity in zero point energy.
- Consistent back-reaction and consequences for cosmology.

The Classical Cosmological Constant and its measurement

Einstein's equations:

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R}} + \boxed{g_{\mu\nu}\Lambda} = 8\pi G \boxed{T_{\mu\nu}}$$

pure geometry

cosmological constant

sources

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$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = 8\pi G \left(T_{\mu\nu} + T^{\Lambda}_{\mu\nu} \right)$$

$$T^{\Lambda}_{\mu\nu} = -\frac{\Lambda}{8\pi G} g_{\mu\nu}$$

$$-T^{\Lambda}_{00} = \rho_\Lambda = \text{an energy density}$$

Einstein's greatest blunder?

Astro measurements:

$$\rho_\Lambda = 0.7 \times 10^{-29} \text{ g cm}^{-3} = 2.8 \times 10^{-47} \text{ Gev}^4/\hbar^3 \text{c}^5.$$

specialize to expanding universe:

metric: $$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a(t)^2 d\mathbf{x}\cdot d\mathbf{x}.$$

time dependent scale factor

$$T_{\mu\nu} = \mathrm{diag}(\rho, p, p, p), \qquad \rho = \text{energy density}, \quad p = \text{pressure}$$

Einstein's equations:

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho,$$

Friedmann eqn.

$$\dot\rho = -3\left(\frac{\dot a}{a}\right)(\rho + p)$$

energy conservation

At the present time t₀ : $a(t_0) = 1, \qquad \dot a/a = H_0 = \text{Hubble constant}$

$$H_0^2 = \frac{8\pi G}{3}\rho_c$$

$$\rho_c = \text{total energy density} = \rho_{\text{rad}} + \rho_m + \rho_\Lambda$$

Composition of the universe:

$$\text{radiation}: \quad p_{\rm rad} = \rho_{\rm rad}/3, \qquad \Longrightarrow \rho_{\rm rad}/\rho_c = \Omega_{\rm rad}/a^4$$

$$\text{non}-\text{relativistic matter}: \quad p_{\rm m} = 0, \qquad \Longrightarrow \rho_m/\rho_c = \Omega_m/a^3$$

$$\text{cosmological constant}: \quad \text{since } T^{\Lambda}_{\mu\nu} \propto g_{\mu\nu}, \quad p_{\Lambda} = -\rho_{\Lambda} \qquad \Longrightarrow \rho_{\Lambda}/\rho_c = \Omega_{\Lambda}$$

Friedmann equ:

$$\boxed{\begin{gathered}\frac{1}{H_0^2}\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Omega_{\rm rad}}{a^4} + \frac{\Omega_m}{a^3} + \Omega_{\Lambda} \\ \Omega_{\rm rad} + \Omega_m + \Omega_{\Lambda} = 1\end{gathered}}$$

The current Universe

Radiation is currently negligible to a very good approximation.

The solution to a(t) depends only on Ω_m and Ω_Λ (which add up to 1):

$$a(t) = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3} \left[\sinh\left(3\sqrt{\Omega_\Lambda} H_0 t/2\right)\right]^{2/3}$$

At early times, $a \sim t^{2/3}$, later times $a \sim e^{tH}$

Measuring the Cosmo. Const.

- Measure the apparent luminosity ℓ of supernova with known absolute luminosity L.
- Infer the (luminosity) distance d.
- d depends on the detailed evolution a(t). Best fit.
- Usually expressed in terms of redshift z rather than t.

$$\ell = \frac{L}{4\pi d^2} \qquad 1 + z = \frac{a(t_0)}{a(t)}$$

$$d = \frac{1}{H_0}(z + \ldots)$$ (small z Hubble law)

Best fit: $\Omega_\Lambda = 0.72$

Only out to z < 2 !

What is the problem?

- The cosmo. const. Λ could simply be a new fundamental constant of nature. No problem.
- the Problem arises when ρ_Λ is identified with a zero point energy density in Quantum Mechanics. I.e. $\rho_\Lambda = \rho_{vac} = \langle vac| \mathcal{H} |vac\rangle$
- Could be the first experimental observation of an effect that involves both gravity and QM. This is important, even though gravity does not need to be quantized here.
- Next: review zero point energy of harmonic oscillators. QFT is just an infinite collection.

Zero point energy of harmonic oscillators

Bosons:

$$S = \int dt \left(\frac{1}{2}\dot{\phi}^2 - \frac{\omega^2}{2}\phi^2 \right) \qquad \dot{\phi} = \partial_t \phi$$

$$\phi \;=\; \left(a e^{-i\omega t} + a^\dagger e^{i\omega t}\right)/\sqrt{2\omega}. \qquad [a, a^\dagger] = 1$$

hamiltonian: $H = \omega(a^\dagger a + \frac{1}{2})$

Fermions: action 1st order in derivatives (Dirac):

$$S = \int \left(i\overline{\psi}\partial_t\overline{\psi} - i\psi\partial_t\psi - \omega\overline{\psi}\psi \right)$$

$$H = \omega(b^\dagger b - \frac{1}{2}), \qquad \{b, b^\dagger\} = 1$$

Opposite sign!

bosonic spectrum: $E_n = \omega\,(n + 1/2)$

Question: Can zero point energy $E_0 = \omega/2$ be measured in QM? (ignore gravity)

No!

- As in classical mechanics, the hamiltonian can be shifted by an arbitrary constant with no measurable consequences.
- Casimir effect? Later.....

The standard field theory calculation:

Second quantization: annihilation operator a_k for each wave-vector k with $\omega_k = \sqrt{(k^2 + m^2)}$:

$$\rho_{\rm vac} = \frac{N_b - N_f}{2} \sum_{\mathbf{k}} \omega_{\mathbf{k}} = \boxed{\frac{N_b - N_f}{2} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \sqrt{\mathbf{k}^2 + m^2}} \approx (N_b - N_f) \frac{k_c^4}{16\pi^2}$$

$$k_c = \mathrm{UV\ \ cut - off}$$

- For k_c = Planck scale = 10^{19} Gev, this is larger than the measured ρ_Λ by 120 orders of magnitude.
- Wrong sign! $N_{b,f}$ = numbers of bosons, fermions. In standard model, $N_f > N_b$

Scrutinizing.....

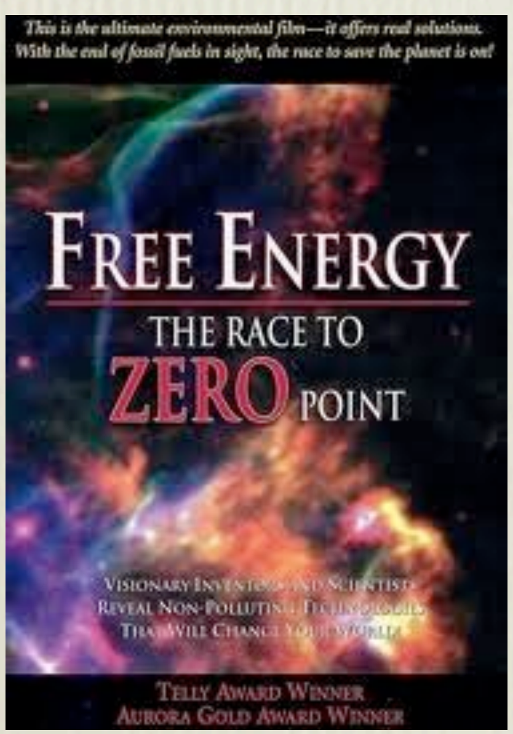
- At least 2 aspects are uncomfortable:
- Is this k_c^4 contribution real? It's a constant and we said energy was only fixed up to an arbitrary constant.
- How does the calculation change in curved space, like an expanding universe?
- To address "Is it real?", re-examine Casimir effect.

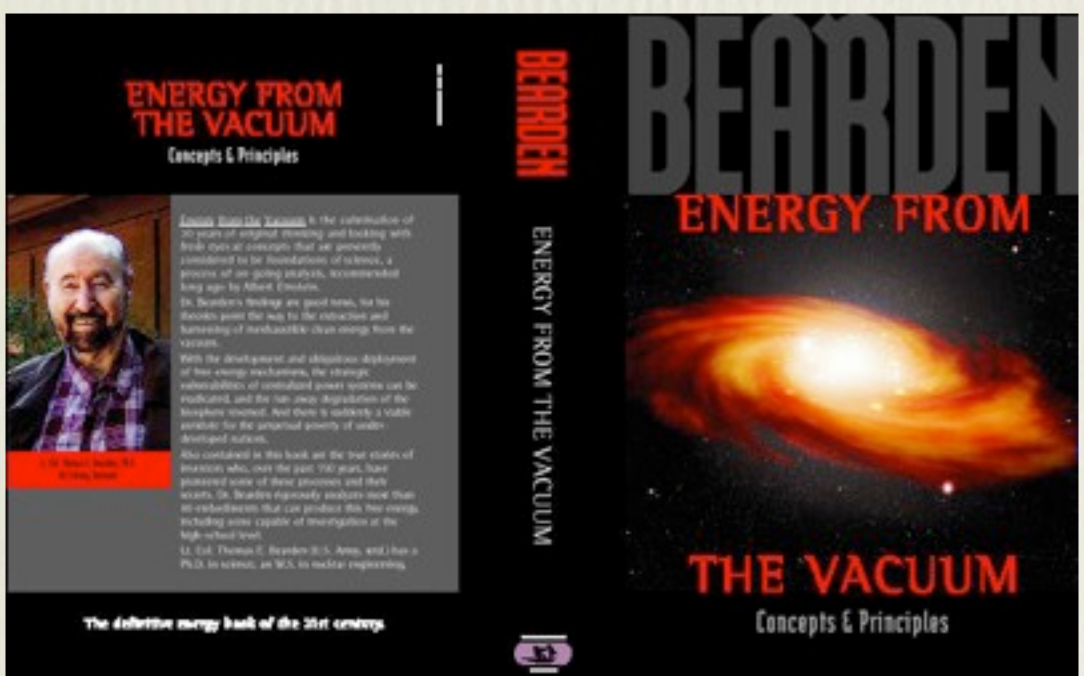
Is it real?

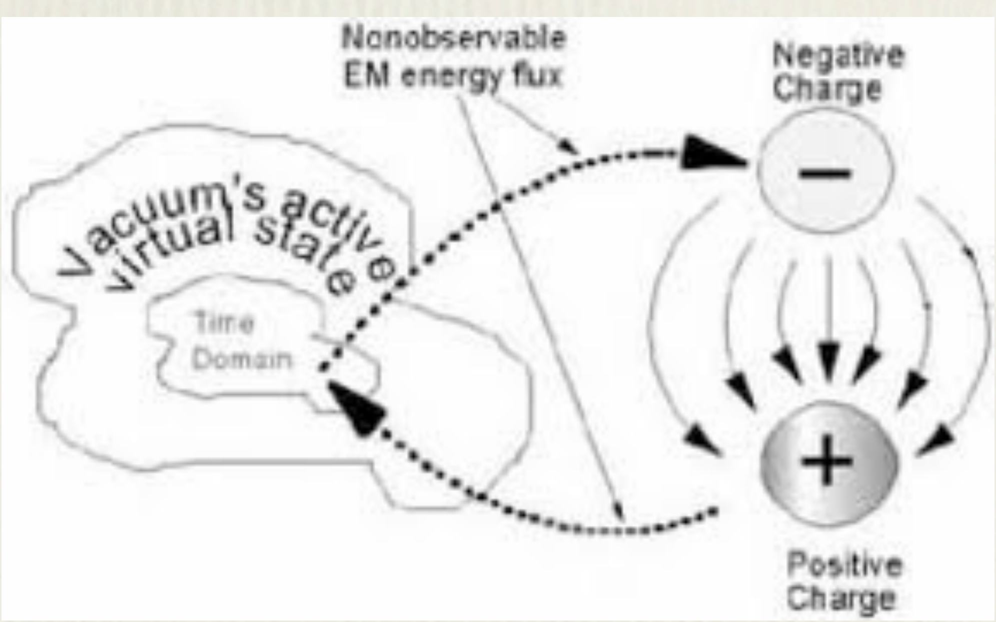
Feynman and Wheeler: “There is enough vacuum energy in a teacup to boil all the Earth’s oceans.”

Schwinger: “....the vacuum is not only the state of minimum energy, it is the state of zero energy, zero momentum, zero charge, zero whatever.”

Would you invest money into developing vacuum energy into the ultimate sustainable resource?

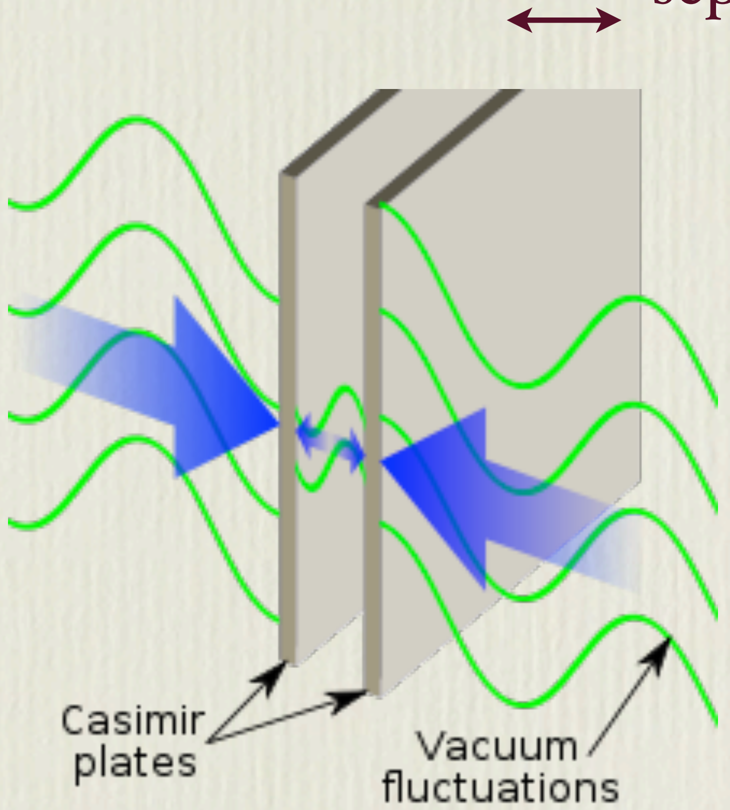






The Casimir effect is real.   But.....

separation = ℓ



Force= $F(\ell) = -dE_{\mathrm{vac}}(\ell)/d\ell.$

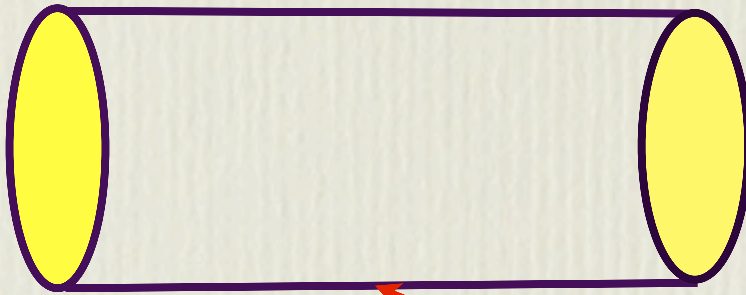
energy density: $\rho^{\mathrm{cas}}_{\mathrm{vac}} = -\pi^2/720\ell^4.$ (derive later)

Note: an arbitrary shift of the vacuum energy does not change the measured force! All that matters is it's l-dependence.

¡Obviously an IR effect!

Cylindrical version of Casimir effect

Just change boundary conditions: join plates at edges to have periodic b.c.



One compactified spatial dimension of circumference β

2 dim'l space + time

Relation to Casimir: $$\rho^{\rm cas}_{\rm vac}(\ell) = 2\rho^{\rm cyl}_{\rm vac}(\beta = 2\ell)$$

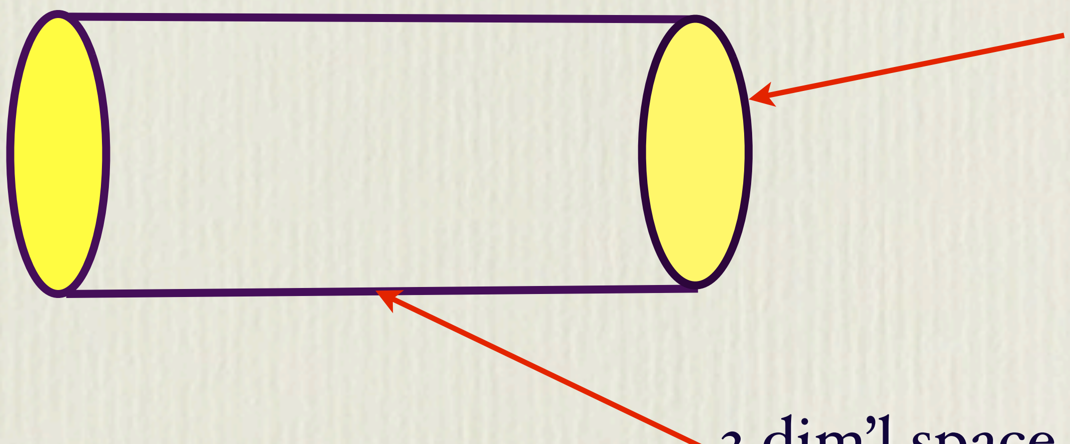
$$\rho^{\rm cyl}_{\rm vac} = \frac{1}{2\beta}\sum_{n\in\mathbb{Z}}\int \frac{d^2\mathbf{k}}{(2\pi)^2}\sqrt{\mathbf{k}^2+(2\pi n/\beta)^2} = -\beta^{-4}\pi^{3/2}\Gamma(-3/2)\zeta(-3)+\text{const.}$$

quantized modes on circle

divergent as UV cutoff $k_c \to \infty$.
We used Riemann zeta function regularization.

Quantum Statistical Mechanics viewpoint.

Passing to euclidean time $t = -i\,\tau$, ρ_{vac} is just the finite temperature free energy on the cylinder with circumference $\beta = 1/T$.



$$= -\beta^{-4}\pi^{3/2}\Gamma(-3/2)\zeta(-3)$$

Quantum Statistical. Mech. gives a very different convergent expression. Problem seems to disappear.

$$\rho_{\rm vac}^{\rm cyl} = \frac{1}{\beta}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\log\left(1 - e^{-\beta k}\right) = -\beta^{-4}\frac{\zeta(4)}{2\pi^{3/2}\Gamma(3/2)} = -\frac{\pi^2}{90}T^4.$$

black body

Aside:

Same as before due to a non-trivial identify connected with the Riemann hypothesis:

$$\xi(\nu) = \xi(1-\nu) \text{ where } \xi(\nu) = \pi^{-\nu/2}\Gamma(\nu/2)\zeta(\nu)$$

$$\Longrightarrow \xi(1/2 + iy) \text{ is real if y is real.} \quad \text{RH : Only zeros of } \zeta(z) \text{ exist at } y \neq 0$$

Some proposed solutions to the CCP

- Two kinds: UV solutions: add structures that are not yet known to exist. IR solutions
- SUSY: originally attractive idea when the CC was thought to be zero. Now ruled out since SUSY scale at least > GeV. (UV)
- Anthropic reasoning*. Still a possibility. Can only be ruled out by discovery of the correct explanation of the Cosmological Constant Problem. (UV).
- Our proposal: an IR effect. Analogy to Casimir effect. Will need fermions to get repulsion.

Our hypotheses and consistency

* $\langle T_{\mu\nu}\rangle = \langle \mathrm{vac}|T_{\mu\nu}|\mathrm{vac}\rangle$ depends on choice of |vac>

* chose |vac> so there is no particle production.

* calculate $\rho_{\mathrm{vac}} = \langle \mathcal{H}\rangle = \langle \mathrm{vac}|\mathcal{H}|\mathrm{vac}\rangle$

* Assume $\langle T_{\mu\nu}\rangle = -\rho_{\mathrm{vac}}\, g_{\mu\nu}$ implies $\dot{\rho}_{\mathrm{vac}} = 0$,

* Impose the stability of empty Minkowski space: zero-point energy is fixed by requiring $\varrho_{\mathrm{vac}} = 0$ when a(t) is constant in time.

* include ϱ_{vac} (a(t)) as a source in Einstein's eqns, and require self-consisent back-reaction.

What to expect.....a preview:

By dimensional analysis with k_c = UV cut-off, there is an 'adiabatic' expansion:

$$\rho_{\text{vac},0} = k_c^4 + \boxed{k_c^2\,\widehat{R}} + \widehat{R}^2 + \cdots$$

UV

both UV & IR

IR

$\widehat{R}$ is related to the curvature and is a linear combination of $(\dot{a}/a)^2$ and $\ddot{a}/a$, depending on the choice of $|\text{vac}\rangle$

✠ UV term is just a constant which we will set to zero using the arbitrariness of zero-point of Energy.

✠ IR term is very, very small in the current universe, so we will neglect it.

Coleman (1990's): "....the cosmological constant is the mass of a box of empty space. You can always fine-tune it to zero. And nobody will say you can't do it, but nobody will applaud you when you do it, either."

✠ According to our principles, we must keep the dominant mixed term, which vanishes in Minkowski space. It IS physically meaningful because it depends on time variation of the scale factor a(t), by analogy to separation of plates in Casimir effect.

The QFT calculation

Scalar field in curved spacetime: $$S = \int dt\, d^3\mathbf{x}\, \sqrt{|g|}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{m^2}{2}\Phi^2\right)$$

$$\Phi = \chi/a^{3/2}.$$

$$= \int dt\, d^3\mathbf{x}\, \frac{1}{2}\left(\partial_t\chi\partial_t\chi - \frac{1}{a^2}\vec{\nabla}\chi\cdot\vec{\nabla}\chi - m^2\chi^2 + \mathcal{A}(t)\chi^2\right)$$

A(t) is like a time-dependent mass²

$$\mathcal{A} \equiv \frac{3}{4}\left(\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\ddot{a}}{a}\right).$$

Expand field in modes: $$\chi = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}}\left(a_\mathbf{k}\, u_\mathbf{k}\, e^{i\mathbf{k}\cdot\mathbf{x}} + a^\dagger_\mathbf{k} u^*_\mathbf{k}\, e^{-i\mathbf{k}\cdot\mathbf{x}}\right), \qquad [a_\mathbf{k}, a^\dagger_{\mathbf{k}'}] = \delta(\mathbf{k}-\mathbf{k}')$$

$$(\partial_t^2 + \omega_\mathbf{k}^2)u_\mathbf{k} = 0, \qquad \omega_\mathbf{k}^2 \equiv (\mathbf{k}/a)^2 + m^2 - \mathcal{A}$$

Solution: (WKB)

$$u_\mathbf{k} = \frac{1}{\sqrt{2W}}\exp\left(i\int^t W(s)ds\right),$$

where W satisfies the differential equation:

$$W^2 = \omega_\mathbf{k}^2 + \frac{3}{4}(\dot{W}/W)^2 - \frac{1}{2}\ddot{W}/W$$

For massive particles in adiabatic approximation: W = ω_k

This gives: $$\rho_{\text{vac},0} = \frac{1}{V}\langle H\rangle = \frac{1}{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\sqrt{\mathbf{k}^2 + m^2 - \mathcal{A}},$$

Introducing UV cut-off k_c : $$\boxed{\rho_{\text{vac}} \approx \Delta N \frac{k_c^2}{16\pi^2}\,\mathcal{A}}$$

$$\Delta N = N_f - N_b$$

One generation of Standard Model : $N_f = N_b = 30,\quad \Longrightarrow$ need at least 2 generations

3 generations : $N_f - N_b = 60$

Fortunately Positive!

This is the correct order of magnitude:

If $k_c \approx 3 \times 10^{18}$ Gev. then $$\boxed{\rho_{\text{vac}} \approx 10^{-29} g/cm^3}$$

Consistent Back-reaction

Since $\rho_{\rm vac}$ depends on time derivatives of a(t), it leads to a back-reaction on the geometry. This must be solved self-consistently.

Including $\rho_{\rm vac}$ in the Friedmann eqn:

$$\left(1-\frac{g}{3}\right)\left(\frac{\dot a}{a}\right)^2-\frac{2g}{3}\frac{\ddot a}{a}=\frac{8\pi G}{3}\left(\rho_m+\rho_{\rm rad}\right)$$

where g is the dimensionless constant:

$$g=\frac{3\Delta N}{8\pi}G\,k_c^2$$

Using Einstein's equations, one can infer the pressure of the vacuum energy:

$$p_{\rm vac}=-\frac{1}{g}\,\rho_{\rm vac}.$$

This is only consistent with energy conservation if g=1.

This implies $\rho_{\rm vac}$ is constant in time.

Dividing by critical density:

$$\frac{2}{3H_0^2}\left[\left(\frac{\dot{a}}{a}\right)^2 - \frac{\ddot{a}}{a}\right] = \frac{\Omega_m}{a^3} + \frac{\Omega_{\rm rad}}{a^4}.$$

In the current universe, $\Omega_{\rm rad}$ is nearly zero.

The solution with no radiation is:

$$a(t) = \left(\frac{\Omega_m}{\mu}\right)^{1/3}\left[\sinh(3\sqrt{\mu}H_0 t/2)\right]^{2/3}.$$

$$\frac{\rho_{\rm vac}}{\rho_c} = \mu$$

which implies that $\mu + \Omega_m = 1$, i.e. μ is just $\Omega_{\rm vac}$

➢ Our vacuum energy mimics a cosmological constant!

➢ The above solution a(t) is indistinguishable from the ΛCDM model of the current universe.

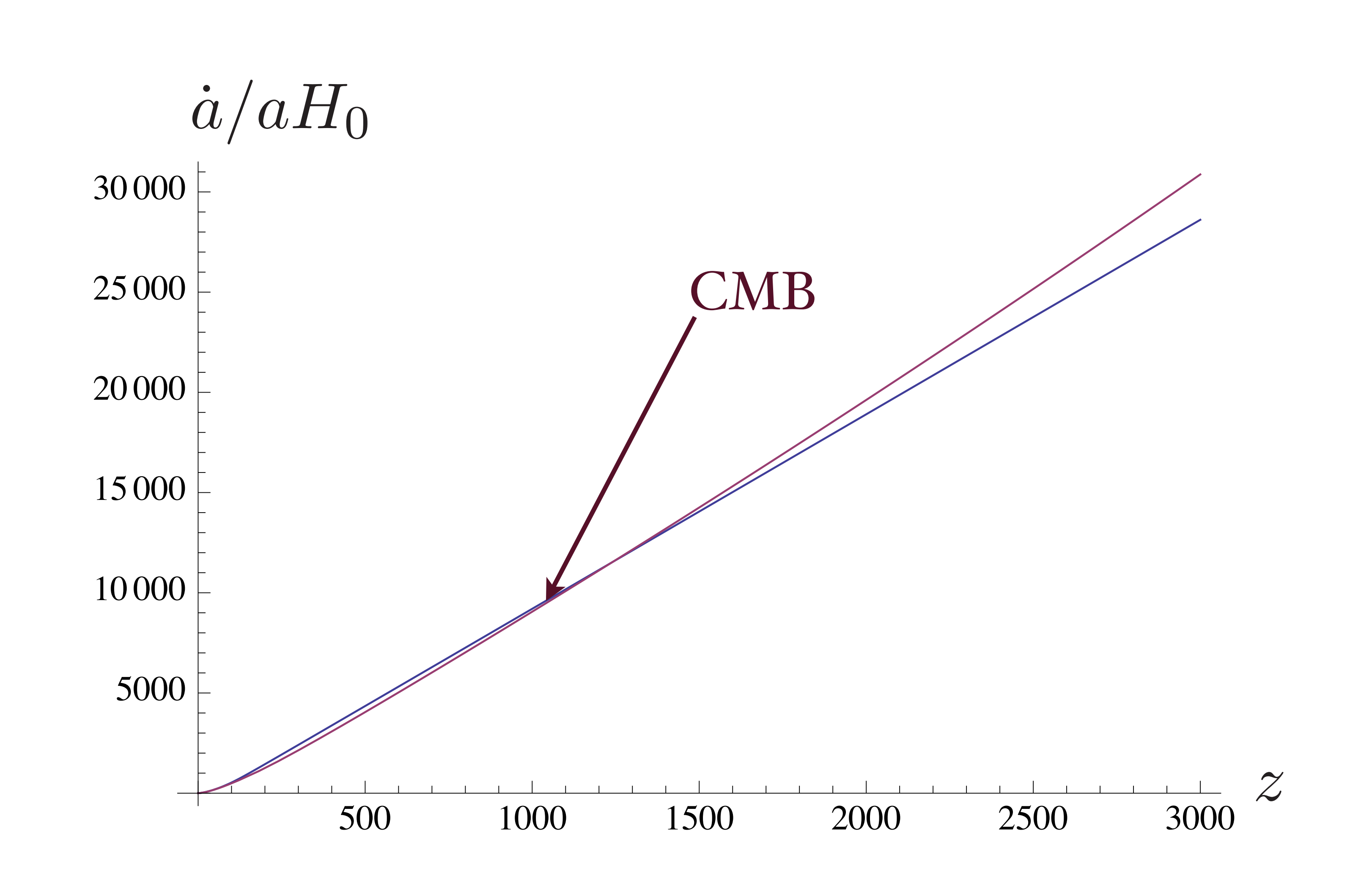
➢ When radiation is present our vacuum energy becomes time-dependent, and thus differs from a simple cosmological constant. This doesn't occur unless there is appreciable radiation, i.e. fairly large redshift z.

➢ Our model agrees with observations at least up to z=100.

Testing the model....

Comparison of our model with standard cosmology, i.e. matter + radiation + cosmological constant.

Expansion rate H as a function of redshift z.



Mimics a cosmological constant!

Massless particles. Inflation?

➢ Another choice of vacuum is consistent with a radiation dominated universe.

➢ Mathematically, there are corrections from the "WKB" formula when particles are massless:

$$u_{\mathbf{k}} = \frac{1}{\sqrt{2W}} \exp\left(i \int^{t} W(s) ds \right),$$

Recall: where W satisfies the differential equation:

$$W^2 = \omega_{\mathbf{k}}^2 + \underbrace{\frac{3}{4}(\dot{W}/W)^2 - \frac{1}{2}\ddot{W}/W}_{\text{Modifies } \omega_k}$$

$$\omega_k \Rightarrow \quad \widehat{\omega}_{\mathbf{k}}^2 \equiv \mathbf{k}^2 - \mathcal{R}a^2/6.$$

$$\mathcal{R} = 6\left((\dot{a}/a)^2 + \ddot{a}/a\right)$$ = Ricci scalar (curvature)

$$\boxed{\rho_{\widehat{\text{vac}}} \approx \Delta N \left[\frac{k_c^2}{96\pi^2}\mathcal{R} - \frac{1}{4608\pi^2}\mathcal{R}^2 \right].}$$

➢ new vacuum energy is constant in time when only radiation is present.

Solution: $a(t) = \left(\frac{\Omega_{\rm rad}}{\nu}\right)^{1/4} \sqrt{\sinh(2H\sqrt{\nu}\, t)},$ $\qquad \frac{\rho_{\widehat{\rm vac}}}{\rho_i} = \nu$

➢ In the very early universe, there was only radiation.

➢ It's vacuum energy behaves like a cosmological constant.

➢ INFLATION is essentially explained by a cosmological constant, but much larger than the presently measured one.

➢ When $H = \partial_t a/a$ is large, must keep the R^2 term in $\varrho_{\rm vac}$

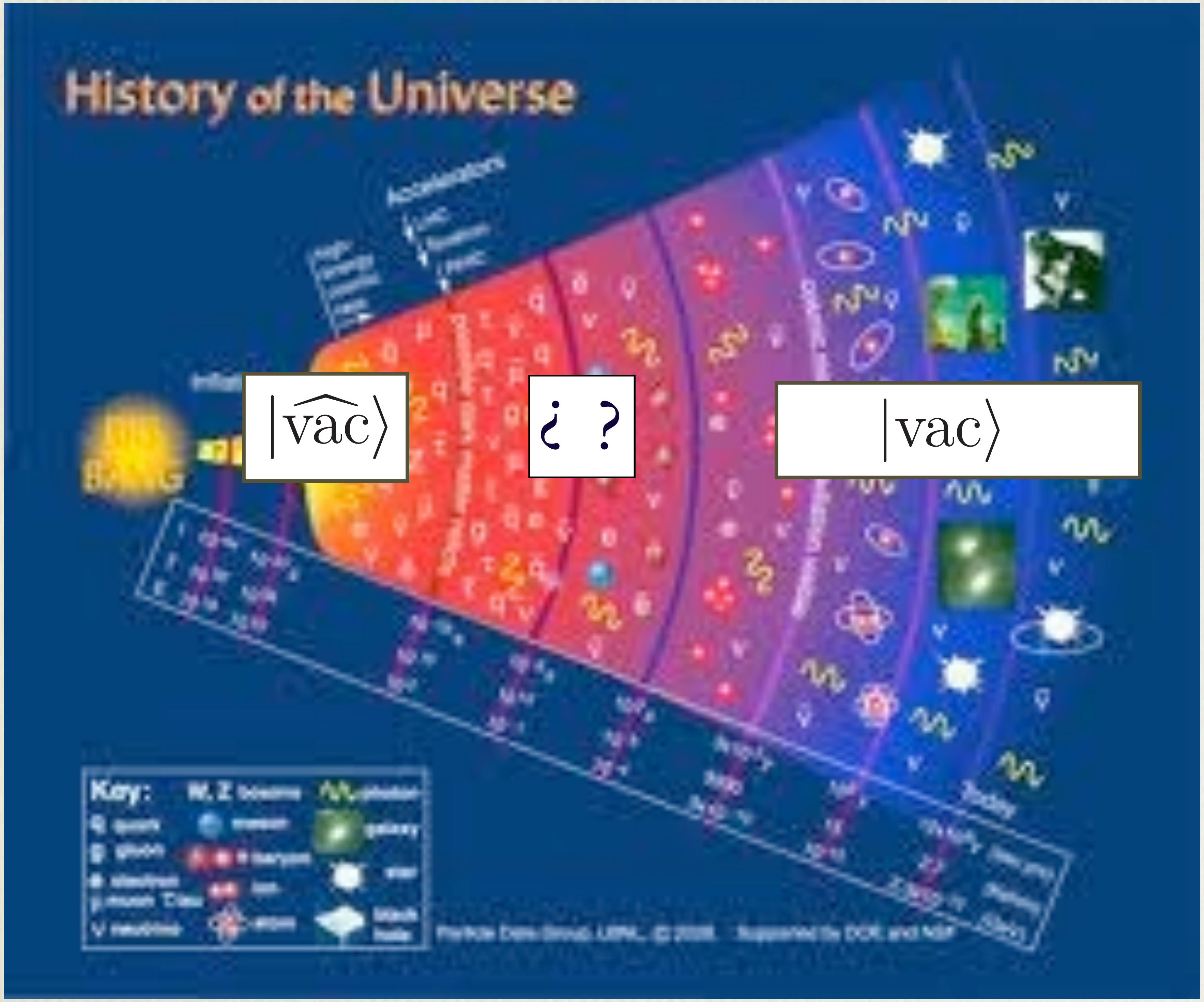
➢ The Friedmann equation is an algebraic equation that fixes $H \sim k_c$ in constantly expanding (de Sitter) space. About the right magnitude.

Roughly, Friedmann implies

$H^2 = (k_c^2 R - R^2)/k_c^2$

$R \sim H \quad \Rightarrow \quad H \sim k_c$

A cosmic scenario.......



(Quantum mechanics as the origin of Gravity?)

Interpretation of g=1? Recall: $$g = \frac{3\Delta N}{8\pi} G\, k_c^2$$

A "derivation" of gravity from quantum mechanics:

* First Law of Thermo: dE = dQ - p dV

* dQ =0 in closed universe (adiabatic)

* Identify internal energy dE = ρ_{vac} dV

* First Law reads becomes the spatial Friedmann eq! $$\left(\frac{\dot a}{a}\right)^2 + 2\,\frac{\ddot a}{a} = -8\pi G\, p.$$

Using energy conservation, can derive 1st Friedmann!

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho,$$

where $G = \frac{8\pi}{3\Delta N k_c^2}$ is emergent

Should we abandon trying to quantize gravity, since it's already a quantum effect?


Thank you!