Cornell University
Laboratory for Elementary-Particle Physics

USPAS course on
Recirculated and Energy Recovered Linacs

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Synchrotron Radiation Effect on the Beam

Jefferson Lab

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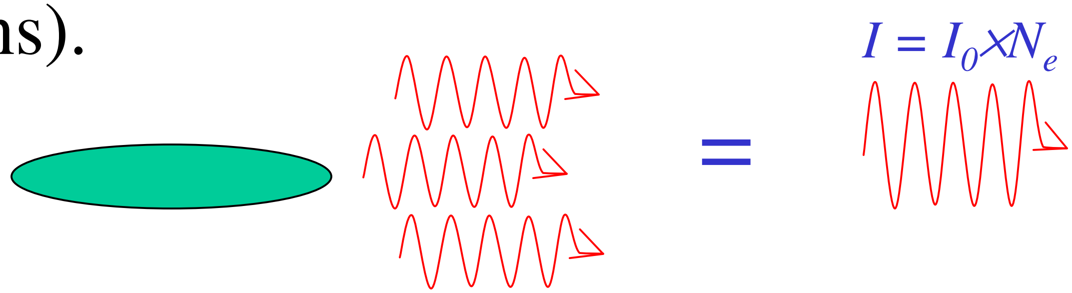
Quantum excitation

Quantum excitation in accelerator physics refers to diffusion of phase space (momentum) of charged particles (mainly e^+/e^-) due to recoil from emitted photons.

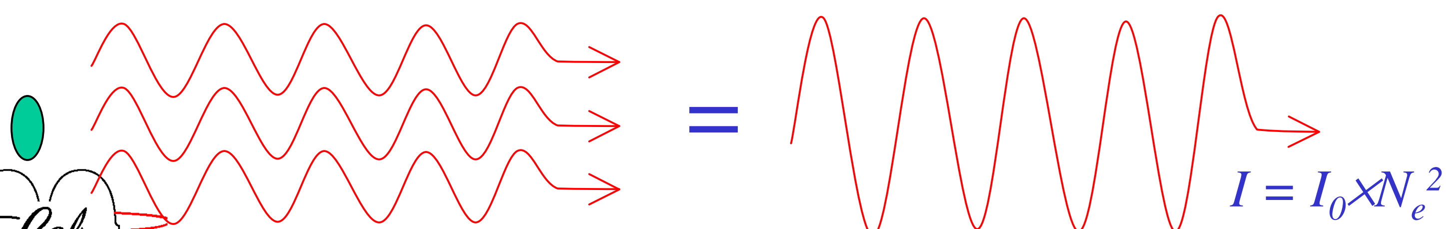
Because radiated power scales as $\propto \gamma^4$ and critical photon energy (divides synchrotron radiation spectral power into two equal halves) as $\propto \gamma^3$, the effect becomes important at high energies (typically $\geq$ 3 GeV) in electron accelerators unless bunches are extremely short.

ISR vs. CSR

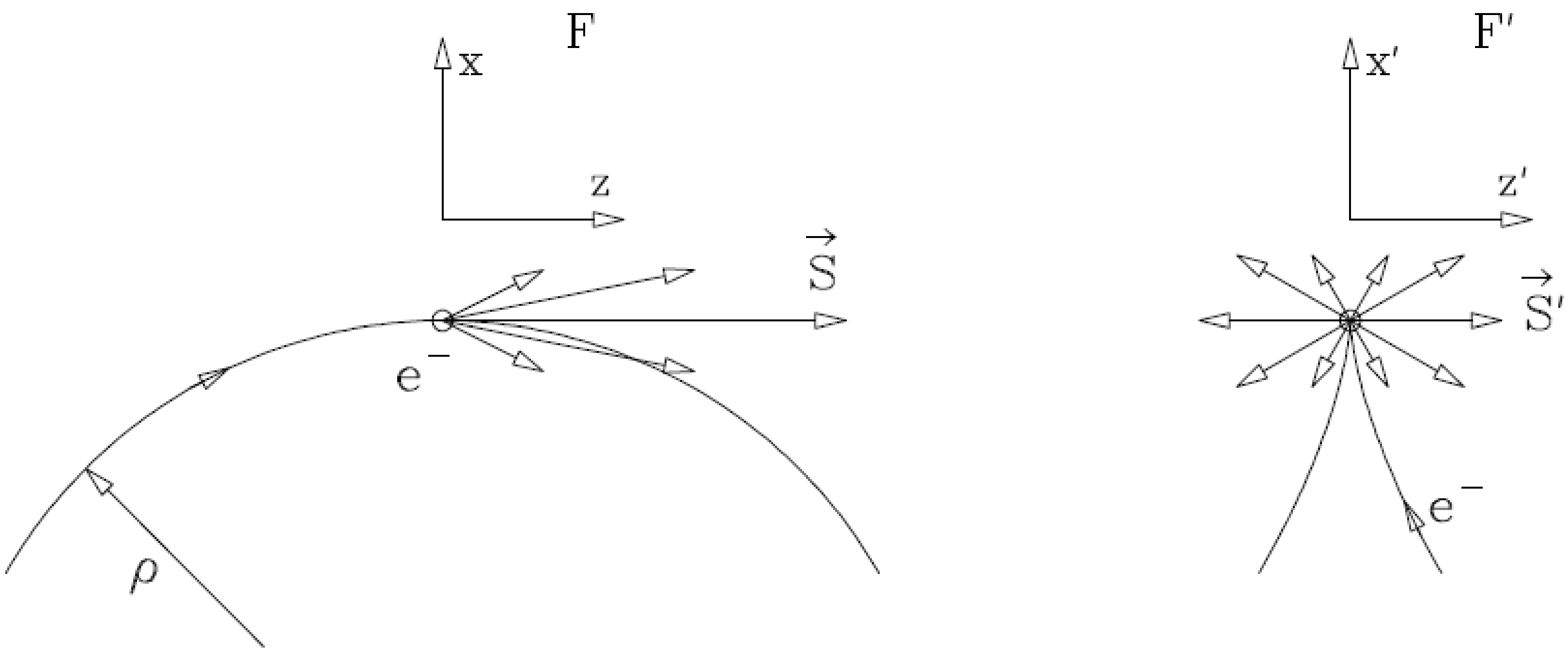
Incoherent synchrotron radiation (ISR): $\lambda << \sigma_z$. The radiation power scales linearly with the number of electrons).



Coherent synchrotron radiation (CSR): $\sigma_z \leq \lambda$. When radiation wavelength becomes comparable with the bunch length (or density modulation size), the radiated power becomes quadratic with peak current. The effect can be important at all energies if the bunch is sufficiently short.

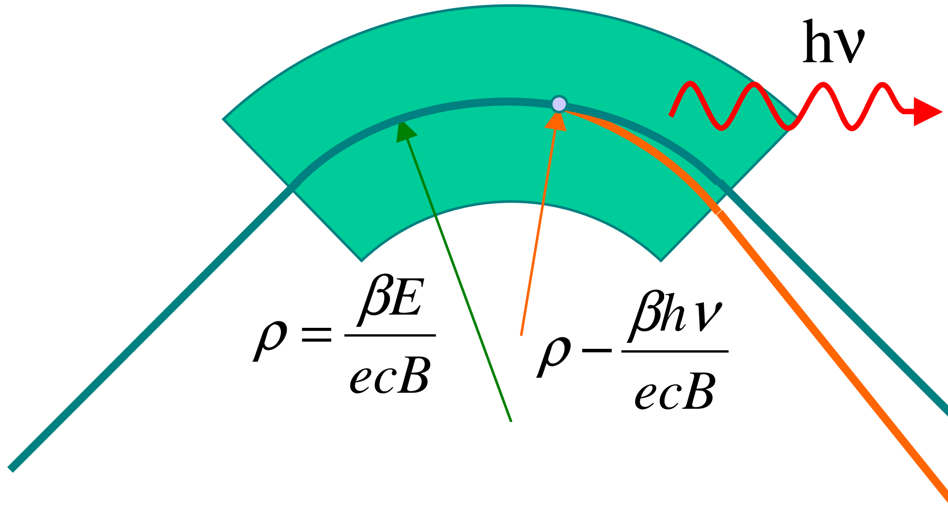


Circular traj. in the “rest” frame



Lorentz back-transform 4-vector $(\omega/c, \vec{k})$

Recoil due to photon emission



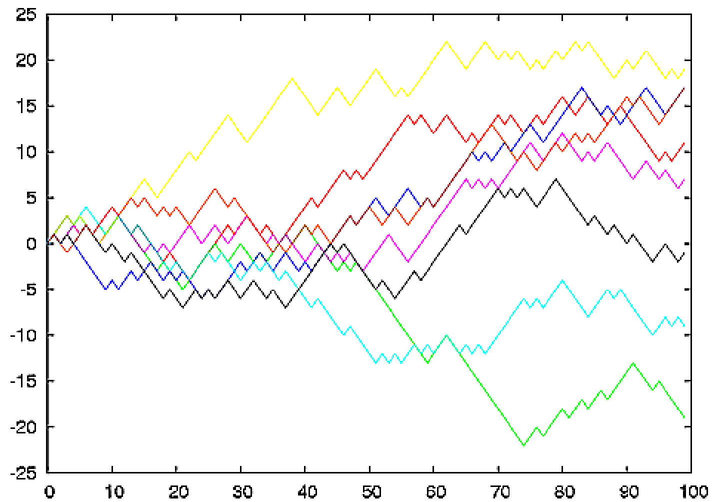
Photon emission takes place in forward direction within a very small cone (~ $1/\gamma$ opening angle). Therefore, to 1^{st} order, photon removes momentum in the direction of propagation of electron, leaving position and divergence of the electron intact at the point of emission.

Energy spread

Synchrotron radiation is a stochastic process. Probability distribution of the number of photons emitted by a single electron is described by Poisson distribution, and by Gaussian distribution in the approximation of large number of photons.

If emitting (on average) N_{ph} photons with energy E_{ph}, random walk growth of energy spread from its mean is

$$\sigma_E^2 = N_{ph} E_{ph}^2$$



If photons are emitted with spectral distribution $N_{ph}(E_{ph})$, then one has to integrate:

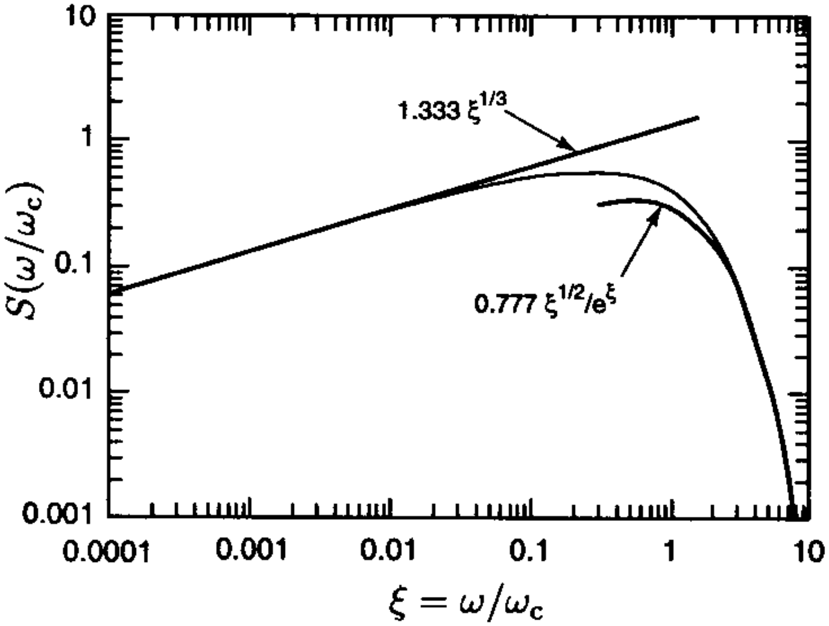
$$\sigma_E^2 = \int E_{ph}^2 N(E_{ph}) dE_{ph}$$

Spectrum of SR from bends

Photon emission (primarily) takes place in deflecting magnetic field (dipole bend magnets, undulators and wigglers). Spectrum of synchrotron radiation from bends is well known (per unit deflecting angle):

$$\frac{dN_{ph}}{d\psi} = \frac{4\alpha}{9}\gamma\frac{\Delta\omega}{\omega}\frac{I}{e}S\left(\frac{\omega}{\omega_c}\right)$$

$$\omega_c \equiv \frac{3}{2}c\gamma^3/\rho$$



Energy spread from bends

$$\sigma_E^2 = \frac{55}{32\sqrt{3}\pi} C_\gamma \hbar c (mc^2)^4 \gamma^7 \int \frac{ds}{\rho}$$

Sands' radiation constant for e⁻: $C_\gamma = \frac{4\pi r_c}{3(mc^2)^3} = 8.86 \cdot 10^{-5} \frac{\mathrm{m}}{\mathrm{GeV}^3}$

For constant bending radius ρ and total bend angle Θ (Θ = 2π for a ring) energy spread becomes:

$$\boxed{\frac{\sigma_E^2}{E^2} = 2.6 \cdot 10^{-10} E^5 (\mathrm{GeV}^5) \frac{1}{\rho^2(\mathrm{m}^2)} \frac{\Theta}{2\pi}}$$

Radiated energy loss:

$$E_\gamma = C_\gamma \frac{E^4}{\rho} \frac{\Theta}{2\pi} \qquad E_\gamma(\mathrm{MeV}) = 0.0886 \frac{E^4(\mathrm{GeV}^4)}{\rho(\mathrm{m})} \frac{\Theta}{2\pi}$$

Energy spread from undulator

$$\sigma_E^2 = \int E_{ph}^2 N(E_{ph}) dE_{ph} \approx N_{ph} \varepsilon_\gamma^2$$

Photon in fundamental $\varepsilon_\gamma = \dfrac{hc}{\lambda_p} \dfrac{2\gamma^2}{(1+\frac{1}{2}K^2)}$ $\quad \varepsilon_\gamma(\mathrm{eV}) = 950 \dfrac{E^2(\mathrm{GeV}^2)}{\lambda_p(\mathrm{cm})(1+\frac{1}{2}K^2)}$

Radiated energy / e⁻ $E_\gamma = \dfrac{4\pi^2 r_c E^2 K^2 L_u}{3\lambda_p^2 mc^2}$ $\quad E_\gamma(\mathrm{eV}) = 725 \dfrac{E^2(\mathrm{GeV}^2) K^2}{\lambda_p^2(\mathrm{cm}^2)} L_u(\mathrm{m})$

Naively, one can estimate $N_{ph} \approx \dfrac{E_\gamma}{\varepsilon_\gamma} = 0.763 \dfrac{K^2(1+\frac{1}{2}K^2)}{\lambda_p(\mathrm{cm})} L_u(\mathrm{m})$

$$\frac{\sigma_E^2}{E^2} \approx 7 \cdot 10^{-13} \frac{E^2(\mathrm{GeV}^2) K^2}{\lambda_p^3(\mathrm{cm}^3)(1+\frac{1}{2}K^2)} L_u(\mathrm{m})$$

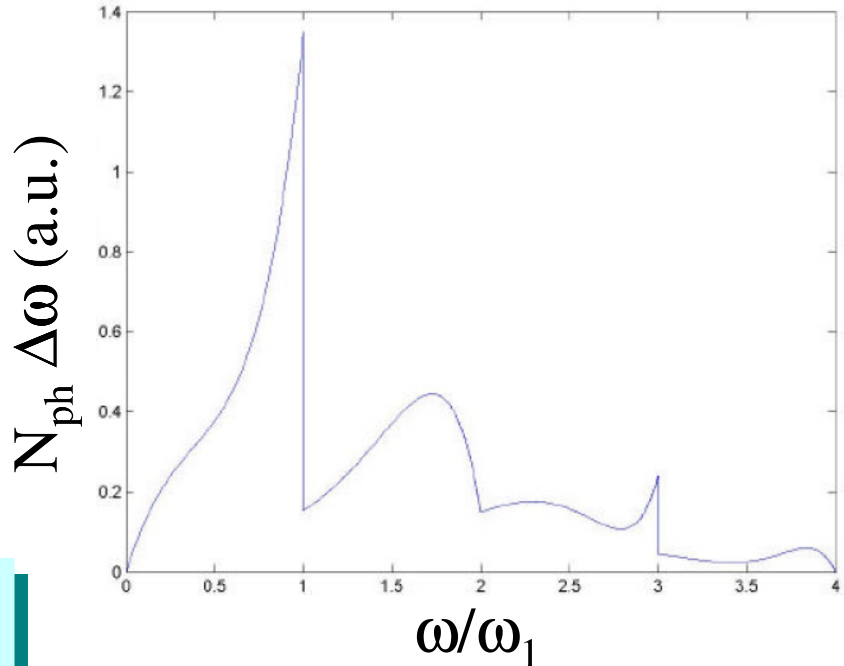
Energy spread from planar undulator

In reality, undulator spectrum is more complicated with harmonic content for K ≥ 1 and Doppler red shift for off-axis emission.

More rigorous treatment gives

$$\frac{\sigma_E^2}{E^2} \approx 4.8 \cdot 10^{-13} \frac{E^2(\mathrm{GeV}^2) K^2 F(K)}{\lambda_p^3(\mathrm{cm}^3)} L_u(\mathrm{m})$$

with $F(K) \approx 1.2K + (1 + 1.33K + 0.4K^2)^{-1}$



Emittance growth

Consider motion: $x = x_\beta + \eta_x \frac{\Delta E}{E}$ $\quad x' = x'_\beta + \eta'_x \frac{\Delta E}{E}$

where $x_\beta = a_x \sqrt{\beta_x(s)} e^{i\psi_x(s)}$

As discussed earlier, emission of a photon leads to:

$$\delta x = 0 = \delta x_\beta + \eta_x \frac{E_{ph}}{E} \qquad \delta x_\beta = -\eta_x \frac{E_{ph}}{E}$$

$$\delta x' = 0 = \delta x'_\beta + \eta'_x \frac{E_{ph}}{E} \qquad \delta x'_\beta = -\eta'_x \frac{E_{ph}}{E}$$

changing the phase space ellipse

$$a_x^2 = \gamma_x x_\beta^2 + 2\alpha_x x_\beta x'_\beta + \beta_x x'^2_\beta$$

$$\boxed{\left\langle \delta a_x^2 \right\rangle = \frac{E_{ph}^2}{E^2} H_x(s)}$$

here $H_x = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta'_x + \beta_x \eta'^2_x$

Emittance growth in bend

$$\sigma_x^2 = \left\langle \left(a_x \sqrt{\beta_x(s)} e^{i\psi_x(s)} \right)^2 \right\rangle_s = \frac{1}{2} a_x^2 \beta_x$$

$$\boxed{\varepsilon_x = \frac{1}{2} \Delta \left\langle a_x^2 \right\rangle = \frac{1}{2cE^2} \int ds \int E_{ph}^2 \dot{N}_{ph}(E_{ph}) H(s) dE_{ph}}$$

In bends:

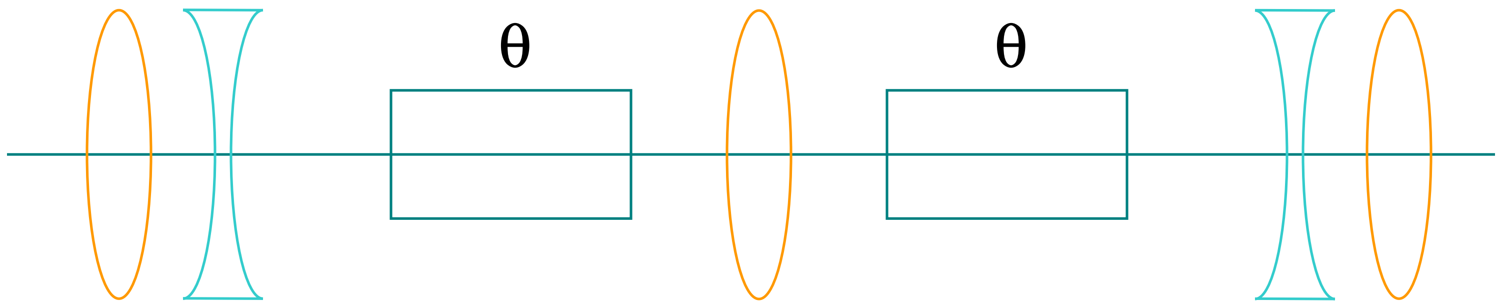
$$\varepsilon_x = \frac{55 C_\gamma \hbar c (mc^2)^2}{64\pi\sqrt{3}} \gamma^5 \int \frac{H ds}{\rho^3}$$

$$\varepsilon_x(\mathrm{m\text{-}rad}) = 1.3 \cdot 10^{-10} \frac{E^5(\mathrm{GeV}^5) \left\langle H \right\rangle(\mathrm{m})}{\rho^2(\mathrm{m}^2)} \frac{\Theta}{2\pi}$$

H-function (curly H)

As we have seen, lattice function H in dipoles ($1/\rho \neq 0$) matters for low emittance.

In the simplest achromatic cell (two identical dipole magnets with lens in between), dispersion is defined in the bends. One can show that an optimum Twiss parameters (α, β) exist that minimize $\langle H \rangle$

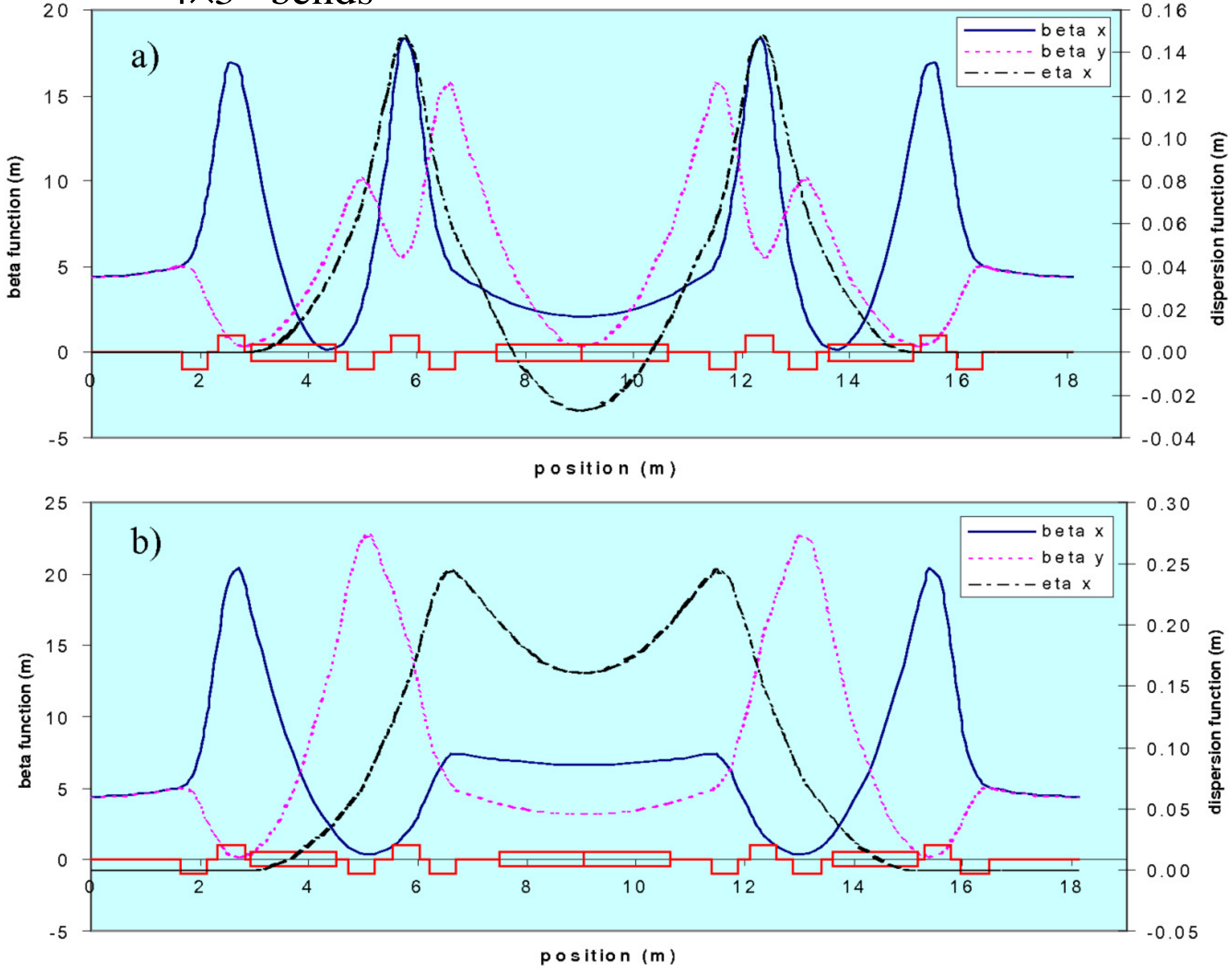


Such optimized double bend achromat is known as a Chasman Green lattice, and H is given by

$$\langle H \rangle = \frac{1}{4\sqrt{15}} \rho \theta^3$$

Example of TBA

4×3°-bends



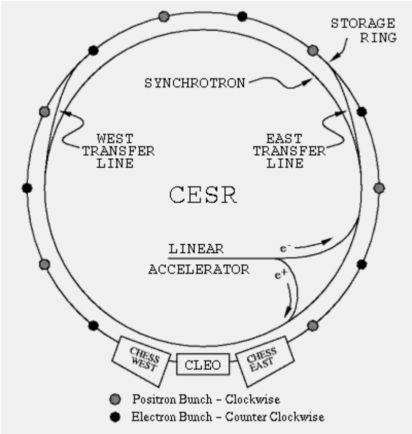
$$\langle H \rangle = 3.6\,\text{mm}$$

$$\langle H \rangle = 9.1\,\text{mm}$$

$$\langle H \rangle_{C-G} \approx 0.66\,\text{mm}$$

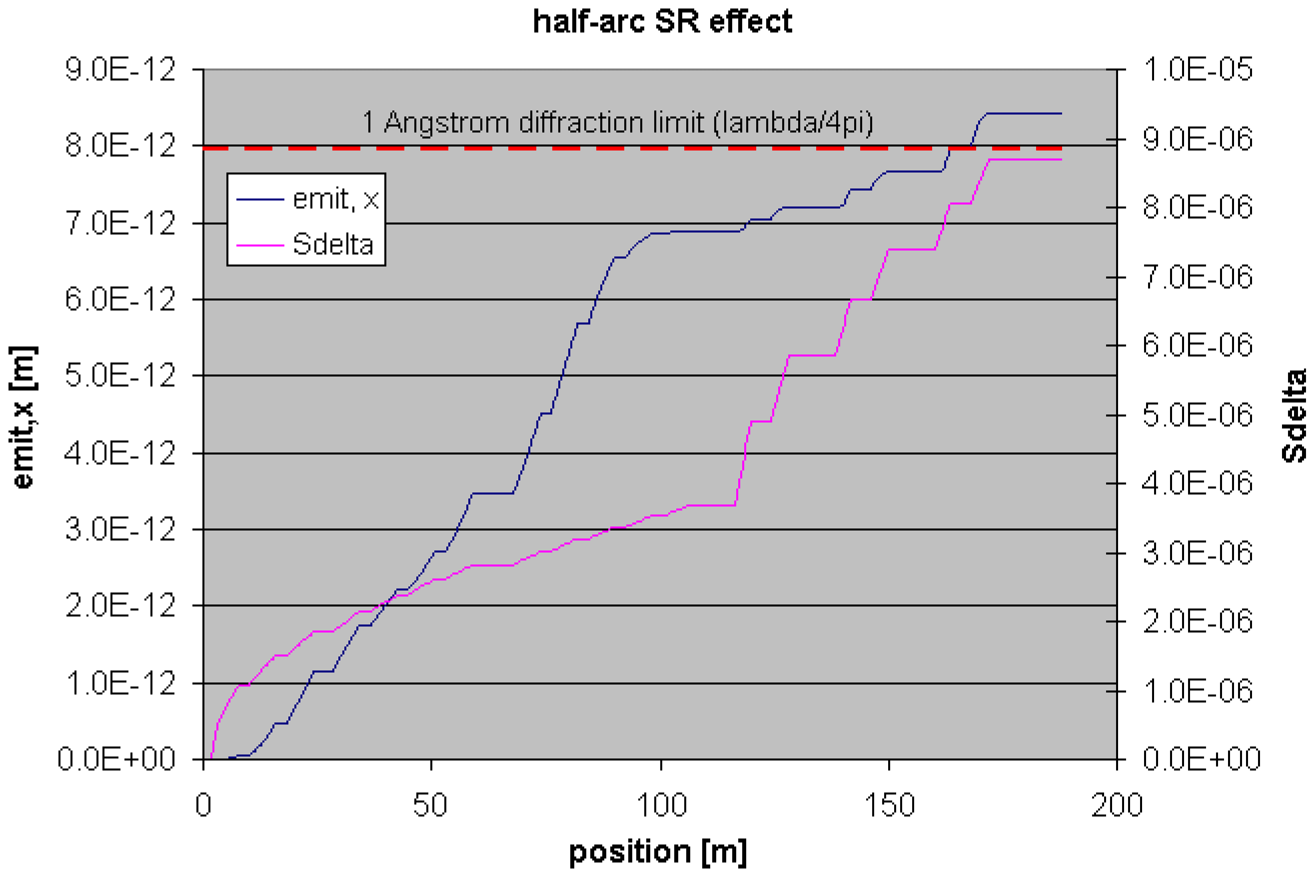
A TBA cell for an ERL recirculating arc: a) $R_{56} = 0$; b) $R_{56} = 2\,\text{cm}$

Example: emittance and energy spread in ¼ CESR



5 GeV ring (768 m) FODO lattice

Energy = 5 GeV



large dispersion section for bunch compression

Emittance growth in undulator

$$\varepsilon_x \approx \frac{1}{2}\frac{\sigma_E^2}{E^2}\langle H \rangle$$

with energy spread σ_E/E calculated earlier.

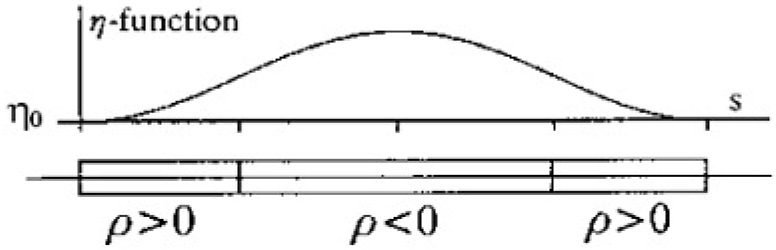
$$\langle H \rangle = \frac{1}{L_u}\int_0^{L_u}(\beta\eta'^2 + 2\alpha\eta\eta' + \gamma\eta^2)ds$$

For sinusoidal undulator field $B(s) = B_0 \cos k_p s$ with $k_p = 2\pi / \lambda_p$

Differential equation for dispersion $\eta'' = \frac{1}{\rho} = \frac{1}{\rho_0}\cos k_p s$

with $\rho_0 = \frac{\gamma}{k_p K}$

Emittance growth in undulator



Dispersion function in one period of a undulator magnet.

$$\eta(s) = \frac{1}{k_p^2\rho_0}(1-\cos k_p s) + \eta_0 \qquad \eta'(s) = \frac{1}{k_p\rho_0}\sin k_p s$$

For an undulator with beam waist (β^*) located at its center

$$\langle H \rangle \approx \frac{\beta^*}{2k_p^2\rho_0^2}\left(1 + \frac{L_u^2}{12\beta^{*2}} + \frac{2\eta_0^2 k_p^2 \rho_0^2}{\beta^{*2}} + \frac{8\eta_0\rho_0}{\beta^{*2}} + \frac{11}{2\beta^{*2}k_p^2}\right)$$

$$\approx \frac{\beta^* K^2}{2\gamma^2}\left(1 + \frac{L_u^2}{12\beta^{*2}} + \frac{2\eta_0^2\gamma^2}{\beta^{*2}K^2} + \frac{8\gamma\eta_0}{k_p K\beta^{*2}}\right)$$

Unless undulator is placed in high dispersion region, contribution to emittance remains small.

Coherent radiation from a bend

$$\frac{d^2 I}{d\omega d\Omega} = [N + N(N-1) f(\omega)] \frac{d^2 I_0}{d\omega d\Omega}, \; f(\omega) = \left| \int \exp\left(\frac{i\omega z}{c}\right) S(z) dz \right|^2, \text{ for } N \to \infty$$

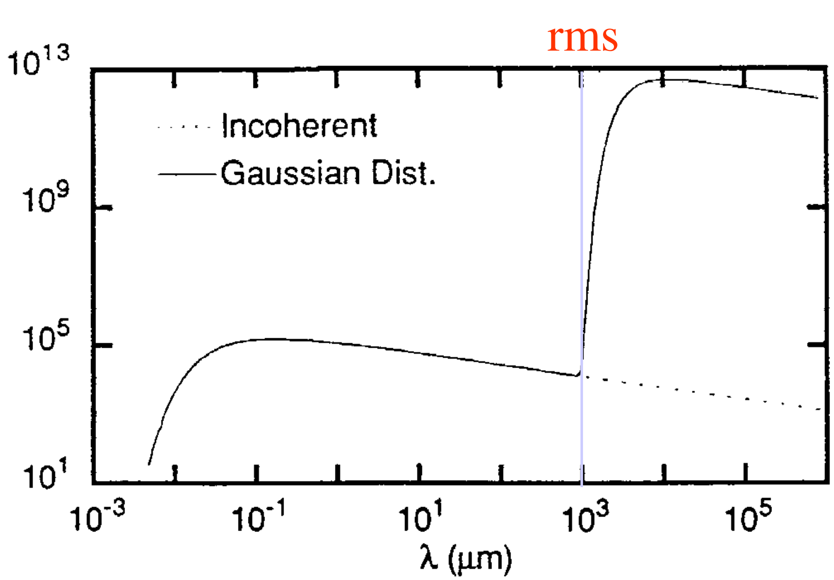
Form factor

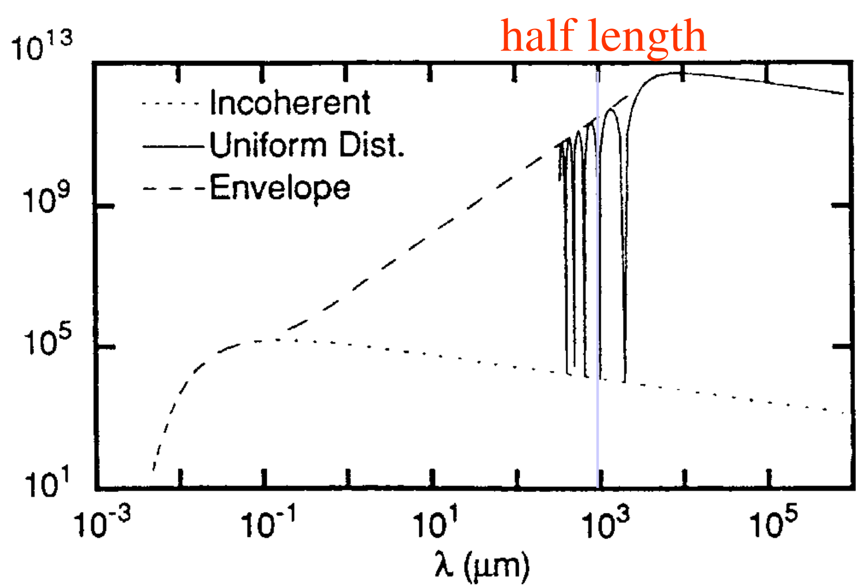
Gaussian

$$f(\omega) = \exp\left(-4\pi^2\sigma^2 / \lambda^2\right)$$

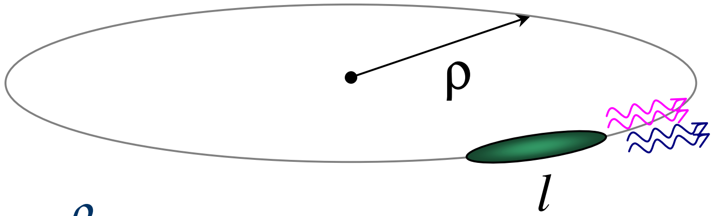
Uniform

$$f(\omega) = \mathrm{sinc}^2(2\pi l / \lambda)$$



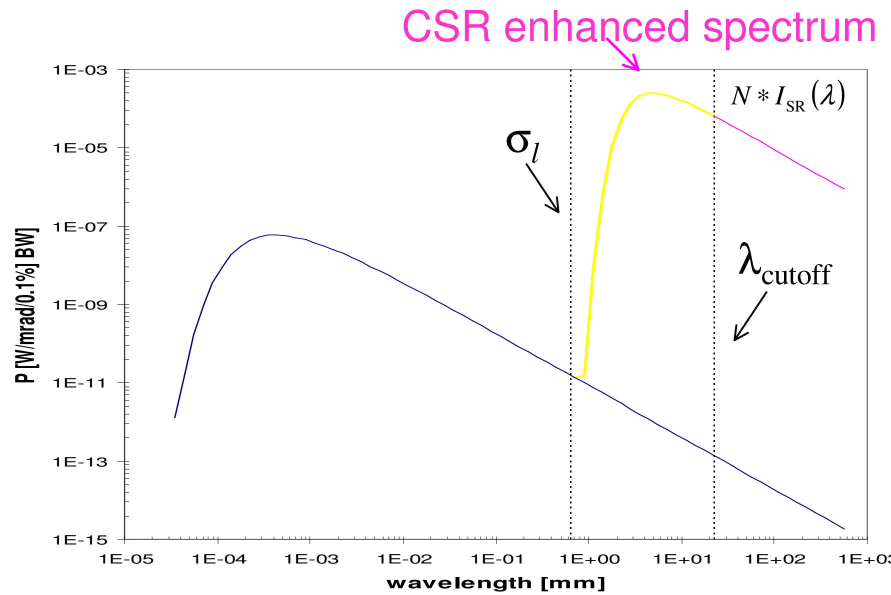


CSR: longitudinal and transverse

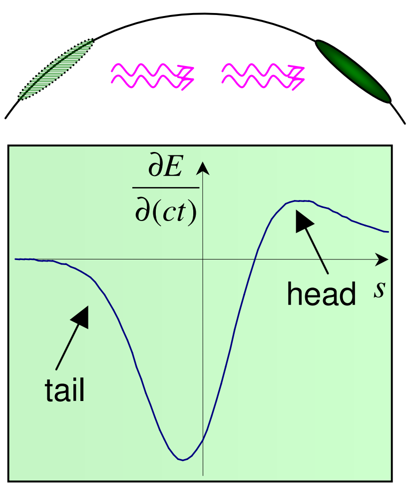


$$\lambda_{SR} \propto \frac{\rho}{\gamma^3} \qquad \lambda_{CSR} \geq l$$

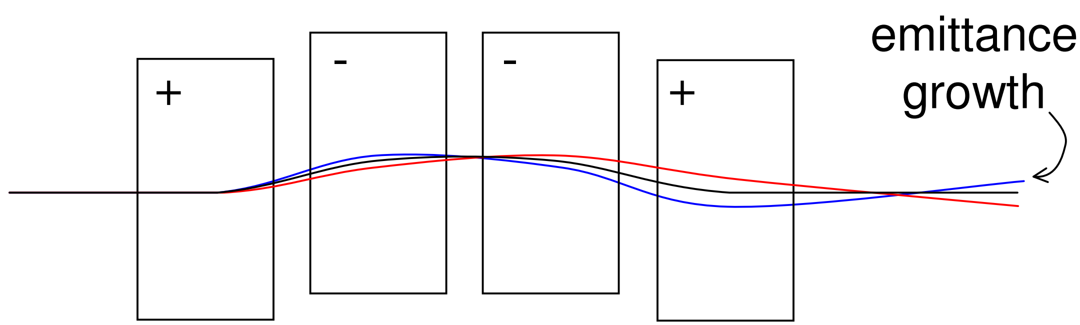
$$P_{SR} \propto N \frac{\gamma^4}{\rho^2} \qquad P_{CSR} \propto N^2 \frac{1}{\rho^{2/3} l^{4/3}}$$



Longitudinal Effect



Transverse Effect



Dispersion ≠ 0

Lienard-Wiechert formula: two e⁻'s

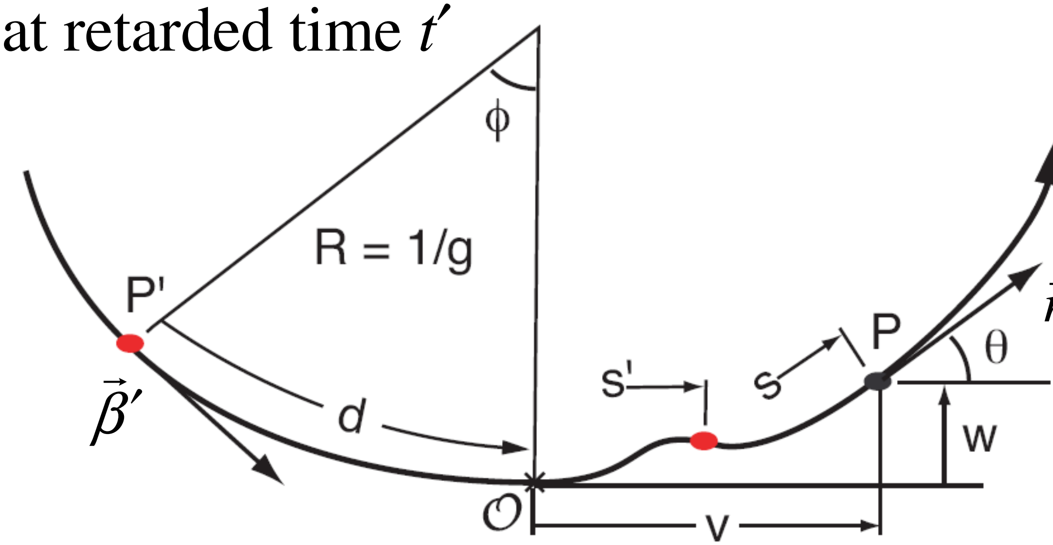
$$\vec{E}(P) = \underbrace{\frac{e}{\gamma^2}\frac{\vec{n}-\vec{\beta}'}{L^2(1-\vec{n}\cdot\vec{\beta}')^3}}_{\text{Coulomb term}} + \underbrace{\frac{e}{c^2}\frac{\vec{n}\times[(\vec{n}-\vec{\beta}')\times\vec{a}']}{L(1-\vec{n}\cdot\vec{\beta}')^3}}_{\text{radiative term}}$$

L – distance between P and P′

P′ – position of (red) particle at retarded time *t*′

$$t - t' = L/c$$

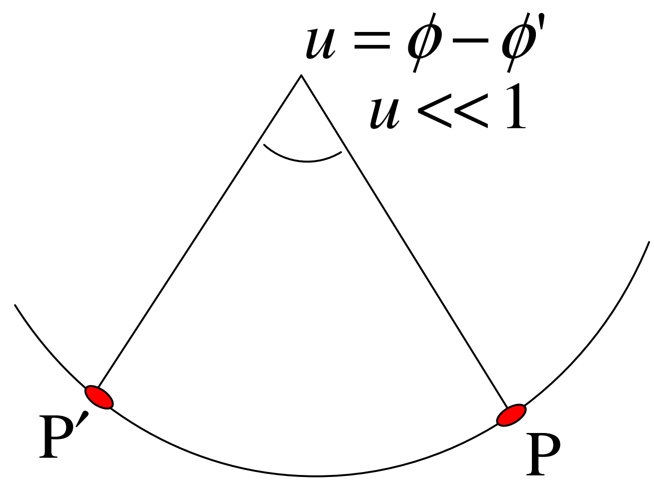
$$s - s' = L_s - \beta L$$



Energy loss in a circle

$$ec\beta\vec{n}\cdot\vec{E}(P)=\frac{e^2c}{R^2}\left[\frac{1-\beta+3u^2/8}{\gamma^2u^2(1-\beta+u^2/8)^3}+\frac{u^2/8-(1-\beta)}{2(1-\beta+u^2/8)^3}\right]$$

$$s-s'=(1-\beta)Ru+\frac{Ru^3}{24}$$



Problem: There is a singularity when u → 0

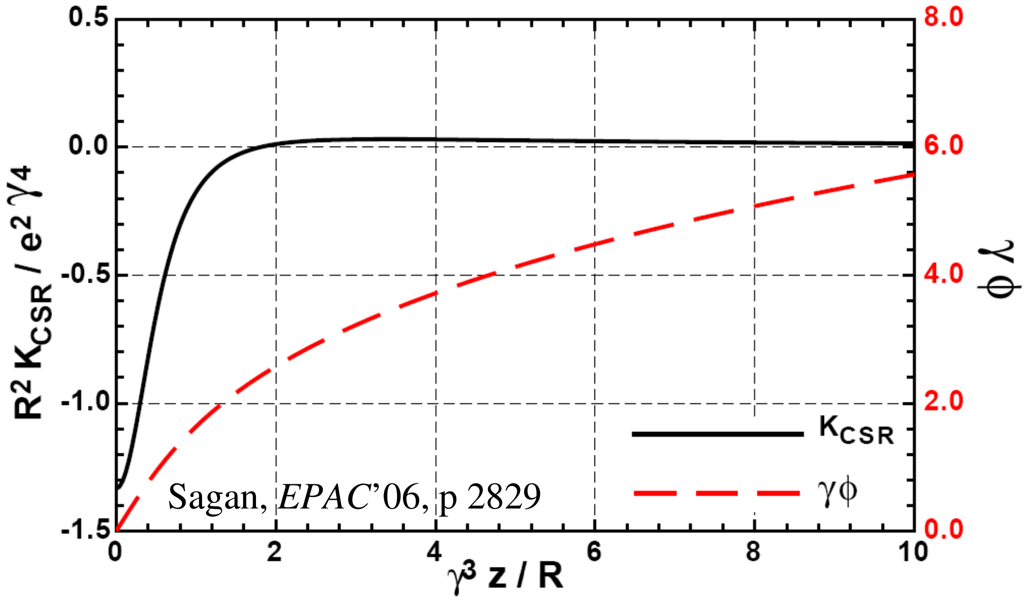
Solution: subtract work due to Coulomb force

$$e\beta\vec{n}\cdot\vec{E}_{sc}(P)=e\vec{\beta}\cdot\frac{e\vec{n}}{\gamma^2(s-s')^2}$$

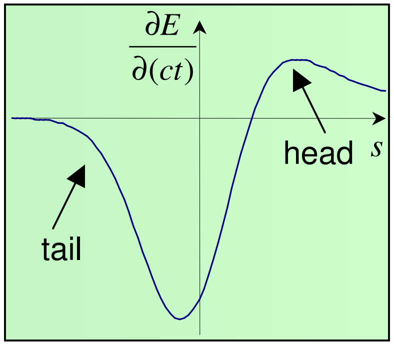
Energy loss in a circle

$$K_{CSR} \equiv (e\beta E)_{CSR} = \frac{e^2}{R^2}\left\{\frac{u^2/8-(1-\beta)}{2(1-\beta+u^2/8)^3}+\frac{1}{\gamma^2 u^2}\left[\frac{1-\beta+3u^2/8}{(1-\beta+u^2/8)^3}-\frac{1}{(u^2/24+1-\beta)^2}\right]\right\}$$

CSR wake



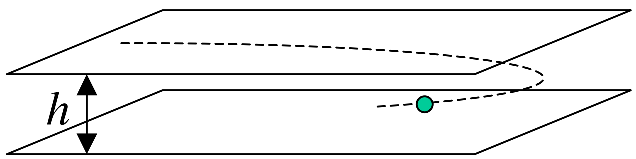
Energy spread for Gaussian



Shielding effect

Radiation at high frequency can be limited by critical wavelength (low energy), or by the bunch form factor.

Low frequency content is cut-off by conducting vacuum chamber (c.f. waveguide modes).



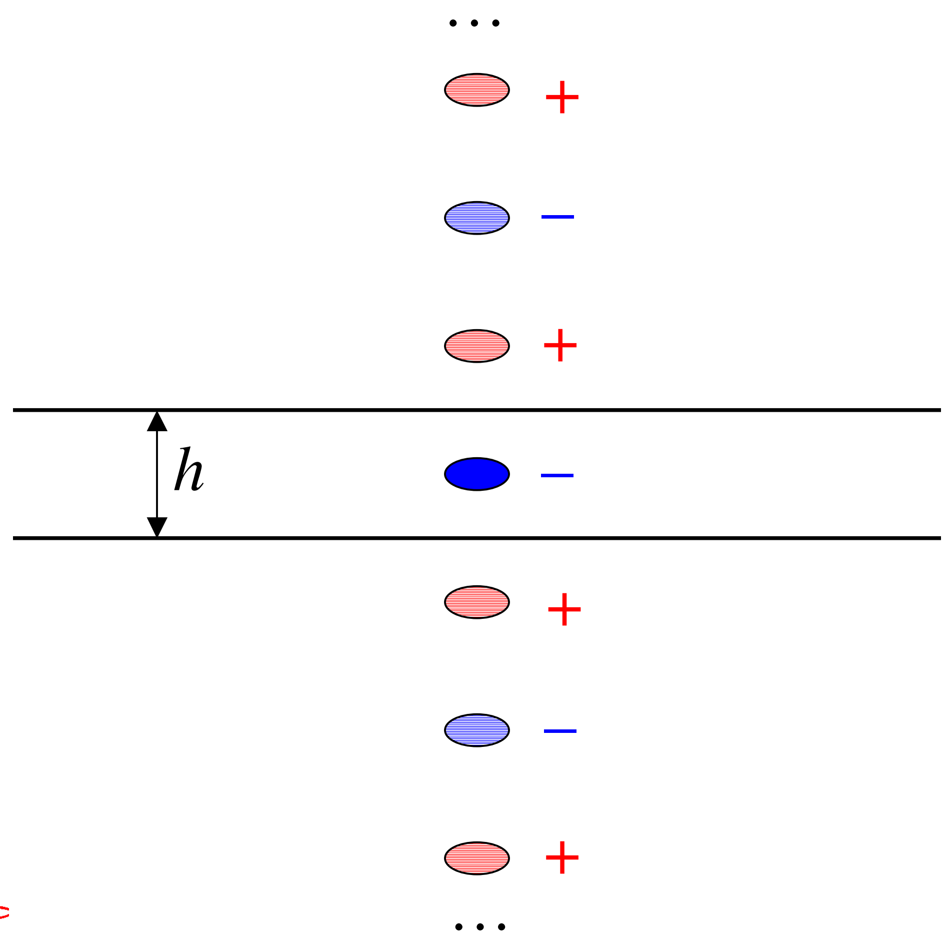
infinite conducting plates

Schwinger predicted for uniform bunch of length l reduction in total CSR power by a factor of

$$\left(\frac{l}{R\sqrt{3}}\right)^{1/3}\left(\frac{h}{2l}\right)$$

Simulation of shielding effect

Simple geometry of two parallel plates is simulated through adding 'infinite' number of image charges of alternating polarity, and direct application of Lienard-Wiechert formula (still need to solve for the retarded time).



Shielding effect on CSR wake

$\sigma_z = 0.3$ mm, $\rho = 10$ m

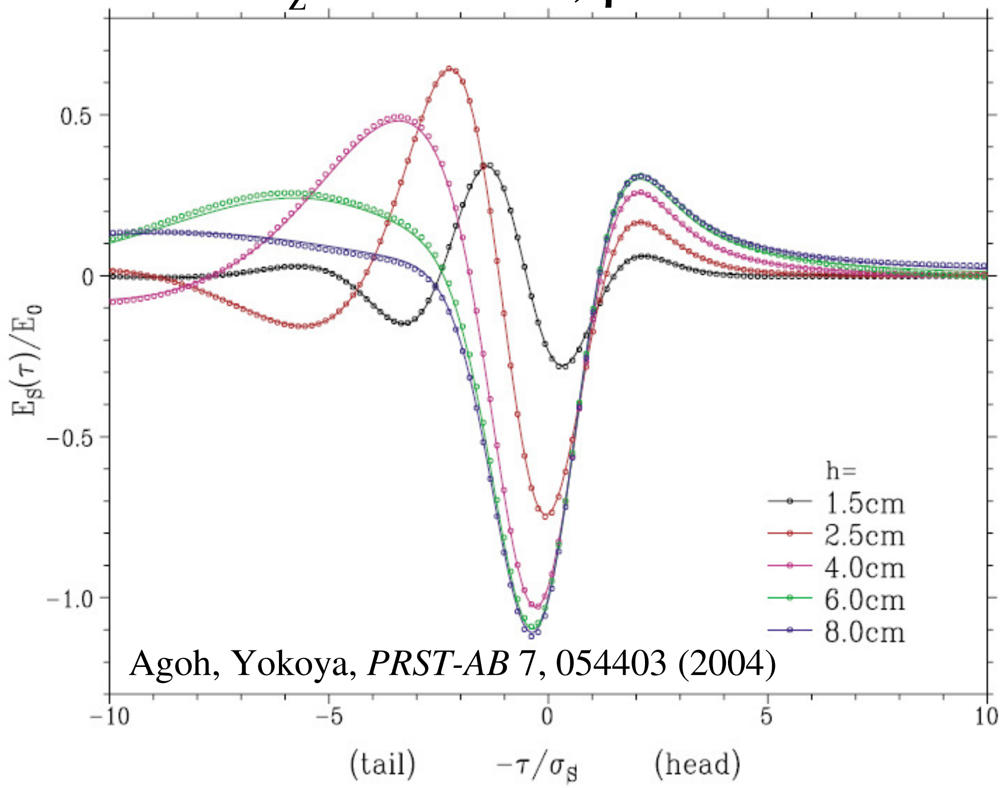
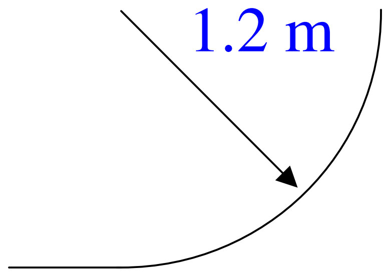
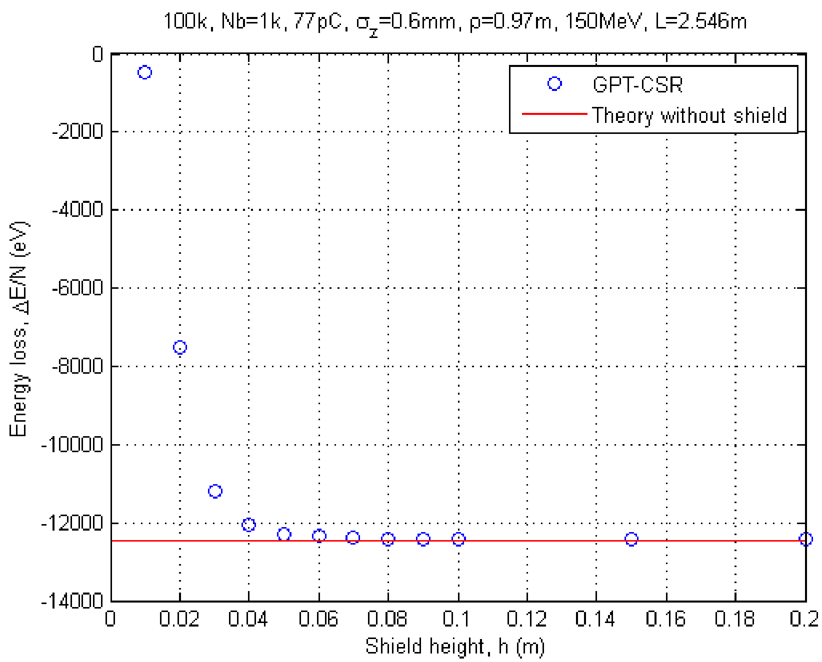


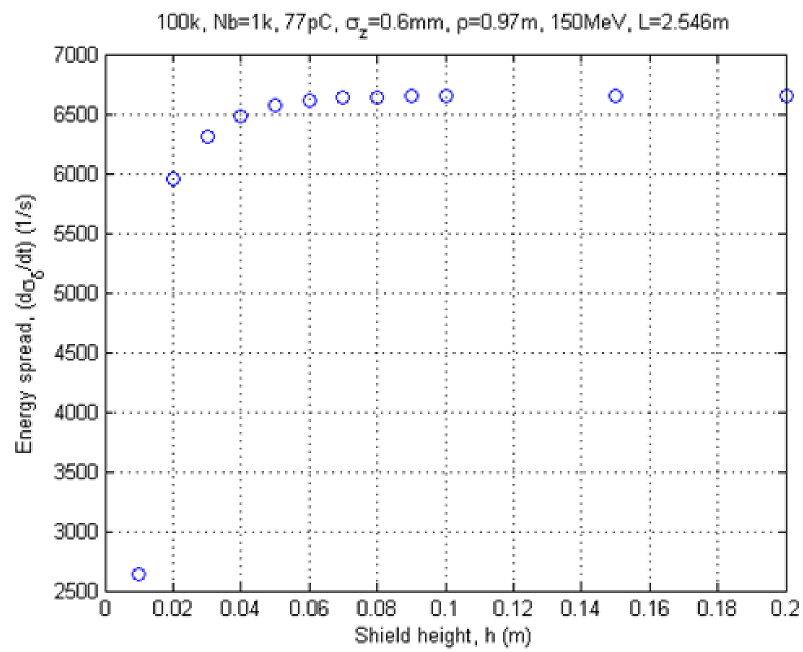
FIG. 2. (Color) The longitudinal electric field E_s in steady state at $x = y = 0$ between infinite parallel plates. Warnock's formula is plotted with solid lines. The dots are the simulation results. The different colors show the gap between the two horizontal parallel plates, $h = 1.5$ cm (black), 2.5 cm (red), 4 cm (magenta), 6 cm (green), and 8 cm (blue), respectively. The width of the chamber is $w = 50$ cm and the length of the magnet is 3 m.

Shielding effect on CSR wake

Simple beam trajectory: a short drift followed by circular trajectory

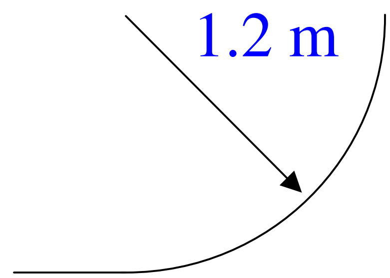


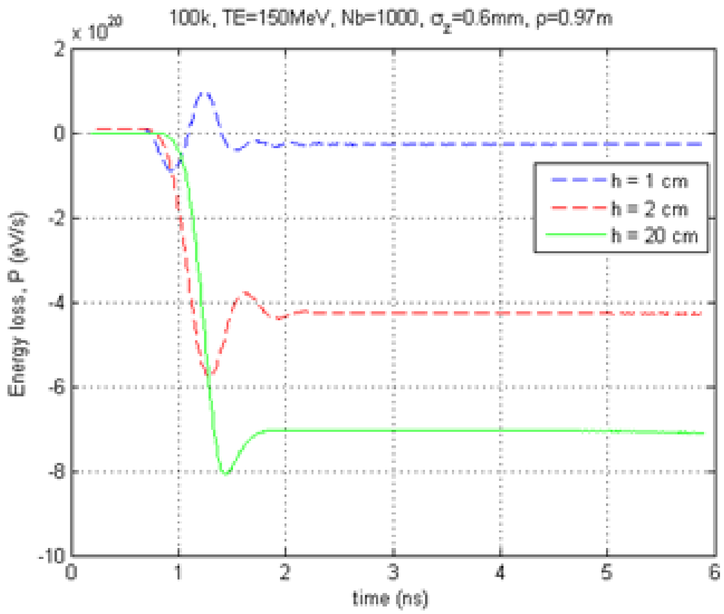


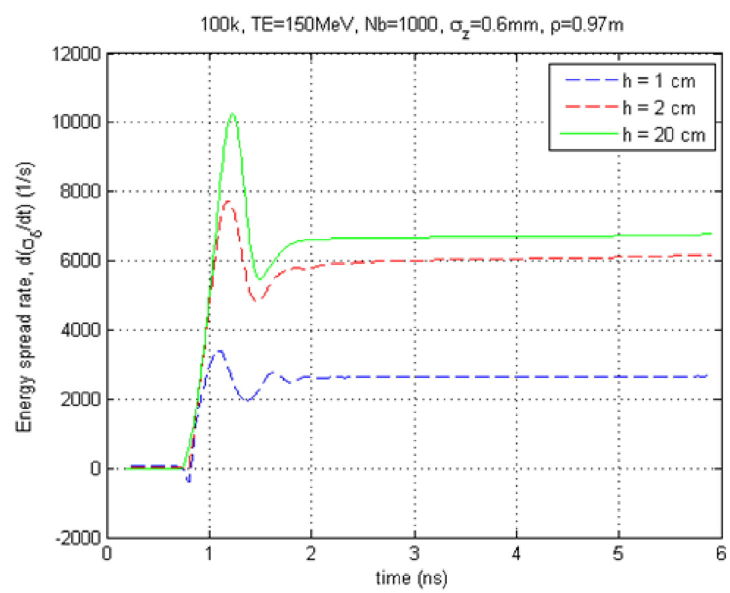


Shielding effect on CSR wake

Simple beam trajectory: a short drift followed by circular trajectory

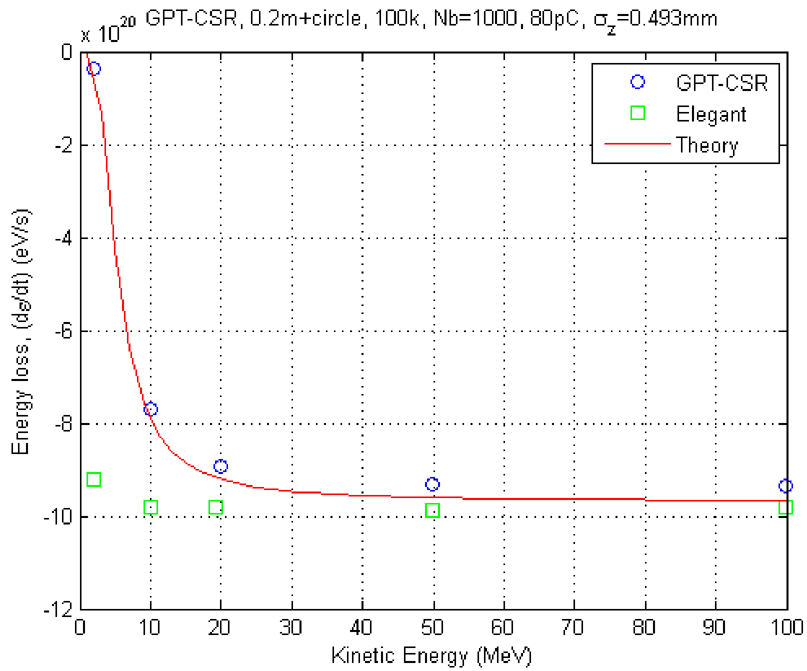






Beam energy effect on CSR

Interesting feature of CSR is its independence from beam energy (e.g. radiated power of coherent fraction). However, at low beam energies (important for the injector), CSR can be substantially weaker, while Coulomb term dominates.



CSR simulation tools

$$\frac{\sigma_x}{\sigma_z} << \left(\frac{R}{\sigma_z}\right)^{1/3}$$ (1D criterion)

- 1D, high gamma approximation: **elegant**
- 1D, correct at low energy, shielding: **BMAD**
- 1D, correct at low energy, shielding, space charge: **GPT (Cornell version)**
- Self-consistent: **Tredi3D, CSRTrack** (one of the routines)
- …

Summary

- Practical tools for evaluating degradation of emittance and energy spread due to synchrotron radiation presented
- ISR is typically negligible for low and medium beam energies (< 3 GeV), can dominate beam line design and higher energies
- CSR is important whenever the bunch is short (bunch compressors), except at very low energies (injector) when the space charge dominates.