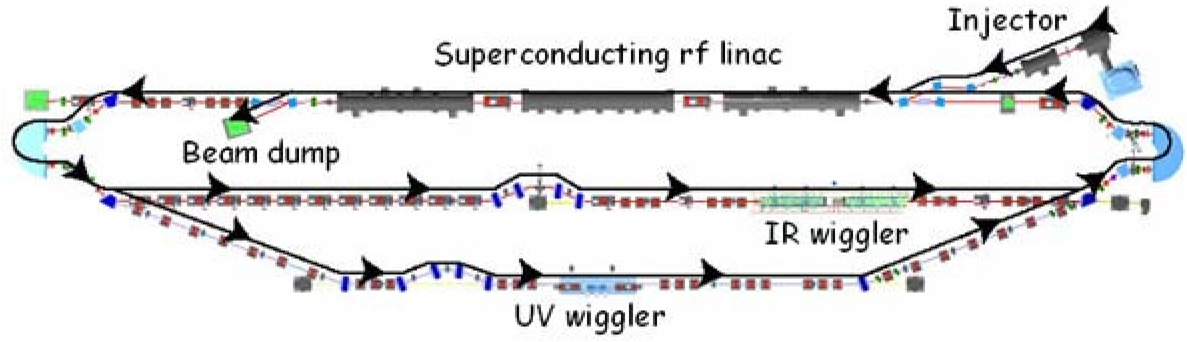
USPAS course on *Recirculated and Energy Recovered Linacs*

Ivan Bazarov, Cornell University

Geoff Krafft and Dave Douglas, JLAB

Computer class: Linear Optics in JLAB: Longitudinal Dynamics and BBU

JLAB IRFEL



Spreadsheet model of JLAB IRFEL includes:

- full first-order optics
- longitudinal phase space visualization
- beam break-up simulations

Downloading the lesson

Download the spreadsheet and bbu code to a *single* write-enabled directory from
http://www.lns.cornell.edu/~ib38/uspas08/

Make sure macros are enabled. Start the spreadsheet (note: it may take a while to initialize all formulas).

Spreadsheet organization

The spreadsheet is organized into three main parts

```
---> elements
  matrices
  products
  twiss
```

layout and lattice control

```
---> bbu
  bbu_latfile
  bbu_homs
  bbu_param
```

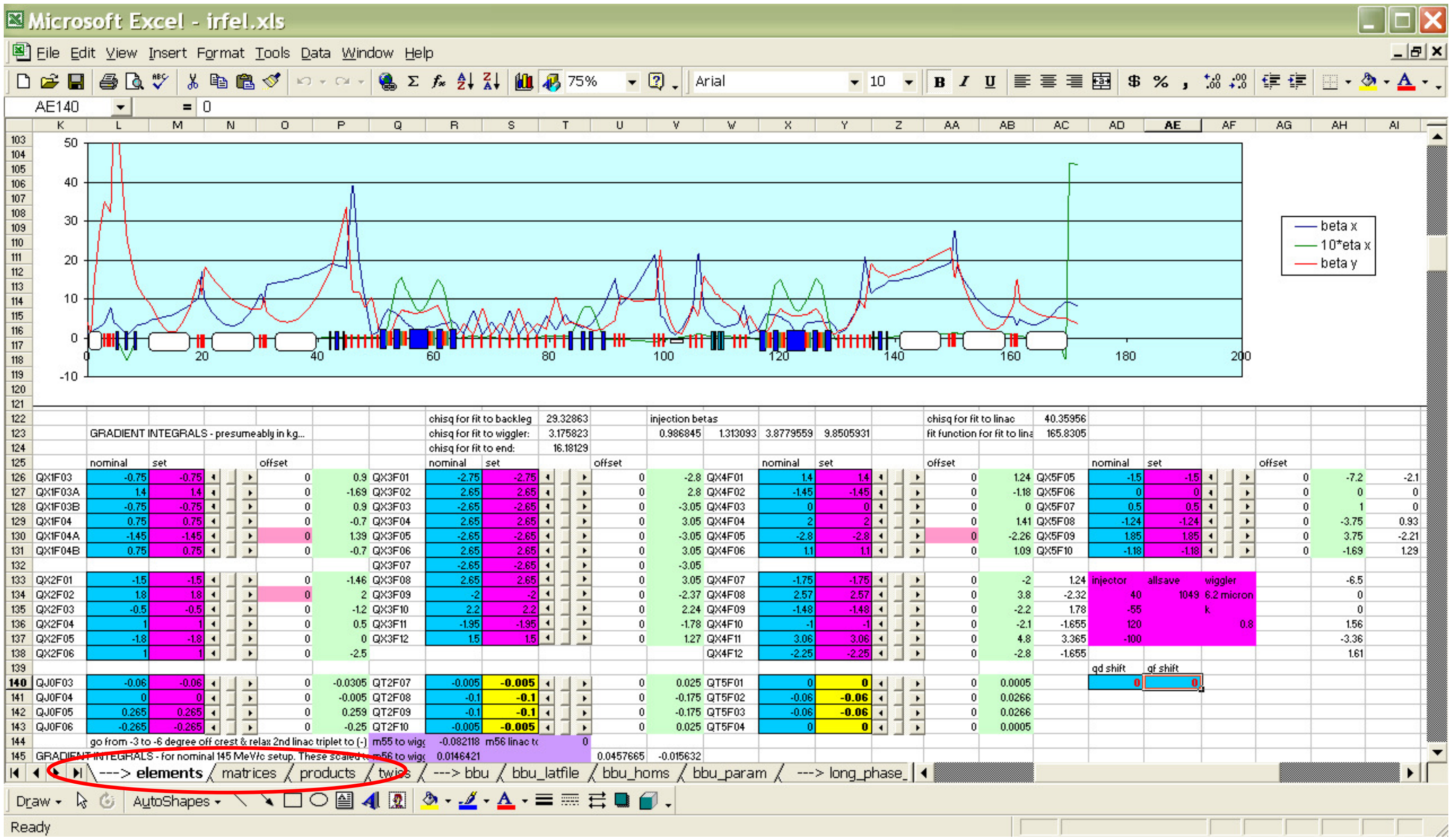
beam breakup simulation

```
---> long_phase_space
  R56
  z
  pz
```

longitudinal phase space

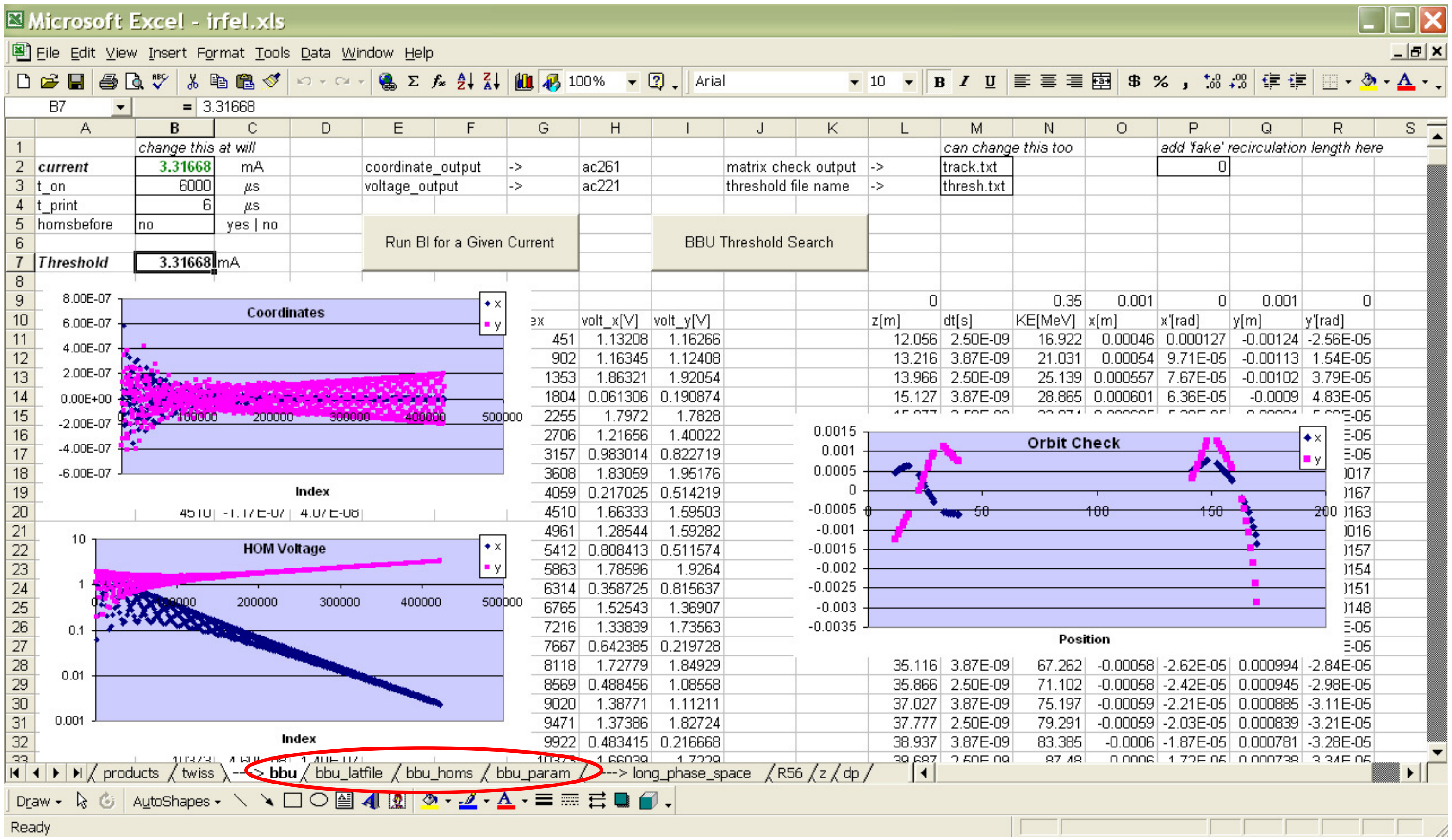
Organization: layout and lattice

`--->` `elements` sheet contains optics controls



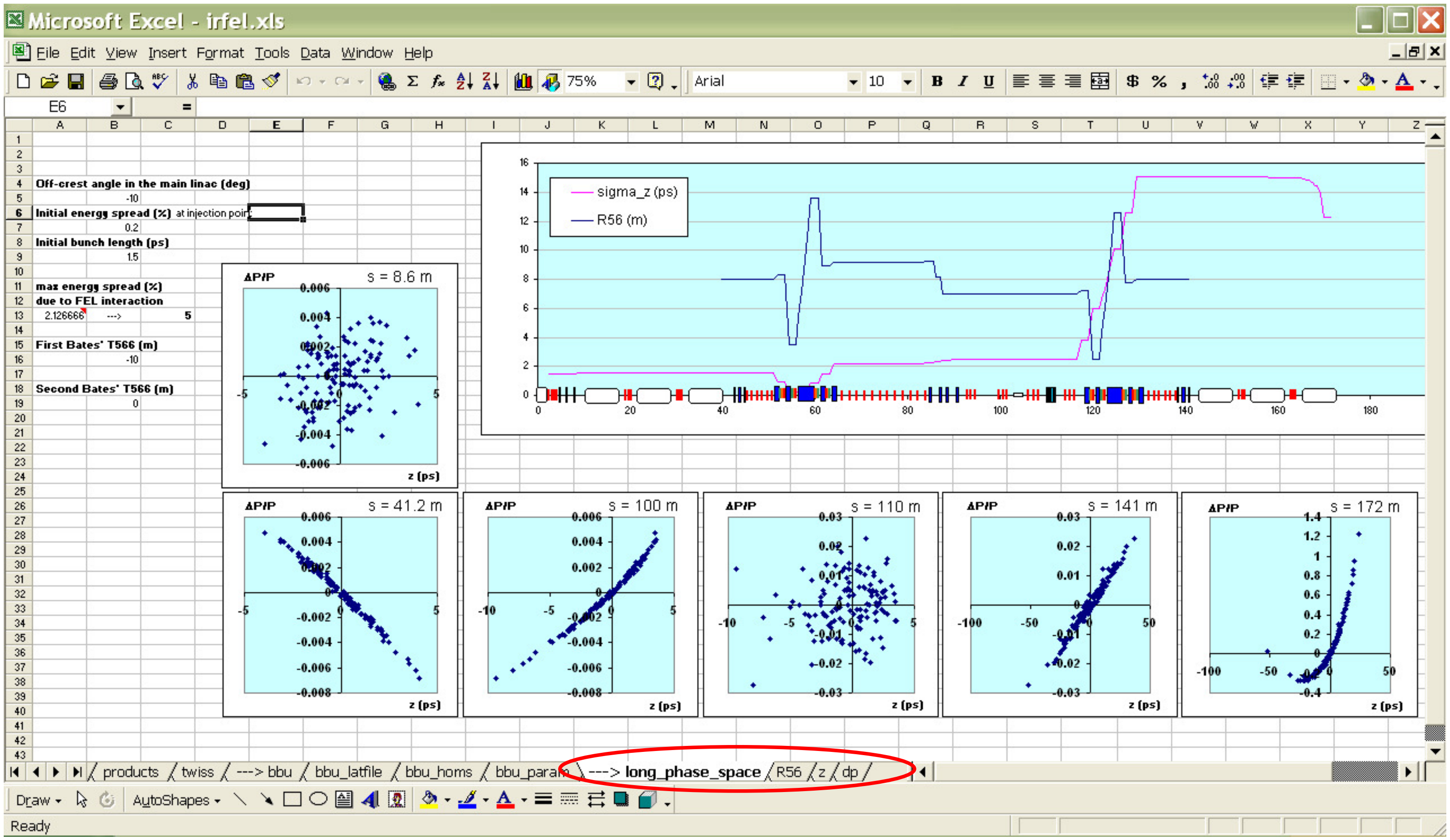
Spreadsheet: beam break-up

`--->` `bbu` controls execution of beam break-up code



Spreadsheet: longitudinal phase space

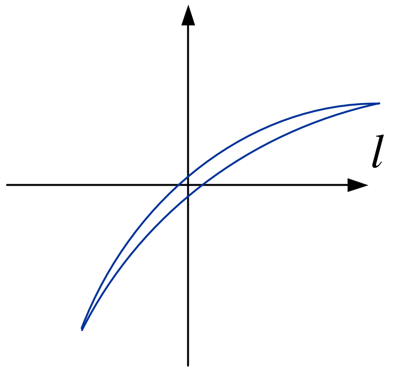
`---> long_phase_space` tracks a bunch in the long. p.s.



A brief overview of longitudinal dynamics

- first- and second-order correlation in longitudinal phase space
- second-order momentum compaction
- requirements for energy recovery

First- and second-order correlations



$$\delta = \delta_0 + \left.\frac{\partial \delta}{\partial l}\right|_{l=0} l + \frac{1}{2!}\left.\frac{\partial^2 \delta}{\partial l^2}\right|_{l=0} l^2 + \ldots$$

$$\delta \cong \delta_0 + \alpha_\delta l + \frac{1}{2}\beta_\delta l^2$$

$$\sigma_\delta = \sqrt{\sigma_{\delta_0}^2 + \alpha_\delta^2 \sigma_l^2 + \tfrac{1}{2}\beta_\delta^2 \sigma_l^4}$$

$$\varepsilon_{\delta\text{-}l} = \sigma_l \sqrt{\sigma_{\delta_0}^2 + \tfrac{1}{2}\beta_\delta^2 \sigma_l^4}$$

$$\alpha_\delta = -\frac{E_{linac}}{E_{final}} k_{RF} \sin\varphi \qquad \beta_\delta = -\frac{E_{linac}}{E_{final}} k_{RF}^2 \cos\varphi$$

$$k_{RF} = 2\pi / \lambda_{RF} = 31.5\ \mathrm{m}^{-1} \text{ for } 1.5\ \mathrm{GHz}$$

After acceleration

after the main linac:

$$\alpha_\delta \approx -k_{RF}\varphi$$

$$\beta_\delta \approx -k_{RF}^2$$

assuming large $E_{final}/E_{injection}$ and small energy spread

energy spread:

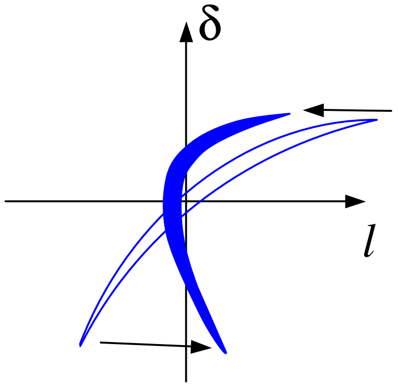
$$\sigma_\delta \approx \alpha_\delta \sigma_l \quad \text{for } |\varphi| > \tfrac{1}{\sqrt{2}} k_{RF}\sigma_l$$

$$\sigma_\delta \approx \tfrac{1}{\sqrt{2}} \beta_\delta \sigma_l^2 \quad \text{for } |\varphi| < \tfrac{1}{\sqrt{2}} k_{RF}\sigma_l$$

longitudinal emittance:

$$\varepsilon_{\delta\text{-}l} \approx \tfrac{1}{\sqrt{2}} \beta_\delta \sigma_l^3$$

Longitudinal transform



$$l^* = l + R_{56}\delta + T_{566}\delta^2$$
$$\delta^* = \delta$$

$$\Rightarrow \quad \alpha_\delta^* = \frac{\alpha_\delta}{1+R_{56}\alpha_\delta} \qquad \beta_\delta^* = \frac{\beta_\delta - 2T_{566}\alpha_\delta^3}{(1+R_{56}\alpha_\delta)^3}$$

$$L = \int \sqrt{(1+x/\rho)^2 + x'^2 + y'^2}\, ds$$

momentum compaction (times the path length):

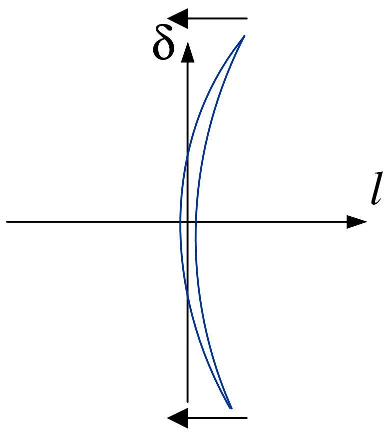
$$R_{56} = \int \frac{\eta}{\rho} ds$$

second-order momentum compaction:

$$T_{566} = \int \left[\frac{\eta_{(2)}}{\rho} + \frac{\eta^2}{2\rho} + \frac{\eta'^2}{2} \right] ds$$

Compression

for maximum compression need $R_{56} = -\frac{1}{\alpha_\delta} \approx \frac{1}{k_{RF}\varphi}$



for maximum compression need

$$T_{566} = \frac{\beta_\delta}{2\alpha_\delta^3} \approx \frac{1}{2k_{RF}\varphi^3}$$

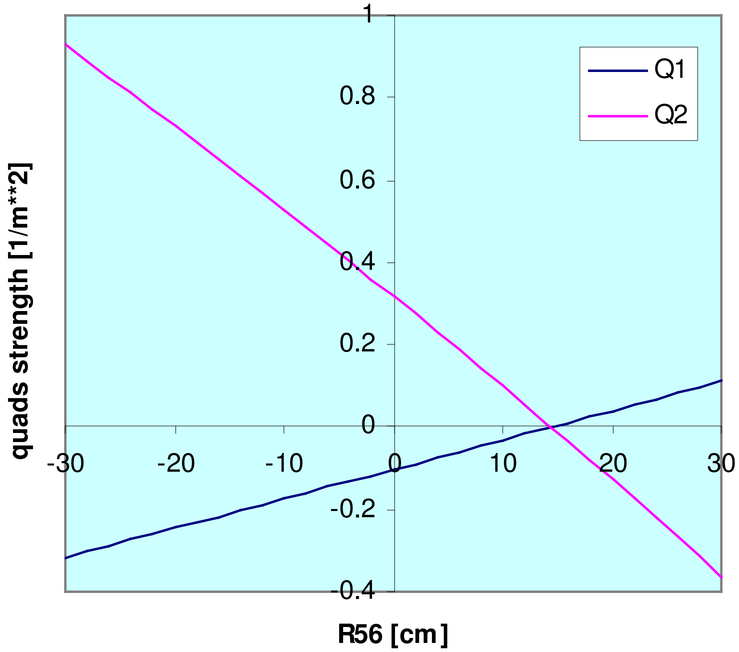
actual (absolute) value of T_{566} can be smaller

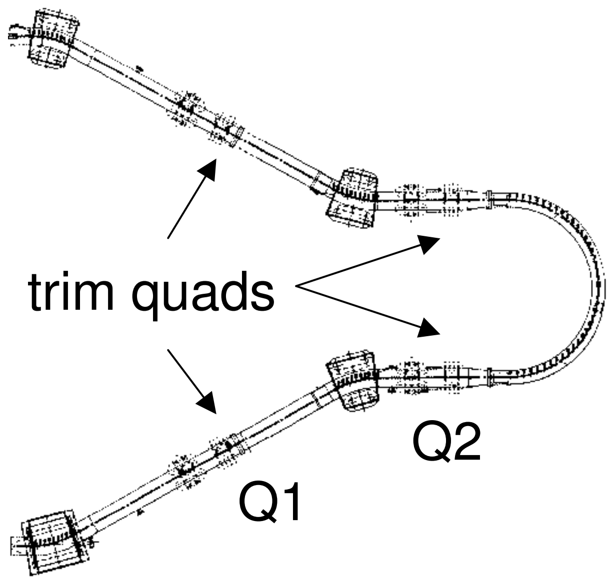
$$\Delta T_{566,\sigma_l^{comp}} = \frac{\sigma_l^{comp}}{\sqrt{2\alpha_\delta^2\sigma_l^2}} \approx \frac{\sigma_l^{comp}}{\sqrt{2k_{RF}^2\sigma_l^2\varphi^2}}$$

no T_{566} is needed beyond a certain off-crest phase angle

$$\varphi > \varphi_{T_{566=0}} = \frac{\sigma_l^2}{\sigma_l^{comp}} \frac{k_{RF}}{\sqrt{2}}$$

Achieving the right values of R_{56}

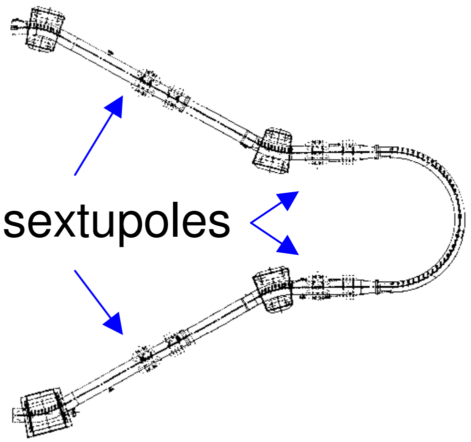




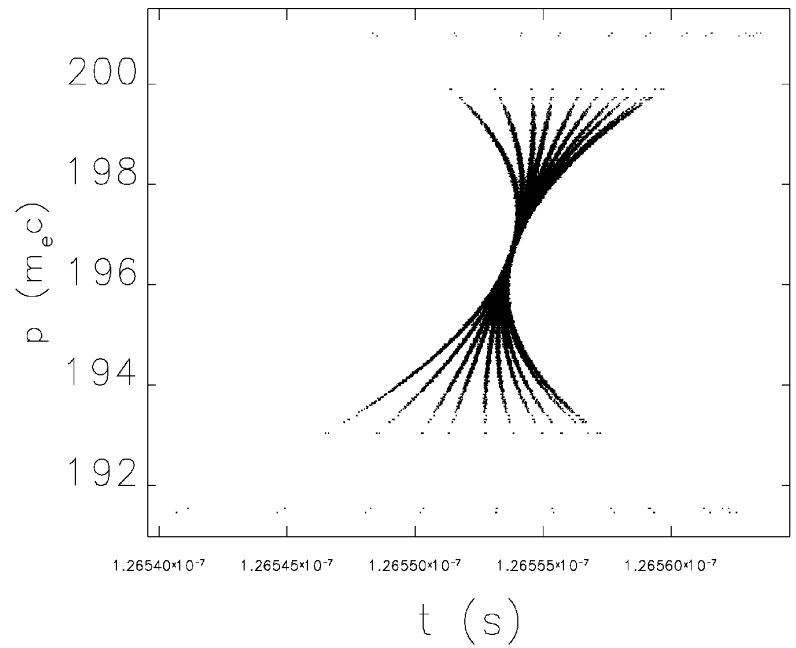
Adjustable R_{56}

$$R_{56} = \int_1^2 \frac{\eta_x}{\rho} ds$$

Achieving the right values of T_{566}



changing sextupoles strength in the Arc…

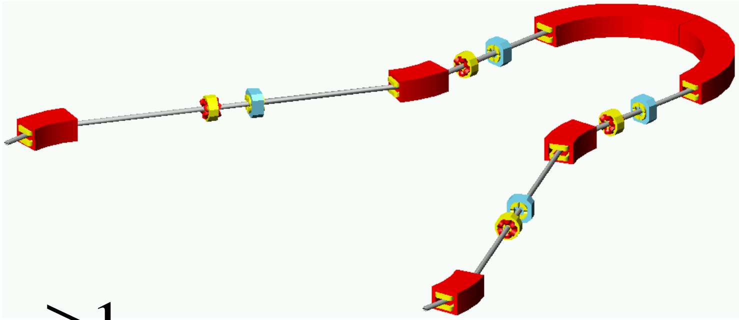


$$T_{566} = \int \left[\frac{\eta_{(2)}}{\rho} + \frac{\eta^2}{2\rho} + \frac{\eta'^2}{2} \right] ds$$

$$\eta''_{(2)} + K(s)\eta_{(2)} = -h + k_1\eta - \tfrac{1}{2}k_2\eta^2 + (h^3 + 2k_1 h)\eta^2 + \tfrac{1}{2}h\eta'^2 + h'\eta'\eta + 2h^2\eta$$

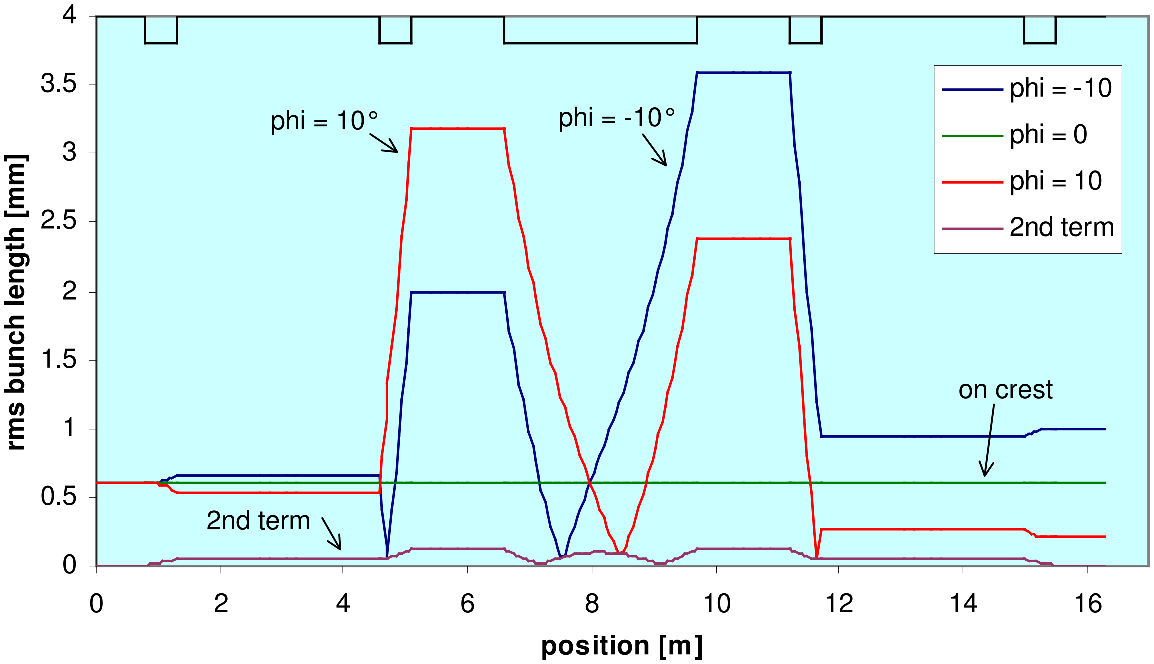
Bunch length in the Bates'

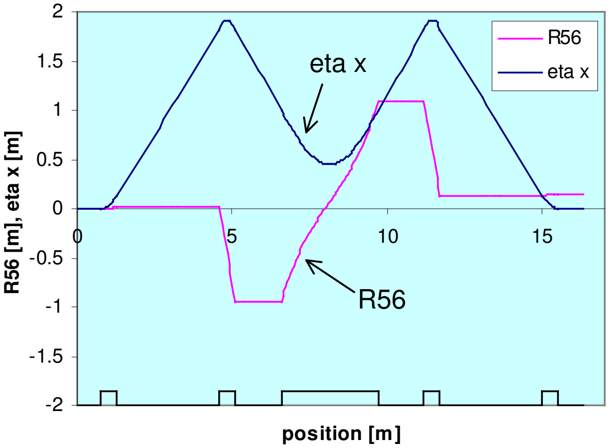
Bunch length in the Arcs:



$$\sigma_{l,t}^2 = \sigma_{l,0}^2\left(1+\frac{\partial\delta}{\partial l}R_{56}\right)^2 + \text{second\_term}^2$$

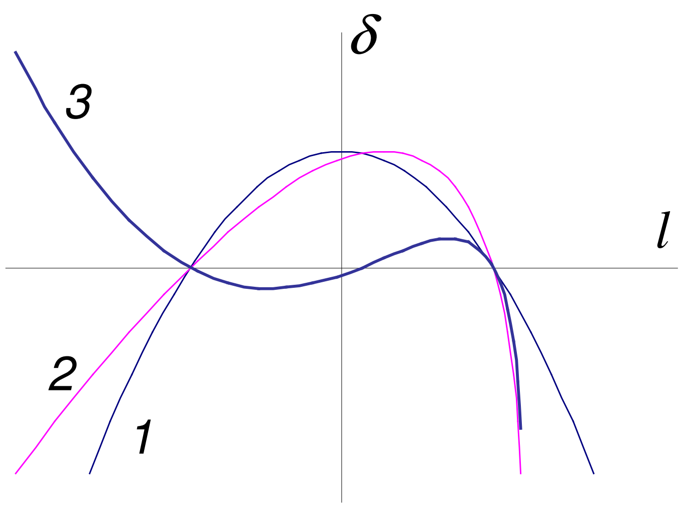
For off-crest of several deg: $\frac{\partial\delta}{\partial l}R_{56} \geq 1$





R_{56} = 14.4 cm
(trim quads off)

Energy recovery



$$\Delta\delta = \beta_\delta l \Delta l$$

$$\Delta l \sim R_{56}\sigma_\delta \qquad \Delta l \sim T_{566}\sigma_\delta^2$$

$$\Delta\delta \sim \sigma_{\delta_{dump}}$$

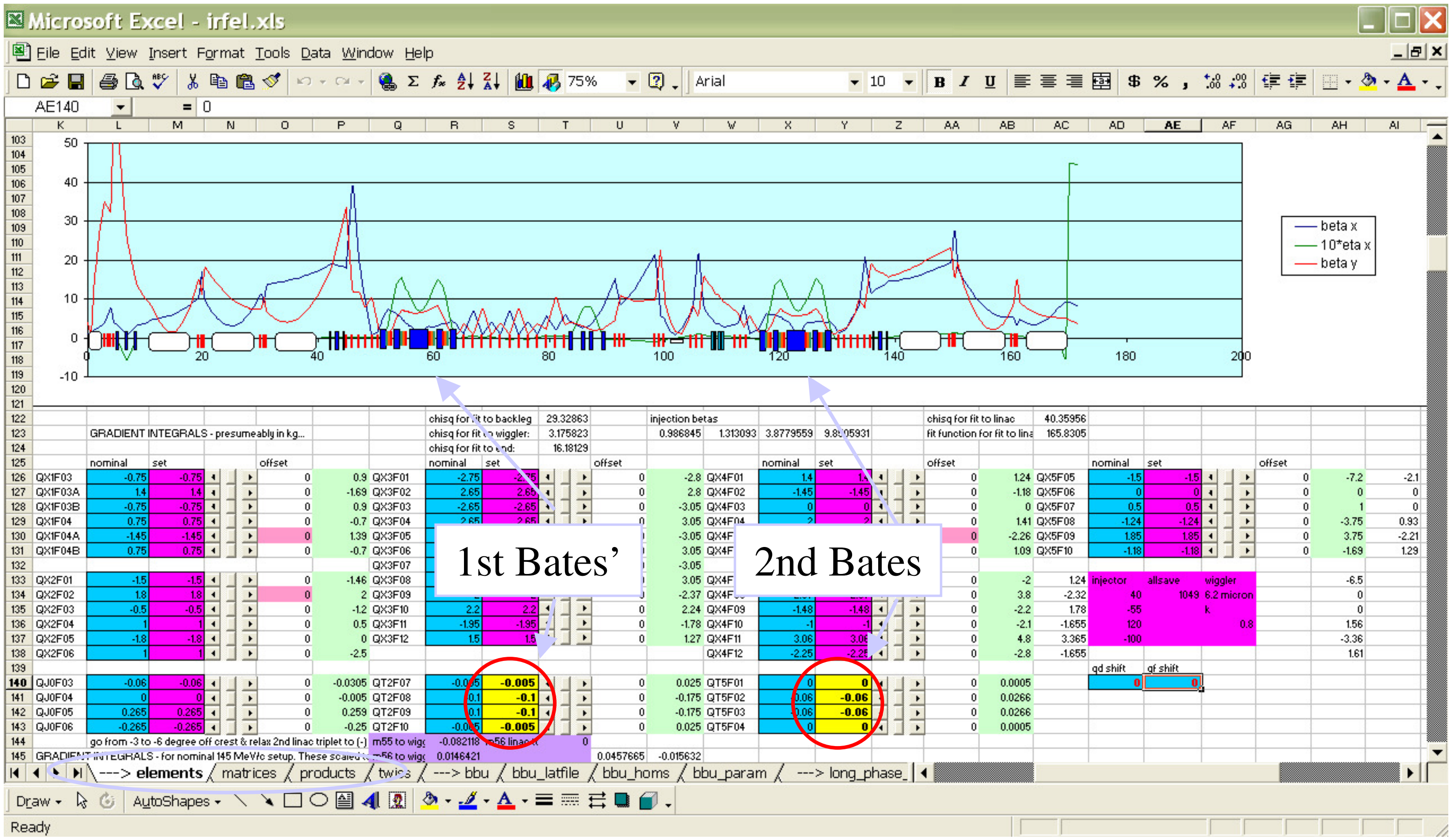
$$R_{56} \sim \frac{\sigma_{\delta_{dump}}}{\beta_\delta^2 \sigma_l^3} \qquad T_{566} \sim \frac{\sigma_{\delta_{dump}}}{\beta_\delta^3 \sigma_l^5}$$

General rule of thumb for successful energy recovery is having the full recirculating arc isochronous to first and second order ($R_{56} = T_{566} = 0$).

In IRFEL, the main difficulty is an additional energy spread generated at the wiggler due to FEL interaction.

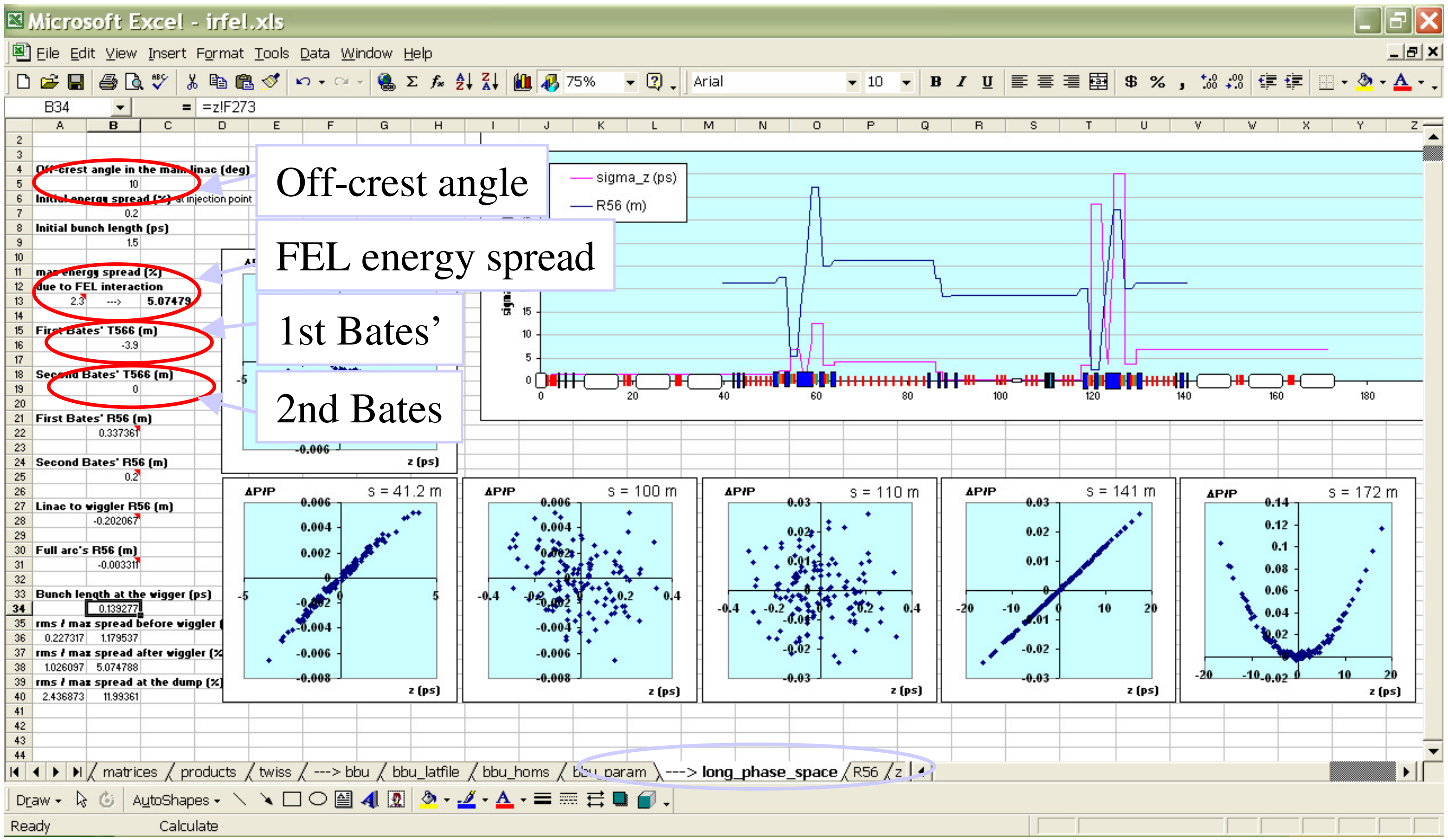
Controlling Bates' quads

`---> elements` sheet, yellow region



Off-crest phase, FEL energy spread and T_{566}

---> long_phase_space sheet



Exercises (longitudinal gymnastics)

• Set the off-crest phase angle in the main linac to 0 and 'turn-off' laser interaction. Observe how the longitudinal phase space looks throughout the accelerator and at the beam dump.

• Set the off-crest phase angle in the main linac to –10°. Achieve the shortest bunch possible at the wiggler location using linear optics only (T_{566} should be 0). Compare calculated R_{56} with the value in the model.

• Use T_{566} to maximally compress the bunch at the wiggler. Compare calculated T_{566} with the value in the spreadsheet. How much shorter is the bunch length when both second- and first-order compaction is used, as opposed to only the first-order compression? Achieve less than 150 fs rms bunch duration.

NOTE: R_{56} from the linac to the wiggler consists comes from two parts: Bates' turn-around and a chicane

Exercises (optional)

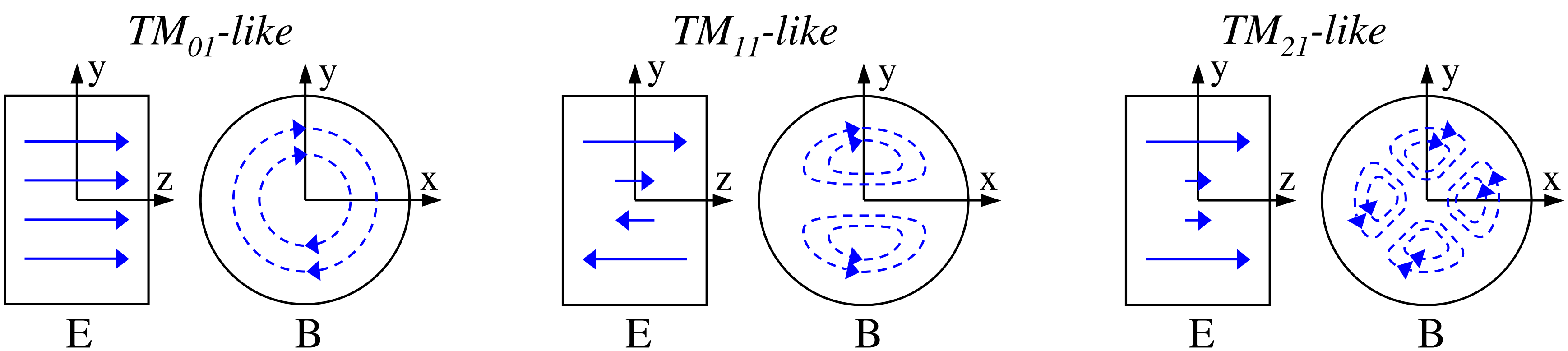
- 'Turn-on' the laser interaction (actual max. energy spread of 5 %). Observe the longitudinal phase space at the dump. Is the beam being successfully recovered?
- Adjust R_{56} in the second Bates' section to minimize energy spread at the dump. Note the smallest energy spread you were able to achieve.
- Use T_{566} to minimize energy spread at the dump. Note the values of R_{56} and T_{566} of the whole recirculating arc that allowed the result. What is the smallest energy spread you were able to achieve? Achieve less than 15 % max energy spread at the dump.

Higher order modes

Two basic concerns:

- Multipass beam breakup (dipoles)
- Resonant excitation of a higher order mode (monopoles)

monopole (m = 0) | dipole (m = 1) | quadrupole (m = 2)



high energy losses, no kick

kick and losses when beam is not centered

kick, coupling and losses when beam is not centered

BBU threshold for a single dipole mode

$t_r \frac{\omega_\lambda}{2Q_\lambda} \ll 1$ — NO → $t_r \frac{\omega_\lambda}{2Q_\lambda} \gg 1$ — NO → use code!

$t_r \frac{\omega_\lambda}{2Q_\lambda} \ll 1$ — YES → $T_{12} \sin\omega_\lambda t_r < 0$

$T_{12} \sin\omega_\lambda t_r < 0$ — NO → $T_{12} \sin\omega_\lambda t_r > 0$

$T_{12} \sin\omega_\lambda t_r < 0$ — YES →

$$I_0 = -\frac{\epsilon}{\mathcal{K}} \frac{2}{T_{12} \sin\omega_\lambda t_r}$$

$T_{12} \sin\omega_\lambda t_r > 0$ →

$$I_0 = \frac{2}{\mathcal{K}|T_{12}|} \sqrt{\epsilon^2 + \frac{1}{n_r^2} \mathrm{Mod}(\omega_\lambda t_r, \pi)^2}$$

$t_r \frac{\omega_\lambda}{2Q_\lambda} \gg 1$ — YES →

$$I_0 = \frac{2}{\mathcal{K}|T_{12}|} \sqrt{\epsilon^2 + \frac{1}{n_r^2} \mathrm{Mod}(\omega_\lambda t_r \pm \frac{\pi}{2}, 2\pi)^2}$$

$I_0 \approx 2\epsilon/\mathcal{K}|T_{12}|$, is *independent* of t_r

$\epsilon = \frac{\omega_\lambda}{2Q_\lambda} t_b$, $\epsilon \ll 1$

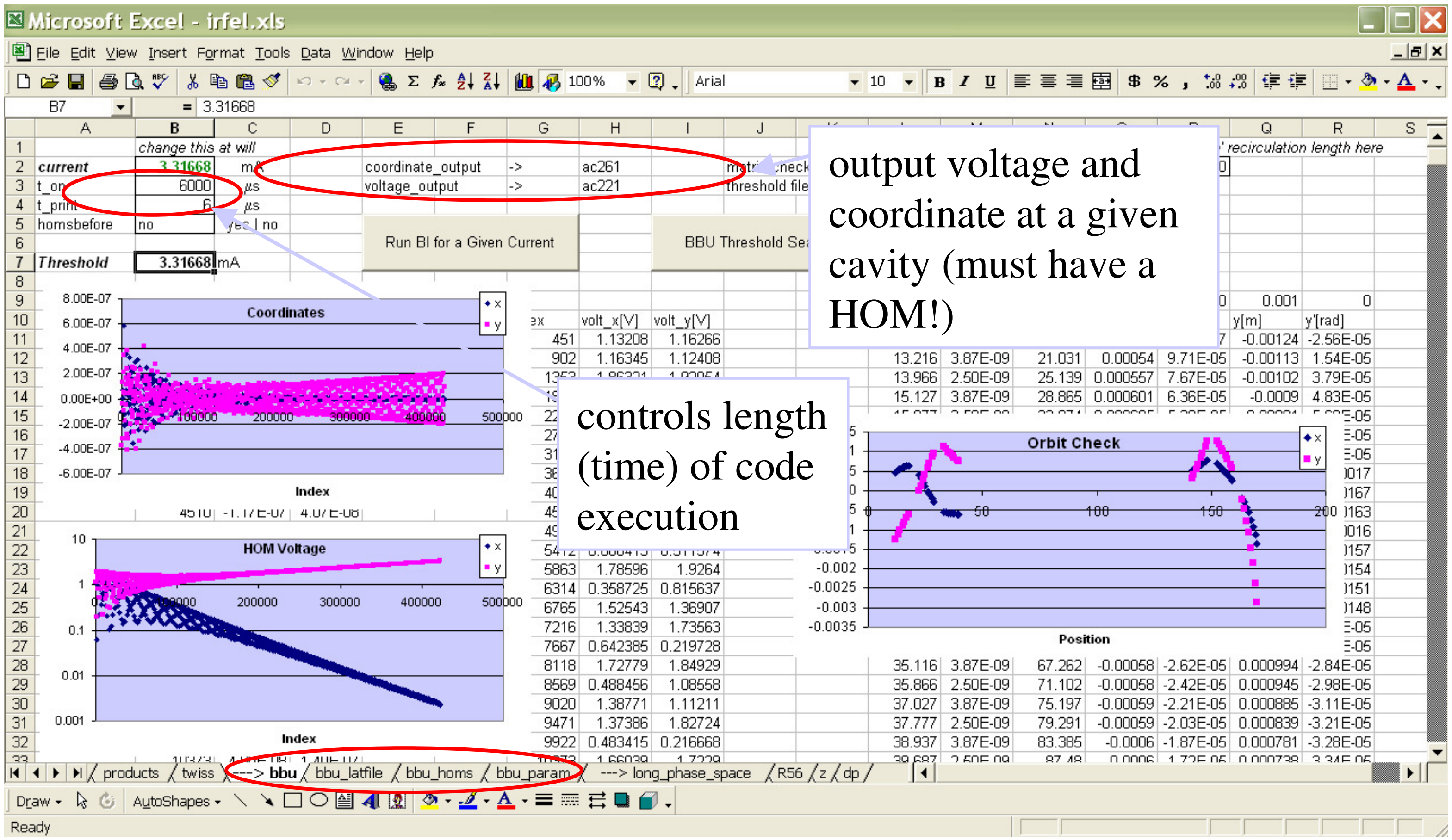
$t_r = (n_r - \delta) t_b$

integer n_r and $\delta \in [0, 1)$

$\mathcal{K} = t_b (e/c^2)(R/Q)_\lambda (\omega_\lambda^2/2)$

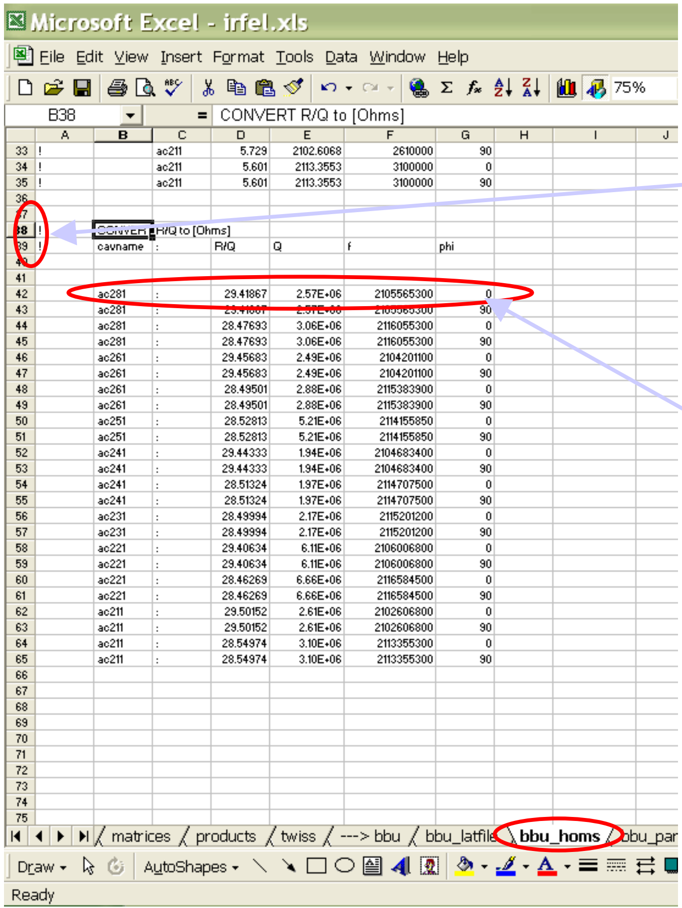
Controlling BBU code

---> `bbu` controls execution of beam break-up code



Controlling BBU code: HOMs

bbu_homs spreadsheet



	A	B	C	D	E	F	G
33	!		ac211	5.729	2102.6068	2610000	90
34	!		ac211	5.601	2113.3553	3100000	0
35	!		ac211	5.601	2113.3553	3100000	90
36							
37							
38	!	CONVERT R/Q to [Ohms]					
39	!	cavname	:	R/Q	Q	f	phi
40							
41							
42		ac281	:	29.41867	2.57E+06	2105565300	0
43		ac281	:	[illegible]	[illegible]	[illegible]	90
44		ac281	:	28.47693	3.06E+06	2116055300	0
45		ac281	:	28.47693	3.06E+06	2116055300	90
46		ac261	:	29.45683	2.49E+06	2104201100	0
47		ac261	:	29.45683	2.49E+06	2104201100	90
48		ac261	:	28.49501	2.88E+06	2115383900	0
49		ac261	:	28.49501	2.88E+06	2115383900	90
50		ac251	:	28.52813	5.21E+06	2114155850	0
51		ac251	:	28.52813	5.21E+06	2114155850	90
52		ac241	:	29.44333	1.94E+06	2104683400	0
53		ac241	:	29.44333	1.94E+06	2104683400	90
54		ac241	:	28.51324	1.97E+06	2114707500	0
55		ac241	:	28.51324	1.97E+06	2114707500	90
56		ac231	:	28.49994	2.17E+06	2115201200	0
57		ac231	:	28.49994	2.17E+06	2115201200	90
58		ac221	:	29.40634	6.11E+06	2106006800	0
59		ac221	:	29.40634	6.11E+06	2106006800	90
60		ac221	:	28.46269	6.66E+06	2116584500	0
61		ac221	:	28.46269	6.66E+06	2116584500	90
62		ac211	:	29.50152	2.61E+06	2102606800	0
63		ac211	:	29.50152	2.61E+06	2102606800	90
64		ac211	:	28.54974	3.10E+06	2113355300	0
65		ac211	:	28.54974	3.10E+06	2113355300	90

comments are denoted by '!' and are ignored; use it to 'disable' HOMs

HOM data that goes into BBU code

Exercises (beam-break up)

- By commenting out modes in `bbu_homs` sheet, determine the worst mode (the one with highest threshold). How does the threshold due to the single worst mode compares to the situation when all modes are present?
- Work with the worst offending mode (for faster computing speed). Slightly change the frequency of the mode and obtain dependence of threshold vs. the mode frequency. Plot the dependency. What is the ratio of max over min threshold that you found in this manner? What is the frequency difference between the two adjacent maxima?
- Add 'fake' 1000 m to the recirculation length (`---> bbu` sheet) and repeat the steps from 2). What is the ratio of max over min threshold in this case? What is the frequency difference between the two adjacent maxima? Try to explain the result.