Homework Problems 2

1. Show that for any two-by-two unimodular real matrix M ($\det(M)=1$), the condition that the eigenvalues of M remain on the unit circle is equivalent to

   $$\left(\frac{\mathrm{Tr}\, M}{2}\right)^2 < 1.$$

   Show the stability condition follows from this condition on M, applied to the single pass longitudinal transfer matrix. Note ρ_l is proportional to E_l .

   Compute the synchrotron phase advance per pass in the microtron as a function of ν and the synchronous phase φ_s.

2. Verify this table from the lectures, for constant K and ρ

	$K < 0$	$K = 0$	$K > 0$
$D_{p,0}(s)$	$\frac{1}{\lvert K\rvert\rho}\left(\cosh\left(\sqrt{\lvert K\rvert}s\right)-1\right)$	$\frac{s^2}{2\rho}$	$\frac{1}{K\rho}\left(1-\cos\left(\sqrt{K}s\right)\right)$
$D'_{p,0}(s)$	$\frac{1}{\sqrt{\lvert K\rvert}\rho}\sinh\left(\sqrt{\lvert K\rvert}s\right)$	$\frac{s}{\rho}$	$\frac{1}{\sqrt{K}\rho}\sin\left(\sqrt{K}s\right)$

3. Verify, using the polytron bender magnet geometry, that

$$\Delta\gamma = \nu \frac{f_c}{f_{RF}} \frac{1}{1-(p/2\pi)\sin(2\pi/p)}$$

This figure may be helpful