Introduction to Diffraction

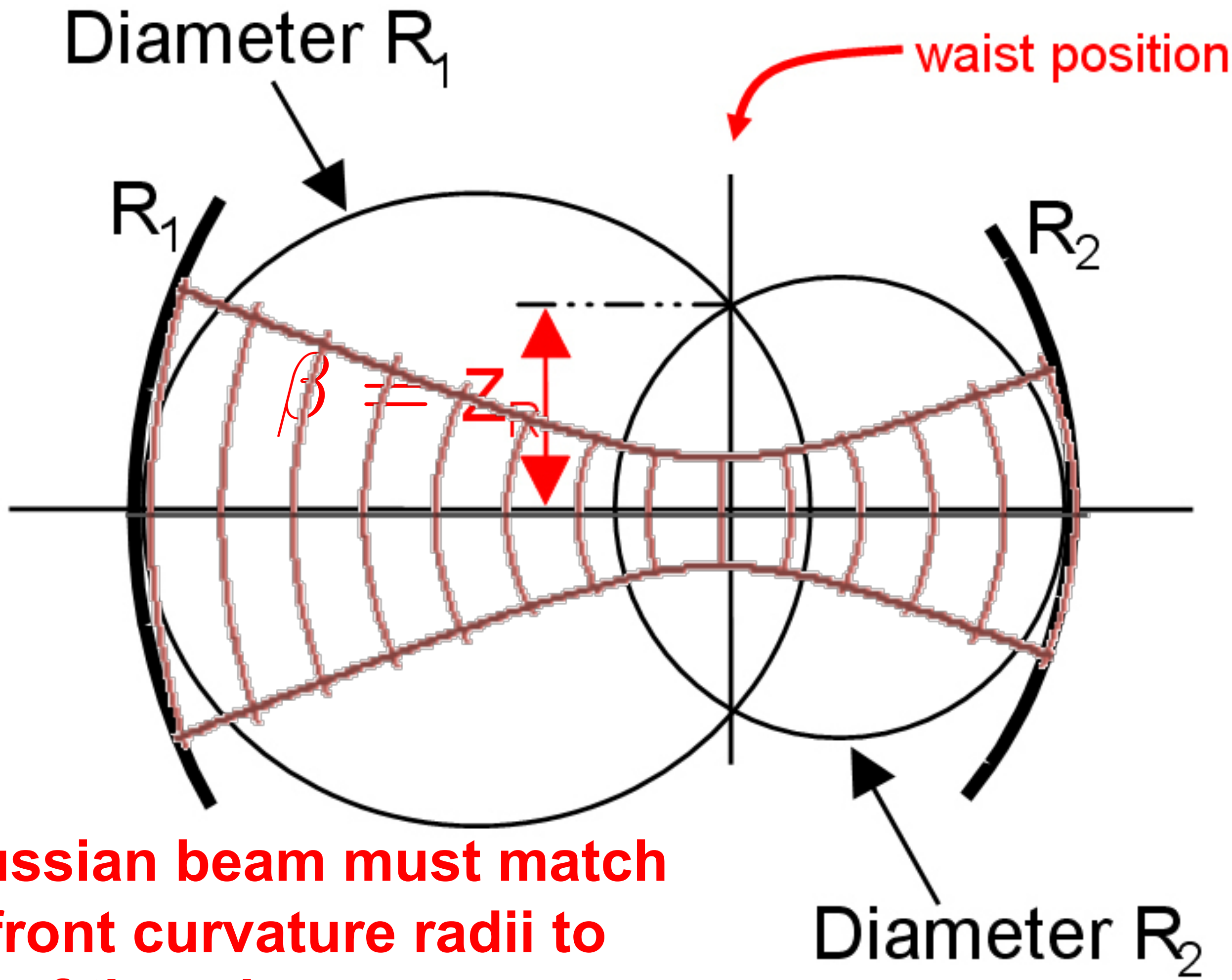
Ivan Bazarov

Cornell Physics Department / CLASSE

Outline

- **Gaussian beams in optical resonators (contd.)**
- **He-Ne laser spectrum (contd.)**
- **Introduction to diffraction**
- **Fresnel vs. Fraunhofer diffraction**

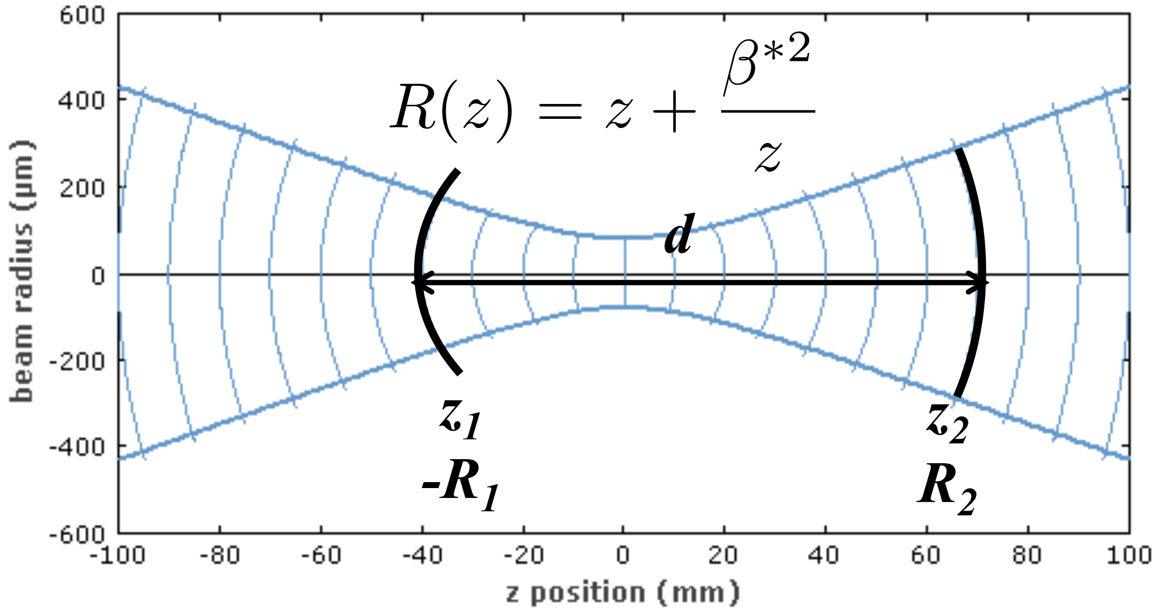
Two-mirror resonator



A Gaussian beam must match wavefront curvature radii to those of the mirrors

Gaussian mode in 2-mirror cavity

Recall Gaussian mode curvature:



$$d = z_2 - z_1$$

$$-R_1 = R(z_1) = z_1 + \frac{Z_R^2}{z_1}, \qquad R_2 = R(z_2) = z_2 + \frac{Z_R^2}{z_2},$$

Solution: Rayleigh range & waist position in the cavity

$$z_1 = \frac{d(d - R_2)}{R_1 + R_2 - 2d},$$

$$z_2 = \frac{d(R_1 - d)}{R_1 + R_2 - 2d},$$

$$Z_R = \frac{d(R_1 + R_2 - d)(R_1 - d)(R_2 - d)}{(R_1 + R_2 - 2d)^2},$$

Recall that the rms waist is given by: $\sigma_x = \sqrt{Z_R \epsilon_x} = \sqrt{Z_R \frac{\lambda}{4\pi}}$

Actual wave equation to solve

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \qquad \mathbf{E}(t, \mathbf{r})$$

Fourier transform

Or equivalently ***Helmholtz equation:***

$$k = \omega / c \qquad \nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0 \qquad \mathbf{E}(\omega, \mathbf{r})$$

This is usually solved using the ***paraxial approximation***

$$r = \sqrt{x^2 + y^2 + z^2} \approx z + \frac{x^2 + y^2}{2z}$$

Infinite number of solutions!

There are ***infinite solutions*** to the Helmholtz equation in free space.

E.g. Hermite-Gaussian TEM_{nm} modes (E-field, not intensity!)

$$E_{nm}(x,y,z) = E_0 \frac{w_0}{w(z)}$$

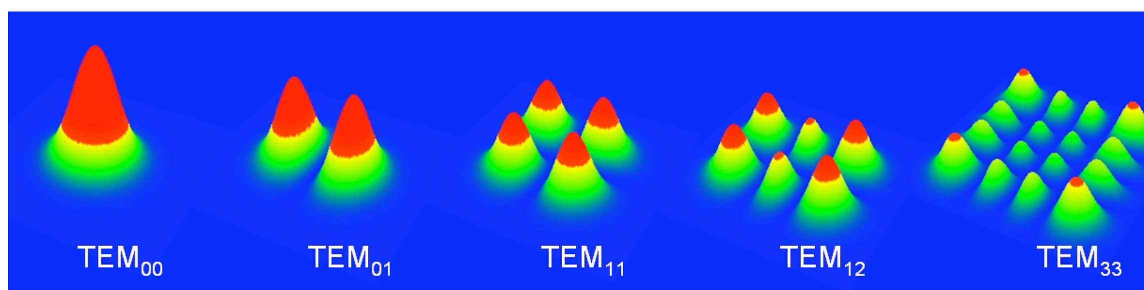
$$\cdot H_n\left(\sqrt{2}\frac{x}{w(z)}\right)\exp\left(-\frac{x^2}{w(z)^2}\right)\cdot H_m\left(\sqrt{2}\frac{y}{w(z)}\right)\exp\left(-\frac{y^2}{w(z)^2}\right)$$

$$\cdot \exp\left(-i\left[kz-(1+n+m)\arctan\frac{z}{z_R}+\frac{k\left(x^2+y^2\right)}{2R(z)}\right]\right)$$

$$w(z) = w_0\sqrt{1+(z/z_R)^2}$$

$$z_R = \frac{\pi w_0^2}{\lambda}$$

$$R(z) = z\left[1+(z_R/z)^2\right]$$

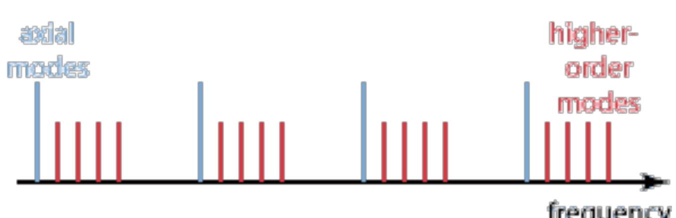


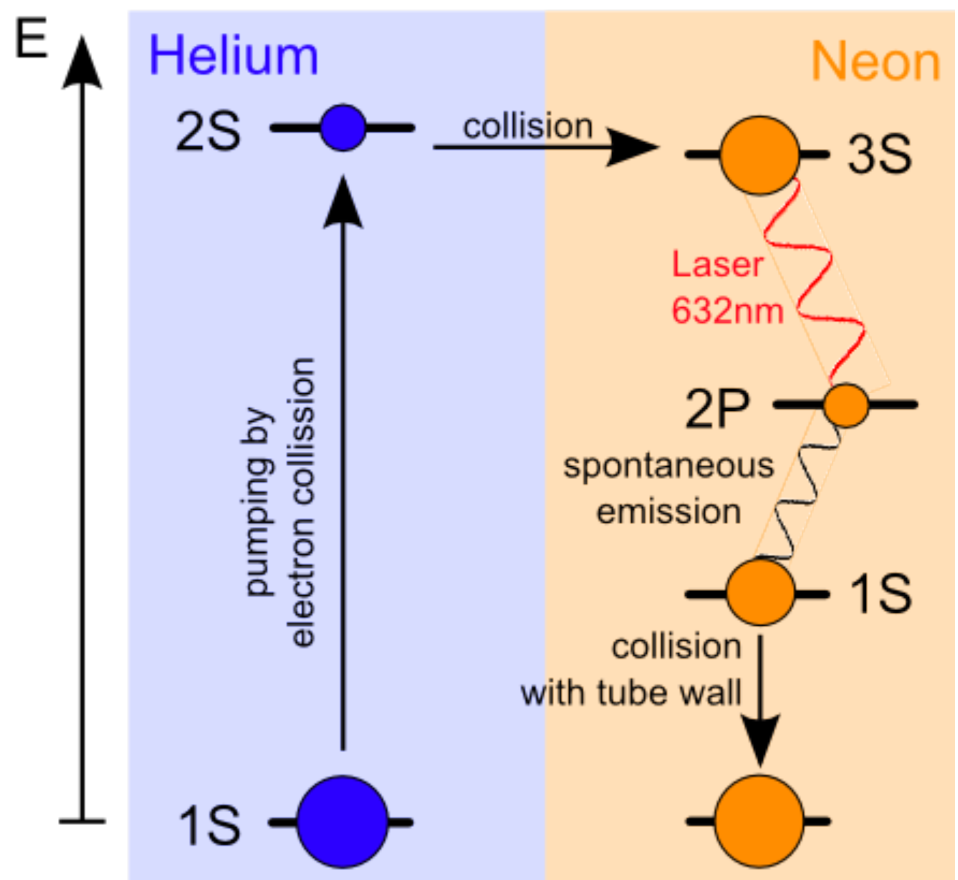
He-Ne laser energy levels

95% of laser power is in the TEM_{00} (Gaussian mode)

Width of the resonance (medium gain) ~1.5 GHz, i.e. only 2 axial modes are typically present

Can use Michelson's interferometer to "see" the individual axial modes

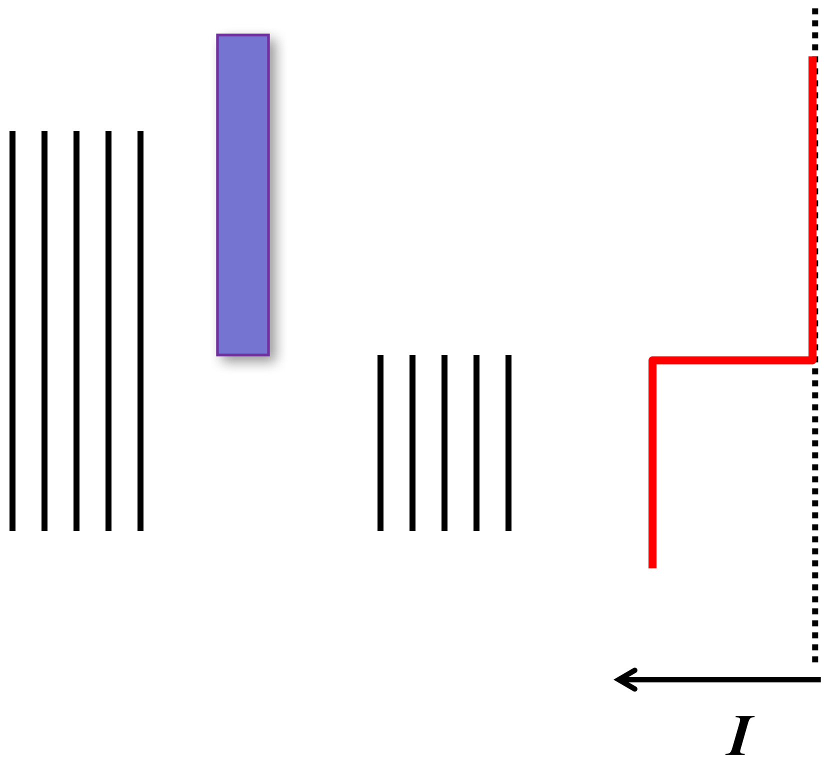




10:1 He-Ne mixture

Diffraction

Geometrical optics…

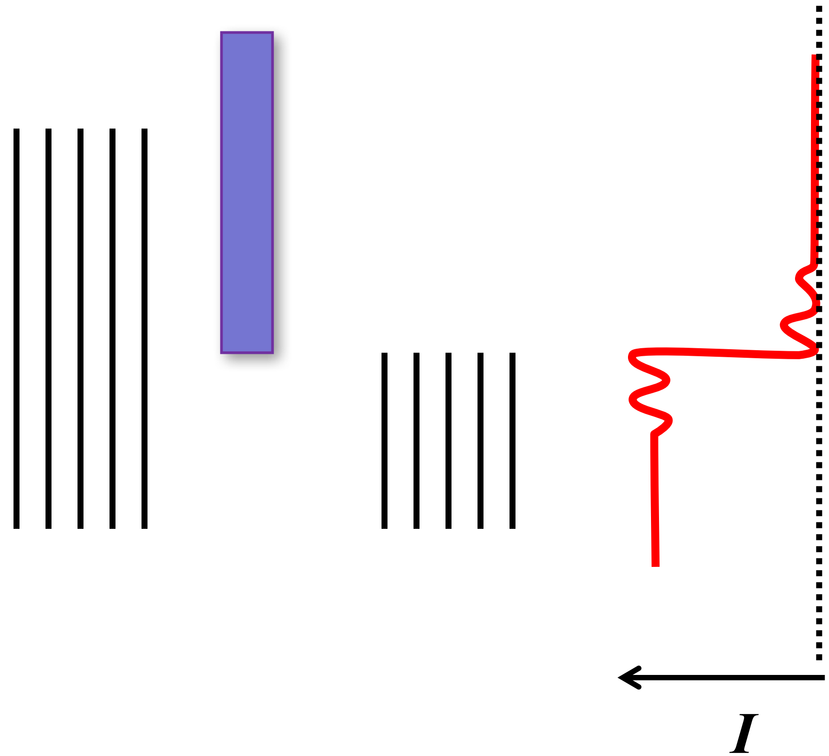


…light can't turn a corner.

Diffraction

Physical optics…

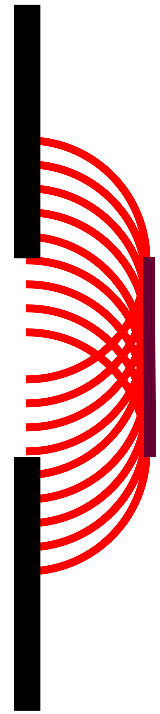
Francesco Maria Grimaldi
(1618 - 1663)



…actually, it can.

Hyugens-Fresnel principle

every point on a wavefront may be regarded as a secondary source of spherical wavelets



The propagated wave follows the periphery of the wavelets.

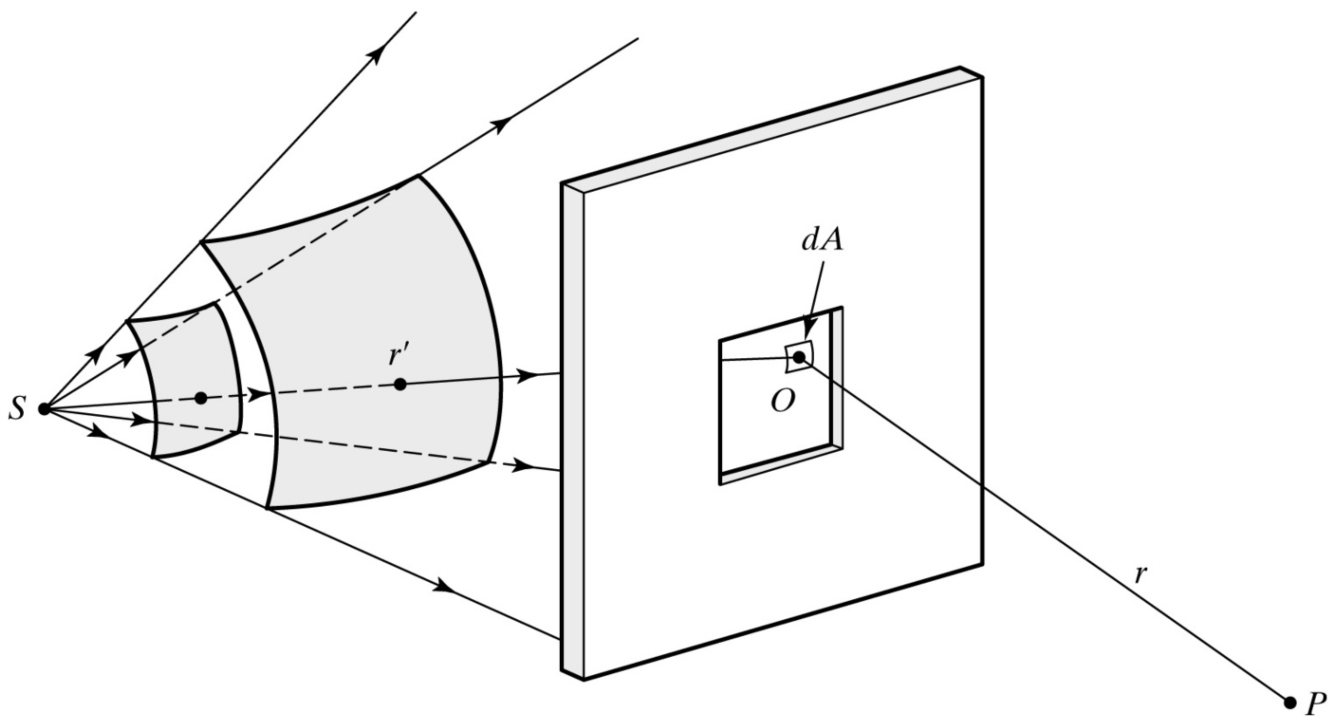
Christaan Huygens
(1629-1695)

Augustin Fresnel
(1788-1827)

Huygens, just add the wavelets considering interference!

Hyugens-Fresnel principle

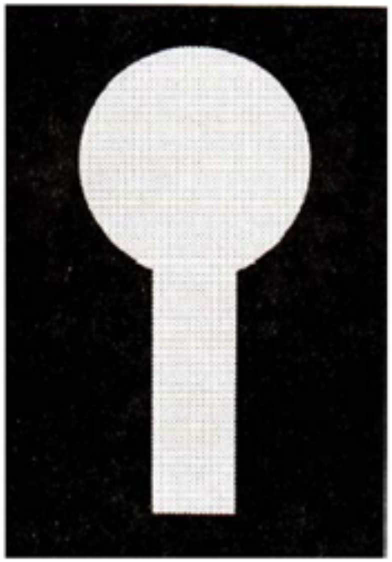
If one perturbs a plane wavefront, the Huygens wavelets will no longer constructively interfere at all points in space. Adding the wavelets by physical optics explains why light can turn corners and create fringes around images of objects.



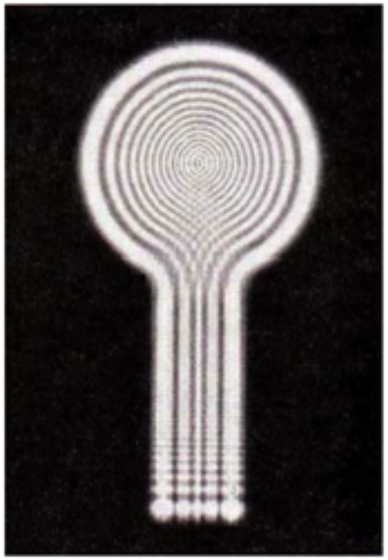
Diffraction: a generic wave phenomenon

Key-hole

Incoherent

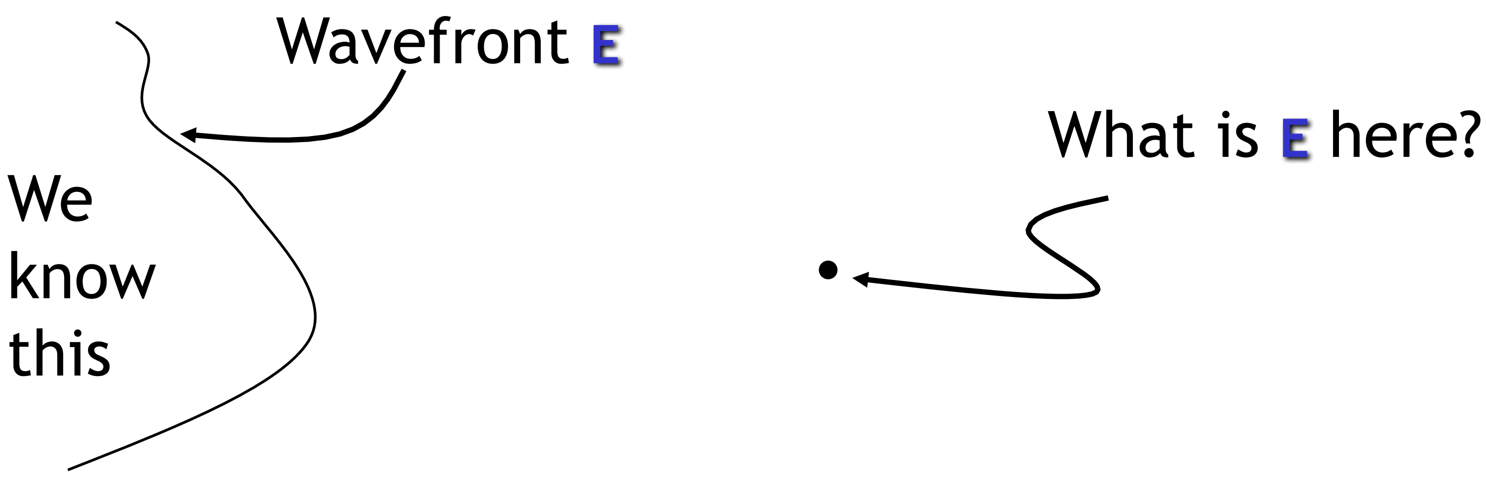


Coherent illumination



Sea waves

Diffraction theory

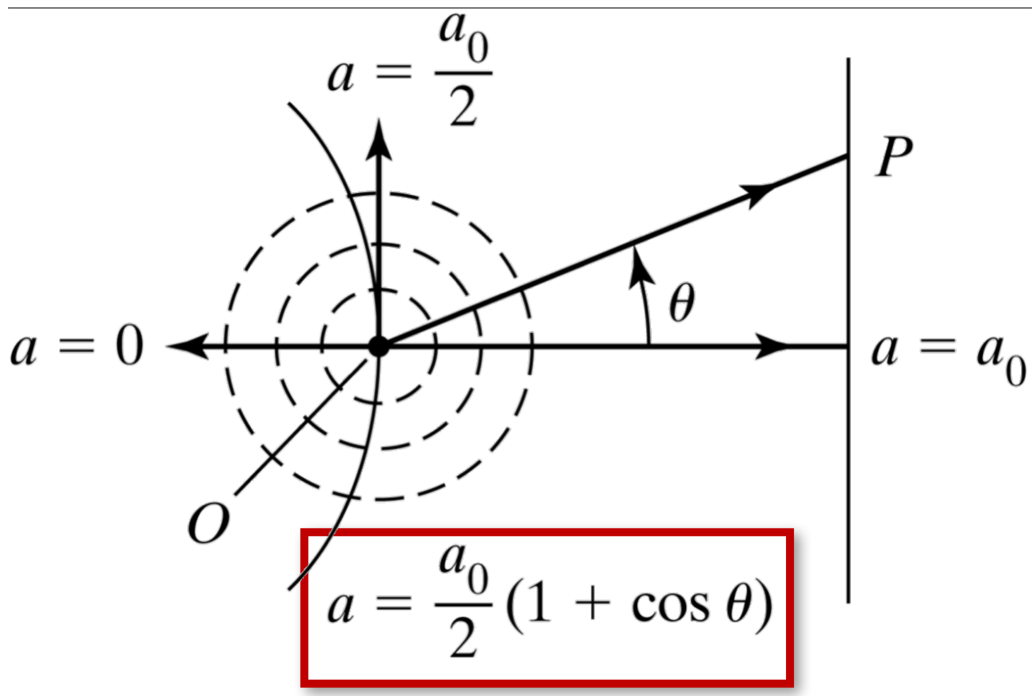


Note: the wavefront is considered to be *perfectly coherent* (i.e. fixed relationship between *E* phases for any 2 points)

Obliquity factor

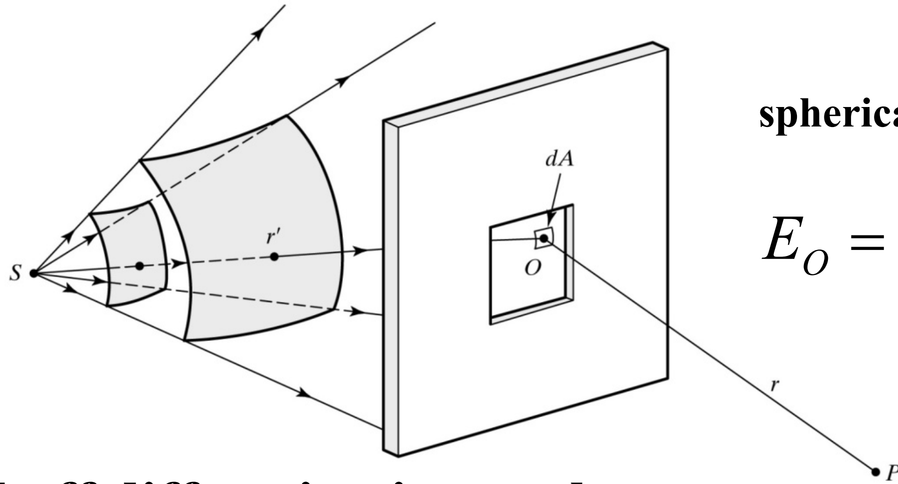
- wavelets propagate isotropically—in forward and ~~reverse~~ directions

- to use the Huygens approach, modify amplitude of wavefront as a function of θ:



$$a = \frac{a_0}{2}(1 + \cos\theta)$$

Calculating the diffracted wave amplitude



spherical waves:

$$E_O = \frac{E_s}{r'} e^{i(kr'-\omega t)}$$

Fresnel-Kirchhoff diffraction integral:

$$E_P = \frac{-ikE_S}{2\pi} e^{-i\omega t} \iint F(\theta) \frac{e^{ik(r+r')}}{rr'} dA$$

phase shift

$$-i = e^{-i\pi/2}$$

obliquity factor

$$F(\theta) = \frac{1+\cos\theta}{2}$$

great for evaluation by a computer

Fresnel vs. Fraunhofer

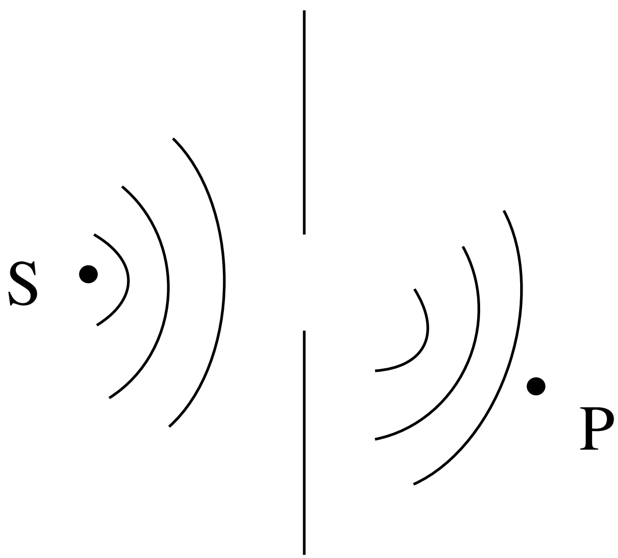
Augustin Fresnel
(1788-1827)

Joseph von Fraunhofer
(1787-1826)

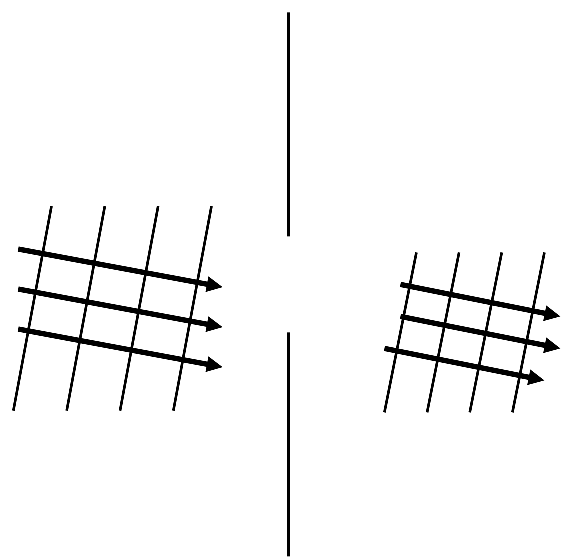
Contemporaries, but not collaborators (nor competitors).

Fresnel vs. Fraunhofer diffraction

- Fresnel regime is the near-field regime: the wave fronts are curved, and their mathematical description is more involved.



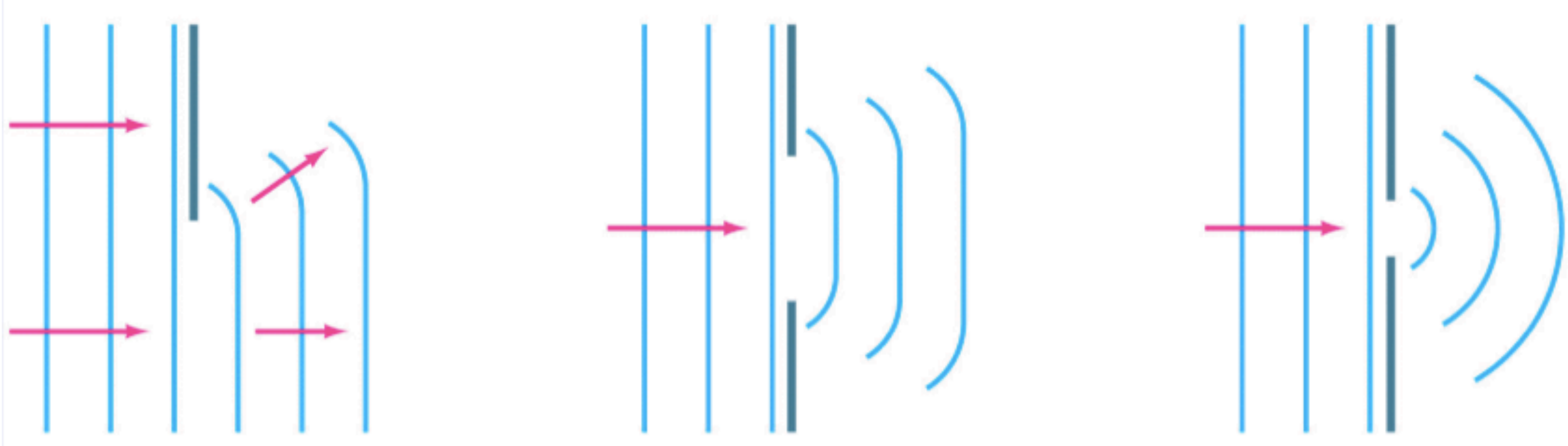
- Very far from a point source, wavefronts almost plane waves.
- Fraunhofer approximation valid when source, aperture, and detector are all very far apart (or when lenses are used to convert spherical waves into plane waves)



Aperture size matters

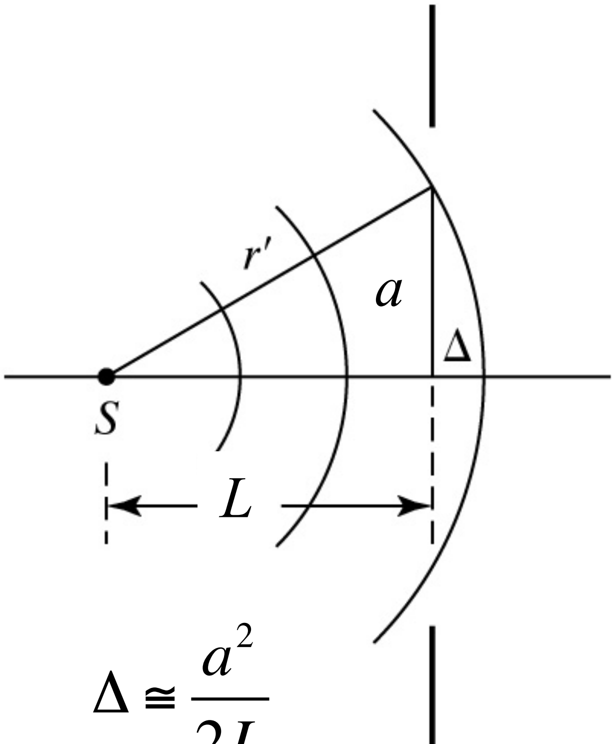
Fraunhofer-like (or "far field")

Fresnel-like (or "near field")



the amount of wavefront curvature decides whether to use Fresnel or the simplified Fraunhofer diffraction

Fresnel number definition



$$\Delta \cong \frac{a^2}{2L}$$

How many half-wavelength are contained in the curvature of the wavefront?

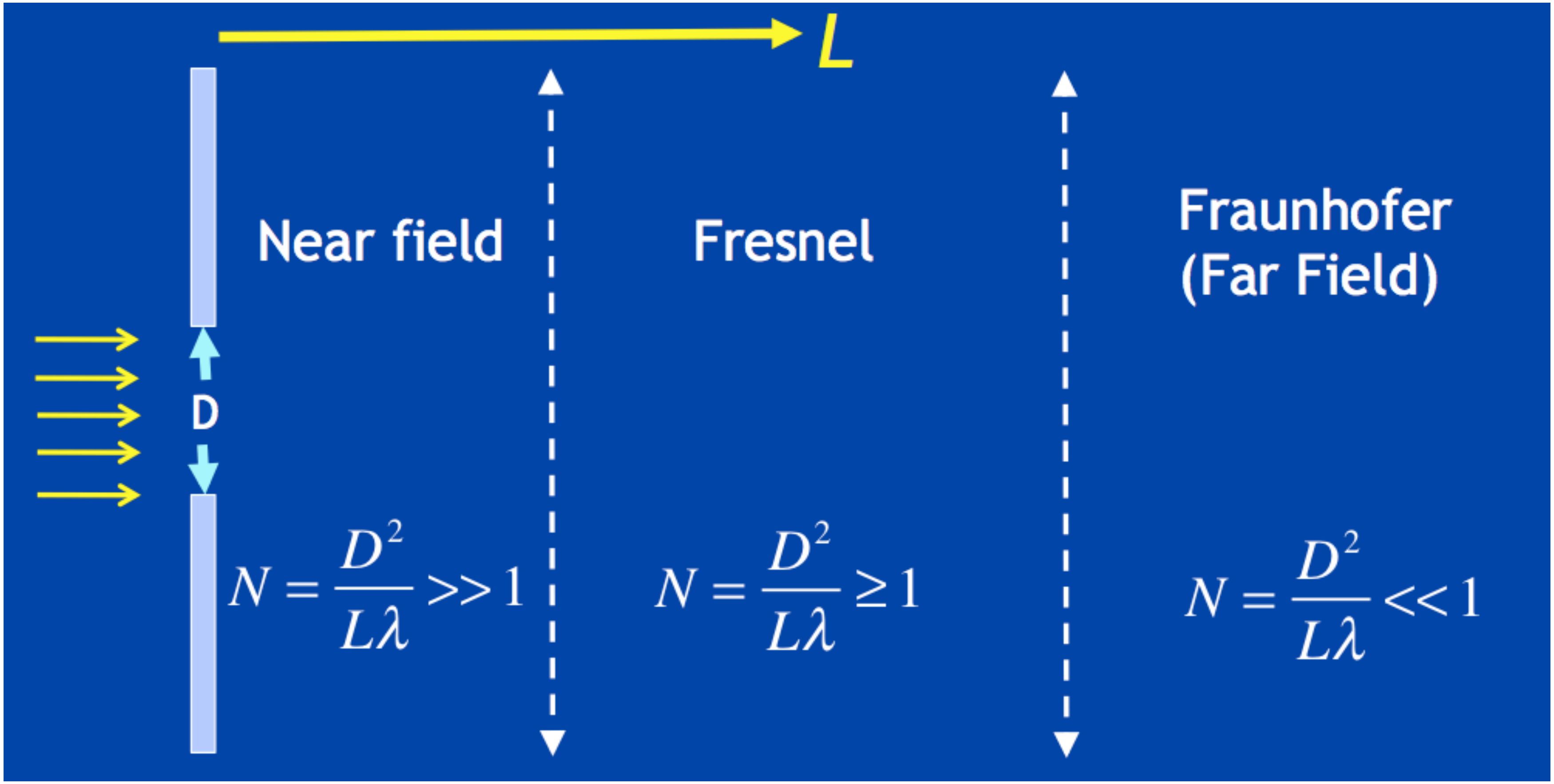
$$N = \frac{\Delta}{\lambda / 2} = \frac{a^2}{\lambda L}$$

aperture size (radius) → a

Fresnel number → N

screen-aperture distance → L

Regions of validity for diffraction calculations



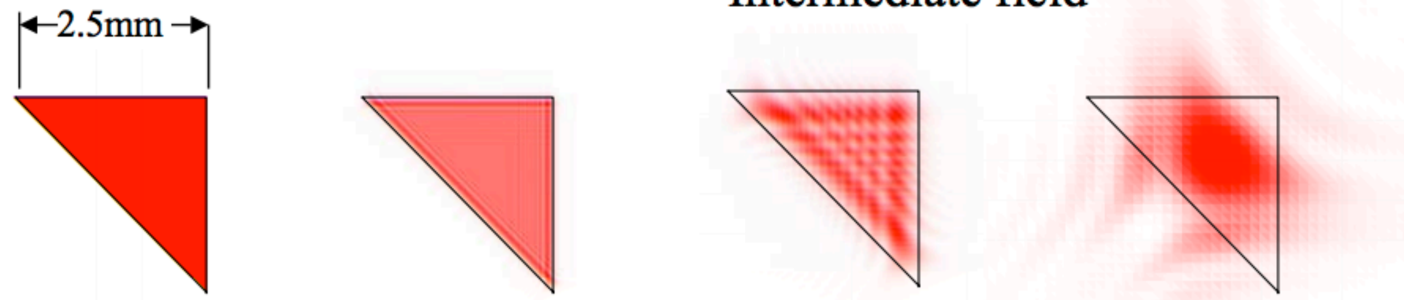
Math simplifies: E_p is simply a 2D FT of the E_s at the aperture!

An example

Pattern on screen at various distances

Near Field

Intermediate field

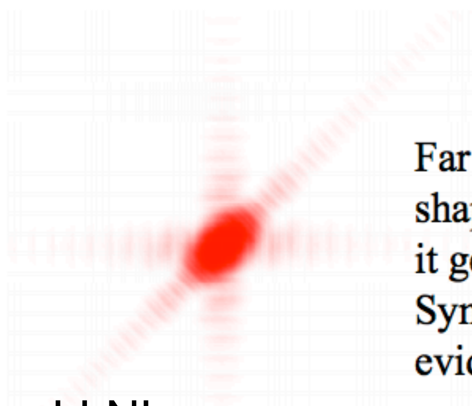


Immediately behind screen

25 mm from screen, bright fringes just inside edges

250 mm light penetrates into shadow region

2500 mm pattern doesn't closely resemble mask



Far field – at a large enough distance shape of pattern no longer changes but it gets bigger with larger distance. Symmetry of original mask still is evident.

Credit: Bill Molander, LLNL

Links/references

http://www.optique-ingenieur.org/en/courses/OPI_ang_M01_C03/co/Grain_OPI_ang_M01_C03.html

http://edu.tnw.utwente.nl/inlopt/overhead_sheets/Herek2010/week7/13.Fresnel%20diffraction.ppt

http://www.ucolick.org/~max/289/Lectures%20Final%20Version/Lecture%203%20Physical%20Optics/Lecture3%20Physical%20Optics_v2.ppt