Optical resonators

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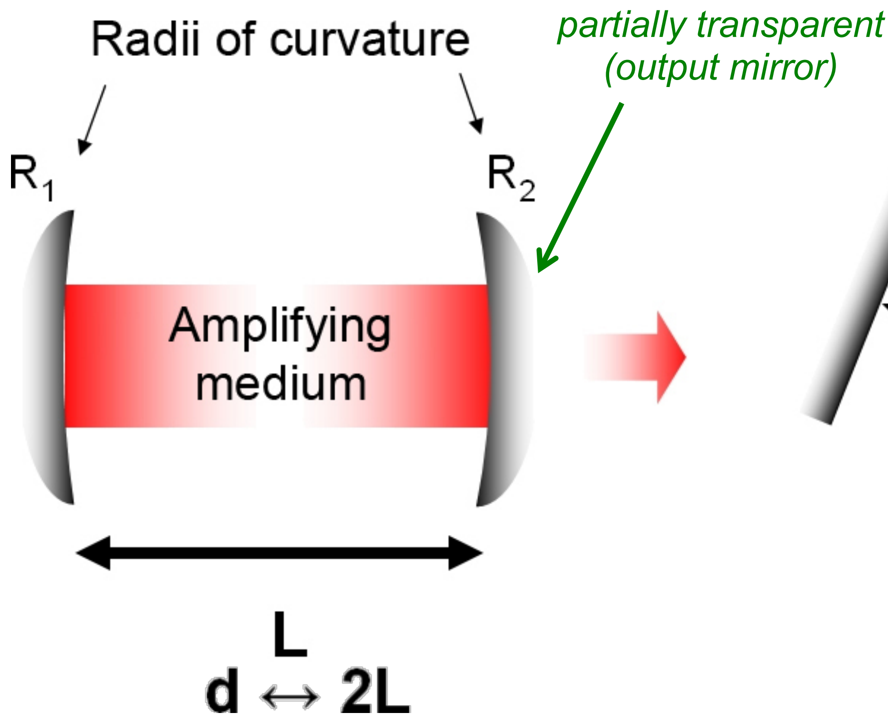
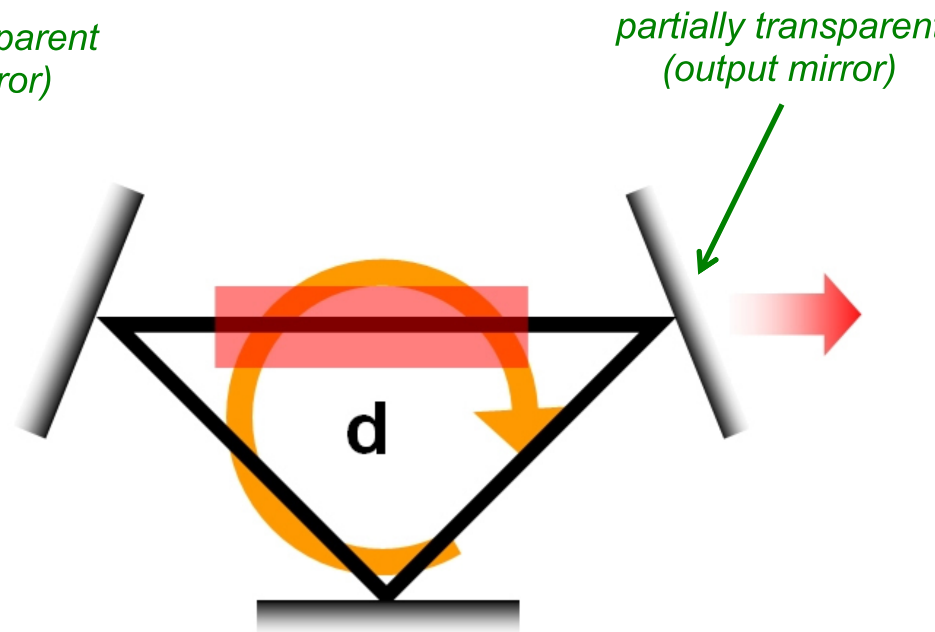
Outline

- **Optical resonator types**
- **Laser spectrum**
- **Stability criterion**

Resonator types



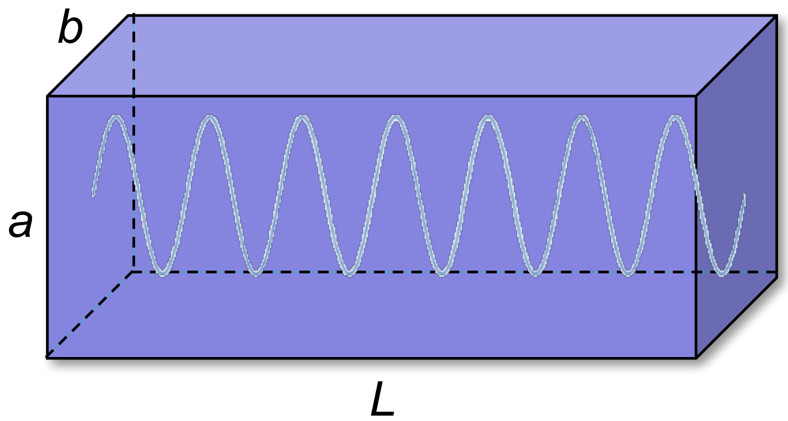


Parallel mirrors (Fabry-Perot) resonator: plane-wave output

Curved mirrors' resonator: Gaussian beam output

"Ring cavity"

3D box resonator modes TEM_{mnq}



$$\nu = \frac{c}{\lambda} = \frac{c}{2\pi}k = \frac{c}{2\pi}\sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$\nu_{mnq} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{q}{L}\right)^2}$$

if $(a, b) \gg L$ and $a \approx b$

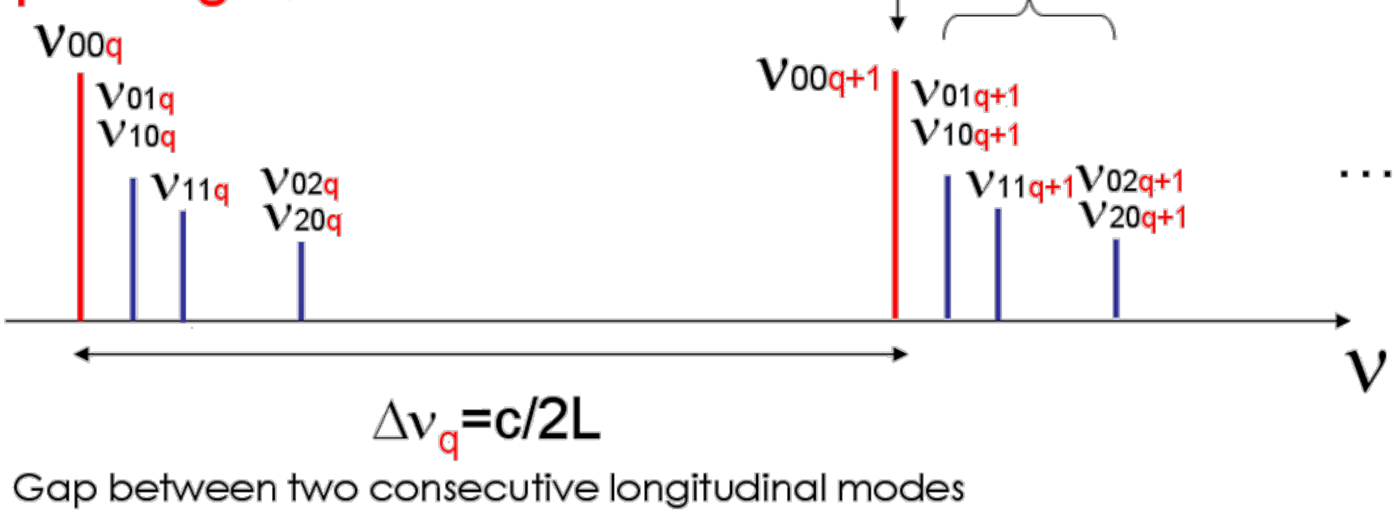
$$\nu_{mnq} \approx q\frac{c}{2L} + (m^2 + n^2)\frac{cL}{4qa^2}$$

Spectrum of a resonator

One mode ν_{mnq} = three parameters m, n, q

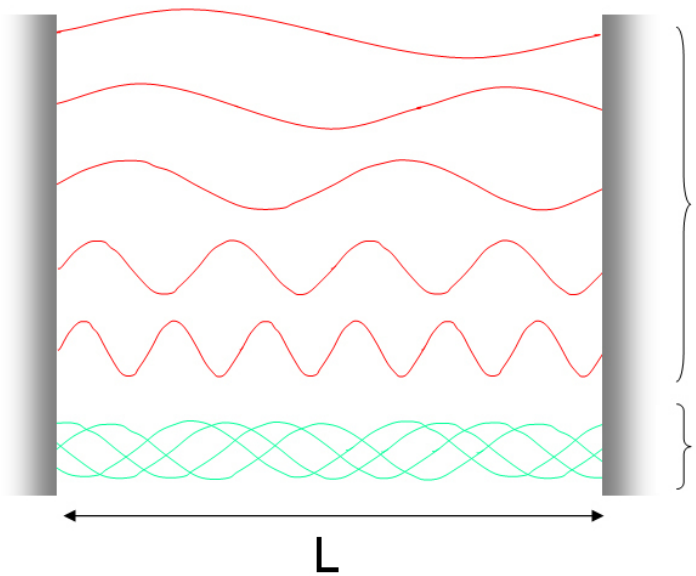
– m, n = transverses modes

– q = longitudinal modes



- E.g. spectral separation for L = 10 cm is $\Delta\nu_q = c/2L$ = 1.5 GHz
- TEM_{00} has the lowest frequency for any given q

Laser spectrum



Eigenmodes

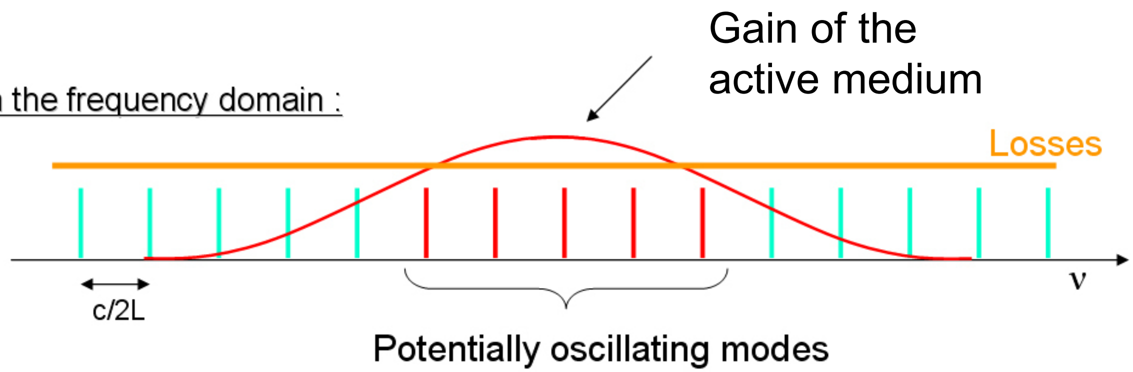
$$2L = q.\lambda$$

$\Leftrightarrow$

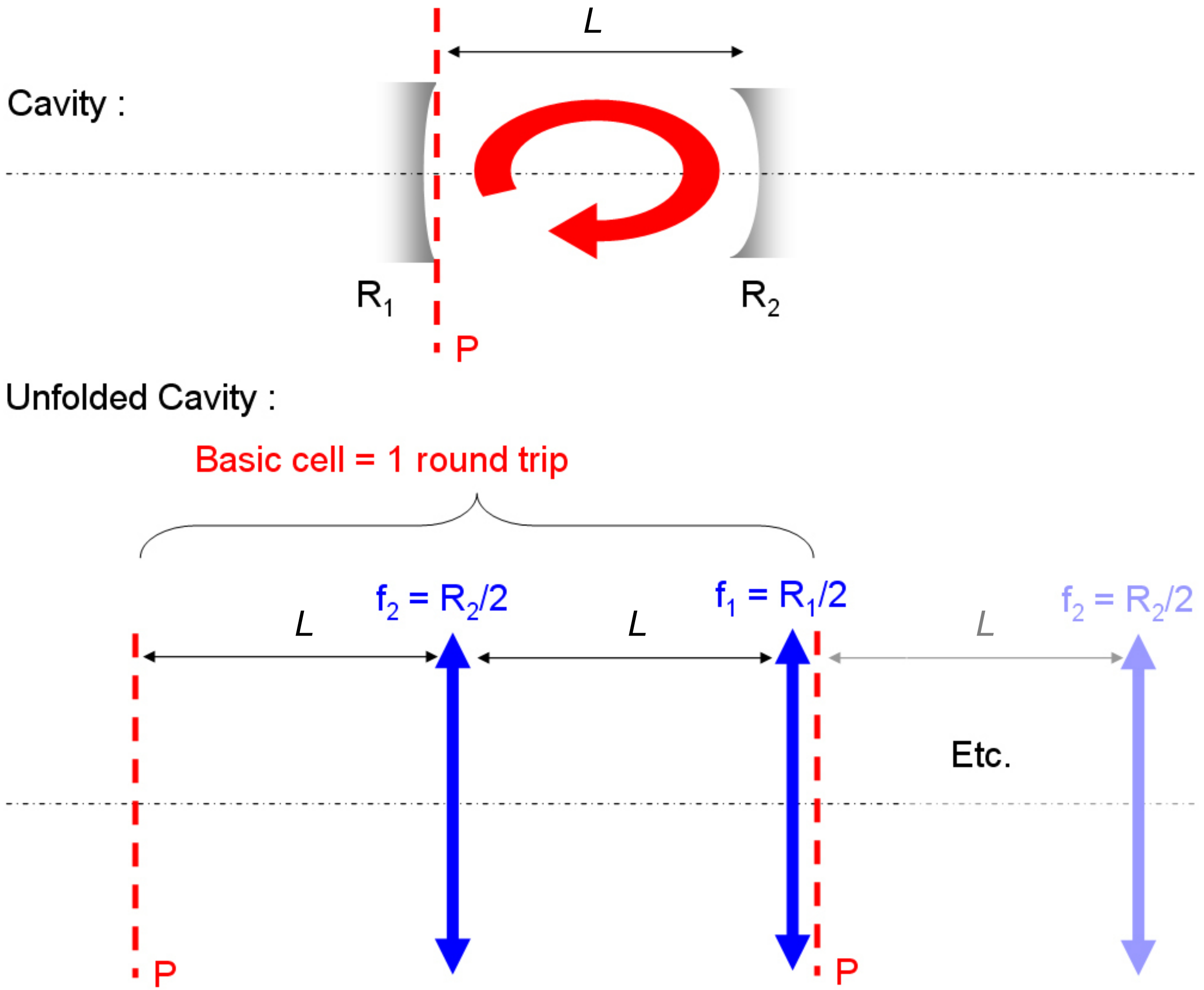
$$\nu_q = q.c/2L$$

After a few roundtrips, the intensity of every *non-resonant* mode goes to zero.

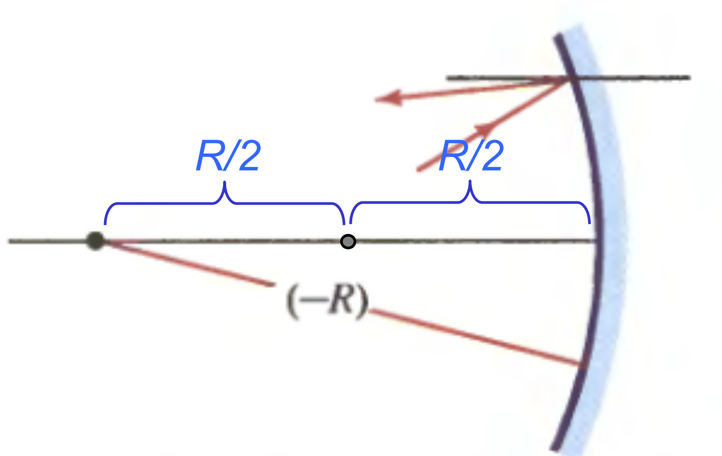
In the frequency domain :



Periodic structure of laser cavity



ABCD matrix of a curved mirror

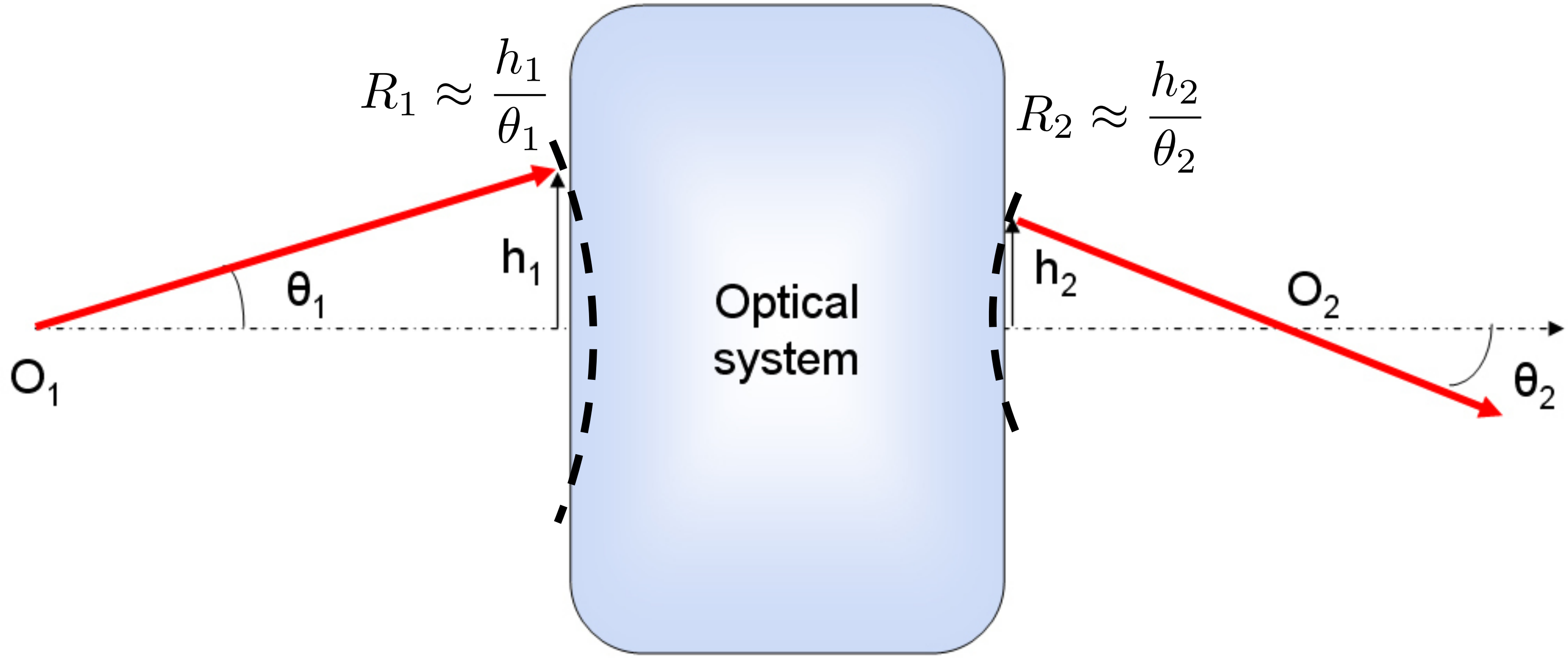


Concave: $R < 0$; convex: $R > 0$

focal length is at $R/2$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$$

ABCD law relating spherical wavefront radii



From:

$$\begin{bmatrix} h_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} h_1 \\ \theta_1 \end{bmatrix}$$

we get:

$$R_2 = \frac{AR_1 + B}{CR_1 + D}$$

ABCD law

Cascade of identical optical elements

$$\begin{bmatrix} y_m \\ \theta_m \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^m \begin{bmatrix} y_0 \\ \theta_0 \end{bmatrix}$$

$$y_{m+1} = Ay_m + B\theta_m$$

$$\theta_{m+1} = Cy_m + D\theta_m$$

Eliminate θ to get:

$$y_{m+2} = 2by_{m+1} - F^2 y_m$$

with

$$b = \frac{A+D}{2}$$

$$F^2 = AD - BC = \det[\mathbf{M}]$$

Solution

Substitute $y_m = y_0 h^m$ into $y_{m+2} = 2by_{m+1} - F^2 y_m$

To obtain:

$$h^2 - 2bh + F^2 = 0,$$

$$h = b \pm j\sqrt{F^2 - b^2}$$

Convenient notation

$$\varphi = \cos^{-1}(b/F)$$

so that

$$b = F\cos\varphi, \sqrt{F^2 - b^2} = F\sin\varphi,$$

and therefore

$$h = F(\cos\varphi \pm j\sin\varphi) = F\exp(\pm j\varphi)$$

Stability condition

$$F = \sqrt{\det[\mathbf{M}]} = 1$$

so, the periodic solution is $y_m = y_{\max} \sin(m\varphi + \varphi_0).$

This requires $\varphi = \cos^{-1} b$ to be real, or

$$|b| \leq 1 \qquad \text{or} \qquad \tfrac{1}{2}|A + D| \leq 1.$$

$$|\mathrm{Tr}[\mathbf{M}]| \leq 2$$

stability criterion

Can also rewrite it as $0 \leq \dfrac{A + D + 2}{4} \leq 1$

ABCD matrix for a 2-mirror optical oscillator

$$M=\begin{pmatrix}1 & 0\\ \frac{-1}{f_1} & 1\end{pmatrix}\begin{pmatrix}1 & d\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \frac{-1}{f_2} & 1\end{pmatrix}\begin{pmatrix}1 & d\\ 0 & 1\end{pmatrix}=\begin{pmatrix}1-\frac{d}{f_2} & d(2-\frac{d}{f_2})\\ \frac{-1}{f_1}-\frac{1}{f_2}+\frac{d}{f_1 f_2} & (1-\frac{d}{f_1})(1-\frac{d}{f_2})-\frac{d}{f_1}\end{pmatrix}$$

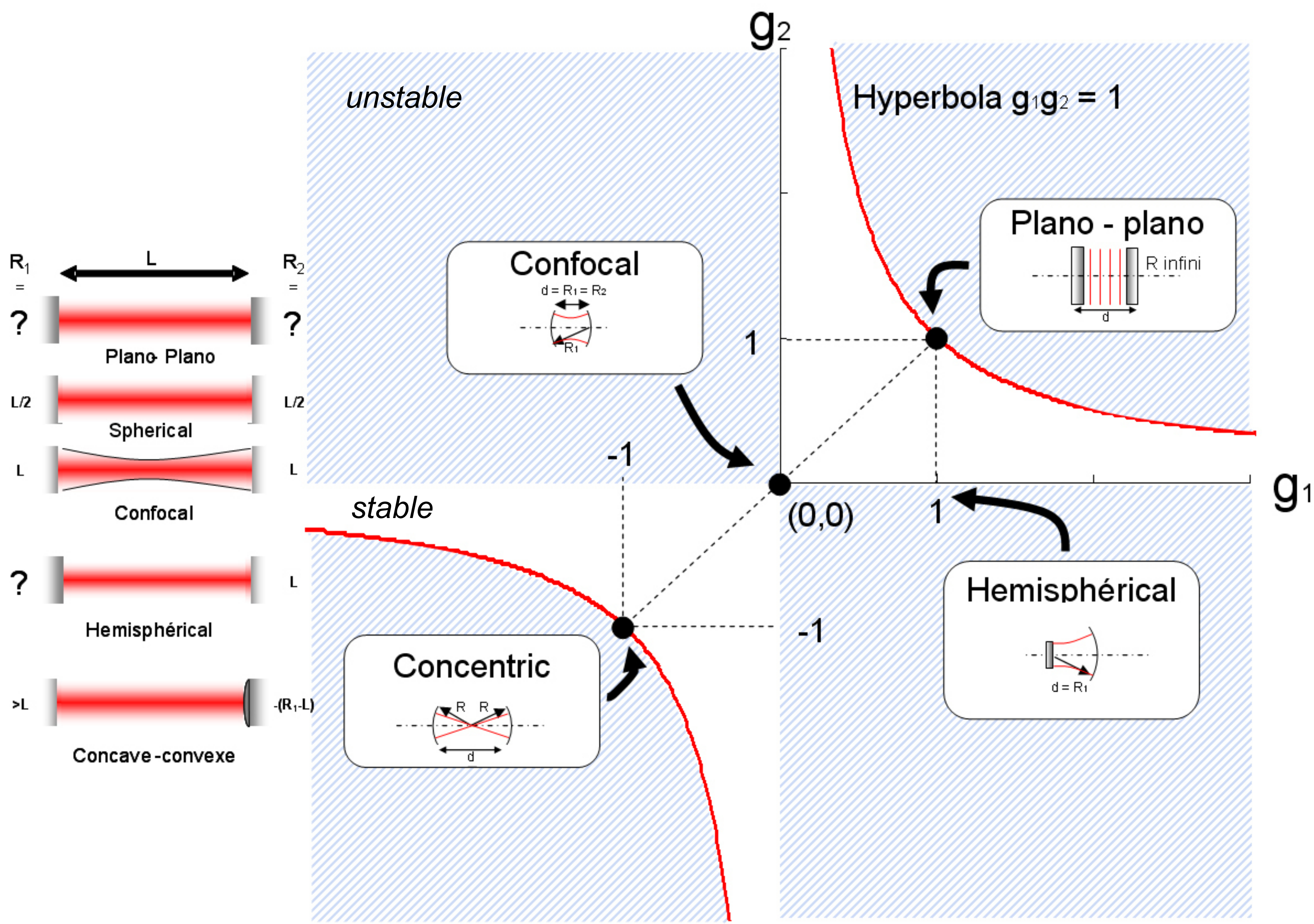
with $d = L$, $f_1 = R_1/2$, $f_2 = R_2/2$, one can show that

$$\frac{A+D+2}{4}=\underbrace{(1-\frac{L}{R_1})}_{g_1}\underbrace{(1-\frac{L}{R_2})}_{g_2}$$

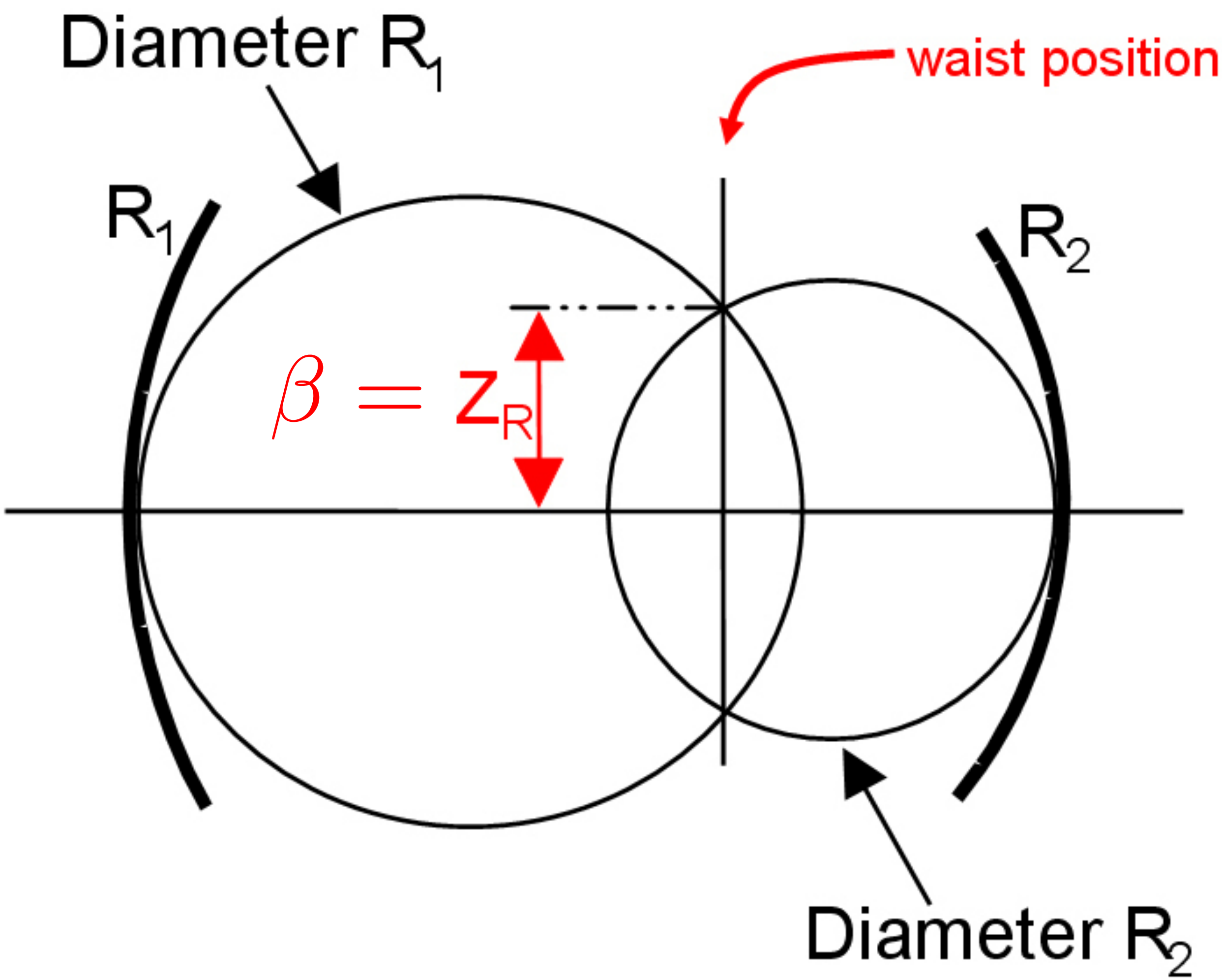
Thus, the stability criterion for an optical resonator is:

$$0 \le g_1 g_2 \le 1$$

Stability of 2-mirror resonator



Graphical trick to determine whether the resonator is stable or not



Links/References

http://www.optique-ingenieur.org/en/courses/OPI_ang_M01_C03/co/Grain_OPI_ang_M01_C03.html

Saleh & Teich, Chapter 1