Nuclei

① Nuclear structure

- atomic number Z
  neutron # N
  mass # $A = N + Z$

Z determines electronic structure → chemistry

Notation : $^{\textcircled{A}}_{\textcircled{Z}}X$ ← chemical element symbol

$\textcircled{A} \rightarrow \bigcirc$ sometimes here

sometimes drop → $\textcircled{Z}$

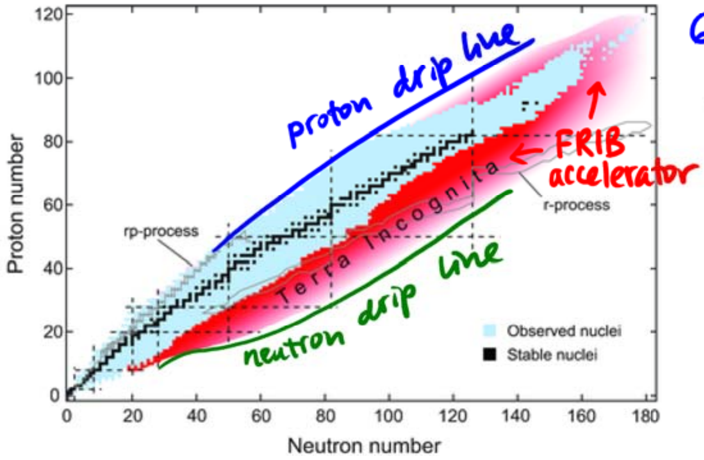
- isotopes : same Z (different N)

e.g. $^{12}_{6}C$ 98.9% } natural abundance
$^{13}_{6}C$ 1.1% }
$^{14}_{6}C$ unstable 5,730 yr half-life

isobars : same A
^{12}B, ^{12}C

isotones : same N (replace p with n in 'isotope')
^{12}B, ^{13}C

isomer : excited nucleus of the same type
(e.g. prior to γ emission) $^{99m}Tc^{(*)}$ "technetium" 6hr half-life } 140keV x-ray (medical app)



② Nuclear stability

due to nuclear force !

$Z \approx N$, for $Z \lesssim 20$

$N \gtrsim Z$, larger Z

"shell" structure Z or N :
2, 8, 20, 28, 50, 82, 126
" magic numbers "

③ Nucleons

* neutron (n) charge : 0

mass : 1.008 u ($\frac{1}{12}$ of ^{12}C atomic mass)

* proton (p) charge : +e

mass : 1.007 u

Rest energy $E = mc^2$, $1u = 931.5\ MeV/c^2$

* Both p and n size : ~1fm (1fm = 10^{-15} m , "fermi")

Q: can a nucleus contain e^-?

(Rutherford: there maybe (A−Z) neutral proton-electron pairs in nuclei)

No. confinement to 10^{-14} m gives too high KE , > 10 MeV

measured β-decay (= e^-) ~ 1MeV

No. magnetic moment of e^- ≫ nuclear moment

$$\mu_n = \frac{e\hbar}{2m_p}$$

"nuclear magneton"

~2000 smaller than μ_B

* nuclear spin I : same for p & n , $I = \frac{1}{2}$

* magnetic moment :

(p) $2.79\,\mu_n$

(n) $-1.91\,\mu_n$

$$\mu_I = \underset{\substack{\uparrow\\ \text{spin projection}\\ \text{quantum number}\\ -I \le m_I \le I}}{\overset{\substack{\text{nuclear g-factor}\\ \downarrow}}{g_I}\, m_I}\, \mu_n$$

* electric dipole moment ($d = q\ell$)

Standard Model prediction : $|d_n| \sim |d_p| \sim 10^{-32}$ e.cm

SUSY prediction : $|d_n| \sim 10^{-28}$ to 10^{-25} e.cm

best experiment so far: $|d_n| < 10^{-26}$ e.cm

Effort to measure $|d_p|$ to $< 10^{-28}$ e.cm level

(proton electrostatic ring accelerator)

④ Nuclei size

Initial data from scattering

closest approach : $\frac{1}{2} m v_\alpha^2 = \frac{(2e)(Ze)}{4\pi\varepsilon_0 \, r}$

Generally:

$r \simeq r_0 A^{1/3}$, with $r_0 \sim 1.2 - 1.5$ fm

compare to atoms, which roughly have same size

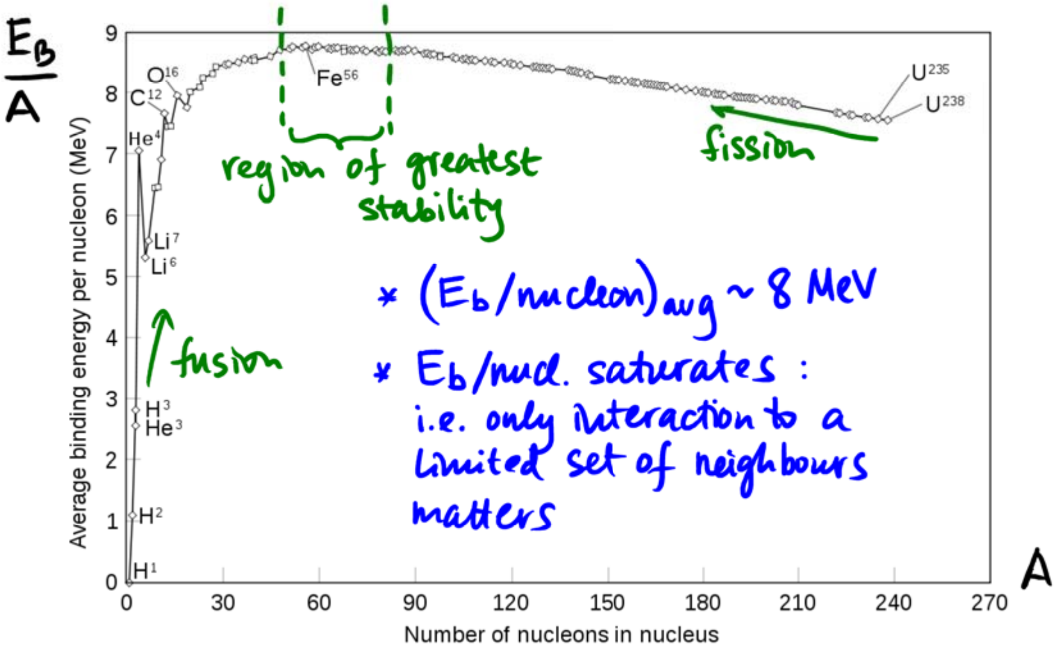
=> different force law

⑤ Binding energy

= mass deficit

$$E_b = (Z M(H) + N m_n - M_A) c^2 = \Delta m c^2$$

mass of a nucleus is always less than the sum individual p's and n's that it consists of.



* $(E_b/\text{nucleon})_{avg} \sim 8$ MeV
* E_b/nucl. saturates : i.e. only interaction to a limited set of neighbours matters

⑥ Nuclear force

- attractive
- short range (~fm)
- depends on spin
- does not depend on charge

} from scattering experiments

- actual force is a sum of Coulomb repulsion (p-p) and nuclear force

E.g. Continuous Electron Beam Accelerator Facility (CEBAF): e^- probe @ 6 GeV smashing nuclei

$$\lambda_{\text{deBroglie}} = \frac{hc}{pc} = \frac{1.24 \text{ keV-nm}}{6 \text{ GeV}} \sim 0.2 \text{ fm}$$

now being upgraded to 12 GeV, $\lambda \sim 0.1$ fm

can see quark structure of nucleons

- nuclear force is mediated by exchange of virtual particles (more later)

Q: where does the energy come from to create virtual particles?

A: nowhere, uncertainty principle allows for large fluctuations of energy if $\Delta t \to 0$

$$\Delta t \lesssim \frac{\hbar}{mc^2}$$

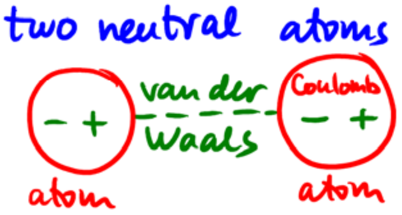
virtual particle mass

$$\text{range} = \Delta t\, c \lesssim \frac{\hbar}{mc} \simeq 2 \text{ fm}, \Rightarrow mc^2 \sim 100 \text{ MeV}$$

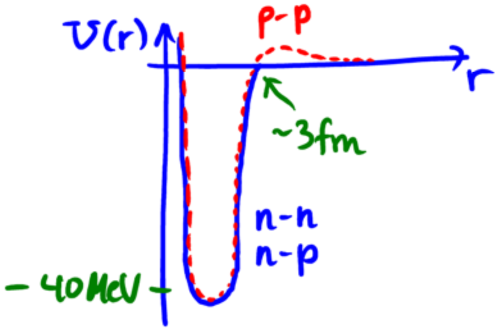
π meson, Yukawa
(pion)

- nuclear force between nucleons: "residual"

similar to Van der Waals force between two neutral atoms



Ex. nuclear potential



- * minimum in potential ~ 40 to 50 MeV
- * drops to zero @ ~3fm
- * strongly repulsive $r < 0.4$fm
- * p-p positive potential barrier ~1 MeV @ 4fm

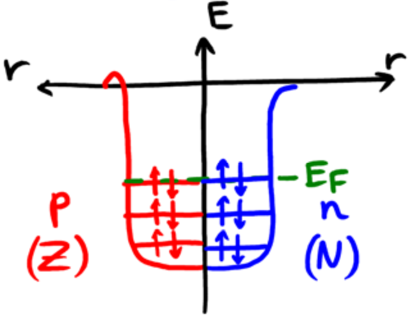
⑦ Nuclear models

- * True theory : must refer to quarks, force b/w them = "strong interaction"
- * Quantum $\underbrace{\text{Chromo}}_{\text{color; gluons carry "color"}}$dynamics => all nuclear structure can be explained in principle

Here, phenomenological models only

a) Fermi gas model

independant fermions moving within net binding potential



low Z

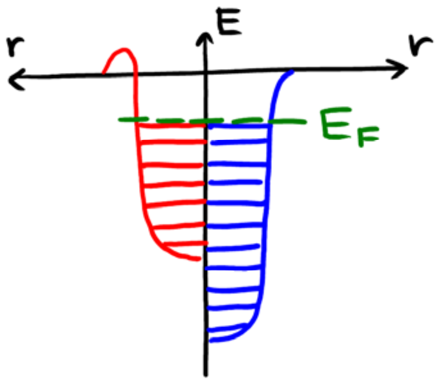
potential for p same as for n } same E_F

Q : What if E_F is different for p's and n's ?

$\underbrace{(m_n - m_p)}_{\text{if free}} c^2 = 1.3$ MeV (small)

A : In this case $(N \neq Z)$, $p \leftrightarrow n$ to level $(E_F)_{n,p}$ out.

Result : $\boxed{N \approx Z}$ explains low Z data



high Z

potential for p higher
(Coulomb repulsion)

$\boxed{N > Z}$

Rough estimate of E_F } → $E_F = \frac{\hbar^2}{2m_n}\left(3\pi^2 \rho_n\right)^{2/3}$ (lecture 12)

$\rho_n = \frac{N}{\frac{4}{3}\pi r^3}$, using $r = \underset{\nwarrow 1.2\,\text{fm}}{r_0} A^{1/3}$

and $N \sim 0.6A$ (neutron rich),

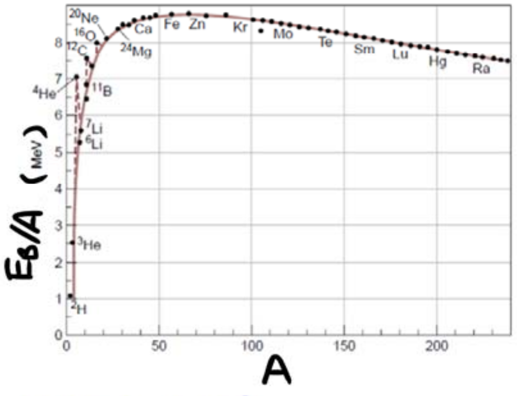
$\Rightarrow \rho_n = 0.083\ \text{fm}^{-3}$ (1n per 12 fm^3)

$\Rightarrow E_F \sim 40\ \text{MeV}$ (from lowest state)

b) Liquid drop model (phenomenological)

$$E_b = \underbrace{\underset{\text{volume term (nuclear force saturation)}}{C_1 A} - \underset{\text{surface term (c.f. surface tension)}}{C_2 A^{2/3}}}_{\text{drop of liquid analogy}} - \underset{\text{Coulomb repulsion for p's}}{C_3 \frac{Z(Z-1)}{A^{1/3}}} - \underset{\text{symmetry of p-n favoured}}{C_4 \frac{(N-Z)^2}{A}}$$

E_b in MeV : $C_1 = 15.7$, $C_2 = 17.8$, $C_3 = 0.71$, $C_4 = 23.6$



← good fit
No explanation for magic numbers

(next lecture)
More models : c) shell model ; d) collective model