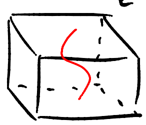
Recap

occupancy: $f(E) = \dfrac{1}{e^{(E-\mu)/kT} \pm 1}$

choose "+" for identical fermions
"−" for — " — bosons
$\mu = \mu(T)$ − chemical potential (N = const)
$\mu = 0$ for photons ($N \neq$ const)

simple system: 3D box with particles



"bose gas" : black-body radiation
"fermi gas" : free electrons in metals

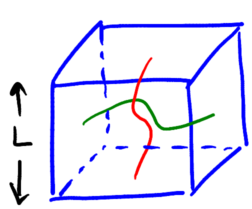
Electron Gas Model of Metals

General properties of metals:

- shiny! Absorb and reradiate visible light
  suggests many e^-'s levels (not discrete lines)
- good conductors of electricity
  electrons free to move around
- good conductors of heat
  charge carriers $\stackrel{?}{=}$ heat exchangers ?

Metal solids

"1st order" : Free electrons in an infinite depth cubic well
valence electrons only!



k_x, k_y, k_z
$+ m_s$ (spin ↑↓)
characterize e^- state

$$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} \quad ; \quad \frac{n\lambda}{2} = L \Rightarrow \frac{n_x \pi}{L} = k_x$$

in 3D : $|k| = \frac{\pi}{L}\sqrt{n_x^2 + n_y^2 + n_z^2}$

Main goal : For large N_{e^-}, want to know

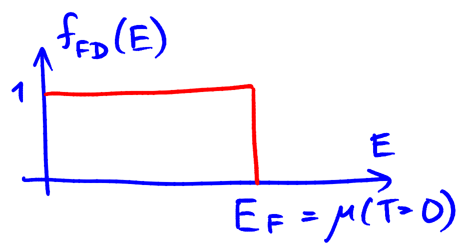
$$n(E)\,dE = \#\text{ of } e^-\text{'s with } E < \text{energy} < E + dE$$

$T \neq 0$ will promote some e^-'s above max $T=0$ energy.
Must use FD distribution !

$$f_{FD}(E) = \frac{1}{e^{(E-\mu)/kT}+1} = \text{prob. to find an electron with energy } E$$

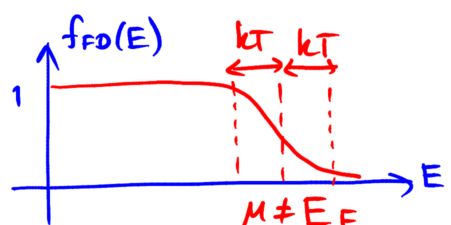
Recall:

ⓐ $T=0 \rightarrow f_{FD}(E) = \begin{cases} 0, & E > \mu(T=0) \equiv E_F \\ 1, & E < E_F \end{cases}$



E_F : Fermi energy
max energy occupied @ $T=0$

ⓑ $T>0$



$\mu \neq E_F$
(but usually close) HW

$$n(E)\,dE = f_{FD}(E) \times \underbrace{g(E)\,dE}_{\#\text{ states with } E<\text{energy}<E+dE}$$

$g(E) =$ "density of states"

"Hotel analogy"

$$\underbrace{n(E)\,dE}_{\substack{\#\text{ guests} \\ E - \text{price}}} = \underbrace{f_{FD}(E)}_{\substack{\text{guest's willingness} \\ \text{pay the price}}} \times \overbrace{g(E)\,dE}^{\#\text{ rooms @ the price}}$$

Here: $g(E) = 2 \times$ (# of wave vectors that give $E < \frac{\hbar^2 k^2}{2m} < E + dE$)

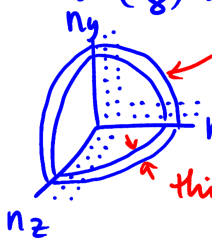
spin

$$\sqrt{\frac{2mE}{\hbar^2}} < k < \sqrt{\frac{2m(E+dE)}{\hbar^2}} \simeq \sqrt{\frac{2mE}{\hbar^2}}\left(1 + \frac{1}{2}\frac{dE}{E}\right)$$

since $k = \frac{\pi}{L}\sqrt{n_x^2 + n_y^2 + n_z^2}$

$$\Rightarrow \frac{L}{\pi}\sqrt{\frac{2mE}{\hbar^2}} < \sqrt{n_x^2 + n_y^2 + n_z^2} < \left(\frac{L}{\pi}\sqrt{\frac{2mE}{\hbar^2}} + \frac{L}{2\pi}\frac{dE}{E}\sqrt{\frac{2mE}{\hbar^2}}\right)$$

Part $\left(\frac{1}{8}\right)$ of a spherical shell in n-space (k-space really)



radius $n = \sqrt{n_x^2 + n_y^2 + n_z^2} = \frac{L}{\pi}\sqrt{\frac{2mE}{\hbar^2}} \equiv r_n$

thickness $\frac{L}{2\pi}\frac{dE}{E}\sqrt{\frac{2mE}{\hbar^2}} \equiv \Delta r_n$

IF L is large (macroscopic) $\Rightarrow$ approximate states in k-space as continuous

$\Rightarrow$ # states in the shell $\approx$ volume $\left(\frac{\text{states}}{\text{volume}}\right)$

$$= \left(\frac{4\pi r_n^2 \Delta r_n}{8}\right)\left(\frac{1 \text{ state}}{\text{unit cube}}\right)$$

$$= g(E)\,dE = \left(\frac{L}{\pi}\right)^3 \cdot \frac{1}{8} \cdot 4\pi \cdot 2\left(\frac{2mE}{\hbar^2}\right)^{3/2} \cdot \frac{1}{2} \cdot \frac{dE}{E}$$

$$= \frac{(2m)^{3/2} E^{1/2} V}{2\pi^2 \hbar^3}, \quad \text{where } V = L^3$$

Now can find E_F: $N = \int_0^\infty n(E)\,dE = \int_0^\infty f_{FD}(E)\,g(E)\,dE$

@ $T=0$ ($f_{FD} = 0, 1$) $\Rightarrow$

$$N = \int_0^{E_F} g(E)\,dE = \frac{V(2m)^{3/2}}{2\pi^2\hbar^3} \underbrace{\int_0^{E_F} \sqrt{E}\,dE}_{\frac{2}{3}E_F^{3/2}}$$

$$\Rightarrow \boxed{E_F = \frac{2\pi^2\hbar^2}{m}\left(\frac{3N}{8\pi V}\right)^{2/3}}$$

only depends on e^- concentration N/V!

* Typical metal : Cu

$N/V = 8.5 \times 10^{28}\ m^{-3} \sim 2\,\text{Å}$ sep

$\Rightarrow E_F = 7\,eV$ (c.f. $kT = \frac{1}{40}eV \ll E_F$!!!)

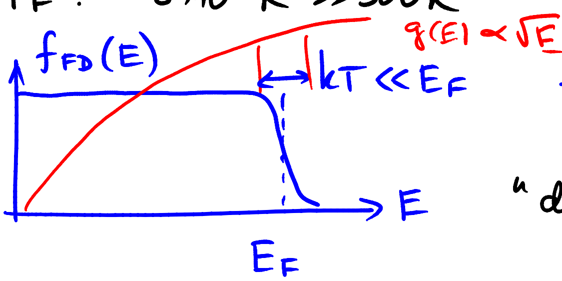
Can also define :

v_F : $E_F = \frac{1}{2} m v_F^2$ (for Cu : $v_F = 1.6 \times 10^6 \frac{m}{s}$ or $\sim \frac{c}{100}$)

p_F : $E_F = \frac{p_F^2}{2m}$

k_F : $p_F = \hbar k_F$

T_F : $8 \times 10^4\,K \gg 300\,K$



e^-'s in metals @R.T. are highly quantum mechanical !

" degenerate Fermi gas "

" Ocean of electrons " : only $\frac{kT}{E_F} \ll 1$ fraction (near the surface) actually can be excited with $\sim kT$ energies.