① Multiparticle states

a) $|1,a\rangle|2,b\rangle$ - two part. state

part 1 $\rightarrow$ state a

part 2 $\rightarrow$ state b

OK for non-identical particles (e.g. n and p)

$|\varphi\rangle|\psi\rangle$ - generally a tensor product (= enlarge wavefcn space)

if two non-interacting particles with same spin state

space repr. of $|1,a\rangle|2,b\rangle \Rightarrow \langle\vec{r}_1|\langle\vec{r}_2|1,a\rangle|2,b\rangle$

$$= \psi_a(\vec{r}_1)\cdot\psi_b(\vec{r}_2)$$

b) exchange operator $\hat{P}$:

$$\hat{P}\,|1,a\rangle|2,b\rangle = |1,b\rangle|2,a\rangle$$

$\hat{P}^2$ - double exchange $\Rightarrow$ identity operator $\hat{P}^2 = 1$

Eigenvalues of $\hat{P}$: ± 1

c) 2 identical bosons

* observables cannot depend on labels
* choose +1 for $\hat{P}$ (Sym.)

$$|a,b;S\rangle = \frac{1}{\sqrt{2}}\left(|1,a\rangle|2,b\rangle + |1,b\rangle|2,a\rangle\right)$$

particles with integer spin

d) 2 identical fermions

* choose −1 for $\hat{P}$ (Antisym.)

$$|a,b;A\rangle = \frac{1}{\sqrt{2}}\left(|1,a\rangle|2,b\rangle - |1,b\rangle|2,a\rangle\right)$$

particles with integer $+\frac{1}{2}$ spin

What if $a=b$ for 2 fermions :

$|a,a;A\rangle = 0 \rightarrow$

No two fermions can be in the same quant. state

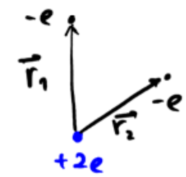
(Pauli exclusion principle $\rightarrow$ Periodic table)

e) exchange interaction: both part. @ $\vec{r}$

$$\psi(a,b;S) = \frac{1}{\sqrt{2}}\left(\psi_a(\vec{r})\psi_b(\vec{r}) + \psi_b(\vec{r})\psi_a(\vec{r})\right) = \frac{2}{\sqrt{2}}\psi_a(\vec{r})\psi_b(\vec{r})$$

$\psi(a,b;A) = 0$ ← "repel"   "attract"

② Ground state of He



$$V(r_i) = -\frac{2e^2}{4\pi\varepsilon_0 r_i}$$

$$\hat{\mathcal{H}} = -\frac{\hbar^2}{2m}\nabla^2_{\vec{r}_1} + V(\vec{r}_1) + $$
$$-\frac{\hbar^2}{2m}\nabla^2_{\vec{r}_2} + V(\vec{r}_2) + $$
$$V_{int}(|\vec{r}_1 - \vec{r}_2|) \quad ;$$

← can be difficult
drop for now

$$\hat{\mathcal{H}}\psi(\vec{r}_1,\vec{r}_2) = E\psi(\vec{r}_1,\vec{r}_2)$$ – stationary states

if Hamiltonian is separable

$$\hat{\mathcal{H}}(\vec{r}_1,\vec{r}_2) = \hat{\mathcal{H}}_1(\vec{r}_1) + \hat{\mathcal{H}}_2(\vec{r}_2)$$

then $\psi(\vec{r}_1,\vec{r}_2) = \psi_a(\vec{r}_1)\psi_b(\vec{r}_2)$

$$\hat{\mathcal{H}}\psi(1,2) = (\hat{\mathcal{H}}_1 + \hat{\mathcal{H}}_2)\psi_a(1)\psi_b(2)$$
$$= E_a\psi_a(1)\psi_b(2) + E_b\psi_a(1)\psi_b(2)$$
$$= (E_a + E_b)\psi(1,2)$$

eigenvalues of $\hat{\mathcal{H}}_1$ and $\hat{\mathcal{H}}_2$

$E = E_a + E_b$ ← sum single-part. energies

Is $\psi(1,2) = \psi_a(1)\psi_b(2)$ good enough?

NO: antisymmetrize (e^- = fermion)

single-particle : $\psi_a(1) = \underbrace{\psi_{n\ell m_\ell}(1)}_{\text{H-like}}\ \underbrace{\chi_\pm(1)}_{\text{spin}}$

$$\psi(1,2;A) = \underbrace{\begin{pmatrix}\text{space}\\ \text{eigenfcn.}\end{pmatrix}}_{\substack{\text{anti}\\ \text{sym}}} \times \underbrace{\begin{pmatrix}\text{spin}\\ \text{eigenfcn.}\end{pmatrix}}_{\substack{\text{sym}\\ \text{anti}}}$$