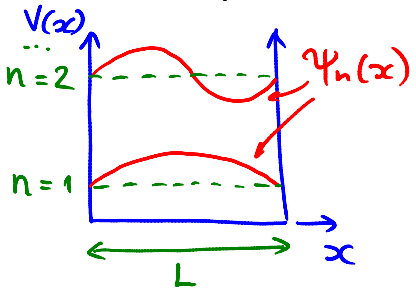
Warmup quiz

① Given a particle in infinite potential well with normalized eigenstates

$$|n\rangle = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)$$

corresponding to energy $E_n = \frac{n^2 h^2}{8mL^2}$



Normalize quantum state

$$|\chi\rangle = |1\rangle + 2i|3\rangle$$

Answer: $|\chi\rangle_{\text{normalized}} = \text{prefactor} \times |\chi\rangle$ with the prefactor equal to

A: $\frac{1}{3}$ C: $-2i$ E: $\frac{i}{\sqrt{5}}$

B: $\frac{1}{\sqrt{3}}$ D: $\frac{1}{\sqrt{5}}$

Soln: $\langle\chi| = \langle 1| - 2i\langle 3|$

$$\langle\chi|\chi\rangle = \langle 1|1\rangle - 2i\langle 3|1\rangle + 2i\langle 1|3\rangle + 4\langle 3|3\rangle$$

$$= 1 + 0 + 0 + 4$$

$$\Rightarrow |\chi\rangle = \frac{1}{\sqrt{5}}|1\rangle + \frac{2i}{\sqrt{5}}|3\rangle$$

also, can multiply the wavefun by $e^{i\alpha}$ with arbitrary $\alpha \in \mathbb{R}$

② Which of the following is true for a superposition of two stationary eigenstates corresponding to $E_1 \neq E_2$ energies?

Answer:

I. QM operators are linear, therefore, the superposition is itself an eigenstate of the Hamiltonian.

II. Since the original two states are stationary, their superposition must also be stationary.

A: only I B: only II

C: I & II D: none of the above

(3) Given a superposition state for particle in a box

$$|\psi\rangle = \frac{1}{\sqrt{5}}\left(|1\rangle + 2i|3\rangle\right)$$

and if the ground energy level is $E_1 = 1eV$, answer the following questions

* What is the expectation value for the energy of this state?

Answer: A: 1eV B: 6.33eV C: 7.4eV

D: 8.5eV E: 9eV

* What is the likely outcome of an energy measurement?

Answer: A: 1eV B: 4eV C: 7.4eV D: 9eV

Soln: $E_1 = 1eV$, $\Rightarrow E_3 = 3^2 \cdot E_1 = 9eV$

outcome of meas.: 1eV or 9eV

with probabilities $P_1 = \frac{1}{5}$ and $P_3 = \frac{4}{5}$

The expectation value $\langle E \rangle = P_1E_1 + P_3E_3 = 7.4eV$