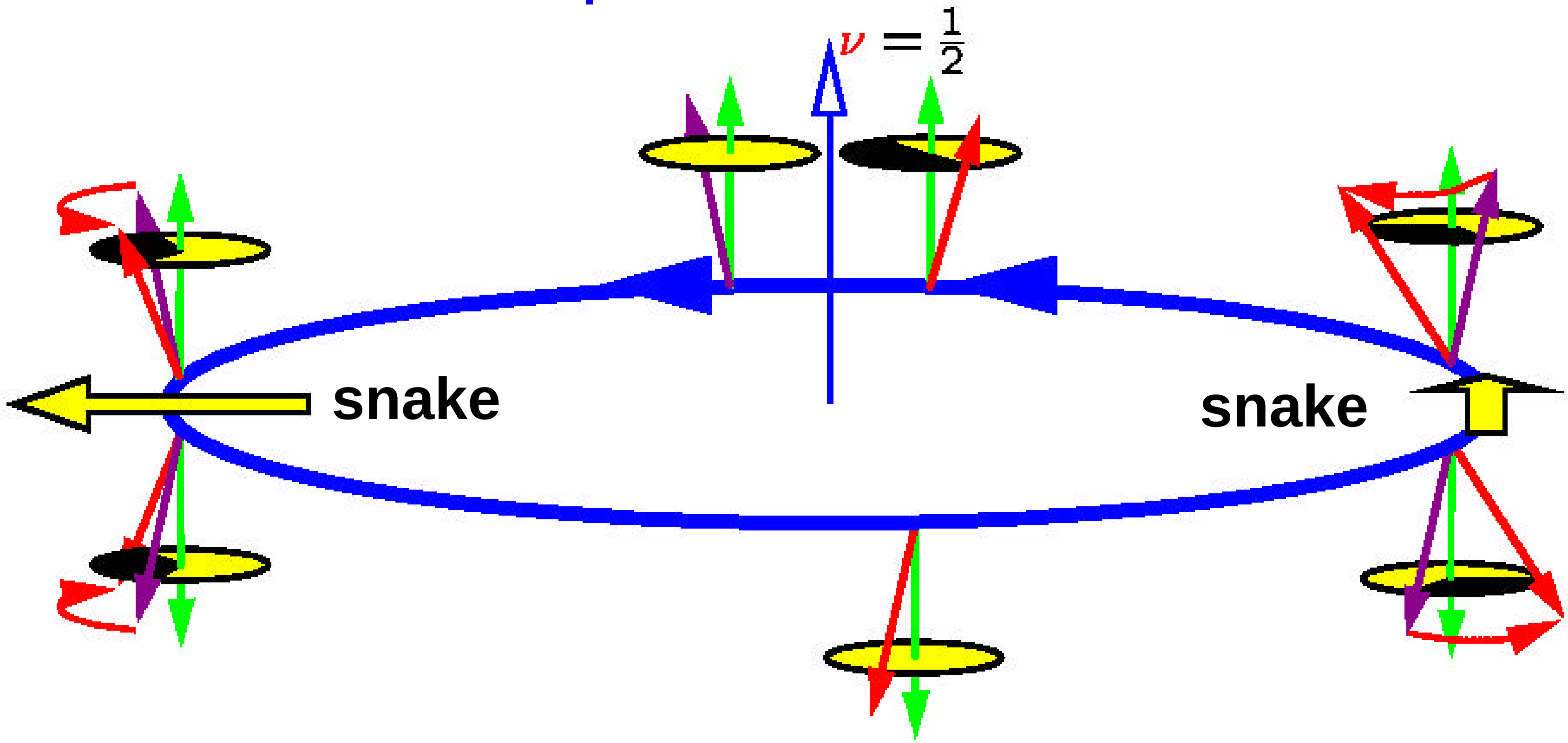
Matching of Siberian Snakes

9 November 2002

AGS Polarization Workshop



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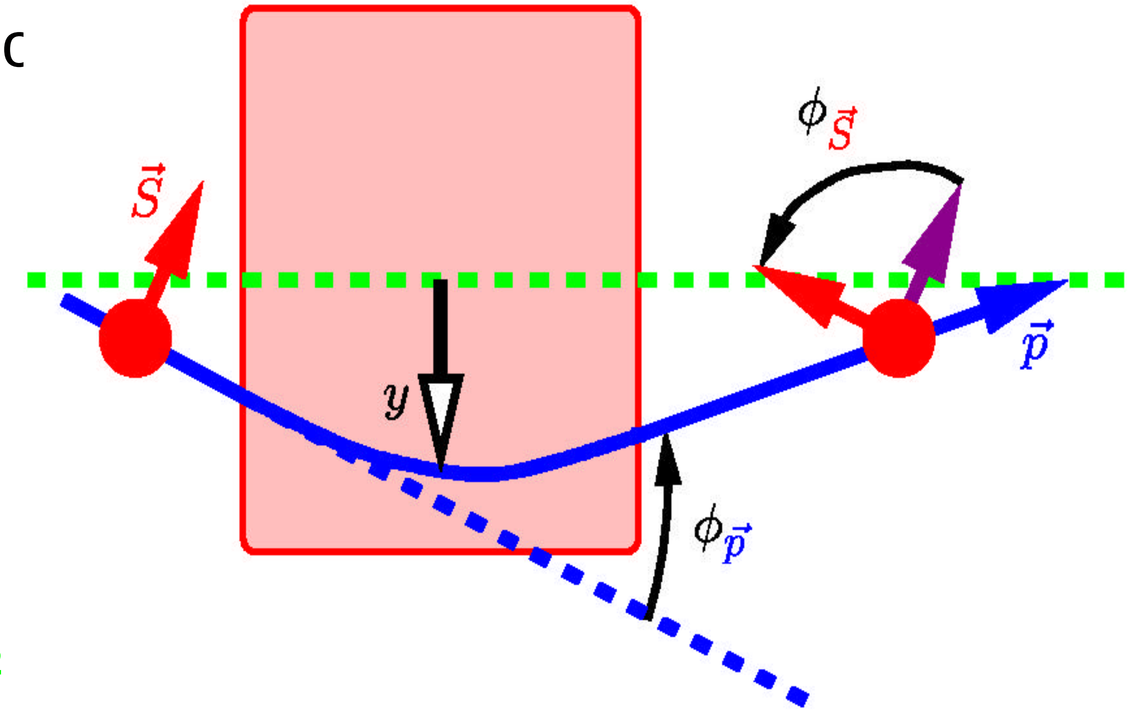
Driven spin perturbation on a trajectory

Integer values of spin-tune n $\pm$ tune n$_y$ lead to coherent
disturbances of spin mo



Remedy:
Siberian Snakes avoid resonances by making the spin-tune n = ½ independent of energy.

$$\phi_{\vec{S}} \propto \phi_{\vec{p}} \propto y = y_0 \sin(\psi_0 + nQ_y)$$

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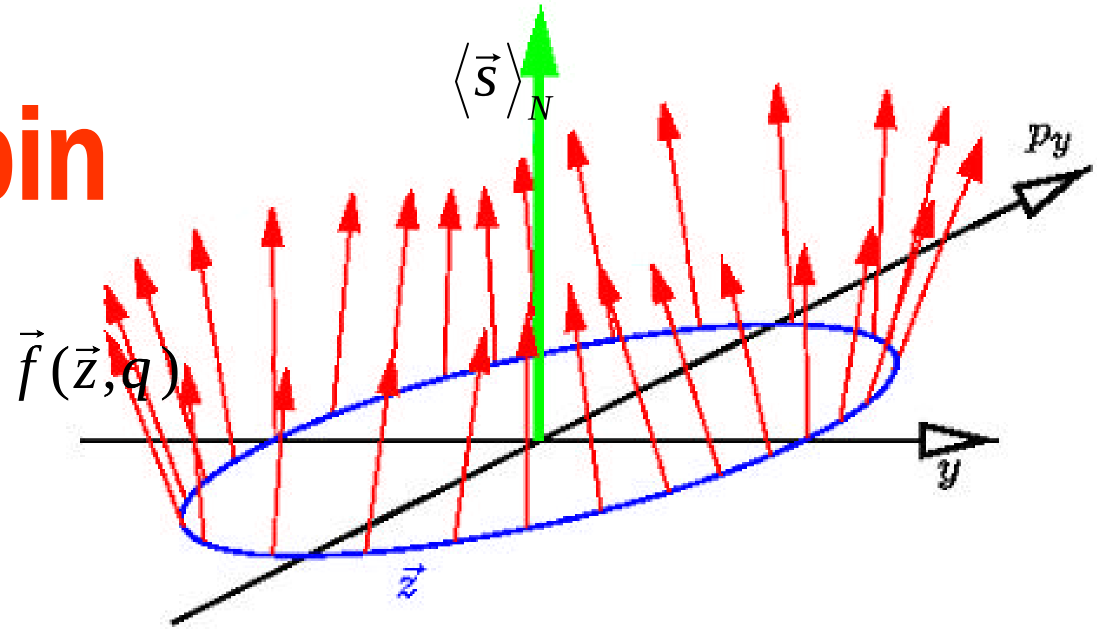
Equation of motion for spin fields



Spin field: Spin direction $\vec{f}(\vec{z},q)$ for each phase space point $\vec{z}$

$$\frac{d}{dq}\vec{S} = \vec{\Omega}(\vec{z},q)\times\vec{S}$$

$$\frac{d}{dq}\vec{f} = \partial_q \vec{f} + [\vec{v}(\vec{z},q)\cdot\partial_{\vec{z}}]\vec{f} = \vec{\Omega}(\vec{z},q)\times\vec{f}$$

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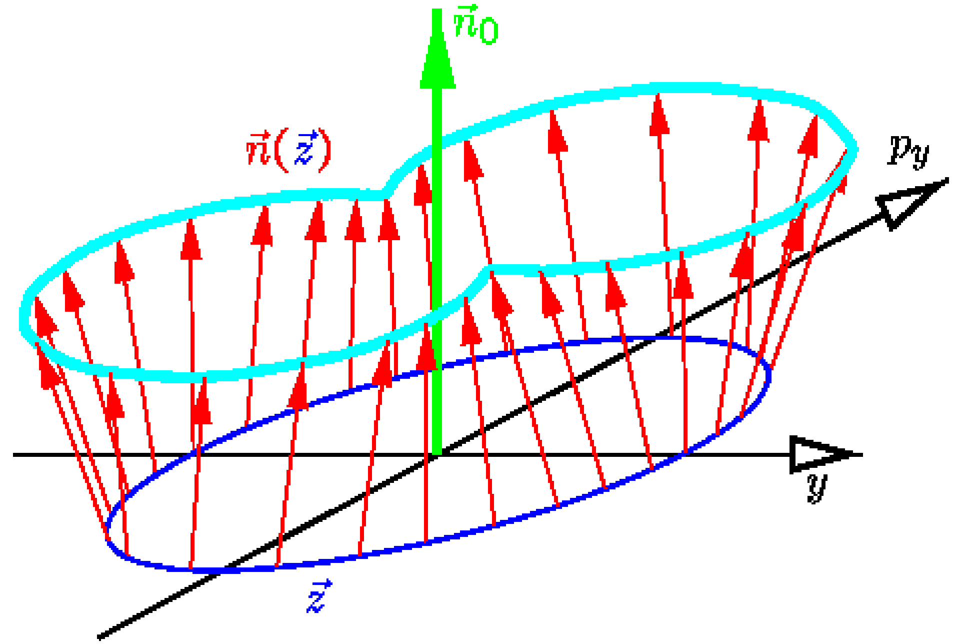
The Invariant Spin Field



A) Maximum polarization: $P_{lim} = \langle \vec{n}(\vec{z}) \rangle_{\text{Phase space}}$

For a large divergence, the average polarization is small, even if the local polarization is 100%.

B) $\vec{n}(\vec{z}) \cdot \vec{S}$ is an adiabatic invariance !

C) $\vec{n}(\vec{z})$ Defines an amplitude dependent spin tune !

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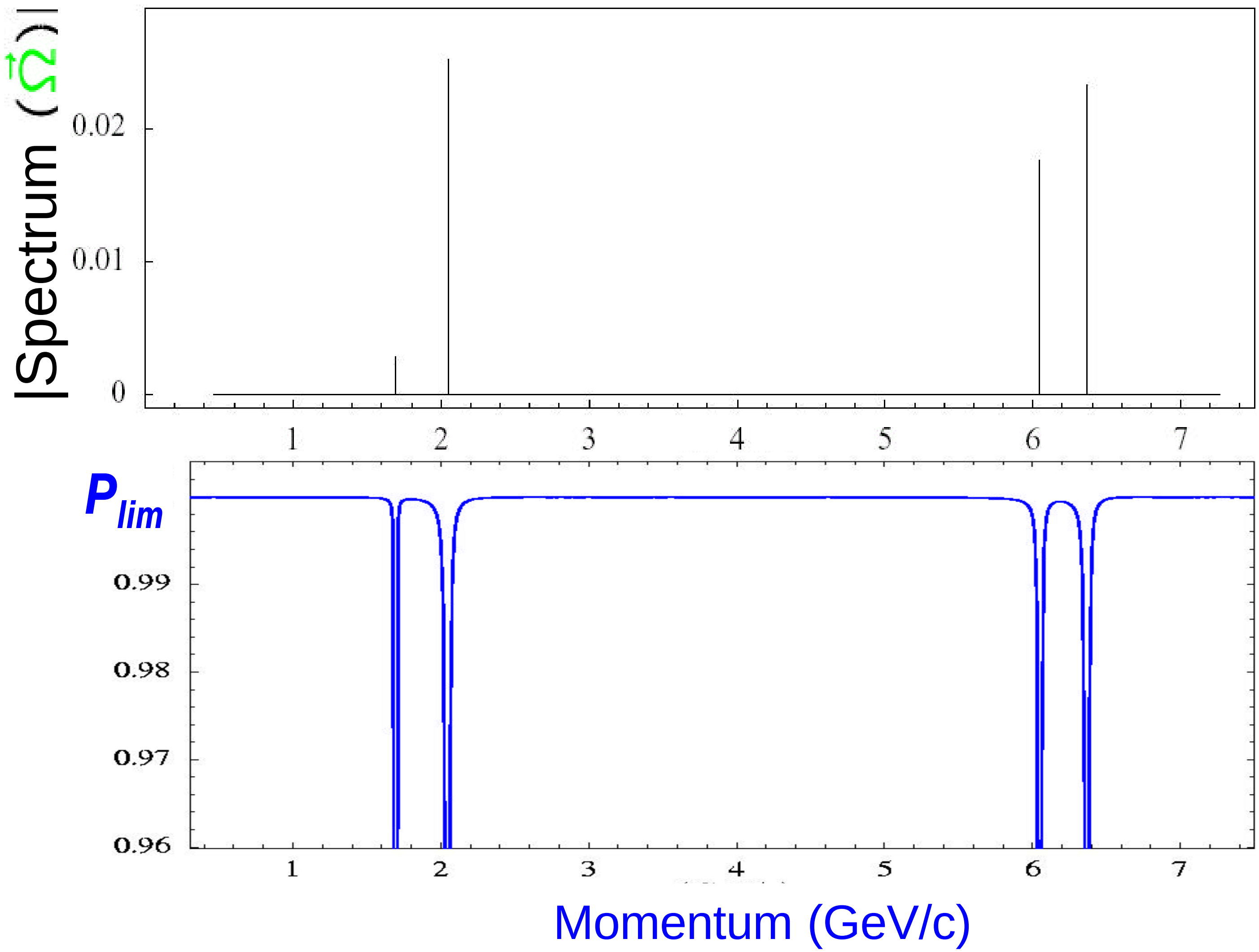
First Order Theories A) DESY III

Isolated resonance model:

Linear spin-field theory:



Low energies: First order theories agree

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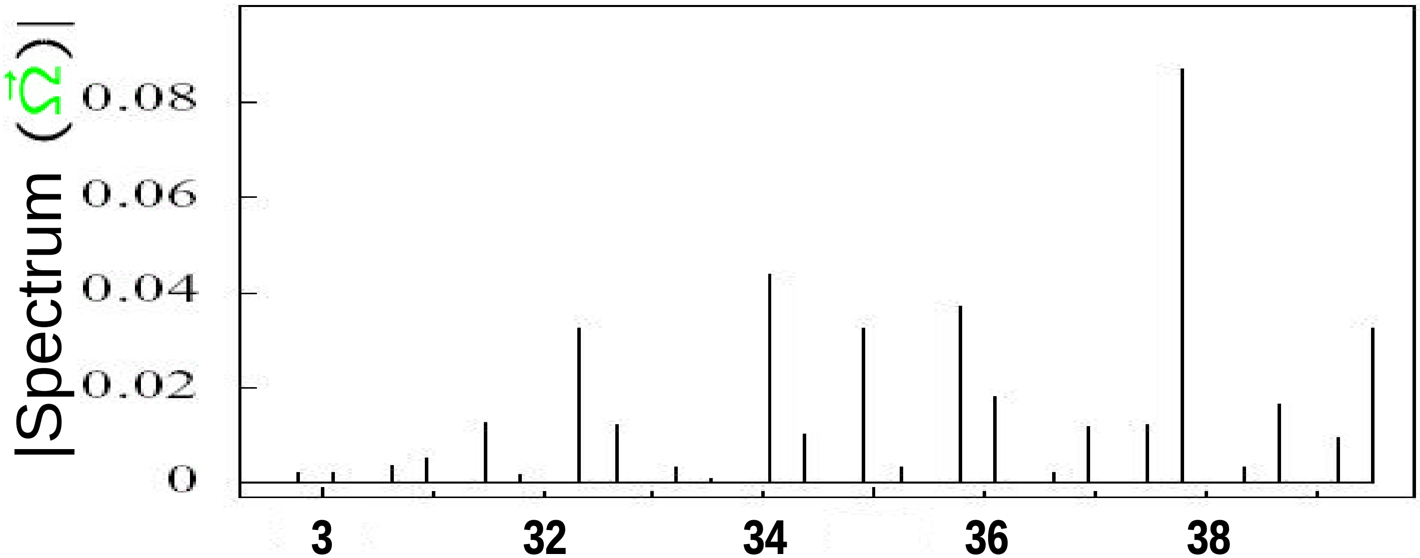
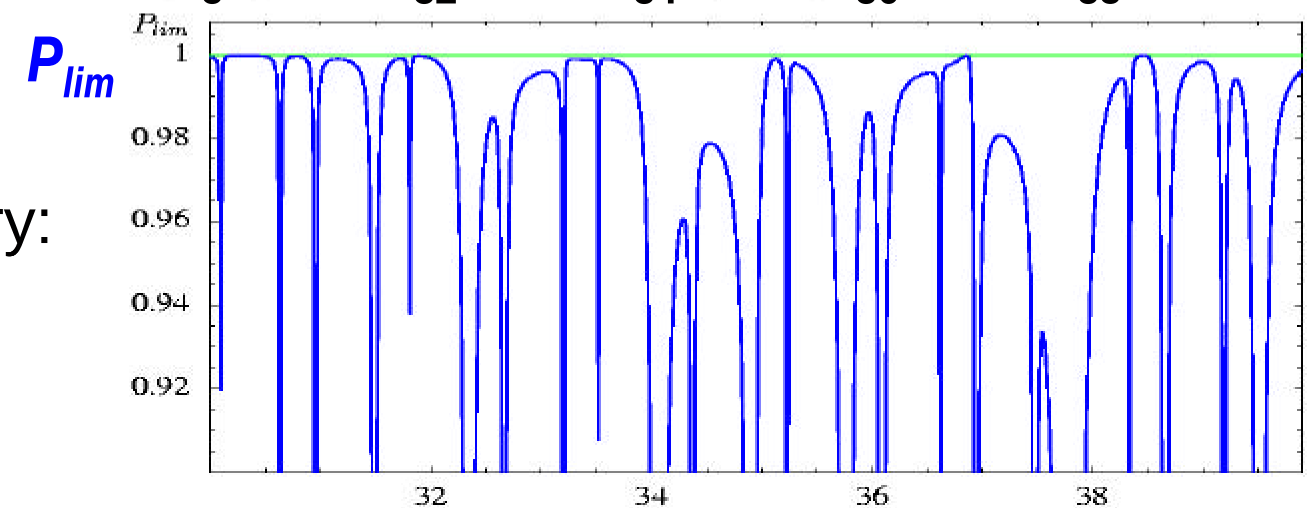
First Order Theories B) PETRA

Isolated resonance model:



Linear spin-field theory:



Momentum (GeV/c)

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Medium energies: resonances still isolated

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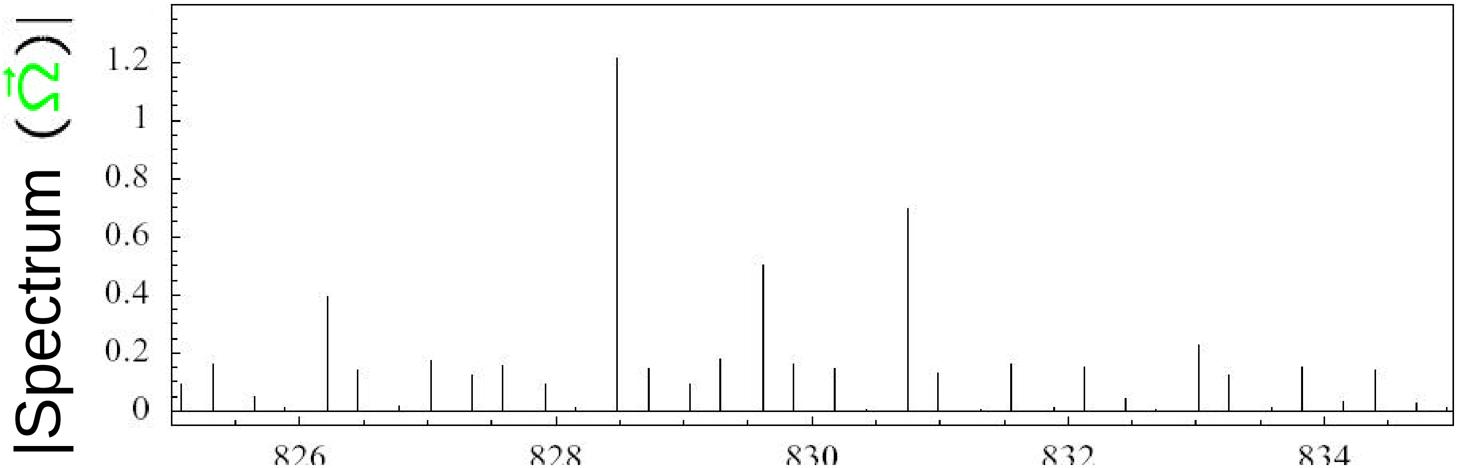
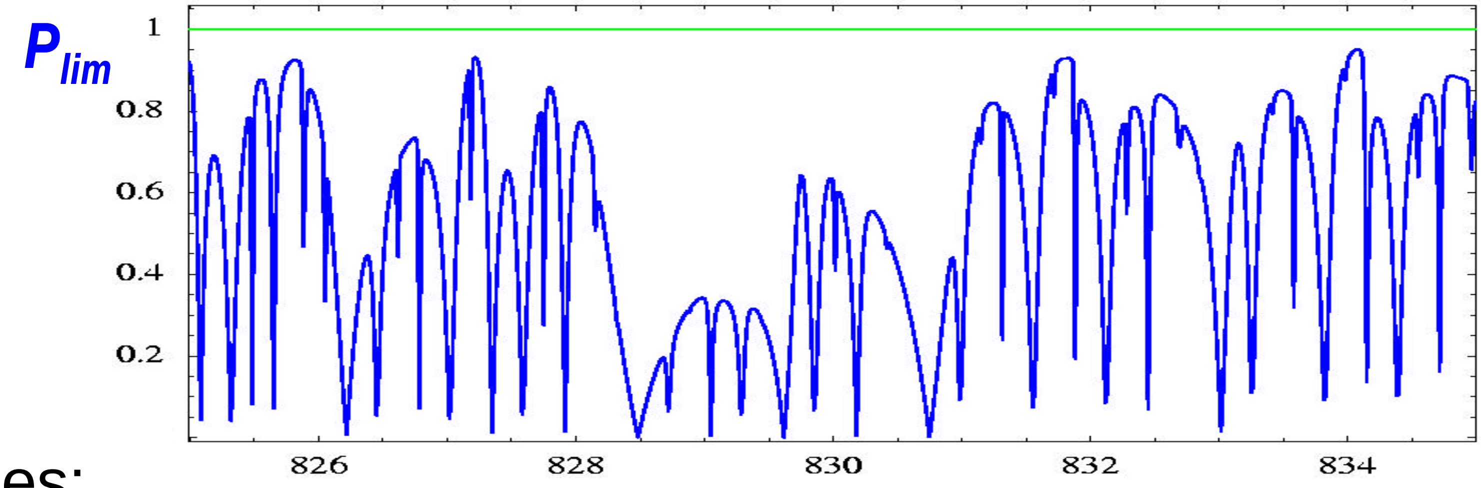
First Order Theories C) HERA

Isolated resonance model:



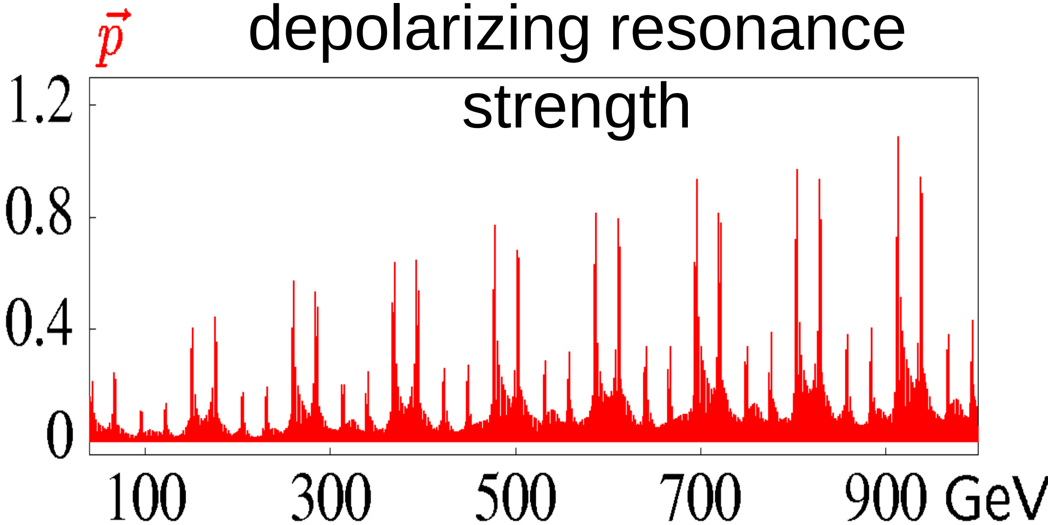
Linear spin-field theory:

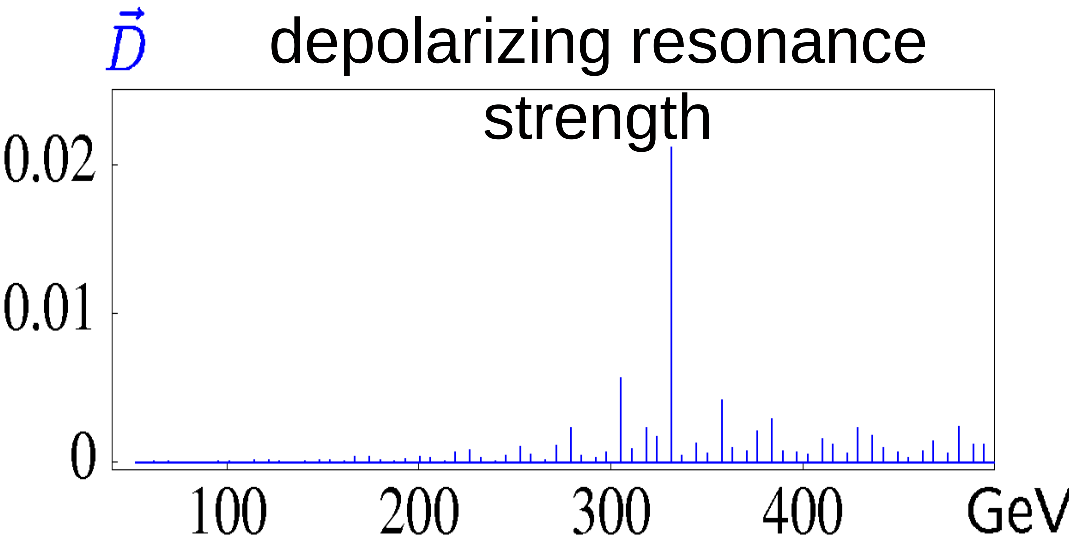


<u>High energies: Resonances are no longer isolated.</u>

Isolated resonance model becomes invalid

Polarized Deuterons





- Resonances are 25 times weaker and 25 times rarer for D than for p
- Transverse polarization could be achieved without Siberian Snakes
- Transverse RF dipoles could be used to rotate and stabilize longitudinal polarization

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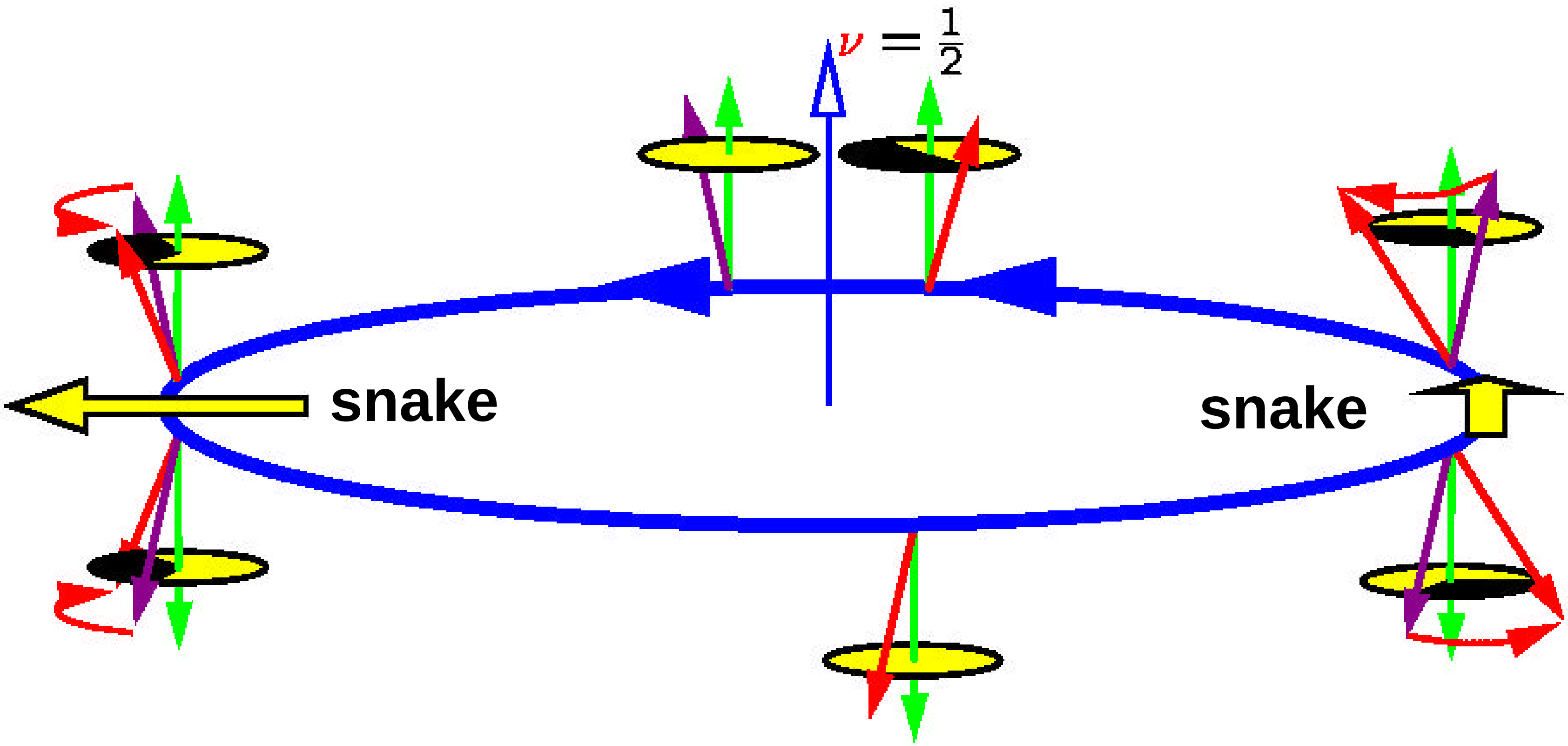
Siberian Snakes

Siberian Snakes rotate spins at each energy ½ times



Freedom: direction of the rotation axis in the horizontal

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CO spin motion with 2N Siberian Snake

$$A = \prod_{j=1}^{2N} i e^{-i\frac{y_j}{2}s_3}(s_1\cos a_j + s_2 \sin a_j)$$

$$= i^N e^{-i\frac{y_{2N}\cdots - y_3 + y_2 - y_1}{2}s_3}\prod_{j=1}^{N}(s_1\cos a_{2j} + s_2\sin a_{2j})(s_1\cos a_{2j-1} + s_2\sin a_{2j-1})$$

$$= i^N e^{-i\frac{\Delta y}{2}s_3}\prod_{j=1}^{N}[\cos(a_{2j} - a_{2j-1}) - i\sin(a_{2j} - a_{2j-1})s_3]$$

$$A = i^N e^{-i\frac{\Delta y + 2\Delta a}{2}s_3}$$

$$\boxed{n_0 = \frac{\Delta y + 2\Delta a}{2p} \qquad \vec{n}_0 = \vec{e}_y}$$

$\Delta y = 0$, to make $_0$ independent of energy

$\Delta a = \dfrac{p}{2}$, to make $_0$=0.5

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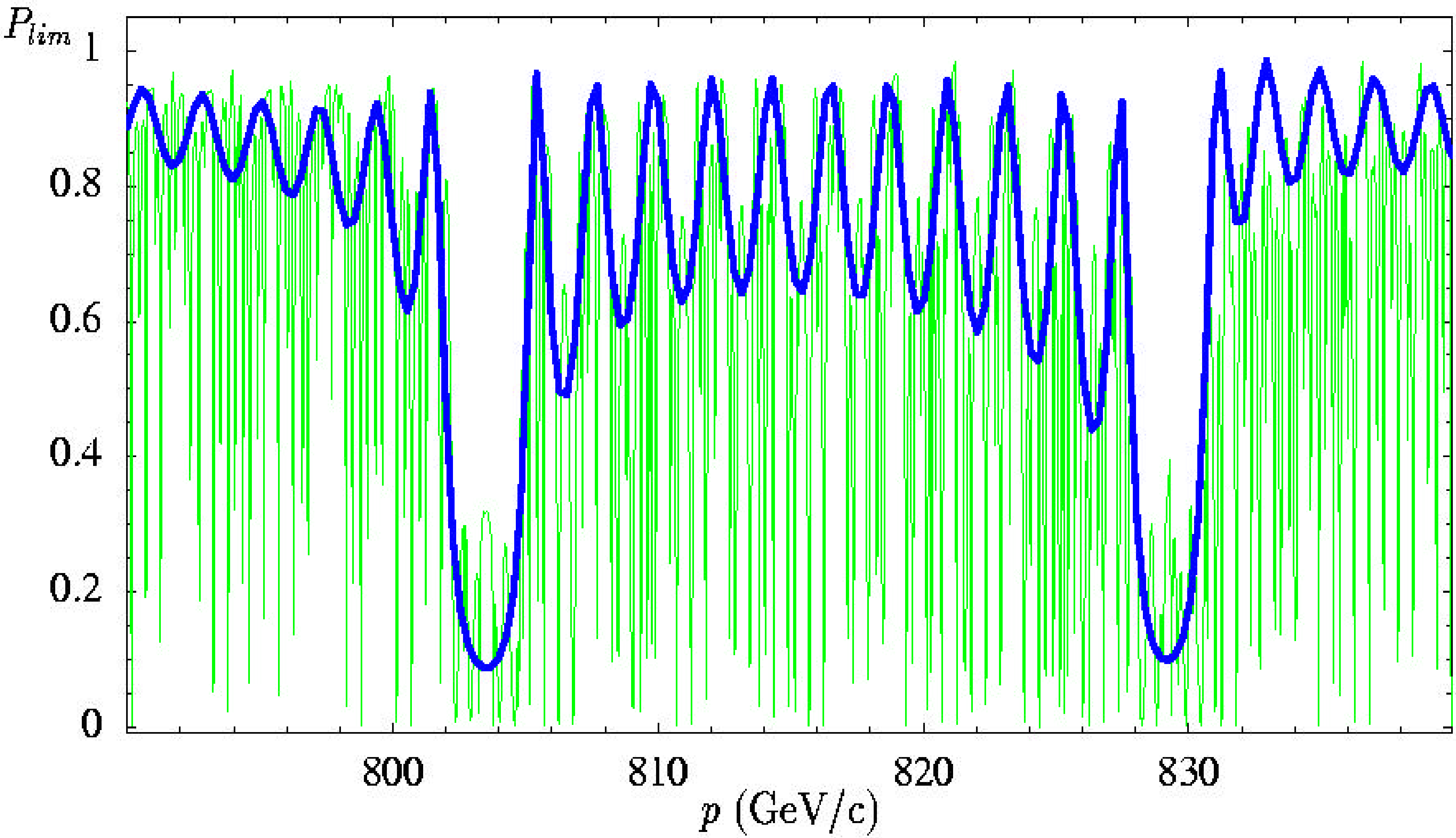
Siberian Snakes and Resonances



Some structure of the 1st order resonances remains after Siberian Snakes have been installed.

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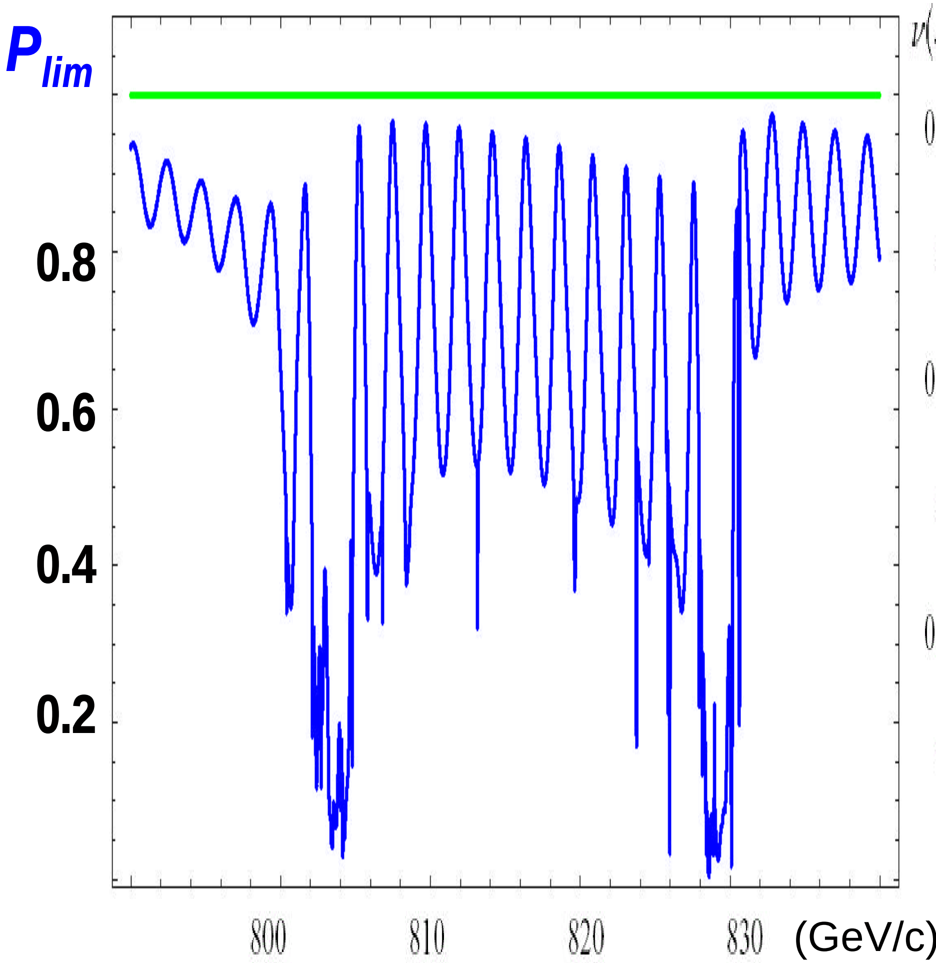
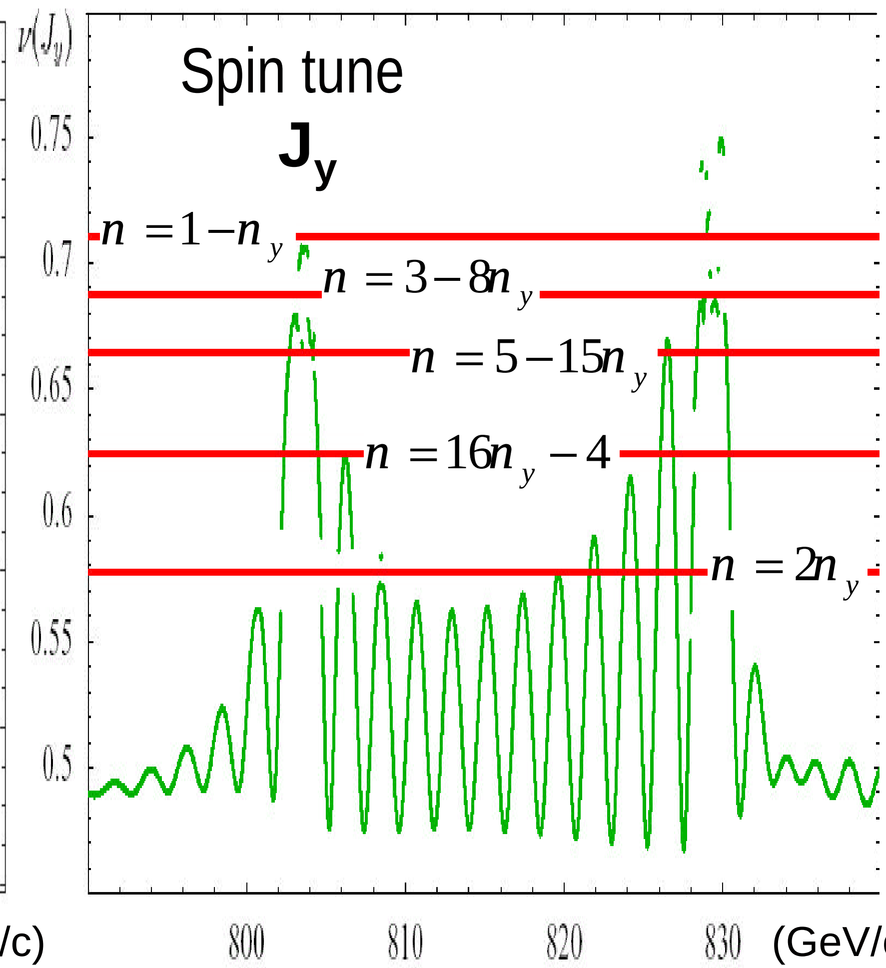
Spin Tune at Higher Order Resonance





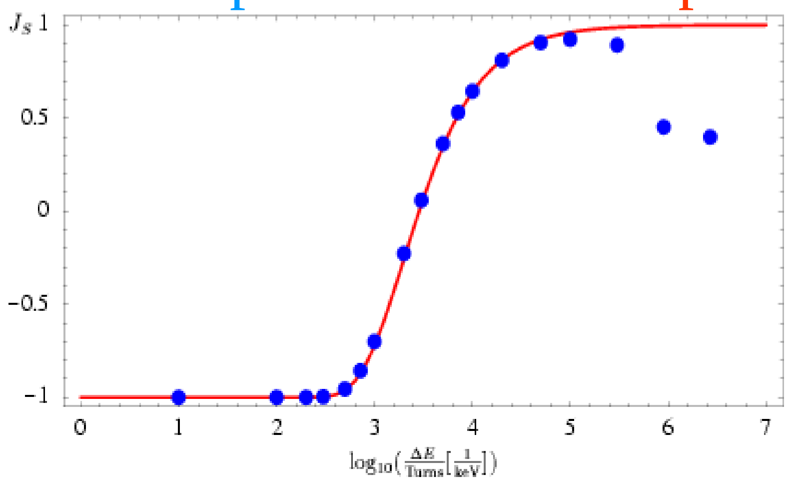
The spin tune deviates from ½ for particles which oscillate around the design trajectory with amplitude J_y.

High Order Resonance Strength

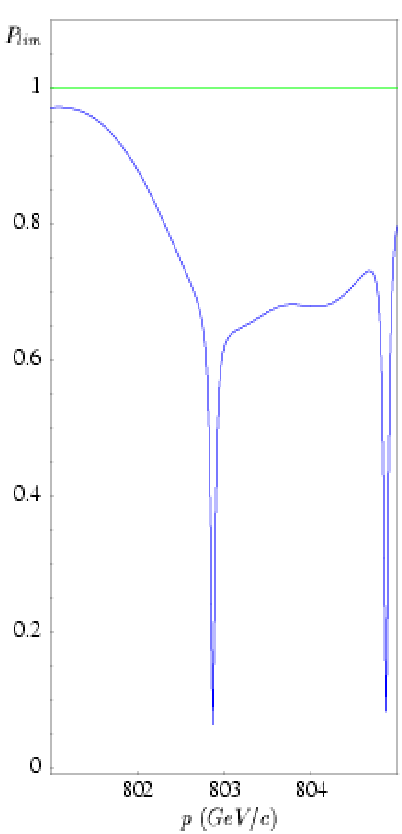
The higher order Froissart-Stora formula

- Resonances up to 19th order can be observed
- Resonance strength can be determined from tune jump.

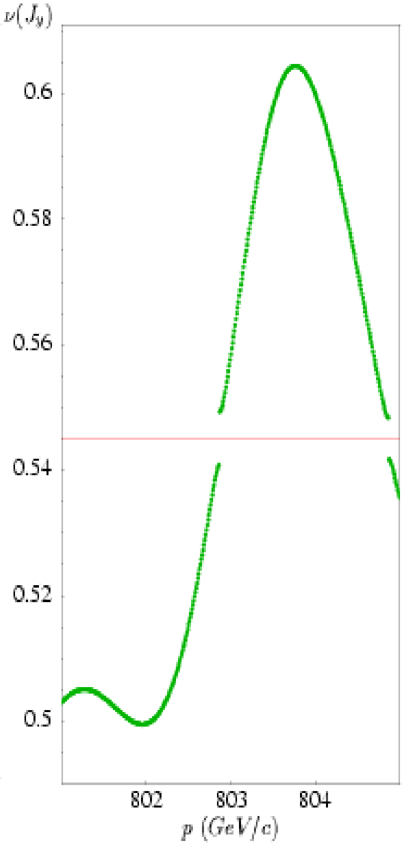
Tracked depolarization as expected



$$P_{\lim} = \langle \vec{n}(\vec{z}) \rangle$$



Spin tune



Computations performed in SPRINT, Hoffstaetter and Vogt, DESY/00

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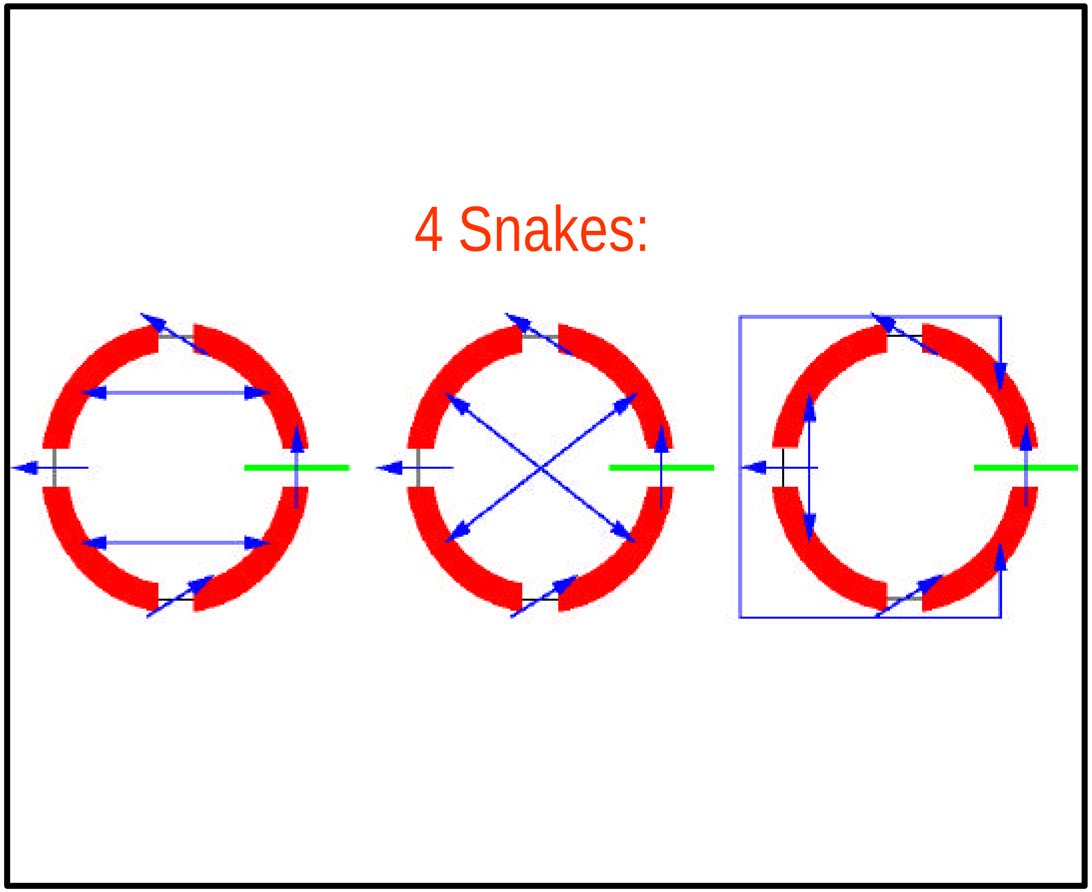
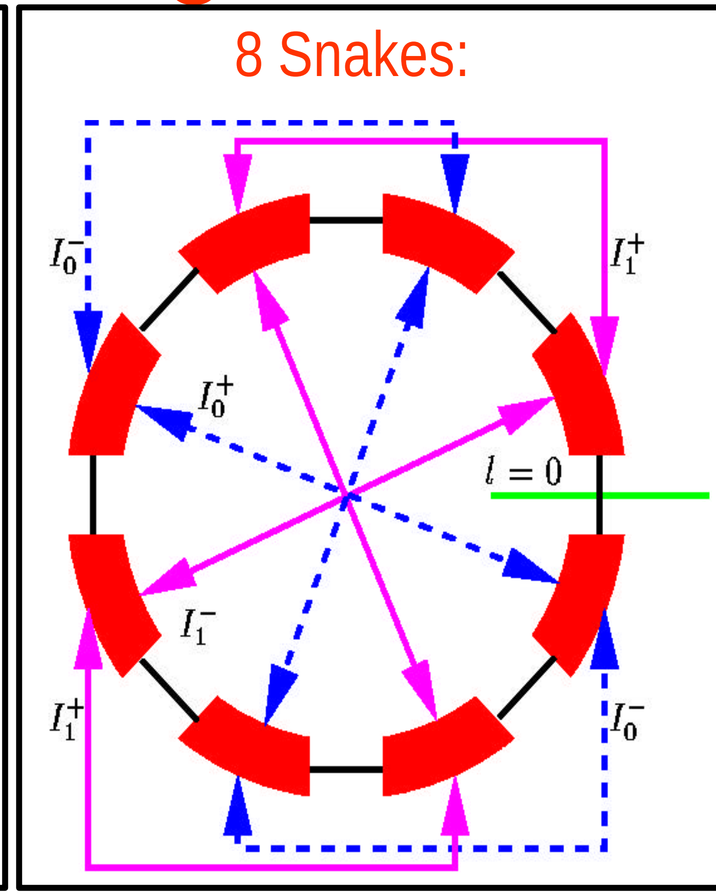
Snake matching





1st Order: 4 harmonics of the spin perturbation in each section.

With 4 snakes only 2 can be compensated

With 8 snakes all 4 can be compensated

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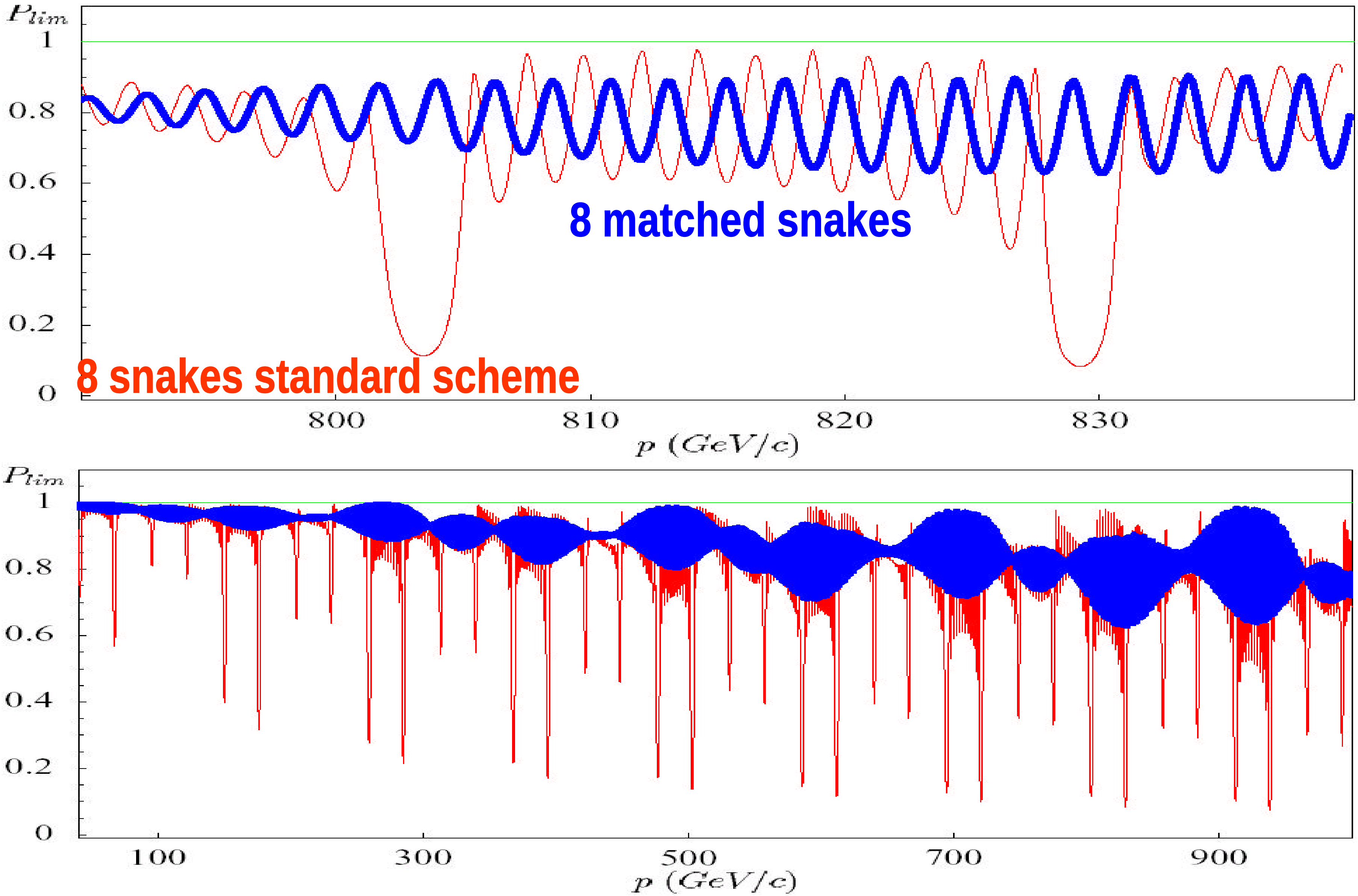
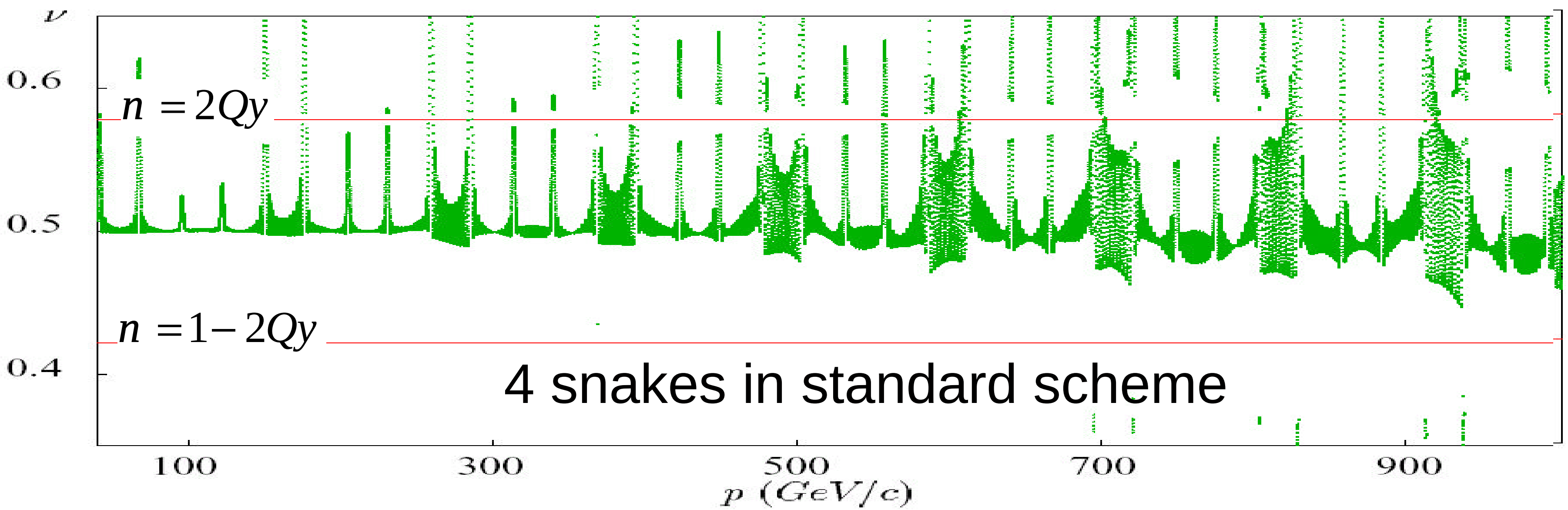
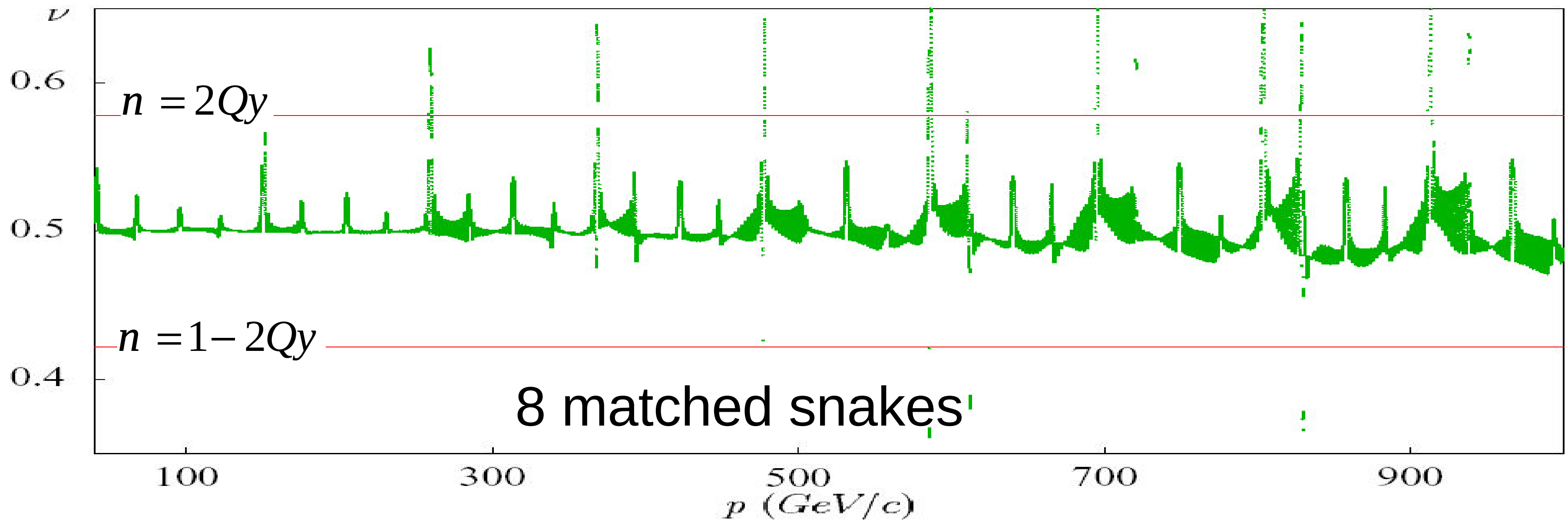
P_{lim} after Snake Matching

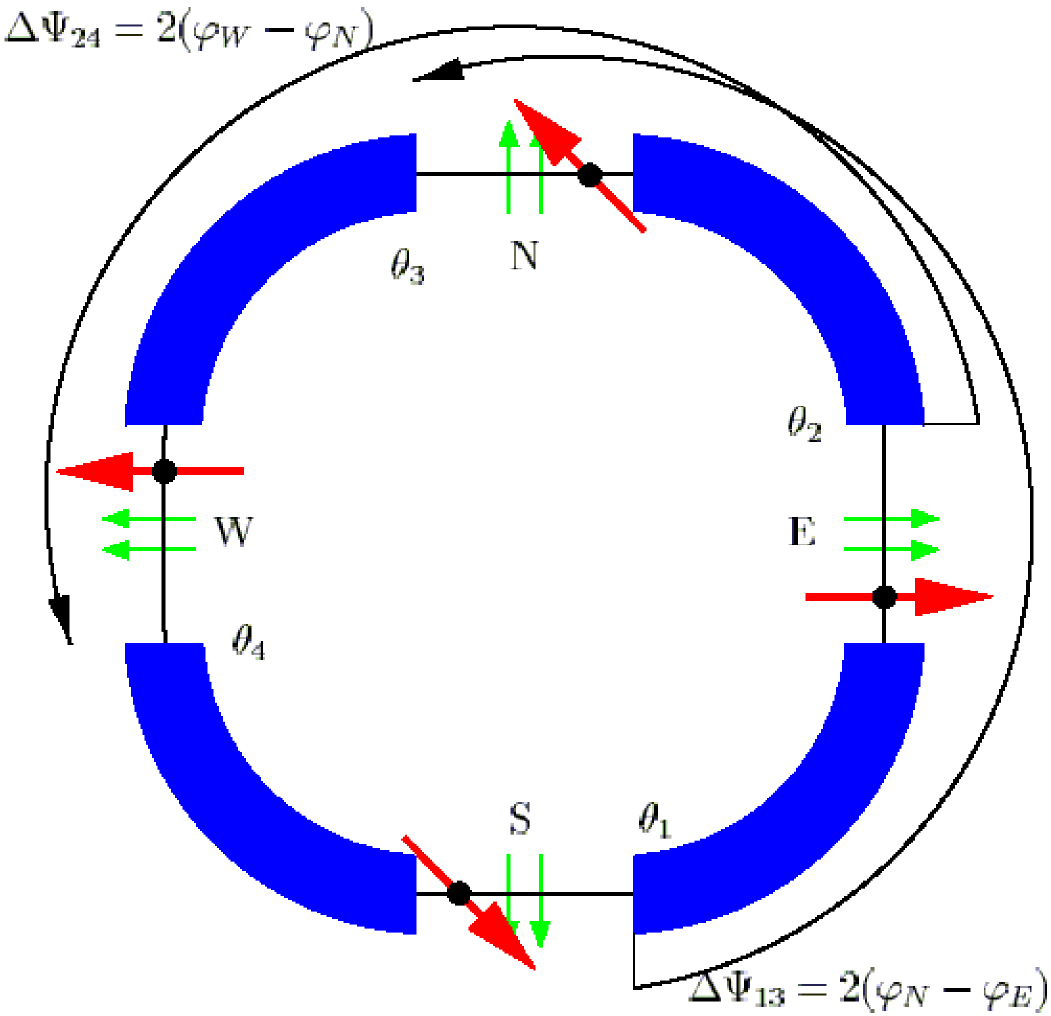


Spin Tune after Snake Matching





Matching the betatron phases



A proper choice of betatron phase advances allows snake matching with 4 snakes

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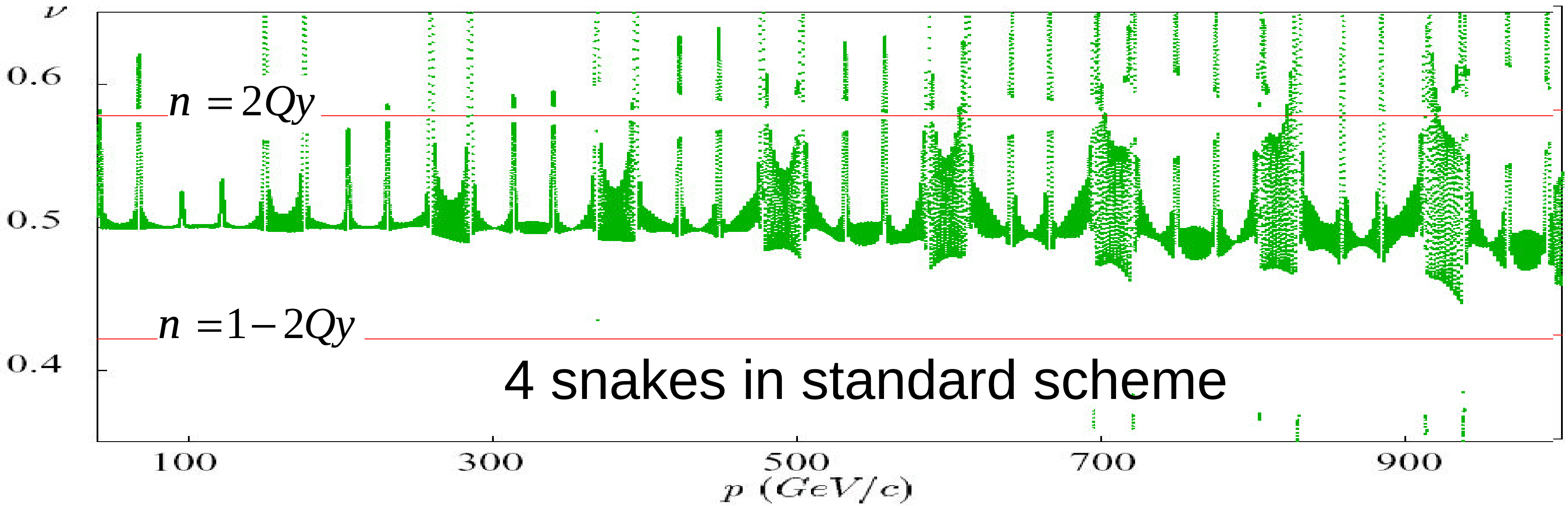
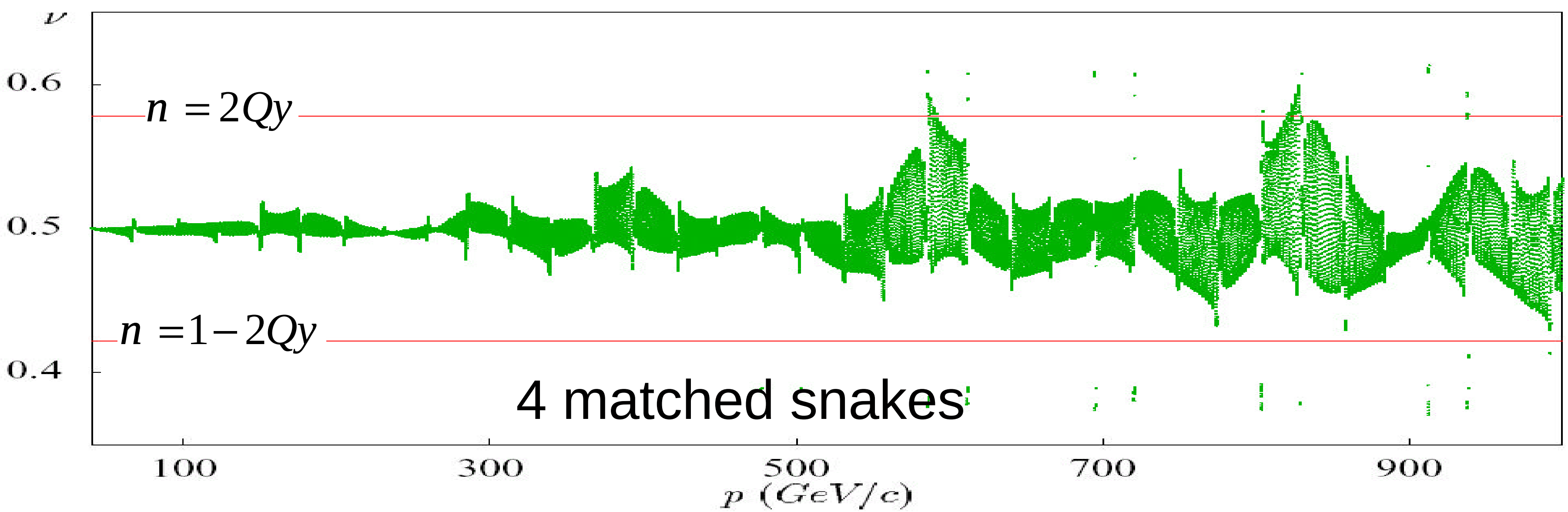
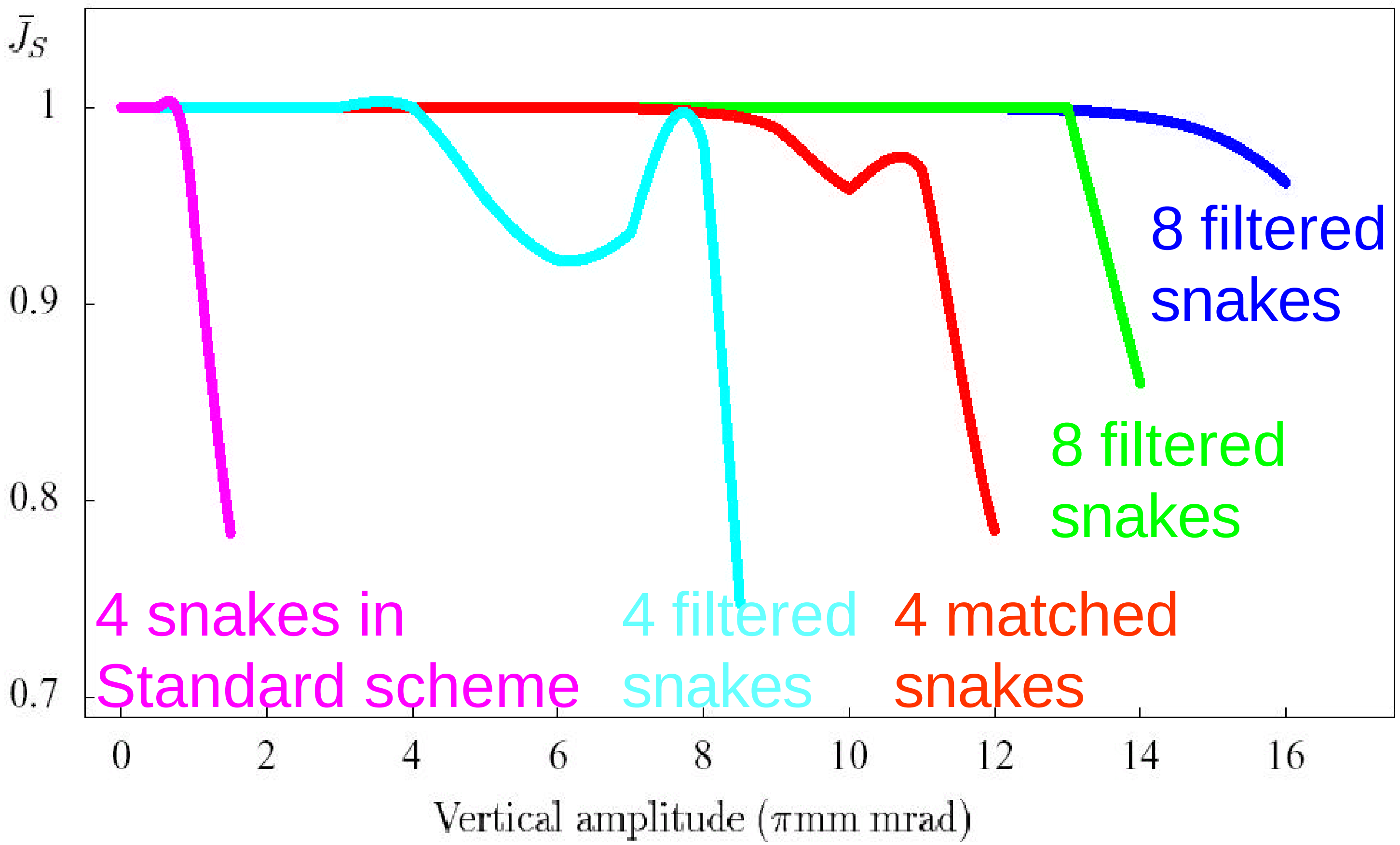
Spin Tune after Snake Matching





Allowed Beam Sizes



Snake matching allows to have significantly larger beams.

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TESLA with Röntgen FEL