

Numerical and theoretical spin stability studies for HERA- p

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Abstract: In a high energy proton ring spin motion depends strongly on the orbital motion in phase space. This affects the acceleration process as well as the storage regime. It is therefore necessary to find a theoretical basis that goes beyond the models that proved to be successful in low energy proton spin dynamics. We present some numerical results of a study based on the concepts of the invariant spin field and the amplitude dependent spin tune. These concepts enable us to identify and classify higher order depolarizing resonances and help us to find means to fight them.

1 Introduction

The motion of the unit spin vector \hat{S} in the rest frame of a relativistic charged particle travelling in electric and magnetic fields is governed by the Thomas-BMT precession equation, which takes the form $d\hat{S}/d\theta = \vec{\Omega}(\gamma, \vec{z}, \theta) \times \hat{S}$ where θ is the azimuthal position in the ring, \vec{z} is the position in phase space and $\gamma = E/m$ is the Lorentz factor.

1.1 The Invariant Spin Field

In the following we will assume the equations of orbital motion in 6-dimensional phase space to be integrable: $\vec{z} = \vec{z}(\vec{J}, \vec{\Psi})$, $\vec{J} = \text{const.}$, $\vec{\Psi}(\theta) = \vec{\Psi}_0 + \vec{Q}\theta$. An arbitrary initial spin \hat{S}_i is mapped to a final spin \hat{S}_f by means of an orthogonal spin transport matrix \underline{R} ,

$$\hat{S}_f = \underline{R}_{\vec{J}}(\theta_f, \theta_i; \vec{\Psi}_i) \hat{S}_i . \quad (1)$$

We define a general spin field on a torus $\vec{J} = \text{const.}$ as a function $\hat{f} : [0, 2\pi)^3 \times \mathbb{R} \mapsto \mathbb{S}_3$ that maps the ‘‘angle space’’ \times azimuthal domain onto the unit sphere and is, if evaluated along each orbital trajectory $\vec{\Psi}_0 + \vec{Q}\theta$, a solution of the T-BMT equation

$$\frac{d}{d\theta} \hat{f}_{\vec{J}} \equiv \left[\partial_\theta + \vec{Q} \cdot \partial_{\vec{\Psi}} \right] \hat{f}_{\vec{J}} = \vec{\Omega} \times \hat{f}_{\vec{J}} . \quad (2)$$

The evolution equation of a spin field known on a given torus $\vec{J} = \text{const.}$ at some arbitrary θ_i reads as

$$\hat{f}(\vec{\Psi}_i + (\theta_f - \theta_i)\vec{Q}, \theta_f) = \underline{R}_{\vec{J}}(\theta_f, \theta_i; \vec{\Psi}_i) \hat{f}(\vec{\Psi}_i, \theta_i) , \quad \forall \theta_f . \quad (3)$$

The invariant spin field (\equiv Derbenev–Kondratenko \hat{n} -axis) [1, 2, 3] is defined to be the special spin field of unit length that is periodic w.r.t. θ , $\hat{n}_{\vec{J}}(\vec{\Psi}, \theta) = \hat{n}_{\vec{J}}(\vec{\Psi}, \theta + 2\pi)$ and it describes the equilibrium polarization distribution of the beam[4]. The \hat{n} -axis can be computed non-perturbatively using Fourier analysis[5], stroboscopic averaging[3] and anti-damping[6]. A special realization of anti-damping in the presence of an RF dipole is described in [7].

It can be shown that the time averaged polarization on some fixed torus $\vec{J}=\text{const.}$ at some azimuth θ and for a given fixed energy $m\gamma$ cannot be greater than the static polarization limit defined by the average of the \hat{n} -axis on that torus evaluated at θ and γ

$$P_{\text{lim}}(\vec{J}, \theta, \gamma) \equiv \left\| \left\langle \hat{n}_{\vec{J}}(\vec{\Psi}, \theta, \gamma) \right\rangle_{\vec{\Psi}} \right\| . \quad (4)$$

The actual polarization on a torus after acceleration to γ depends on the history of the acceleration process. In our ramp-simulations we therefore choose an ensemble of N spins $\{\hat{S}_j(\theta_{\text{inj}}, \gamma_{\text{inj}}) \equiv \hat{n}_{\vec{J}}(\vec{\Psi}_j, \theta_{\text{inj}}, \gamma_{\text{inj}})\}_{1 \leq j \leq N}$ at injection energy, then ramp the ensemble to γ and define the “ramped polarization” as the combination of the ensemble average and the average over a sufficiently large number of turns at *constant* final γ

$$P_{\text{rmp}}(\vec{J}, \gamma, \theta) \equiv \left\| \left\langle \left\langle \hat{S}_j(\theta, \gamma) \right\rangle_{\text{turns}} \right\rangle_j \right\| \leq P_{\text{lim}}(\vec{J}, \theta, \gamma) . \quad (5)$$

If the acceleration process were an adiabatic evolution through stationary states, then $\hat{S}(\theta, \gamma) = \hat{n}_{\vec{J}}(\vec{\Psi}(\theta), \theta, \gamma)$ and we would obtain the limiting case $P_{\text{rmp}} = P_{\text{lim}}$.

1.2 The Amplitude Dependent Spin Tune

In order to obtain a complete action-angle representation of spin motion [8] we must assign to each point in phase space an orthonormal coordinate system $(\hat{n}_{\vec{J}}, \hat{u}_{\vec{J}}^1, \hat{u}_{\vec{J}}^2)$ which is periodic in $\vec{\Psi}$ and θ and in which the spin precession rate around \hat{n} is constant along each trajectory on the torus $\vec{J}=\text{const.}$ and independent of starting azimuth and orbital phases. The spin phase advance per turn in this frame divided by 2π is called the spin tune $\nu(\vec{J}, \gamma)$. For each arbitrary initial spin \hat{S}_i at $(\vec{\Psi}_i, \theta_i = 0)$ we can compute the spin action $I = \hat{S}_i \cdot \hat{n}(\vec{\Psi}_i, 0)$ and the spin phase $\Phi_0 = \arctan(\hat{S}_i \cdot \hat{u}^2(\vec{\Psi}_i, 0) / \hat{S}_i \cdot \hat{u}^1(\vec{\Psi}_i, 0))$. Then the spin motion can be written in the form

$$\hat{S}(\theta) = \sqrt{1 - I^2} \Re \{ (\hat{u}^1 + i\hat{u}^2) e^{-i(\nu\theta + \Phi_0)} \} + I \hat{n} \quad (6)$$

which simply describes a rotation around \hat{n} with constant rate. The spin tune can be computed by an averaging method[9] and the Fourier analysis[5] method returns it automatically while computing the \hat{n} -axis. The spin tune on the closed orbit $\nu_0(\gamma)$ is just $\nu(\vec{0}, \gamma)$. In the purely vertical field on the closed orbit of a perfectly flat ring $\nu_0 = G\gamma$ where $G = g/2 - 1 \approx 1.7928$ is the gyromagnetic anomaly of the proton.

The vectors \hat{u}^1, \hat{u}^2 as well as the amplitude dependent spin tune are not unique. We can use another periodic system \hat{u}_k^1, \hat{u}_k^2 which differs from \hat{u}^1, \hat{u}^2 by some uniform rotation of $2\pi(k_0 + \vec{k} \cdot \vec{Q})$ per turn around \hat{n} . The corresponding spin tune will be $\nu + k_0 + \vec{k} \cdot \vec{Q}$. But there is normally only one branch for which $\lim_{\vec{J} \rightarrow \vec{0}} \nu(\vec{J}, \gamma) = \nu_0(\gamma)$. For this branch we define the *spin tune spread* inside some torus $\vec{J}=\text{const.}$ as $\max_{J'_i \leq J_i} \nu(\vec{J}', \gamma) - \min_{J'_i \leq J_i} \nu(\vec{J}', \gamma)$. If $\nu_0 = 1/2$ as is enforced by Siberian Snakes in all our simulations, all potential resonance positions $\kappa = [nQ_y]$

are symmetric around $1/2$, where we have introduced the brackets $[]$ to mean the *fractional part*. In this case a resonance crossing at κ can occur if $\max_{J'_i \leq J_i} |\nu - 1/2| > |\kappa - 1/2|$. If the spin tune is in resonance with the orbital tunes: $\nu(\vec{J}, \gamma) = m_0 + \vec{m} \cdot \vec{Q}$, we can find a coordinate system in which an arbitrary spin does not precess. Then the \hat{n} -axis is not unique. Moreover near to strong resonances \hat{n} is a strongly varying function of $\vec{\Psi}$ and P_{lim} can be small.

2 Special Features of HERA- p

The HERA proton ring consists of 4 arcs and 4 straight sections. In the arcs the p -ring is located *above* the e^\pm -ring. The p - and e^\pm -beams are only brought to collision in the South (**S**) and North (**N**) straight sections. Therefore these straights include mini-beta regions and magnets for horizontal beam separation. The East (**O**) straight contains the HERMES internal target experiment which only uses the e^\pm -beam. The β -functions for the p -ring are rather relaxed in the East. Nevertheless, the East straight is potentially designed for colliding beam experiments and hence both beams are separated only horizontally. Finally, in the West (**W**) straight the p -beam is used in the HERA-B fixed target experiment and has rather high β -functions and negative horizontal dispersion at the IP.

In order to have both beams in one horizontal plane at the O-,S-,N-IPs there are sections made from interleaved horizontal and vertical bends (and quadrupoles) at the ends of the arc octants OL,OR,SL,SR,NL and NR, where L/R means left/right of the IP when looking in the outwards radial direction. The vertical bends are located inside of the combined matching-dispersion suppressor-sections. The HERA arcs *outside* the vertical bend sections consist of 24 FODO-cells of which the inner 18 (with QP42/QP40 magnets) are strictly periodic and the outer 2×3 already belong to the matching sections. In the vertical bend sections and just outside of them there is generally enough space (10–14m) to place Siberian Snakes, whereas putting snakes in the centres of the arcs would require additional hardware modifications.

We conclude that HERA- p is *not* a flat ring so that the on-orbit spin tune $\nu_0 \neq G\gamma$ and \hat{n}_0 is strongly energy dependent in the unmodified machine! In order to make \hat{n}_0 vertical in the arcs the concept of *flattening* snakes [10, 11] was introduced whereby 6 radial Siberian Snakes are placed at the symmetry points of the 6 vertical bend sections, transforming their on-orbit spin transport maps into the maps of radial snakes themselves. These 6 *distributed snakes*[12] then cancel themselves pair wise since they are separated by straight sections only. There are also some advanced ideas[12] on *combining* flattening-, main-snakes and 90° spin rotators. 3 out of 4 straight sections are surrounded with vertical bend sections, which immediately implies a superperiodicity $P = 1$! Nevertheless the lattice has a “very approximate” superperiodicity $P = 4$. Neglecting the fact that the O- and W-straights are themselves slightly left/right asymmetric, one finds an approximate mirror symmetry w.r.t. the O-W-axis. So an optimal scheme for placing the 4 main snakes should reflect this level of symmetry.

3 Simulations

In all of our simulations we work with the 1996 set of optics and with an unperturbed machine. For the effect of misalignments see [13]. In figure 1 we plot the static polarization limit P_{lim} and the amplitude dependent spin tune ν vs. the reference momentum p_0 computed with the new[5]

SODOM algorithm as implemented in SPRINT. Here the layout of the snake scheme is motivated by the approximate fourfold superperiodicity of HERA- p . The scan is done for purely vertical orbit motion on an invariant ellipse that corresponds to a beam size of 2.5σ , i.e. an emittance of 25π mm mrad. P_{lim} is wildly oscillating between 0–90% in certain regions showing that the average opening angle of the invariant spin field is strongly varying with energy. The spin tune on that torus also varies strongly with energy (0.4–0.65). With these large spin tune variations and with a fractional tune of $[Q_y] \approx 0.2725$ several resonances $\nu = k_0 + kQ_y$ with $k = \pm 2, \pm 9$ are met when scanning the energy. We see that the spin tune performs rather large symmetrical jumps across the resonance lines indicating strong excitation of these resonances. As expected, P_{lim} is particularly wild and its local maxima are particularly small in the neighbourhood of spin-orbit resonances. In order to obtain and maintain a polarized beam in HERA- p at an

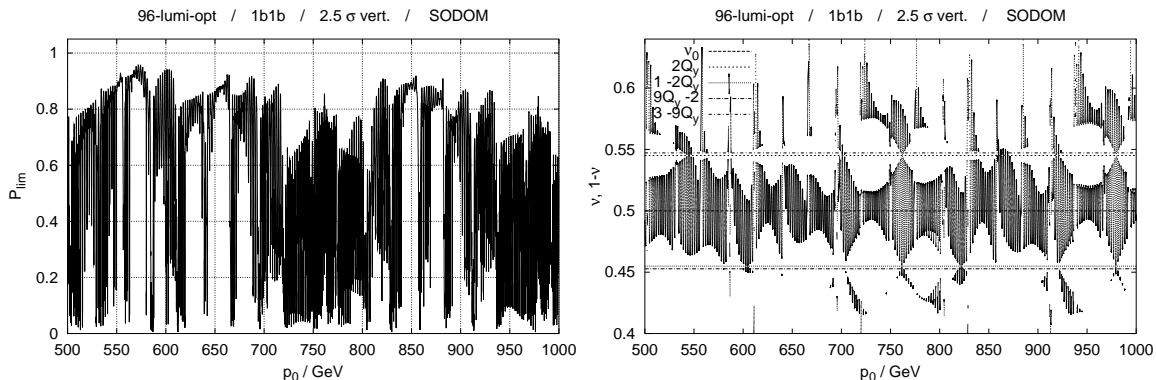


Figure 1: P_{lim} (left) and ν (right) for the 1996 HERA- p luminosity optics in the momentum range from 500GeV to 1TeV evaluated on a phase space torus which corresponds to a vertical beam size of 2.5σ . There was no horizontal or longitudinal amplitude. The lattice contains interleaved radial (O, W) and 45° (S, N) snakes plus 6 radial flattening snakes in the centres of the vertical bend sections ($2 \times O, 2 \times S, 2 \times N$). The tune is $Q_y \approx 32.2725$.

energy $\approx 820\text{GeV}$, spin motion has to be controlled much more effectively than is possible with a “classical” snake scheme and the old tunes. Recent studies[14] have shown that simply adding more snakes does not always improve the spin stability. A snake scheme has to be found that reduces the large spin tune spread and the orbital tunes have to be optimized in order to allow for the biggest possible spin tune spread *without* reducing the dynamic aperture of the ring too drastically. This was done indirectly with the method of filtering[15] which involves maximizing the average over energy of the linearized static polarization limit $\langle P_{\text{lim}}^{(1)} \rangle_{40-820\text{GeV}}$ by choosing optimal snake angles. The scheme obtained by filtering (see figure caption 3 for the snake angles) reflects the real symmetry properties of HERA- p . Figure 2 shows the possible resonant spin tune positions $\kappa = [nQ_y]$ for $|n| < 19$ and $0.27 < [Q_y] < 0.305$. The normal luminosity tunes are $Q_x \approx 31.292$ and $Q_y \approx 32.297$ so that the design orbit spin tune with snakes $\nu_0 = 1/2$ is rather close to the 5-th order resonance conditions $5[Q_y] - 1 \approx 0.485$ and $2 - 5[Q_y] \approx 0.515$. This vertical tune is surely not optimal for ramping a polarized beam since the allowable magnitude of spin tune variations is limited to $< \pm 0.15$. With $[Q_y]$ below 0.285 and above 0.31 the dynamic aperture is strongly reduced but there is enough space close to the $[Q_y] = 2/7 \approx 0.2857$ orbital resonance. Thus the following simulations are for modified tunes $Q_x \approx 31.291$ and $Q_y \approx 32.286$ and for the optimized/filtered snake scheme. In figure 3 P_{lim}

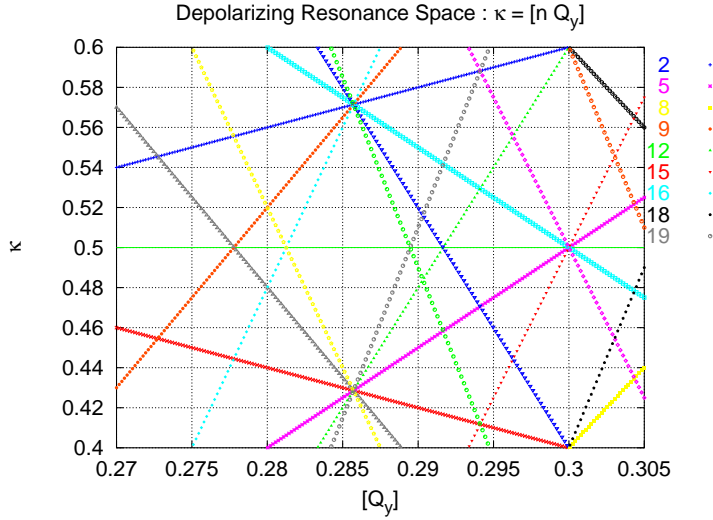


Figure 2: *The possible resonant spin tune positions $\kappa = [nQ_y]$ for $|n| < 19$ as a function of the fractional vertical tune $[Q_y]$. Note the rather big gap around $\kappa = 1/2$ close to the 7-th order orbital resonance $[Q_y] = 2/7 \approx 0.2857$.*

and ν are plotted on the same phase space torus as in figure 1 under the optimized conditions and for the complete HERA- p ramp file sequence. The snake scheme, although optimized for maximal P_{lim} , reduces the maximum spin tune spread to $\leq \pm .04$ which is nicely inside the window $|\nu - \nu_0| \leq .045$ at $[Q_y] = 0.286$ so that no resonance conditions are fulfilled for pure vertical motion on the whole acceleration cycle. Obviously the energy dependence of P_{lim} is much weaker even up to 820GeV and maxima of $P_{\text{lim}} \approx 95\%$ can be found close to 820GeV.

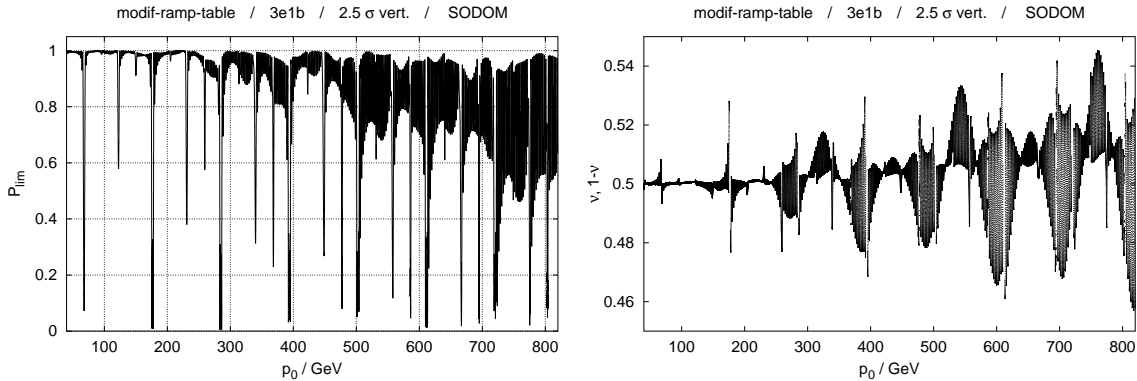


Figure 3: P_{lim} (left) and ν (right) for a file sequence from injection- (*hpin40*) via intermediate- (*hpsz300*) to the separation optics (*hpse820*) holding the vertical tune close to $[Q_y] = 0.286$. The “main” snakes are, from O to N: longitudinal, -45° , radial, $+45^\circ$. The flattening snakes are as in figure 1. Again the vertical phase space is excited to 2.5σ whereas horizontal and longitudinal planes are not excited at all.

Figure 4 finally shows the results of typical ramp simulations with the optimized snake scheme and tunes. P_{rmp} is plotted on a torus with 2σ (left) and 2.5σ (right) beam size in *all 3*

planes. Due to the additional effect of the horizontal and longitudinal motion P_{lim} is expected to be a little less than with purely vertical motion. Thus P_{rmp} is quite close to P_{lim} over the whole acceleration process in the case of 2σ . Unfortunately for 2.5σ , polarization is lost already around 400 GeV at certain residual resonance structures. Similar simulations with solely 2.5σ vertical motion have shown that polarization is conserved up to ≈ 800 GeV. Furthermore in ramp simulations with large horizontal and longitudinal amplitudes but moderate vertical amplitude the polarization survives up to 820 GeV. There is a particularly nasty residual resonance structure at about 804 GeV[14] and about 2.5σ which we did not manage to circumvent yet.

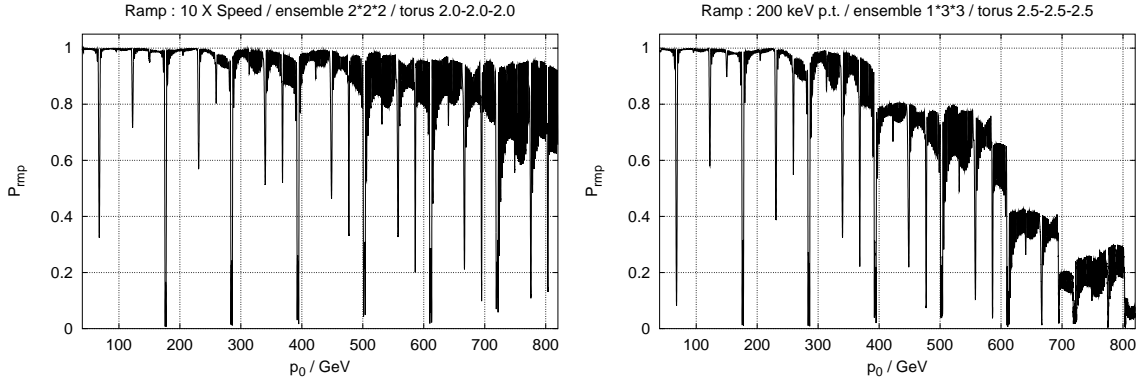


Figure 4: P_{rmp} for a typical acceleration process using the file sequence as in figure 3. (Left): The particles are on a phase space torus with 2σ in all three phase space planes. (Right): Same as left but 2.5σ in all three planes.

4 Conclusion and Outlook

If we neglect the effects of misalignments and non-linear orbital motion, then with a properly filtered/optimized scheme including 4 main- and 6 flattening snakes and with carefully chosen orbital tunes, it seems possible to accelerate polarized protons up to or close to 820 GeV within emittances which correspond to approximately 2σ in all three phase space planes. By restricting one or two phase space planes to moderate amplitudes, beam sizes of up to 2.5σ in the other plane(s) could be accelerated without loss of polarization. This success shows that *static* calculations of $P_{\text{lim}}(\vec{J}, \gamma)$ and $\nu(\vec{J}, \gamma)$ provided very useful guidance for identifying and fighting potentially dangerous regions in energy and orbital action.

Since the limits seem to be due to the spin tune spread during the acceleration and the opening angle of the \hat{n} distribution at the working energy and since both effects vanish for $\vec{J} \rightarrow \vec{0}$, electron cooling in PETRA or HERA[16] should be pursued. An emittance reduction to $1/5$ in the vertical plane alone would already help a lot since spin motion becomes particularly sensitive to the horizontal and longitudinal degrees of freedom only at high vertical amplitudes.

More simulations must be done including misalignments, non-linear orbit motion, collective phenomena, and sources of spin diffusion like intra-beam scattering and eventually the electron cooling itself.

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