DESY-M-96-14 and
Proceedings EPAC96
Barcelona 1996

THE PHASE SPACE DEPENDENT SPIN POLARIZATION DIRECTION IN THE HERA PROTON RING AT HIGH ENERGY

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Abstract

In very high energy storage rings the equilibrium spin polarization direction can vary strongly from point to point in phase space. Thus, unless counter measures are taken, the average polarization of the beam can be seriously limited. In this paper we present estimates of this limitation for the HERA proton ring as a function of beam energy and other system parameters, and discuss methods to minimize it. For this effort it has been essential to obtain a trustworthy approach to computing the polarization distribution. For this purpose we augment the Normal Form approach in various orders with an innovative method which constructs this quantity from spin phase space tracking data.

1 INTRODUCTION

Following the attainment of longitudinal polarization in the HERA electron ring and its use by the HERMES experiment [1], the possibility of obtaining polarized protons at 820GeV at HERA is being studied.

In contrast to electrons there is no practical self polarization mechanism for protons. Thus the polarized proton beam must be accelerated after creation in a polarized source. This method has been tested at several accelerators. However, polarized protons have never been accelerated to more than about 25GeV. To achieve polarized proton beams in RHIC (250GeV), HERA (820GeV), and the TEVATRON (900GeV), special techniques are needed [2, 6]. Problems with beam energies higher than 400GeV are already mentioned in [3].

As a first step in addressing the novel problems appearing at high energies, we have concentrated on analyzing whether a high energy polarized beam can stay polarized for a sufficiently long time. To answer this question it is important to know the equilibrium distribution of spins in phase space, i.e. the distribution which does not change from turn to turn.

2 THE EQUILIBRIUM POLARIZATION DIRECTION

In horizontal dipoles, spins precess only around the vertical field direction. The quadrupoles have vertical and horizontal fields and additionally cause the spins to precess away from the vertical direction. The strength of the spin precession and the precession axis in machine magnets depends on the trajectory and the energy of the particle. Thus in one turn around the ring the effective precession axis can deviate from the vertical and will depend on the initial position of the particle in six dimensional phase space. From this it is clear that if an equilibrium spin distribution exists, i.e. if the polarization vector at every phase space point is periodic in the machine azimuth, it will vary across the orbital phase space. This field of equilibrium spin directions in phase space does not change from turn to turn when particles propagate through the accelerator, although after each turn the particles find themselves at new positions in phase space. These directions, which we denote by the unit vector $\vec{n}(\vec{z},\theta)$, where $\vec{z}$ denotes the position in the six dimensional phase space of the beam and θ is the generalized azimuth, was first introduced by Derbenev and Kondratenko [4] in the theory of radiative electron polarization. Note that $\vec{n}(\vec{z},\theta)$ is usually not an eigenvector of the spin transfer matrix at some phase space point since the spin of a particle changes after one turn around the ring, but the eigenvector would not change. When a particle at an initial phase space point $\vec{z}_i$ is transported to a final point $\vec{z}_f$ during one turn around the storage ring, the periodicity condition can be written using the orthogonal spin transport matrix $\underline{R}(\vec{z}_i,\theta)$ as

$$\underline{R}(\vec{z}_i,\theta)\vec{n}(\vec{z}_i,\theta) = \vec{n}(\vec{z}_f,\theta)\ . \qquad (1)$$

Thus once we know this direction $\vec{n}(\vec{z},\theta)$, the phase space dependent polarization $p(\vec{z},\theta)$ in this direction, and the phase space density function $\rho(\vec{z},\theta)$ we have a complete specification of the polarization state of a beam of spin $1/2$ particles. The maximizing of the polarization of the ensemble implies two conditions; the polarization $p(\vec{z},\theta)$ at each point in phase space should be high and the polarization vector $\vec{n}(\vec{z},\theta)$ at each point should be almost parallel to the average polarization vector of the beam.

According to the T–BMT equation, the rate of spin precession is roughly proportional to $a\gamma$ where $a=(g-2)/2$ is the anomalous part of the spin g factor and γ is the Lorentz factor. At very high energy, as for example in the HERA proton ring, it could happen that on average $\vec{n}(\vec{z},\theta)$ deviates by tens of degrees from the phase space average of $\vec{n}$. Thus even if each point in phase space were 100% polarized the average polarization could be much smaller than 100%. Clearly it is very important to have accurate and efficient methods for calculating $\vec{n}(\vec{z},\theta)$ and for ensuring that the spread of $\vec{n}(\vec{z},\theta)$ is as small as possible.

3 COMPUTATIONAL TECHNIQUES

3.1 Straight Forward Polarization Tracking

One can try to get information about the spread of the equilibrium spin directions over phase space by tracking a completely polarized beam for many turns. This is illustrated in figure 1. Particles at 100 different phases at a normalized one sigma vertical emittance of 4πmm mrad and zero longitudinal and horizontal emittance have been tracked through HERA for 500 turns while the beam was initially 100% polarized parallel to the equilibrium spin direction on the closed orbit, $\vec{n}_0 = \vec{n}(0,\theta)$. Similar kinds of tracking results have been presented in [5]. Since this polarization distribution is not the equilibrium distribution, the averaged polarization exhibits a strong beat. This figure also shows that when spins at phase space coordinate $\vec{z}$ are initially parallel to $\vec{n}(\vec{z},\theta)$, the averaged polarization stays constant. Therefore, by starting simulations with spins parallel to the $\vec{n}$-axis one can perform a much cleaner analysis of beam polarization in accelerators.

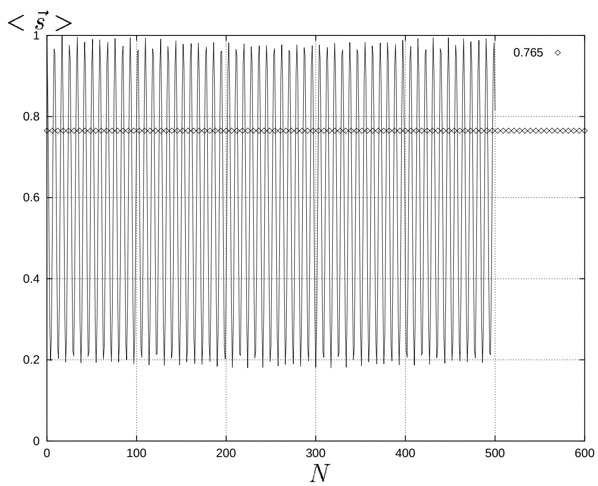

Figure 1: Propagation of a beam that is initially completely polarized parallel to $\vec{n}_0$ leads to a fluctuating average polarization. For another beam that is initially polarized parallel to the periodic spin solution $\vec{n}$ the average polarization stays constant, in this case equal to 0.765.

3.2 The Linear SLIM Theory

A well established and often justified procedure is to linearize the orbit motion. When dealing with spins, one can introduce the components α and β of the spin perpendicular to $\vec{n}_0$. Then one can linearize with respect to these angles. However, since the spins in a very high energy accelerator such as HERA are spread widely over phase space, these components can no longer be considered to be small and this linearization is not justified. Nevertheless the technique has been used to get a first indication that the equilibrium polarization in the HERA proton ring could be limited due to a large spread of the opening angle [6].

3.3 Nonlinear Normal Form Analysis

Nonlinear normal form theory provides solutions of equation (1) beyond first order by computing the Taylor expansion of $\vec{n}(\vec{z},\theta)$ with respect to $\vec{z}$. Theoretically the evaluation order is not limited, but computational errors typically limit the normal form approach to fifth order. Experience has shown that except when the spread of polarization directions is small, this method diverges and a non-perturbative method is needed. When the spread has already been reduced by some means, this method has successfully been applied in the filtering method mentioned below.

3.4 Stroboscopic Averaging

However, in order to handle the general case we have developed a new method for obtaining $\vec{n}$ called stroboscopic averaging[7]. It is based on multi–turn tracking and the averaging of the spin viewed stroboscopically from turn to turn. Since this innovative approach only requires tracking data, it is fast and very easy to implement in existing tracking codes. Probably the main advantage over other methods is the fact that stroboscopic averaging does not have an inherent problem with either orbit or spin orbit resonances due to its non-perturbative nature. This allows the behavior of the periodic spin solution close to resonances to be analyzed.

The stroboscopic average has a very simple physical interpretation which illustrates its practical importance. If a particle beam is approximated by a phase space density, disregarding its discrete structure, then we can associate a spin field $\vec{f}(\vec{z},\theta_0)$ with the particle beam at the azimuth θ_0. If one installs a point like 'gedanken' polarimeter at a phase space point $\vec{z}_0 = \vec{z}(\theta_0)$ and azimuth θ_0, then this polarimeter initially measures $\vec{f}(\vec{z}_0,\theta_0)$. When the particle beam passes the azimuth θ_0 after one turn around the ring, the polarimeter measures

$$\underline{R}(\vec{z}(\theta_0-2\pi),\theta_0-2\pi)\cdot\vec{f}(\vec{z}(\theta_0-2\pi),\theta_0)\ . \qquad (2)$$

After the beam has traveled around the storage ring N times and the polarization has been measured whenever the beam passed the 'gedanken' polarimeter, one averages over the different measurements to obtain $<\ \vec{f}\ >_N$. In [7] it has been proven that this average is parallel to $\vec{n}(\vec{z}_0,\theta_0)$. If the particles of a beam are polarized parallel to $\vec{n}(\vec{z},\theta_0)$ at every phase space point, then the spin field of the beam is invariant from turn to turn due to the periodicity property in equation (1). But in addition, even for beams which are not polarized parallel to $\vec{n}$, we see that the polarization observed at a phase space point $\vec{z}$ and azimuth θ_0 is still parallel to $\vec{n}(\vec{z},\theta_0)$, if one averages over many measurements taken when the beam has passed the azimuth θ_0.

4 POLARIZATION IN HERA WITHOUT SIBERIAN SNAKES

In figure 2 we display how the equilibrium polarization in HERA as it is currently operating varies around the two sigma vertical phase space ellipse at 818GeV. This distribution could only be computed with stroboscopic averaging, as implemented in the program SPRINT. The wide variation of spin directions illustrates clearly that no polarized beam could be stored in HERA currently, even if a completely polarized beam were delivered at high energy.

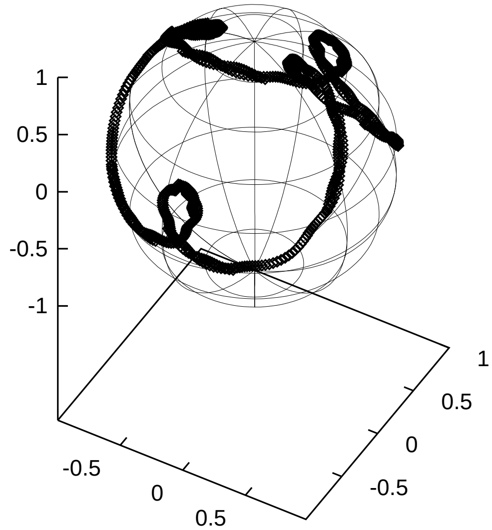

Figure 2: Variation of $\vec{n}(\vec{z})$ over a 2σ vertical ellipse in the current HERA proton ring.

5 POLARIZATION IN HERA WITH SIBERIAN SNAKES

Since the HERA proton ring incorporates vertical bends, conventional schemes of choosing most effective Siberian Snakes are no longer directly applicable. We therefore used the following automatic filtering method which tries all possible Siberian Snake combinations and eliminates unfortunate choices in various stages of complexity to illustrate that the spin distribution can be significantly improved by an appropriate configuration of Siberian Snakes. The procedure works as follows:

1. Find all snake combinations for a flat ring which lead to a spin tune of $\nu = 1/2$.
2. Compute the spin tune on the closed orbit for the non–flat HERA ring and filter on small spin tune deviations away from $\nu = 1/2$.
3. Compute the 1st order opening angle for the non–flat HERA proton ring and filter on small opening angles of the equilibrium spin distribution.
4. Compute the opening angle by 3rd and 5th order normal form theory for the snake combinations which are left after all these filters.

The polarization distribution over the two sigma vertical phase space for the choice of Snakes found by this nonlinear filtering algorithm and calculated using SPRINT is displayed in figure 3. Now obviously the polarization is rather tightly bundled and a polarized beam can be stored in HERA. Further optimization might even improve the situation. It therefore seems fair to state that the variation of $\vec{n}(\vec{z}, \theta)$ over phase space at 818 GeV in HERA will not render the storage of polarized protons in HERA impossible.

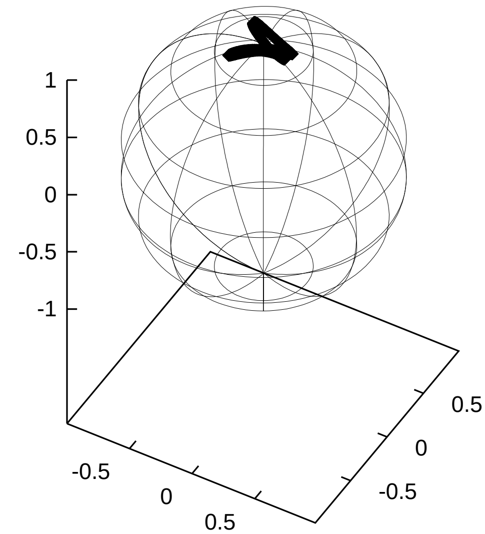

Figure 3: Variation of $\vec{n}(\vec{z})$ over a 2σ vertical ellipse after installation of the best Siberian Snakes found by the filtering procedure.

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