Coherent Synchrotron Radiation Modeling in the ERL

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In high energy X-Ray production machines, such as the proposed Energy Recovery Linac (ERL) at Cornell, precise modeling of Coherent Synchrotron Radiation (CSR) and its effects on a electron beam are crucial in developing an efficient and well understood system. To achieve a high quantity of X-Ray production, short bunch lengths and high charge density bunches are desired, both worsening the effects of CSR on the electron bunches. Simulation programs such as TAO and ELEGANT have been developed in an attempt to model the effects of CSR on various characteristics of the beam. However, due to differences in the algorithms used to calculate CSR, the output values of these programs differ slightly. While it is advantageous to determine if CSR effects on our beam are within acceptable limits, these differences also demonstrate a need to compare the two programs and determine if TAO is a sufficient simulation program to be used in modeling the ERL. Ultimately, it is found that beam modeling in TAO compares reasonably well with ELEGANT in most cases, considering the rough approximation used in ELEGANT. In other cases, the difference are so large tat a further analysis of the algorithms and their implementation is necessary. Using this approximation, the effects due to CSR in the full ERL were found to be tolerable in three of the five proposed working modes. In the Short bunch mode, the simulated CSR emittance growth in the currently proposed lattice is too large.

I. INTRODUCTION

The proposed Energy Recovery Linac, which will act as a short pulse high-coherence and high-spectral-brightness X-Ray source, will require smaller particle bunch lengths and larger charge densities than in conventional ring based x-ray sources. However, as bunch lengths get shorter, the effects of particles within the bunch interacting with each other become more prominent. The effect of Coherent Synchrotron Radiation (CSR) on the bunch as a whole must then be considered in order to give an accurate approximation of the beam's dynamics. Simulation programs such as TAO and ELEGANT attempt to model this behavior by calculating CSR effects using two separate computing algorithms. TAO and ELEGANT have to be compared to determine the more effective in both calculation accuracy and computing time. Thus we analyze difference in the beam's energy as a function of position in the system, emittance, and bunch length. Due to the local development of TAO at Cornell University, this program would be the preferred to be used in beam modeling of the ERL. With these outputs, it can now be determined whether the current ERL layout operates within acceptable limits when accounting for CSR effects on the particle bunch.

II. CSR EFFECTS ON THE PARTICLE BEAM

To understand the algorithms used to model the effect of CSR on our bunch of particles, the CSR force between particles must first be discussed.

It can be generally stated that all charged particles that accelerate emit radiation. When these high-energy particles are moving at relativistic velocities, this radiation is deemed "synchrotron radiation". For non-relativistic particles, the power of the radiation emitted is small by a particle of charge e, mass m_0, and momentum p, or specifically

$$P_e = \frac{e^2}{6\pi\epsilon_0 m_0^2 c^3}\left(\frac{dp}{dt}\right)^2 \tag{1}$$

meaning that, with the large denominator dominating the power-emitted term, ignoring this radiation would not greatly change our modeling of particles moving at non-relativistic velocities. Here t characterizes the laboratory time, c the speed of light, and ϵ_0 the susceptibility of free space. However, as the particle's velocity approaches the speed of light, a Lorentz transformation must be applied to our momentum term, leaving

$$P_e = \frac{e^2 c}{6\pi\epsilon_0}\frac{1}{(m_0c^2)^2}\left[\left(\frac{dp}{dt}\right)^2 - \frac{1}{c^2}\left(\frac{dE}{d\tau}\right)^2\right] \tag{2}$$

This means that the radiated power depends greatly on the angle between $\overrightarrow{v}$ and $\frac{d\overrightarrow{v}}{d\tau}$. This emitted power is at a maximum when the two vectors are perpendicular (when the particle is moving around a circular path with radius R)[4]. Approximating our expression for the circular, relativistic case, we can determine a relationship between the radiation emitted and the initial energy of the charge particles

$$P_e = \frac{e^2 c}{6\pi\epsilon_0}\frac{1}{(m_0c^2)^4}\frac{E^4}{R^2} \tag{3}$$

This expression gives us the relation that the emitted radiation is proportional to E^4[4]. So in high energy X-Ray production lattices such as the ERL, Synchrotron Radiation becomes an unignorable factor in modeling our beam.

When analyzing our particle bunch in the near-field picture, the effect of one particle's emission on another within the bunch results in an adding "coherent effect" if the total bunch length is on the order of the emitted radiation wavelength. This "coherent effect" results in the emitted radiation being proportional to N_p^2, the square of the number of particles within the bunch. This effect can lead to "microbunching" when the radiation interacts back on the bunch. This can result in a greater emittance of the beam and to in increase to the bunch's energy spread. As bunch length gets smaller, the ratio of this Coherent SR to Incoherent SR gets larger, demonstrating the need to account for these effects in our beam in the ERL.

III. CSR KICKS BETWEEN TWO PARTICLES

To correctly model the CSR effect on a particle bunch moving through the ERL, we must first look at the effect on a single particle's orbit due to an emitted radiation field. The Lienard-Wiechert formulae for the absorbed electric and magnetic fields model this effect.

They contain a "near-field", or space charge term, and a radiative term. In ELEGANT the space charge term is neglected, and in TAO the two terms are treated separately to avoid divergences. In our analysis, however, we have switched off the space charge term in Tao in order to achieve a better comparison to ELEGANT.

IV. COHERENT RADIATION CALCULATION APPROXIMATIONS-ELEGANT AND TAO

Difficulty arises in trying to evaluate the CSR force, K_{CSR}, as the distance between a radiative particle and a test particle becomes smaller due to a singularity. TAO and ELEGANT deal with this difficulty differently.

ELEGANT's approximation approach, identifies four cases in which the two particles can be found in the lattice in which CSR effects would be prominent:

1)*Source Particle (P') not in bending element, Kicked Particle (P) in bending element*

2)*Both Particles in bending element*

3)*Both Particle not in bending element*

4)*P' in bending element, P not in bending element*

These cases are shown in Fig. (1). For each case approximations are made that allow an analytic evaluation [1].

However, these approximations limit the flexibility of the calculation. In considering four unique cases, specific restrictions on z are necessary. This limits the cases that can be considered accurately, especially for large bunch lengths and lower values of γ. Furthermore, the approximations are such that CSR effects due to any transverse distribution of the particles cannot be accounted for [3].

Similar approximations are used in TAO, especially in that the transverse distribution is not taken into account. However, it does not become necessary to separate the four cases in Fig (1) [1].

V. COMPARISON BEAM PARAMETERS

To analyze the effect of CSR kicks on the beam, certain characteristics of the particle bunch were plotted to: *a)*determine if the current ERL design operates within acceptable limits of the CSR effect. *b)*compare deviation in output values between the two programs.

In order to understand why certain characteristics of the beam are compared, it is helpful to define and discuss their resulting behavior due to CSR effects.

The four beam characteristics that were compared were:

1)**Energy deviation as a function of lattice position:**

When radiation is emitted when the particles pass through a bend, this radiation can act on other particles in the bend, increasing or decreasing their energy. This can change the average energy as well as the distribution of energies within the bunch. When outputted from TAO and ELEGANT, this energy loss is expressed as a percent change from the initial energy of the system.

2)**Energy Spread**

When the CSR force adds energy to some particles, and subtracts from others, the spread of energies within the beam changes.

3)**Bunch length:**

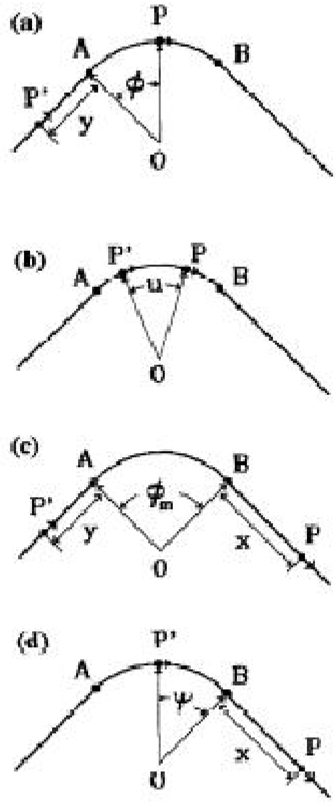

FIG. 1: Four unique cases used in ELEGANT approximations where ϕ and φ are angles of separation between the two particles. [2]

Plotting the distribution of the beam as a function of position, the change in the rms bunch length, σ_{s}., can be shown. Going through a bend, higher energy particles traverse a longer path while the lower energy particles traverse a shorter one. Together with an RF cavity that increases the energy in the front of the bunch, but reduces it in the back of the bunch, this path difference can lead to a compression in the bunch length. As CSR is emitted from the bunch and "kicks" in the front and back are taken into account, a similar change of the bunch length can occur.

4)**Transverse emittance of x and y:**

Defining x and y as being orthogonal to the direction of travel of the bunch, each particle can be plotted in phase space with x vs x', its horizontal slope, and y vs y', its vertical slope.

Both transverse position and velocity of the particles can increase when particles that have experienced different CSR kicks follow different trajectories through bending magnets. This results in an increase of the area in phase space and thus the horizontal emittance. Studying this emittance growth is crucial in evaluating the efficiency of x-ray production in the undulator sections.

VI. RESULTS

Three separate sections of the ERL lattice (a specified series of accelerator elements such as dipoles, quadrupoles, etc) were analyzed:

1) *Proposed ERL Turnaround Section:* curved section following initial injection and first Linac of the system.

2) *Proposed ERL High Energy Loop:* North and South arc section of CESR following the second Linac of the system.

3) *Undulator section with compressed bunch in ERL:* The section of the ERL lattice following the old CESR ring in which the particle bunch is compressed. This section is composed of a series of undulator sections acting as x-ray sources for the experimental beam lines. This is in shown in the Northern Arc of Fig (2).

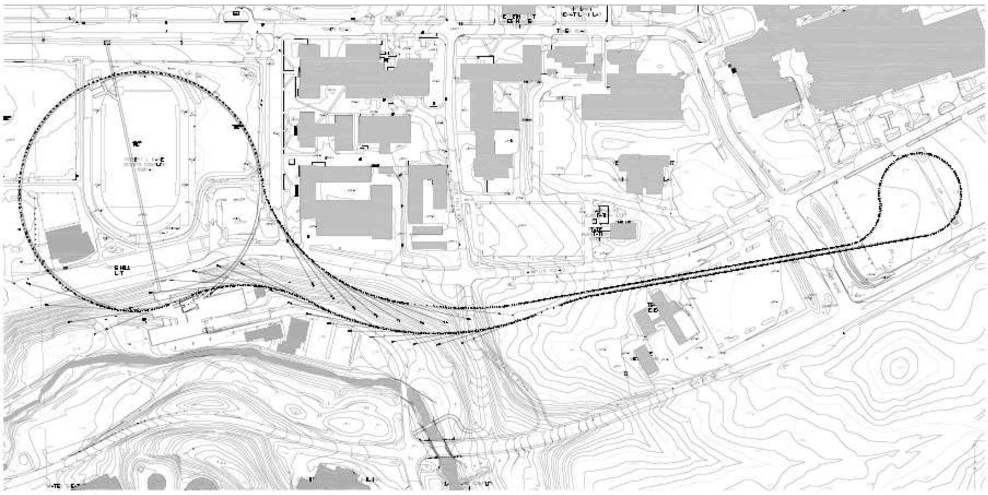

FIG. 2: ERL lattice modeled in Tao and Elegant: The turnaround section to the right of the figure and the High Energy loop to the left of the figure. The Short bunch section is located along the beam lines of the Northern Arc

The ERL turnaround section and the High energy loop were looked at under three proposed operating modes, listed in Fig (3), to see if varying initial parameters, such as bunch lengths and emittance values, would cause a more drastic effect due to CSR. The Short Bunch analysis was performed with a notably larger bunch charge and higher emittance value. These initial parameters can be seen in Fig (3).

As an initial analysis, the effect of the space charge term on the CSR calculation in Tao was probed to determine how drastically it would contribute to the final result, causing output values to deviate from Elegant. This was shown to have a very small contribution to CSR effects.

This effect was then not calculated for the remainder of our analysis to allow a closer comparison to Elegant.

The output plots from the analysis of the turnaround section can be seen in Fig (4) and Fig (5).

While the plot of energy spread with operating mode A shows differences between Tao and Elegant, on the whole, the CSR contribution is the Turnaround section is seen to be small, with only a fraction of a MeV of power being lost from an initial 2.505 MeV and only

		Short term			Long Term	
MODES	Flux	High Coherence	Short Pulse	Ultra High Coherence	Ultra Short Pulse	Units
Energy	5	5	5	5	5	GeV
Macropulse Current	100	25	1	100	1	Milli-A
Bunch Charge	77	19	1000	77	10000	PicoC
Repetition Rate	1300	1300	1	1300	0.1	Mhz
Transverse emittance (norm rate)	0.3	0.08	5	0.06	5	mm.mrad
Transverse emittance (geometric at 5 GeV)	31	8.2	511	6.1	511	Picom
Bunch Length (rms)	2000	2000	50	2000	20	Femtosec
Introbunch energy spread (fractional rms)	2e-4	2e-4	3e-3	2e-4	3e-3	
Beam power	500	125	5	500	5	MW
Beam loss	< 1	<1	< 1	< 1	< 1	?A

FIG. 3: Parameter list which was used in the three cases analyzed. These cases were Flux (A), Coherence (B), and Ultra High Coherence (D)

TABLE I: Significance of the space charge term in Tao calculations. Shown is the percentage difference between the case with and without the longitudinal space charge term.

Beam Parameter	**Turnaround** Difference (percent)	**High Energy Loop** Difference (percent)
Beam Energy	.649	1.15
Bunch length	.026	0
Energy Spread	.030	.018
X Normalized Emittance	.010	.714

a insignificant increase to transverse emittance occurs. It was seen in both the Turnaround section and the High Energy loop that the Y Normalized emittance remained constant and therefore is not included in our discussion. For bunch length and transverse emittance, since each changes normally through our system with no CSR accounted for, it proved more advantageous for our discussion to only focus on the contribution of the change due to CSR. Thus each is plotted as the difference of the CSR case vs the case with no CSR calculated.

In the High Energy Loop, shown in Figure (6) and Figure (7), it proved advantageous to not only determine the maximum change in our beam parameter due to CSR effects, but to analyze specifically the beam's properties in the undulator sections. If, the emittance in increased in these sections, x-ray spectral brightness is reduced, thus undermining the overall effectiveness of the machine. For this reason, the average values and peak values of bunch length change and emittance change in the undulators were also calculated in addition to the maximum change in the lattice. By analyzing these undulator values in particular, we can hope to get a better understanding of the "state" of our beam as it undergoes the x-ray emission desired by the system.

Similar to the results for the Turnaround section, the High energy loop produces small changes in total energy as well as only little change in energy spread. Bunch length was also seen to change negligibly. While the emittance growth in the undulator sections was notably higher than in the Turnaround section, this reported average of roughly 11 % is deemed tolerable.

In the short bunch analysis, more drastic changes in beam parameters can be seen. With a shorter bunch length and larger charge more CSR generation is to be expected and the changes to transverse emittance as well as bunch length are very significant. Most notably, an energy loss of 11 - 16 MeV from the 5 GeV beam is simulated by the end of the undulator section. These results seem trustworthy since Tao and Elegant returned similar values.

VII. CONCLUSIONS OF COMPARISON

It was determined that, under the current operational mode goals specified in table (3), that Cases A & B of the short term goals, and Case D of the long term goals demonstrated no traumatic effects due to CSR in either the Turnaround section or the High Energy loop. For the short bunch operation of Case C, however, due to the high emittance growth, bunch length growth, and unacceptable energy losses in the undulator sections, it was determined that more analysis needs to be performed. Therefore, the more extreme case (E) was not analyzed. An effort should be made to construct a lattice that reduces the CSR effect for high charge, short bunch operation.

However, it is important to note that the CSR analysis discussed in this paper contain approximations that contribute to overwhelming differences in some of the output values. A closer analysis of especially the rather new Tao algorithm is therefore necessary. Because the linacs were not simulated in this setup, a realistic phase space distribution could not be generated and propagated through the lattice. Instead, an ideal Gaussian distribution with no momentum chirp was used. This idealization undoubtedly would effect our final results. The presented CSR analysis should thus be repeated for more realistic phase space distributions.

The algorithms used in both Tao and Elegant also do not take into account the effect of Incoherent Synchrotron Radiation (ISR), which can be very significant for the designed ERL lattice. Further analysis should be performed including this contribution as well.

While performing the here presented brief synopsis of the beam dynamics, computing multiple variables as a function of position in the ERL lattice, we also obtained a comparison of the algorithms used by Tao and Elegant. Because Tao has been recently implemented, it proved advantageous to benchmark and compare the CSR algorithms of Tao to an already established simulation program such as Elegant. Demonstrating that the extra space charge term used in Tao's computation had minimal effect, the comparison between the two programs demonstrated the difference in CSR algorithms alone. Results from the two codes proved to be reasonably close in many cases, with the two differing up to a factor of 2. In some cases, however, the differences were so large that a closer investigation is needed.

VIII. ACKNOWLEDGMENTS

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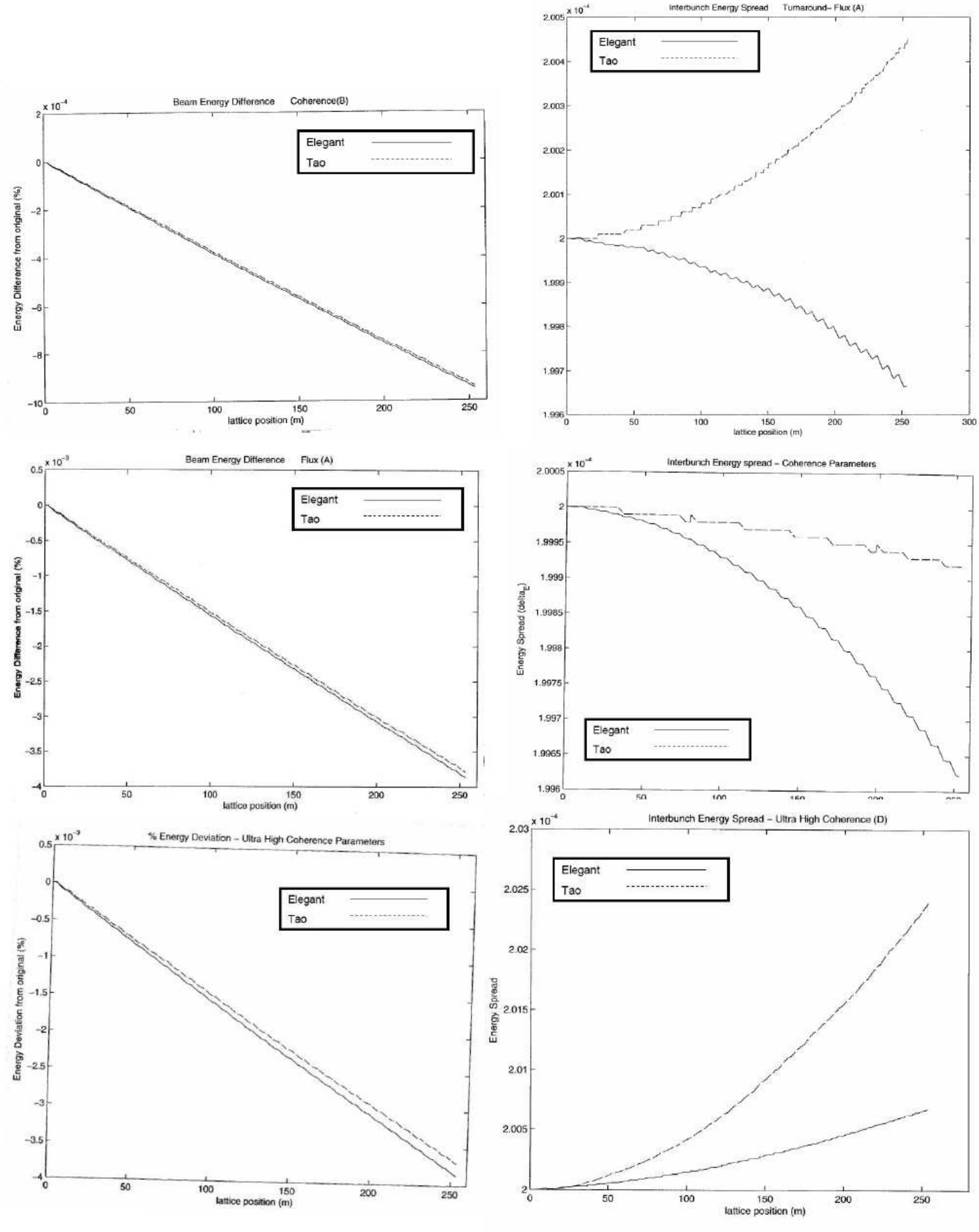

FIG. 4: Plots of Energy Spread and Energy Difference vs lattice position in the Turnaround Section for each of the three operating modes (A)-top, (B)-middle, (D)-Bottom.

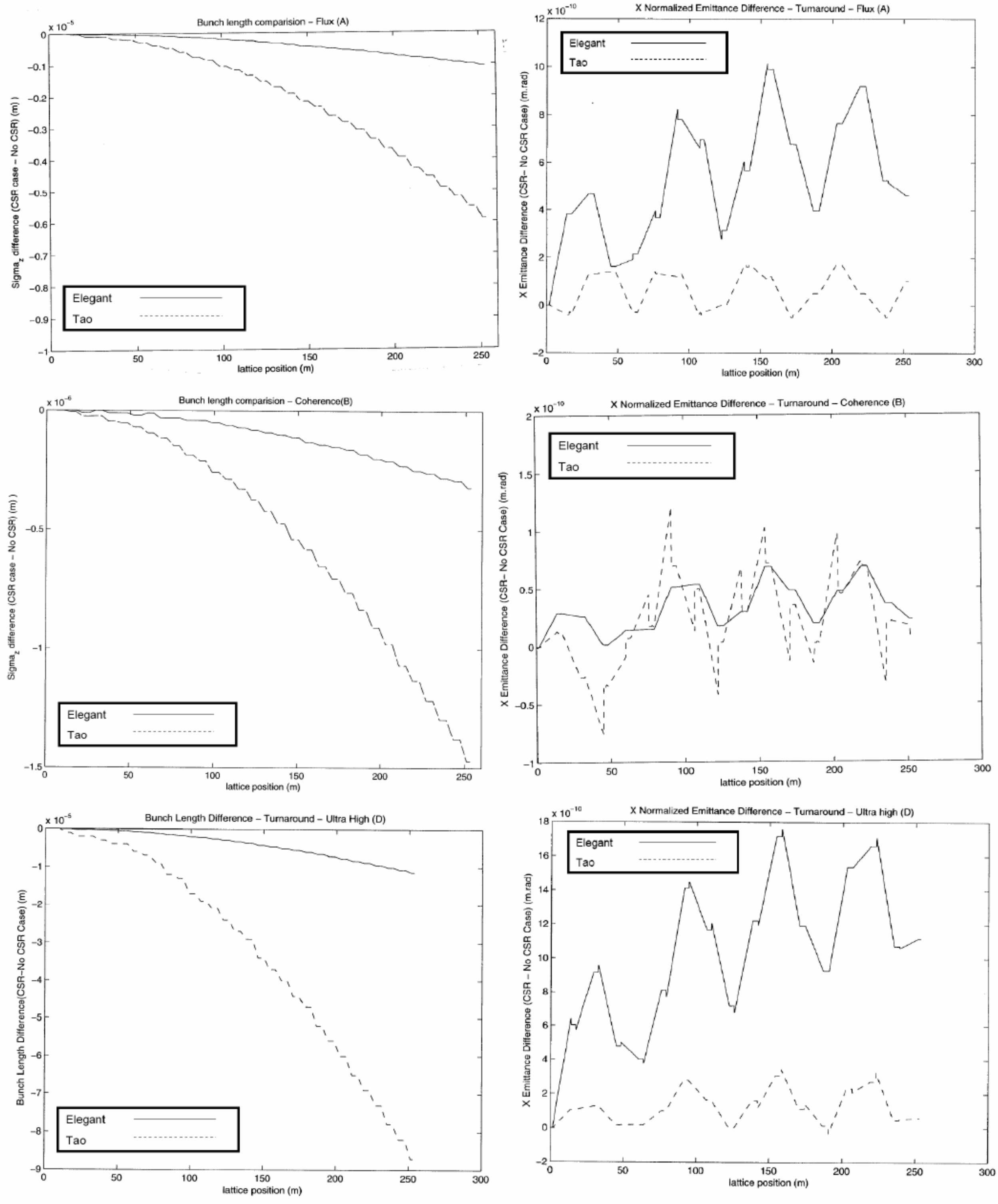

FIG. 5: Plots of bunch length and emittance vs lattice position in the Turnaround Section for each of the three operating modes (A)-top, (B)-middle, (D)-Bottom.

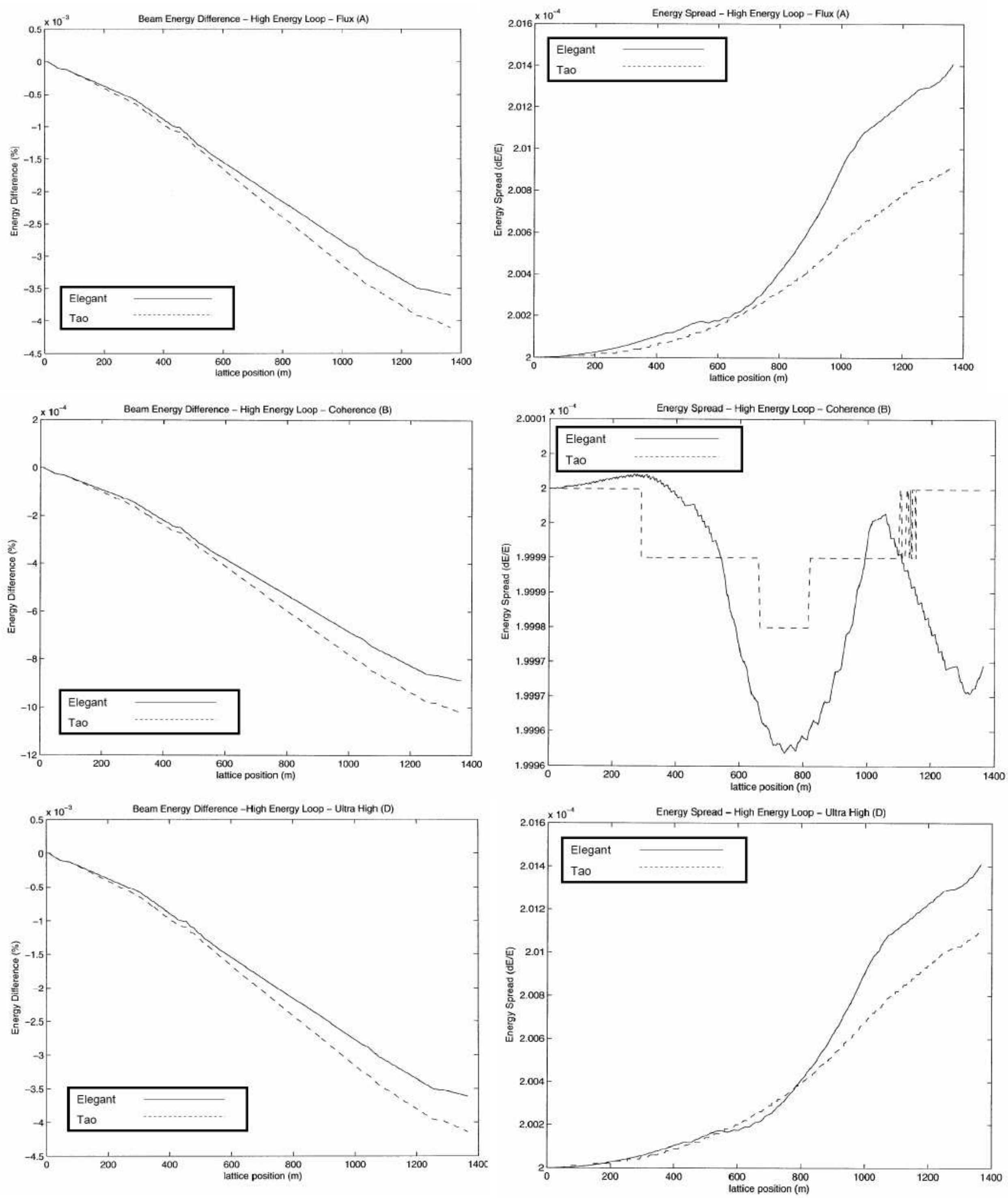

FIG. 6: Plots of Energy Spread and Energy Difference vs lattice position in the High Energy Loop for each of the three operating modes (A)-top, (B)-middle, (D)-Bottom.

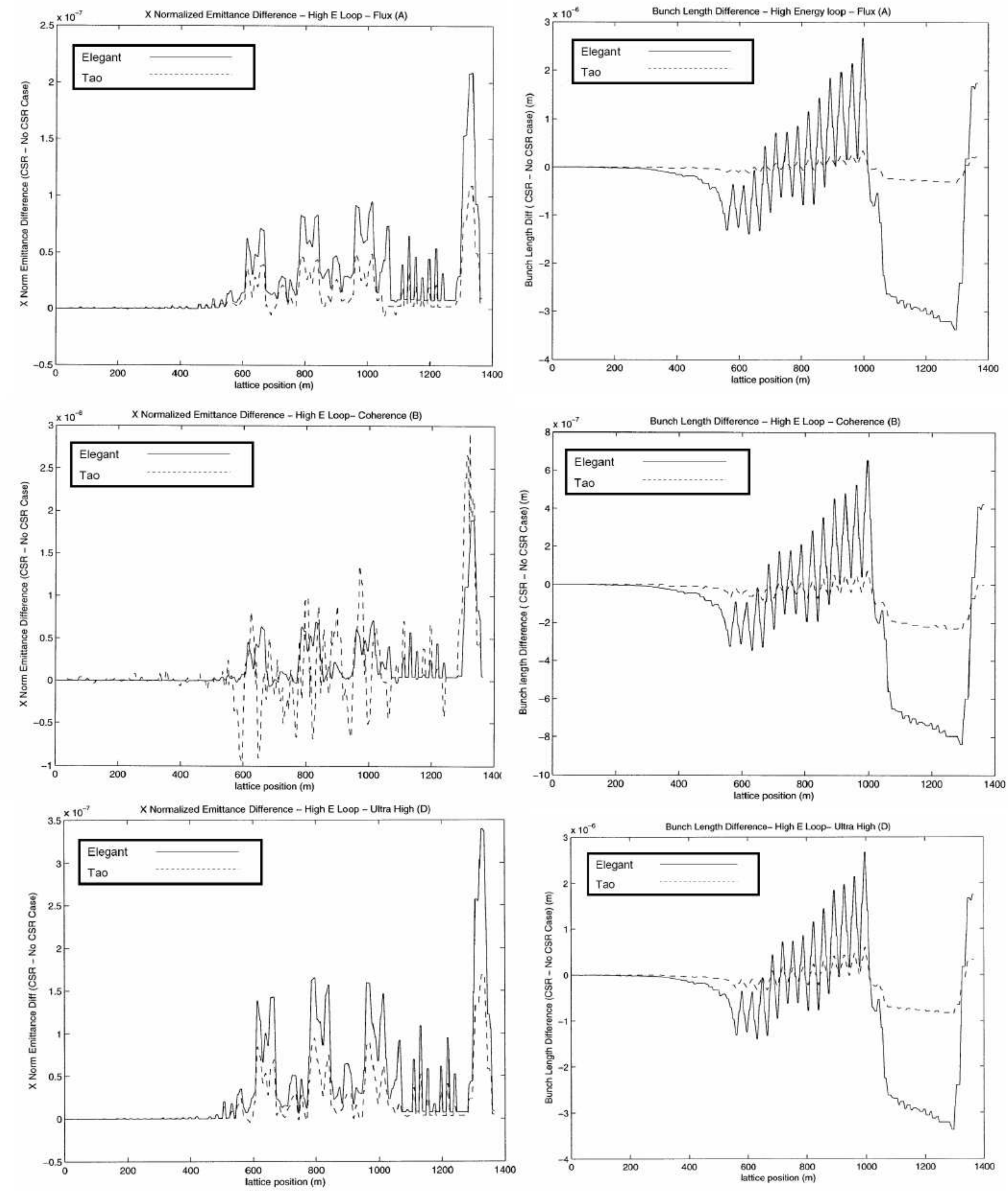

FIG. 7: Plots of Bunch length and X Normalized Emittance vs lattice position in the High Energy Loop for each of the three operating modes (A)-top, (B)-middle, (D)-Bottom.

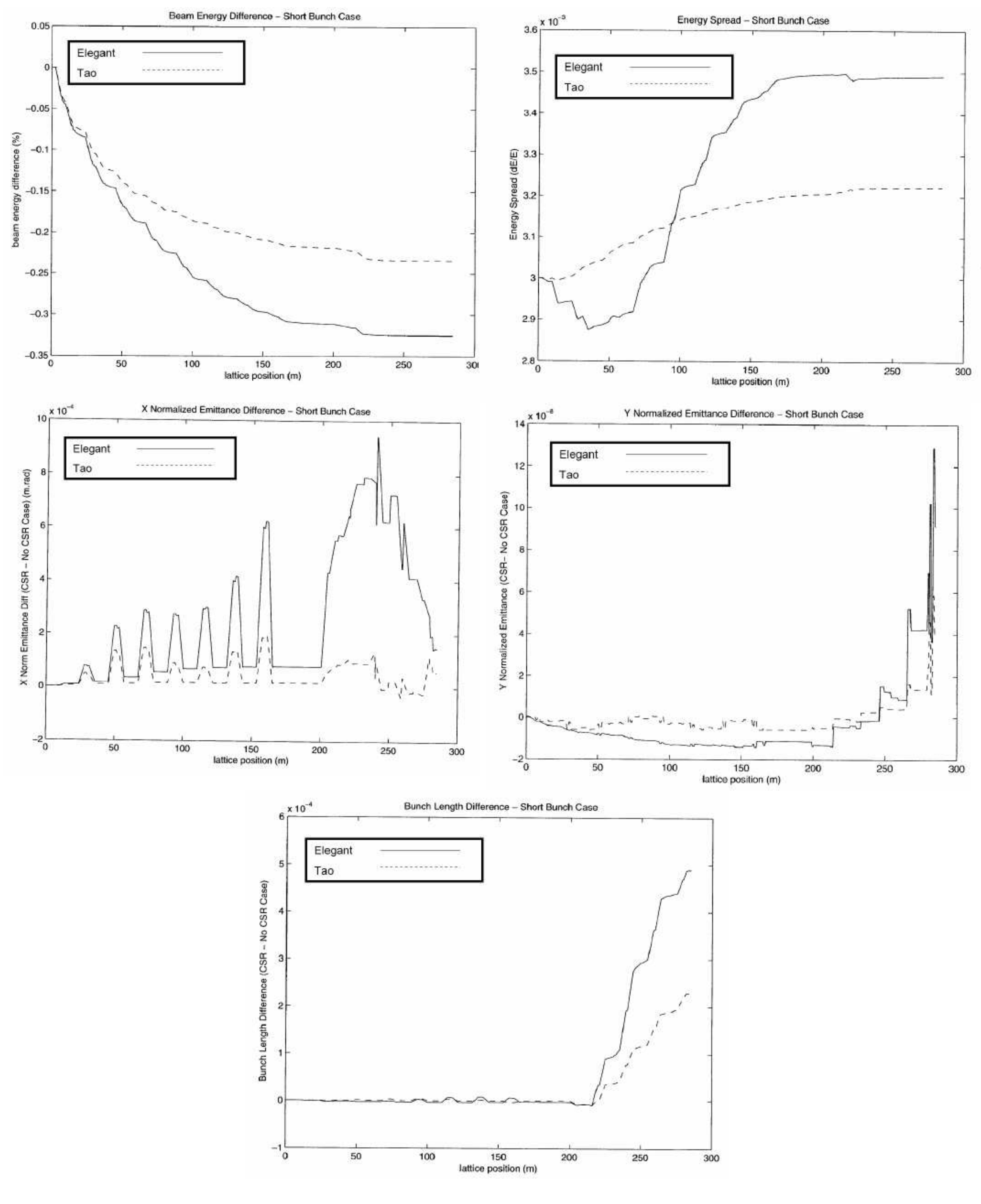

FIG. 8: Plots of compared beam parameters vs lattice position in the North Arc undulator section of the ERL under the "Short Bunch" operating modes.

ENERGY DIFFERENCE			
		Tao	Elegant
		CSR contribution (MeV)	CSR contribution (MeV)
	Case	Max deviation in lattice	Max deviation in lattice
Turnaround	A	0.094	0.097
	B	0.0235	0.0237
	D	0.095	0.179
High Energy Loop	A	0.205	0.181
	B	0.051	0.045
	D	0.207	0.181
Short Bunch	C	11.25	16.25

ENERGY SPREAD							
			Tao			Elegant	
			CSR contribution (%)			CSR contribution (%)	
	Case	Max in undulators	Average in undulators	Max deviation in lattice	Max in undulators	Average in undulators	Max deviation in lattice
TA	A	none	none	0.225	none	none	-0.167
	B	none	none	0.1	none	none	-0.188
	D	none	none	0.34	none	none	0.12
High Energy Loop	A	0.5	~0	0.46	0.5	~0	0.704
	B	~0	~0	0.01	~0	~0	0.013
	D	0.5	~0	0.55	0.5	~0	0.705
Short Bunch	C	6.8	3.76	7.7	16.7	6.7	16.7

FIG. 9: Energy Outputs showing the CSR effect on the beam for each of the four operating modes

BUNCH LENGTH							
			Tao			Elegant	
			CSR contribution (%)			CSR contribution (%)	
	Case	Max in undulators	Average in undulators	Max deviation in lattice	Max in undulators	Average in undulators	Max deviation in lattice
TA	A	none	none	-1	none	none	-1.83
	B	none	none	-0.242	none	none	-0.05
	D	none	none	-14.3	none	none	-2
High Energy Loop	A	0.33	0.16	-0.058	0.01	-0.02	-0.567
	B	0.33	0.16	-0.036	0.33	0.16	-0.14
	D	0.33	0.16	-0.142	0.01	-0.025	-0.567
Short Bunch	C	673.3	366.7	1533	658.5	329.9	3333

			X NORMALIZED EMITTANCE				
			Tao			Elegant	
			CSR contribution (%)			CSR contribution (%)	
	Case	Max in undulato rs	Average in undulators	Max deviation in lattice	Max in undulators	Average in undulators	Max deviation in lattice
TA	A	none	none	0.058	none	none	0.33
	B	none	none	0.156	none	none	0.088
	D	none	none	0.516	none	none	2.97
High Energy Loop	A	1.69	0.088	70	3.96	1.79	36.6
	B	3.69	1.8	21.88	4.1	1.98	3375
	D	11.28	5.4	266.7	11.01	7.8	566.7
Short Bunch	C	340	240	3400	1583	1041	19000

FIG. 10: Chart of Bunch length and Emittance difference results with change in σ_s and normalized emittance as the contribution percentage from CSR alone