Evaluation of Beam Position Monitors in the Nonlinear Regime

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Abstract

We present an algorithm for processing BPM signals and extracting orbit and betatron phase advance that is valid for large closed orbit distortions. We calculate the BPM button response using Green's reciprocity theorem and numerical solution of Laplace's equation in two dimensions. The difference between the calculated signals and the measured signals is minimized on the ¤y to to yield bunch position. We extend this technique for calculating the betatron phase advance.

INTRODUCTION

Typical Beam Position Monitors (BPMs) consists of four button-type electrodes mounted ¤ush with, and electrically isolated from, the surface of the beam pipe.

The BPM buttons are connected to electronics that process and record signals which are a function of the distance between the button and the passing bunch. The four signals from each BPM are used to determine the beam position and betatron phase advance.

Our efforts to improve the beam position measurements by including the nonlinear BPM response is motivated by CESR's pretzel orbits, where electron and positron beams avoid parasitic collisions by following separate paths with large displacements from the central axis of the beam pipe.

BACKGROUND

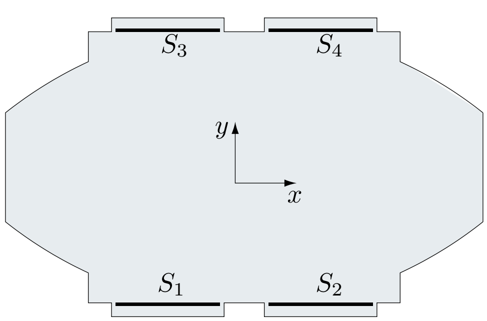

Figure 1: Arrangement of buttons in CESR arc BPMs

Many accelerators, including CESR, have traditionally assumed a linear relationship between the beam position and the BPM button signals. Given four signals S_i $(i = 1, \ldots, 4)$ from buttons arranged as in Fig. 1, the transverse beam position is given approximately by

$$x = k_x \frac{(S_2 + S_4) - (S_1 + S_3)}{\Sigma_i S_i} \tag{1}$$

$$y = k_y \frac{(S_3 + S_4) - (S_1 + S_2)}{\Sigma_i S_i} \tag{2}$$

where $k_{x,y}$ are scale factors set by the geometry of each BPM type. This evaluation of BPM signals is often called the *difference-over-sum* method.

Equations (1-2) provide an estimation of the bunch position in relatively few arithmetic operations, but they are of limited use, since they yield only the linear part of the signal dependence for bunches near the center of the BPM. In the next section, we describe our technique for accurately calculating button signals, but let us £rst use those results to illustrate the limitation of the linearized formulae.

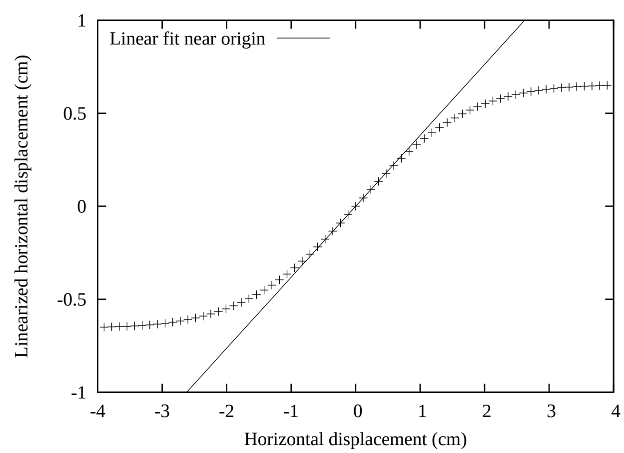

Figure 2: Linearized horizontal position measurement in an arc BPM. At a typical pretzel amplitude of 1.5 cm, the linearized formula shows signi£cant disagreement with a linear £t.

In Fig. 2, the four button signals were calculated numerically for a beam at different horizontal displacements. The four signals were combined according to Eq. (1) and plotted against the known horizontal displacement. The slope near the origin gives k_x^{-1}, but the linear relationship breaks down noticeably at approximately 1 cm, and beyond 2 cm the relationship fails completely. Because pretzel orbits in CESR are typically as large as 1.5 cm, this is precisely what has hindered accurate measurements under colliding beam conditions until the improvements described in this paper were implemented.

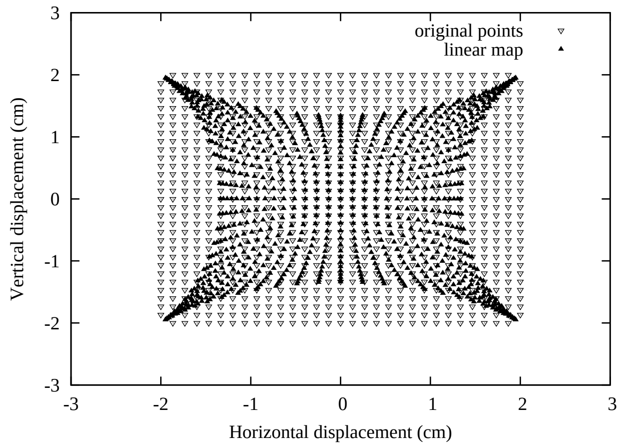

Figure 3: Linearized map distortion in CESR interaction-region BPM with approximately circular cross-section.

The problem of this nonlinearity is made even more evident in two dimensions. Figure 3 the mapping of those same points under Eq. (1-2). The BPM shows the characteristic *pincushion* distortion, which increases with distance from the origin.

In CESR, betatron phase measurements rely on a related assumption about the linearity of the button signals. The betatron phase is measured by shaking the beam at a sideband of the betatron frequency. For each detector, the phase for each button is calculated by electronically comparing AC signal on that button to the phase of the shaking. The phases of the four buttons are weighted equally and averaged [1]. Large closed orbit distortions make unweighted averaging undesireable, and we later show an improvement to this technique.

AN IMPROVED SYSTEM FOR POSITION AND PHASE MEASUREMENT

In order to overcome the limitations described, a new system has been implemented with two major components: realistic numerical models of the button response, and an ef£cient algorithm for inverting the model to yield beam position.

Numerical Calculation of BPM Response

For accurate beam position measurements, a function is required that expresses the bunch location (x, y) as a nonlinear function of the button signals. Since the four button signals lead to two coordinates (and a scale factor), the problem is over-constrained, and this function cannot be obtained directly. The inverse (button signals from beam position), however, is readily obtainable by standard numerical techniques.

The most accurate and direct method of numerical solution is to simulate the bunch in a three-dimensional BPM, calculating the electromagnetic £elds, and from them, the charge on the buttons. The simulation could be repeated for different beam locations, and the £elds recalculated until enough solutions were accumulated to describe the behavior over the entire BPM. However, this is very computationally intensive, and can be avoided by the methods that follow.

Two-Dimensional Approximation For ultra-relativistic bunches in a beam pipe with constant cross section, the approximate electromagnetic £elds can be calculated using a two dimensional formalism [2, 3]. Assuming the bunch has negligible transverse extent, the charge distribution of the bunch may be written, in the lab frame, as

$$\rho = \delta(\mathbf{r} - \mathbf{r_0}) \sum_k \rho_k \cos\left(k(z - vt)\right) \qquad (3)$$

where the longitudinal dependence in z has been written as a Fourier expansion. Transforming to the reference frame of the bunch, the charge density and electric potential are written

$$\rho^* = \delta(\mathbf{r} - \mathbf{r_0}) \sum_k \frac{\rho_k}{\gamma} \cos(kz^*/\gamma), \qquad (4)$$

$$\Phi^* = \Phi(\mathbf{r}) \sum_k \frac{\phi_k}{\gamma} \cos(kz^*/\gamma). \qquad (5)$$

We write Poisson's equation $\nabla^2 \Phi^* = \rho^*$ in the bunch frame as

$$\left(\nabla_\perp^2 - \frac{k^2}{\gamma^2}\right) \Phi(\mathbf{r}) \phi_k = \delta(\mathbf{r} - \mathbf{r_0}) \rho_k \qquad (6)$$

where $\nabla_\perp^2$ is the two-dimensional transverse Laplacian. For bunches with length σ_l without appreciable longitudinal substructure, ρ_k is only relevant for $k \le \frac{1}{\sigma_l}$. The characteristic distance over which $\Phi(\mathbf{r})$ changes is the diameter a of the beam-pipe so that the order of magnitude estimate $|\nabla_\perp^2 \Phi(\mathbf{r})| \approx \frac{1}{a^2} |\Phi|$ can be made. For suf£ciently long bunches and suf£ciently large values of γ, the relevant values of k/γ can be neglected, i.e. when $\frac{1}{\gamma^2} \ll (\frac{\sigma_l}{a})^2$ and the solution is described by the two dimensional, electrostatic case

$$\nabla_\perp^2 \Phi(\mathbf{r}) = \frac{\rho_k}{\phi_k} \delta(\mathbf{r} - \mathbf{r_0}). \qquad (7)$$

Since we only need $\Phi(\mathbf{r})$ up to a multiplicative factor, we don't worry about the constant coef£cients on the right-hand side.

Green's Reciprocity Theorem Rather than perform a separate calculation of the button signals for many beam positions, we use Green's reciprocity theorem to calculate the button signals for all (x, y) inside the BPM with a single numerical calculation. This theorem states that the surface charge σ on a button due to a test charge at (x, y) is proportional to the potential at that same position when the test charge is absent and the button is excited by a potential $\mathcal{V}$.

Suppose we have two scalar potentials ϕ_1 and ϕ_2 in a volume V bounded by a surface S. Associated with the potentials are volume charge density ρ_i and surface charge density σ_i. *Green's reciprocity theorem* states that

$$\int_V \phi_1 \rho_2 \, dV + \oint_S \phi_1 \sigma_2 \, da = \int_V \phi_2 \rho_1 \, dV + \oint_S \phi_2 \sigma_1 \, da \, . \tag{8}$$

Connecting this result to the case of a BPM, imagine ϕ_1 corresponds to the potential when a single button is excited with a potential $\mathcal{V}$ and all other surfaces are grounded. We can calculate the potential $\phi_1(x, y)$ by numerical solution of Laplace's equation. For the second potential ϕ_2, we ground all surfaces and put a charge distribution $\rho_2(x, y)$ inside the BPM.

We plug the two cases into Eq. (8) and observe that the third integral vanishes because there is no volume charge for the £rst case ($\rho_1 = 0$ in V). The fourth integral vanishes because we grounded the beam pipe and the buttons ($\phi_2 = 0$ on S). Since $\mathcal{V}$ can be pulled out of the second integral, what remains is just the total charge on the button, labeled q_b, giving

$$\int_V \phi_1(x, y) \rho_2(x, y) \, dV = -\mathcal{V} q_b. \tag{9}$$

If ρ_2 is a point charge q located at (x_0, y_0), then the integral in Eq. (9) picks out the value $\phi_1(x_0, y_0)$. We arrive at the £nal relation

$$q_b = -\frac{q\phi(x_0, y_0)}{\mathcal{V}}, \tag{10}$$

remembering that $\phi(x, y)$ and $\mathcal{V}$ refer to the two different con£gurations.

Therefore, since the signal on a button is proportional to the induced surface charge on that button q_b, $\phi(x_0, y_0)$ is the solution to the problem of calculating the button signal, up to a multiplicative constant, as a function of the bunch location.

Calculation of the Button Signals Based on the previous arguments, we use *POISSON* to solve the boundary value problem for $\phi(x, y)$. For the two-dimensional boundary, we take a slice at the longitudinal midplane of each BPM. The £rst button is set (arbitrarily) at 10 volts and all other surfaces are grounded. *POISSON* computes the solution to Laplace's equation on a mesh which we use to construct continuous interpolating polynomials.

Using the geometric symmetry of the BPMs, re¤ections or rotations of the coordinates for the excited button in the £rst calculation of $\phi_1(x, y)$ give the signals $\phi_i(x, y)$ on the other three buttons.

Realtime Inversion

For beam position measurements, we start with button signals S_i and seek the location (x, y) of the beam. The result $\phi_i(x, y)$ from the *POISSON* calculation must be inverted, and since we have four constraints (four buttons) and three parameters (position (x, y), and a scale factor) we proceed by *£tting* the calculated button signals to the measured signals. We minimize the merit function

$$\chi^2 = \sum_{i=1}^{4} \frac{(q\phi_i(x, y) - S_i)^2}{\sigma_i^2}, \tag{11}$$

where $\phi_i(x, y)$ is the signal on the i^{th} button and the σ_i are the uncertainties in the measured signals (which we take to be the same for all four buttons). The factor q is proportional to the beam current and could be used for beam loss studies.

Phase Measurements

We can improve our measurement of the betatron phase advance between BPMs by incorporating our knowledge of the nonlinear button response. In this measurement, the beam is excited to small oscillations around its equilibrium position (x_0, y_0). Let the phase and amplitude of the AC signal on the i^{th} button be represented by the complex number $\mathbf{C}_i$, and let the phase and amplitude of the horizontal and vertical components of the oscillatory beam motion be represented by complex numbers $\mathbf{A}_x$ and $\mathbf{A}_y$, respectively. To £rst order, their relationship is given by

$$\mathbf{C}_i = r_{i,x} \mathbf{A}_x + r_{i,y} \mathbf{A}_y, \tag{12}$$

where the $r_{i,(x,y)}$ are the derivatives of the signal on the i^{th} button with respect to x (or y) evaluated at (x_0, y_0).

Given the measured $\mathbf{C}_i$, we calculate $\mathbf{A}_x$ and $\mathbf{A}_y$ by minimizing

$$\chi^2 = \sum_{i=1}^{4} \frac{1}{\sigma_i^2} |r_{i,x} \mathbf{A}_x + r_{i,y} \mathbf{A}_y - \mathbf{C}_i|^2. \tag{13}$$

Since the σ_i depend on the closed orbit deviation and the values of $\mathbf{A}_{x,y}$, the minimization must also be performed iteratively. The horizontal and vertical phase advance is then given by the complex phase of $\mathbf{A}_x$ and $\mathbf{A}_y$. Whenever a horizontal excitation creates a vertical amplitude, or vice versa, this method is used in CESR to compute the coupling coef£cients also.

REFERENCES

[1] D. Sagan, R. Meller, R. Littauer, and D. Rubin. Betatron phase and coupling measurements in the cornell electron/positron storage ring. *Physical Review Special Topics - Accelerators and Beams*, 3, 2000.

[2] J.H. Cupérus. Monitoring of particle beams at high frequencies. *Nuclear Instruments and Methods*, 145:219–231, 1977.

[3] S. Krinsky. Measurement of the longitudinal parameters of an electron beam in a storage ring. In M. Month and S. Turner, editors, *Frontiers of Particle Beams; Observation, Diagnosis, and Correction*. Springer-Verlag, Berlin, 1989.