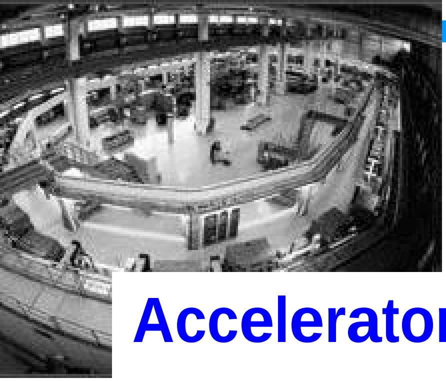
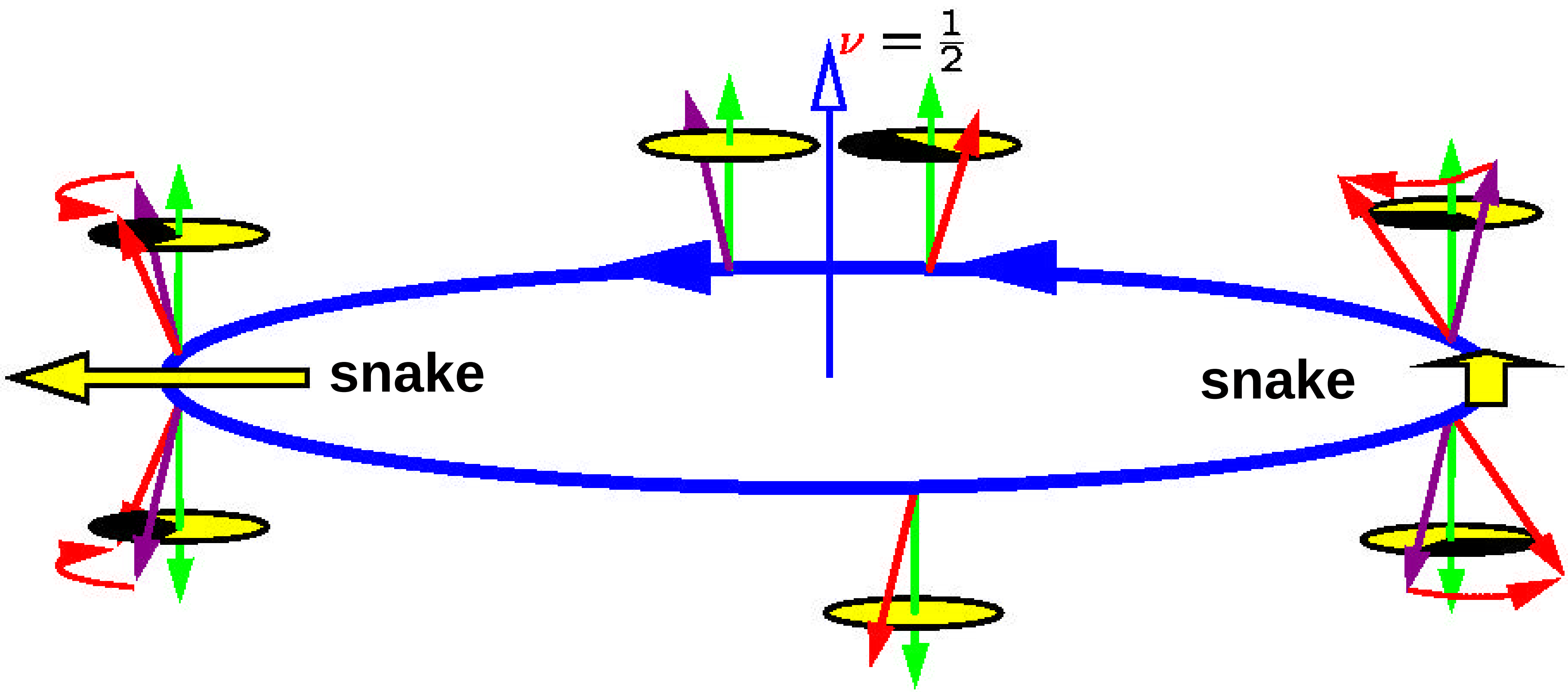


COSY Summer School 2002

Accelerator Rings with Polarized Beams and Spin Manipulation



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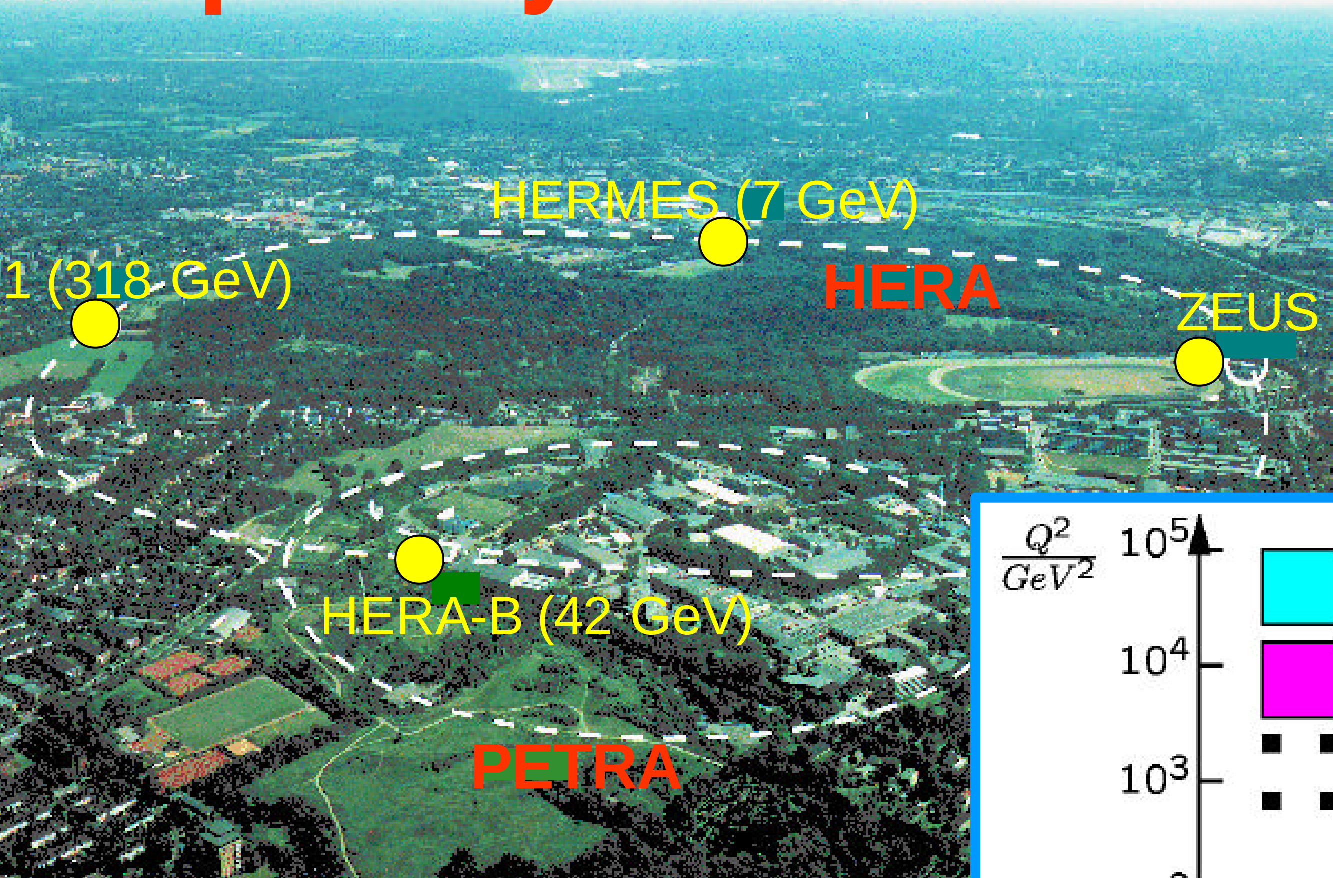
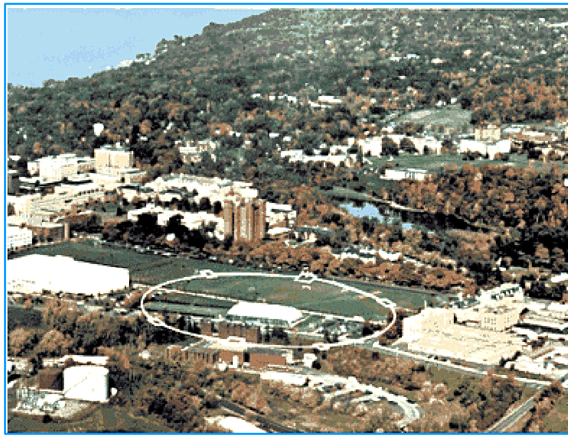
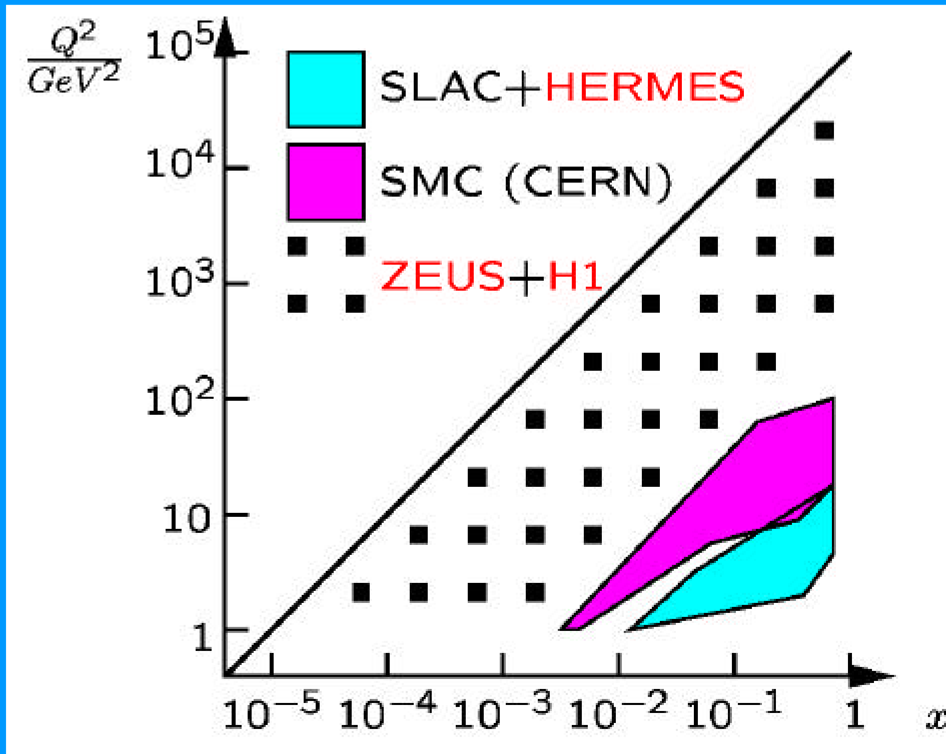
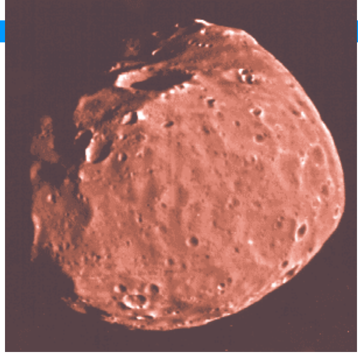
Spin Physics in HERA

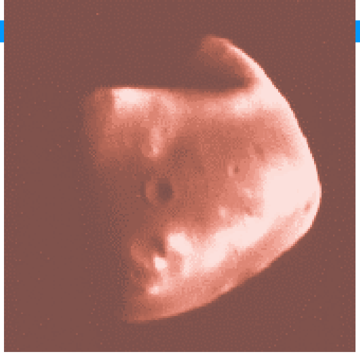


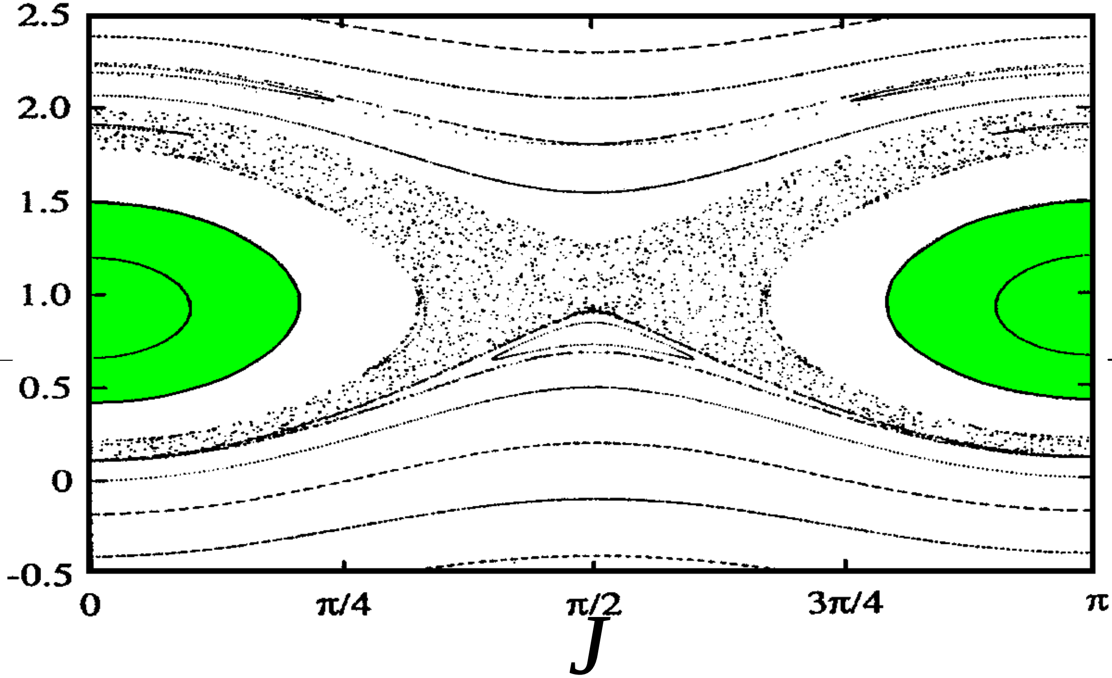


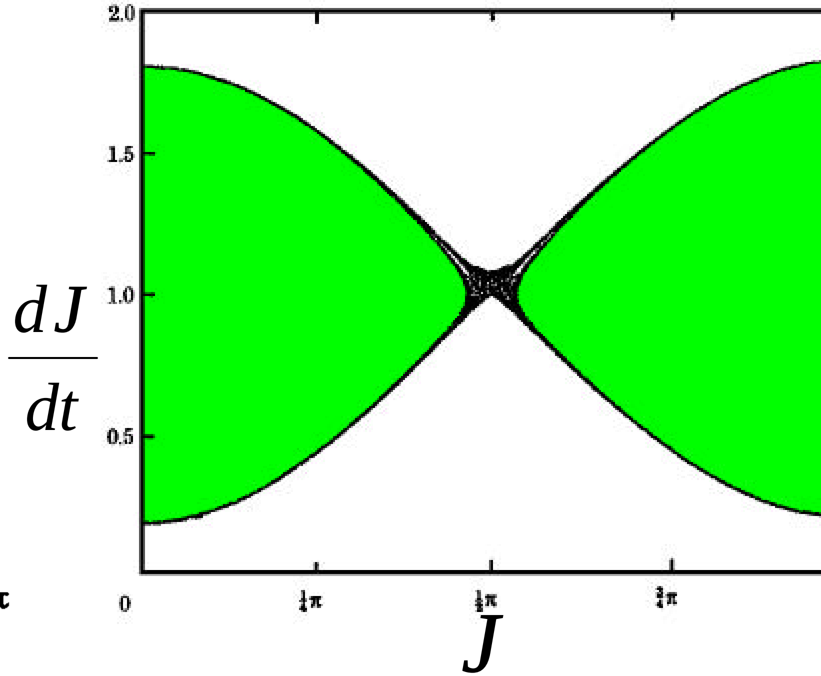




Phobos und Deimos







- Alle Monde waren vor Dämpfung in die Spin-Bahn-Resonanz chaotisch
- In den chaotischen Bereichen hatten alle Monde instabile Rotationsachsen

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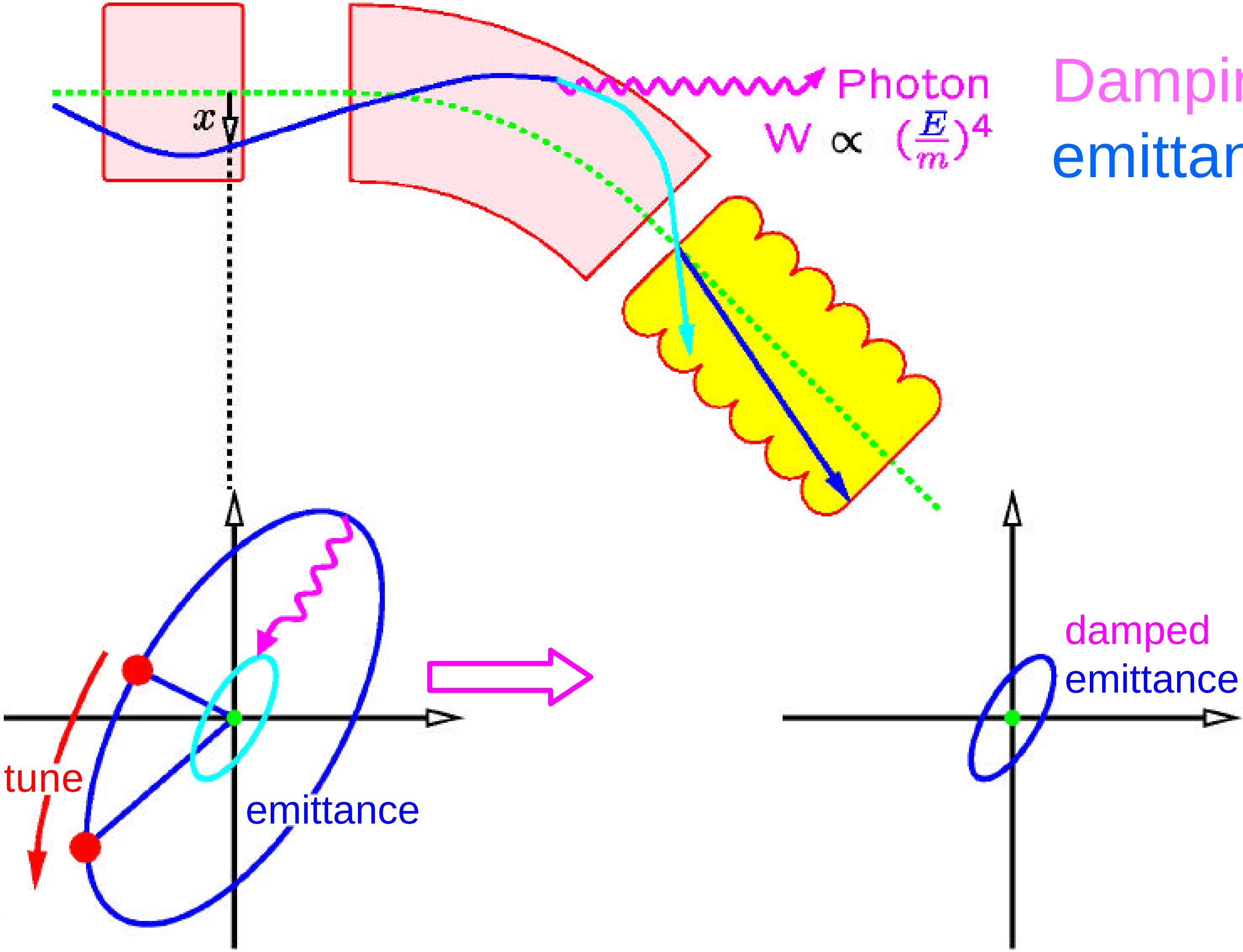
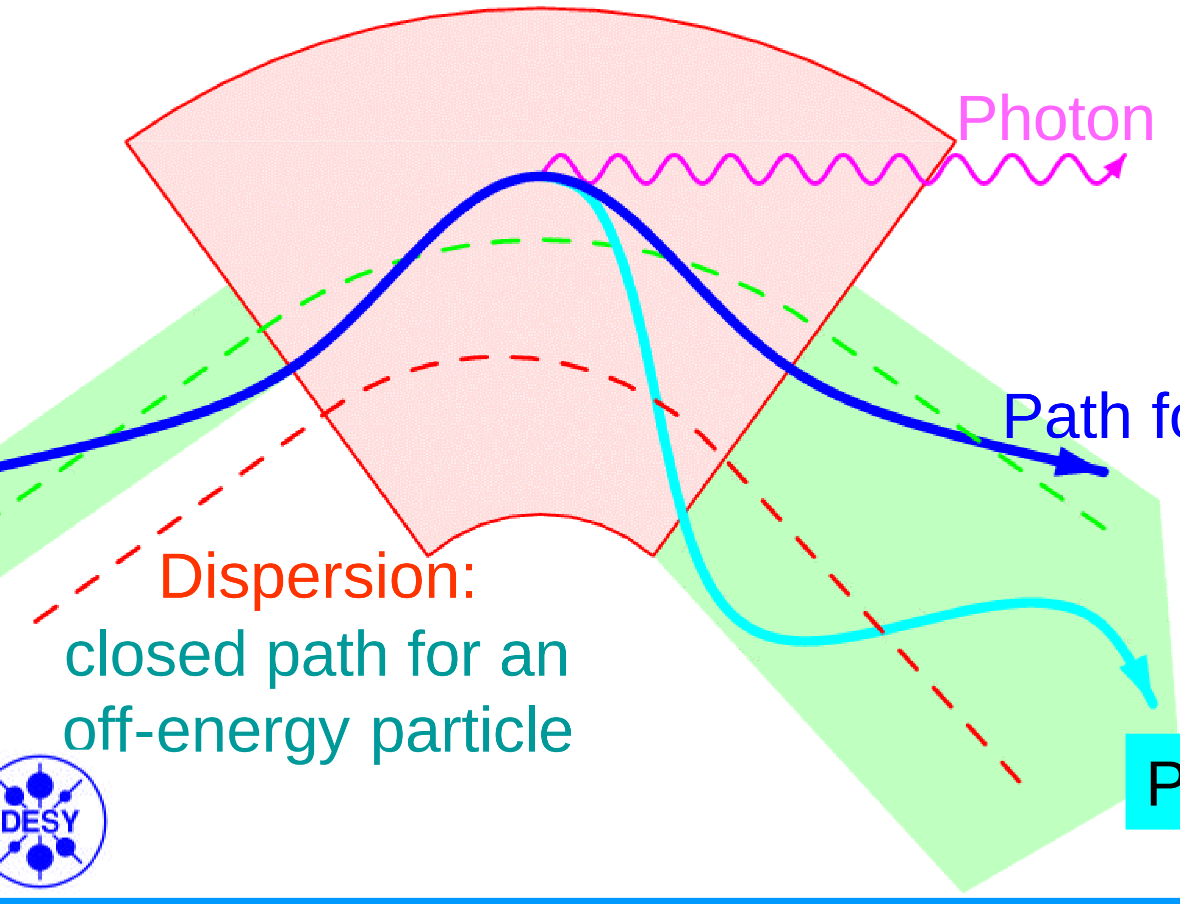
The Electron Beam

Damping of the emittance



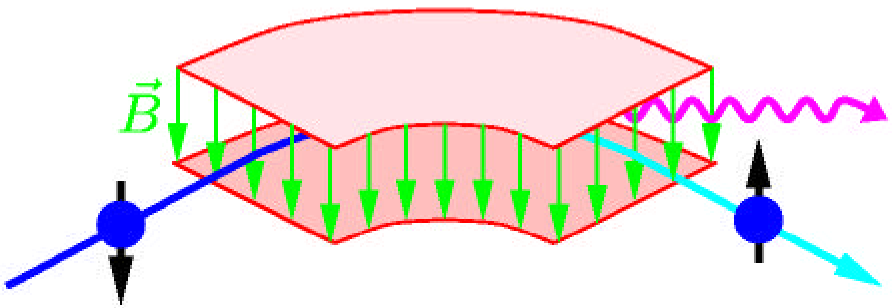
Generation of the Emittance

Stronger focusing → Smaller dispersion → Smaller emittance

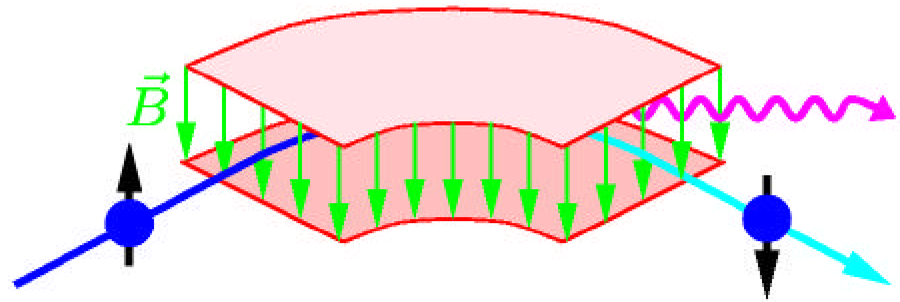


Self Polarization of the Electron Beam

Each 10^{10}-th photon flips the spin of the electron



In HERA every 38.5 minutes



In HERA every 16.2 hours

Ideal ring: equilibrium polarization 92.38%

HERA: routine operation with 60-65% polarization

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First longitudinal lepton polarization

VEPP	1970	80%	0.65 GeV
ACO	1070	90%	0.53 GeV
VEPP-2M	1974	90%	0.65 GeV
VEPP-3	1976	80%	2.0 GeV
SPEAR	1975	90%	3.7 GeV
VEPP-4	1982	60%	5.0 GeV
CESR	1883	30%	5.0 GeV
DORIS	1983	80%	5.0 GeV
PETRA	1982	70%	16.5 GeV
LEP	1993	57%	47 GeV
HERA	1994	70%	27.5 GeV

(longitudinal)

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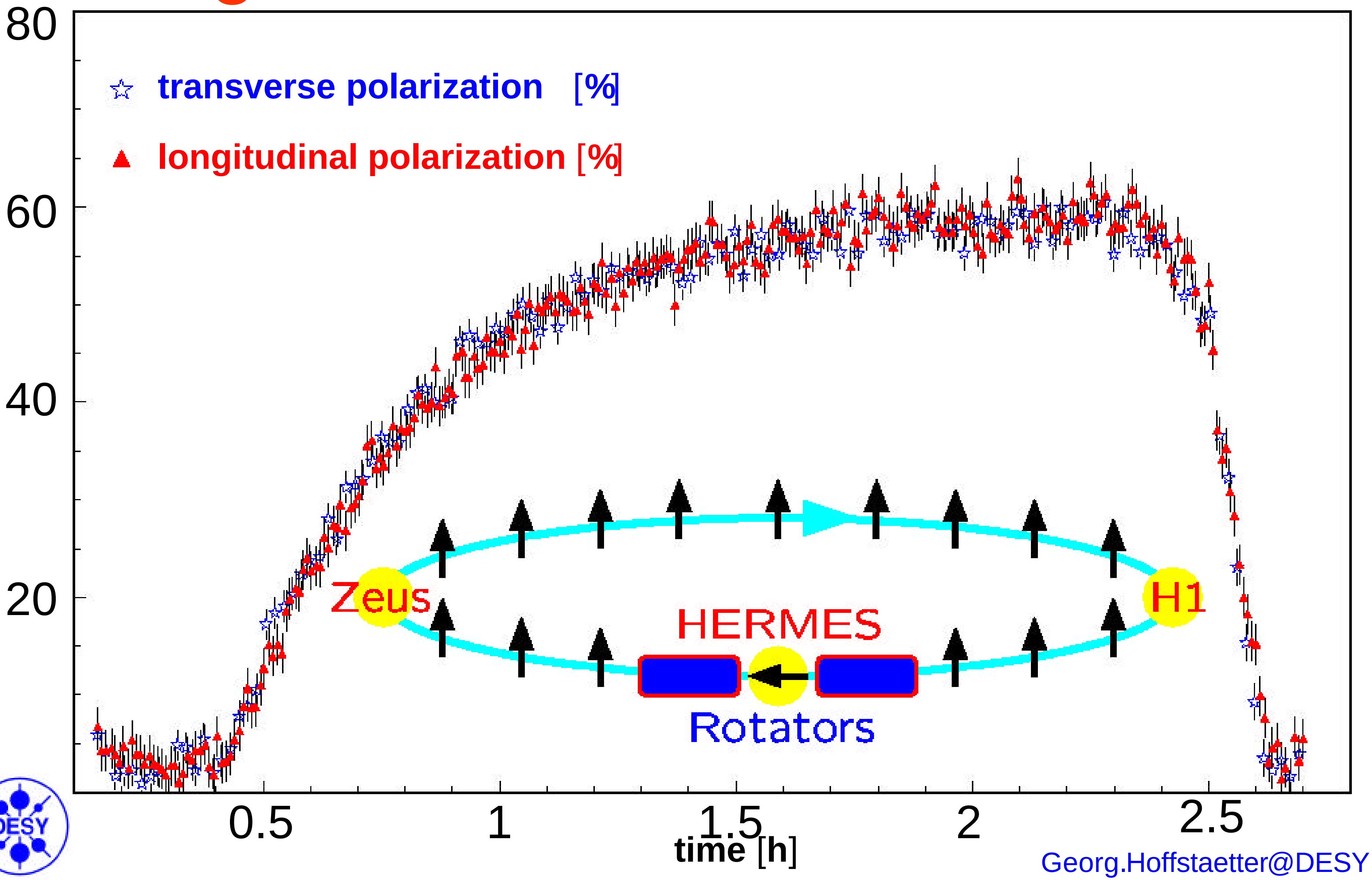
Longitudinal Electron Polarization



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Polarized Proton Beams

- Resonance excitation by the Stern-Gerlach Effect

➡ requires extremely difficult phase space gymnastics

- Spin flip by scatterin of polarized electrons

➡ very long polarization time

- Spin filter with polarized target (FILTEX at TSR)

➡ very long polarization times and for low energies

- Acceleration of polarized protons from rest

RHIC	100 GeV/c
AGS	25 GeV/c
ZGS	12 GeV/c
COSY	3.65 GeV/c
SATURN II	3.6 GeV/c
IUCF	0.7 GeV/c
PSI Cyclotron	0.59 GeV/c

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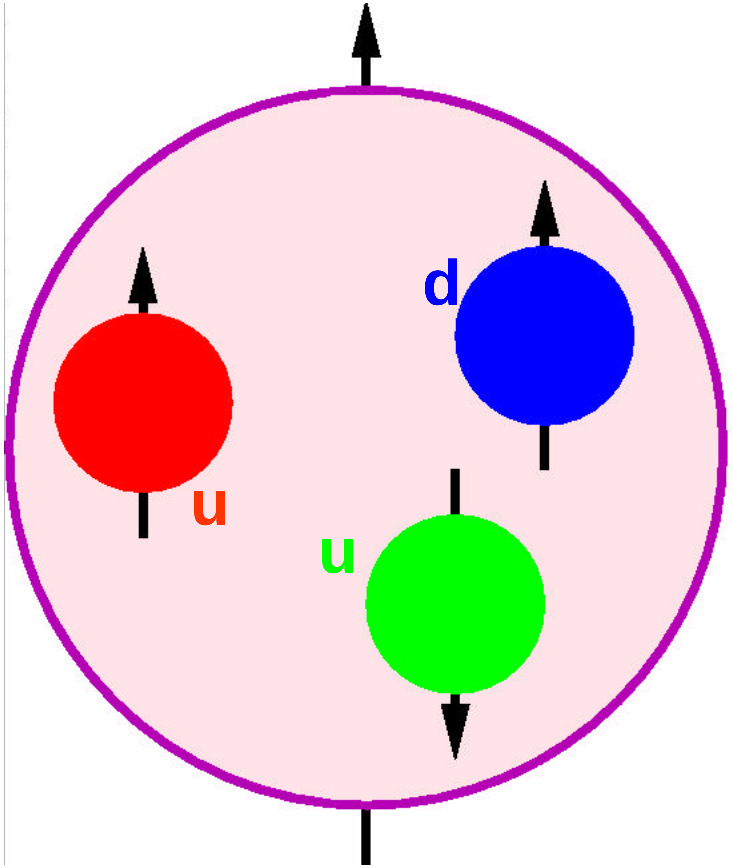
The Structure of the Proton



Proton:

Ground state of a system of two u and one d quark

Then one should find:

- Proton momentum = Sum of quark momenta
- Proton spin = Sum of quark spins

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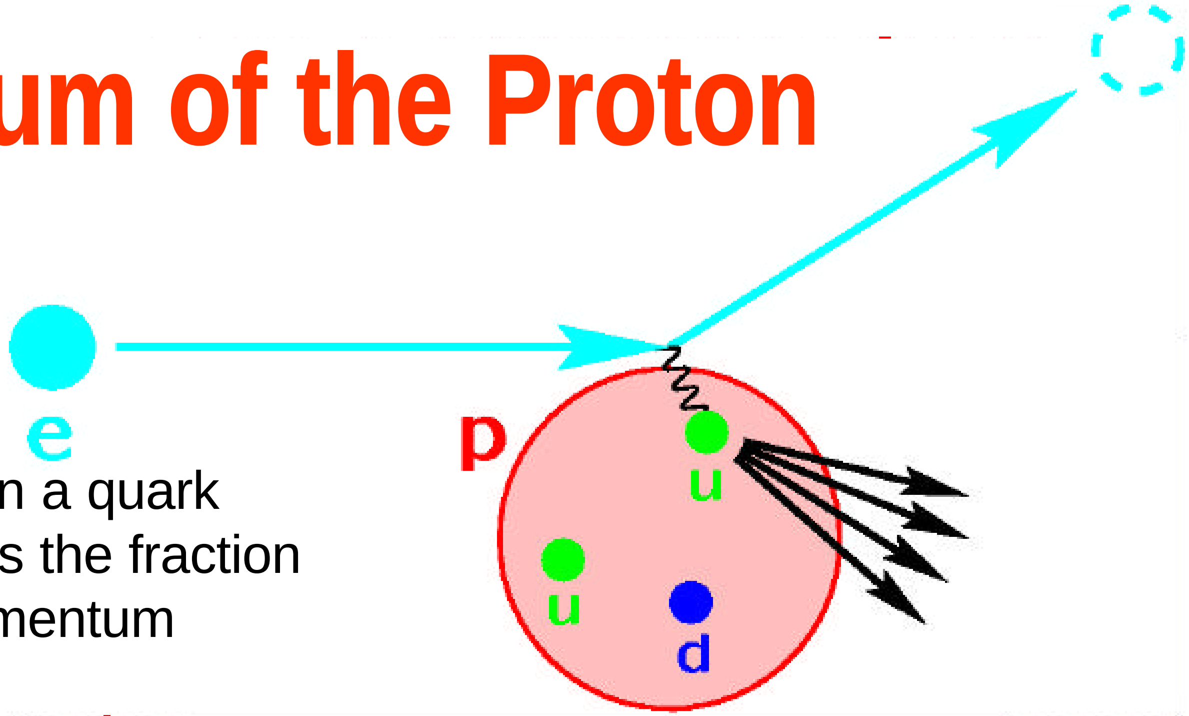
The Momentum of the Proton



q(x):
Probability of scattering on a quark or anti-quark which carries the fraction x of the proton's total momentum

The momentum puzzle

$$\int_0^1 x \cdot q(x) \cdot dx = 0.5$$

Gluons carry 50% of the proton's momentum

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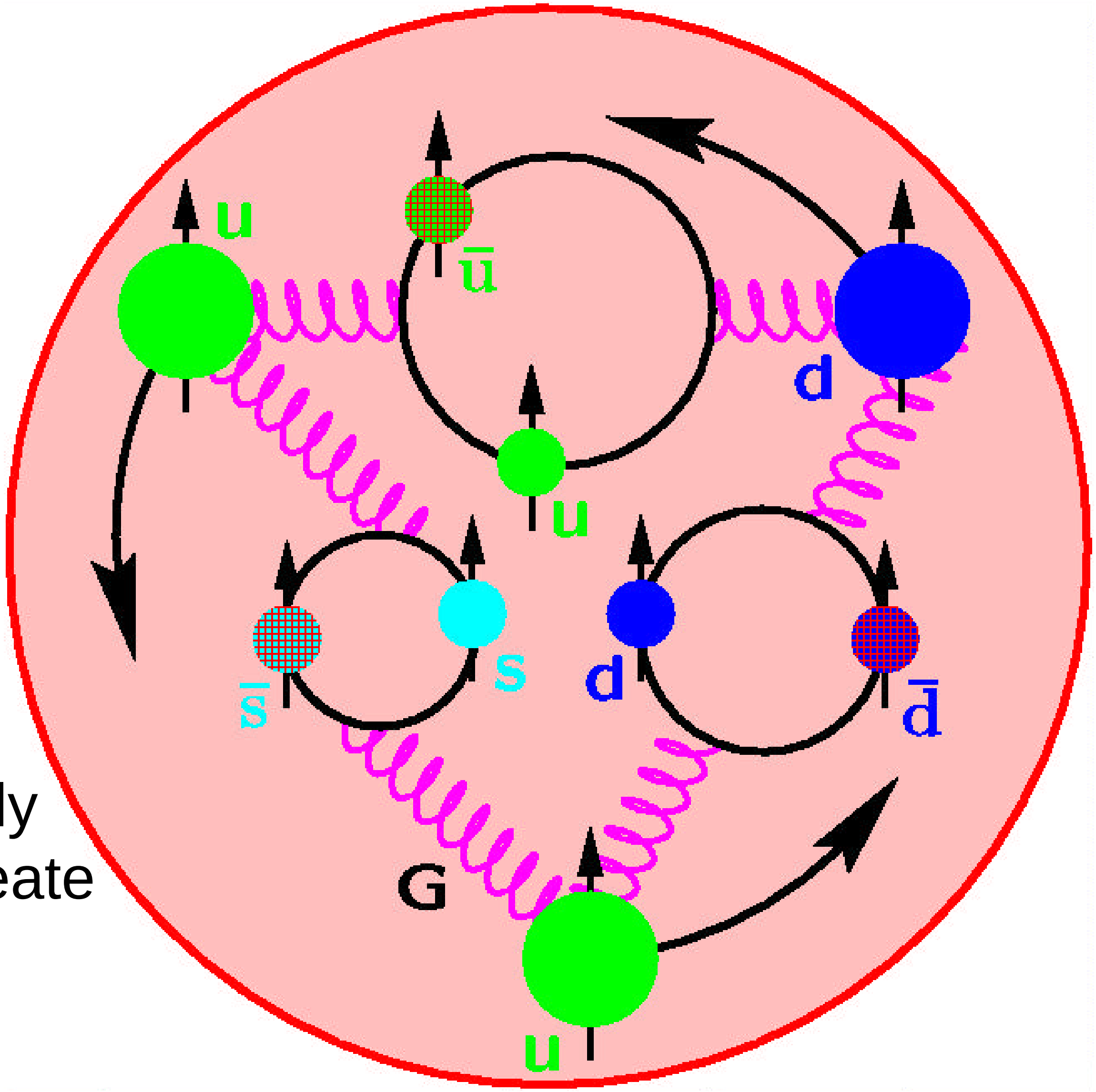
QCD Model of the Proton



The proton is a highly relativistic, bound state of **u** and **d** quarks. Additionally gluons, as field quanta, create quark / anti-quark pairs.

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The Spin Puzzle

S : quark contribution to the proton spin

DG : gluon contribution to the proton spin

D$L_{q,G}$: angular momentum contribution

$$\frac{1}{2} = \frac{1}{2}\Sigma + \Delta G + \Delta L_{q,G}$$

Measurements at SLAC-CERN-DESY and integration:

$$\int_0^1 \ldots \cdot dx \approx 0.27$$

Quarks carry only 20-30% of the proton's total spin

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Goals

- Determination of S, DG, D$L_{q,G}$
- Spin contribution of the individual quark types
- Test of QCD
- Scattering between polarized photon and proton
- How relativistic is the proton ?

Experiment:

Scattering of polarized proton beams on polarized electron beams

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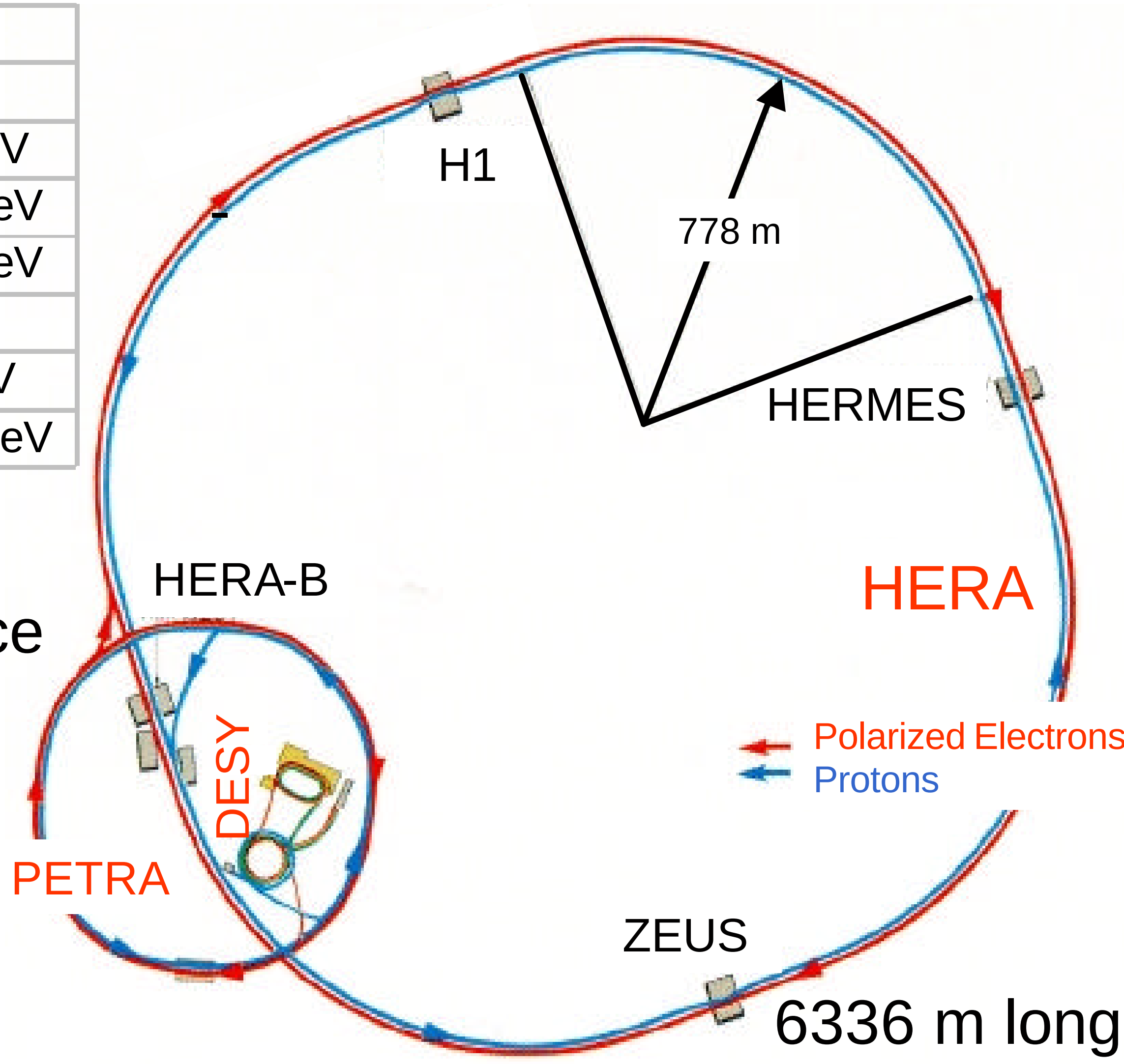
Polarized Protons at DESY

HERA and its Pre-Accelerator Chain

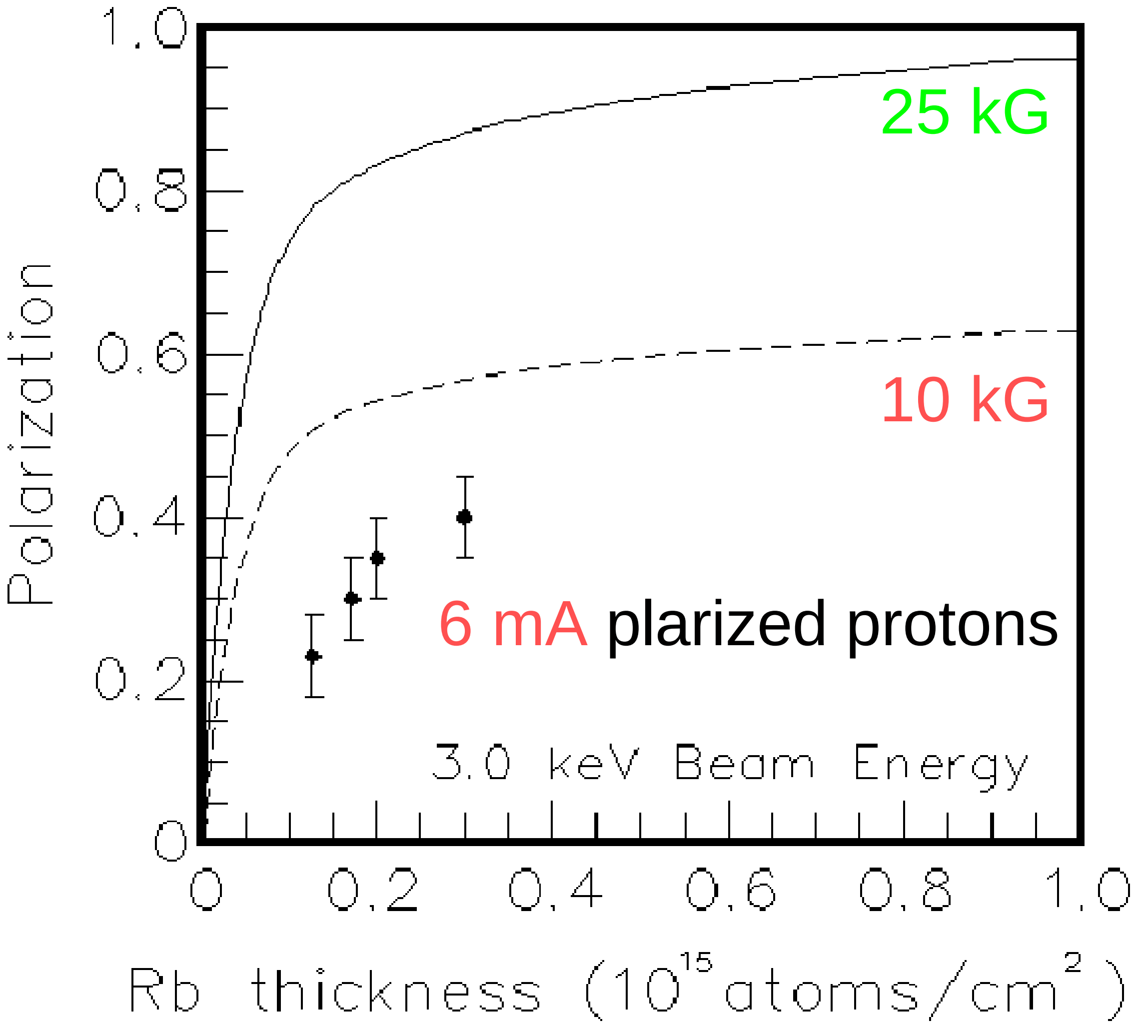
	Protons	Electrons	
20 keV	Source	Source	150 keV
750 keV	RFQ	Linac II	450 MeV
50 MeV	Linac III	Pia	450 MeV
8 GeV	DESY III	DESY II	7 GeV
40 GeV	PETRA	PETRA	12 GeV
920 GeV	HERA-p	HERA-e	27.5 GeV

Challenges:

- Polarized 20 mA H^- source
- Acceleration
- Storage at 920 GeV
- Polarimetry



Polarized H$^-$ source



But there is potential for higher polarization and for higher current

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The Equation of Spin Motion

Restframe: $\frac{d\vec{s}}{dt'} = \frac{gq}{2m}\vec{s}\times\vec{B}'$

Search for 4-vector EOM like: $\frac{d}{dt}U^m = \frac{q}{m}\cdot F^{mn}U_n \iff \frac{d}{dt}\vec{p} = q\cdot(\vec{v}\times\vec{B}+\vec{E})$

Spin 4-Vector: $S^m = (S_0,\vec{S}) = LorentzTrafo[(0,\vec{s}),\vec{b}] = (g\vec{b}\cdot\vec{s}, g\vec{s}_{\parallel}+\vec{s}_{\perp})$

$\Rightarrow S^m U_m = S_0 cg - \vec{S}\cdot\vec{v}g = 0,\quad \frac{d}{dt}(S^m U_m) = U_m\frac{d}{dt}S^m + S^m\frac{d}{dt}U_m = 0$

Allow linear dependence on velocity, acceleration, spin, and fields:

$$\frac{d}{dt}S^m = \cancel{\mathrm{a}\cdot U^m} + \cancel{\mathrm{b}\cdot\frac{d}{dt}U^m} + \cancel{\mathrm{c}\cdot S^m} + \cancel{\mathrm{d}\cdot F^{mn}}U_n$$

$$+\,\mathrm{e}\cdot F^{mn}S_n - \frac{1}{c^2}\left(S_n\frac{d}{dt}U^n\right)U^m + \frac{\mathrm{e}}{c^2}(S_h F^{hn}U_n)U^m$$

$$\mathrm{e} = \frac{gq}{2m}$$

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The Thomas BMT-Equation

$$\frac{d}{dt}\vec{s} = \vec{\Omega}_{BMT}(\vec{r},\vec{p})\times\vec{s}$$

$$\vec{\Omega}_{BMT}(\vec{r},\vec{p}) = -\frac{q}{m}[(\frac{1}{g}+G)\vec{B} - \frac{G\vec{p}\cdot\vec{B}}{g(g+1)m^2c^2}\,\vec{p} - \frac{1}{mc^2 g}(G+\frac{1}{g+1})\vec{p}\times\vec{E}]$$

$$G = \frac{g-2}{2} = \begin{cases} \text{Protons} & G = 1.79 \\ \text{Deuterons} & G = -0.143 \\ \text{Electrons} & G = 0.00116 \end{cases}$$

$$\frac{d\vec{p}}{dt} = (\frac{-q}{m\gamma})\{\qquad \vec{B}_\perp \qquad\} \times \vec{p}$$

$$\frac{d\vec{S}}{dt} = (\frac{-q}{m\gamma})\{(G\gamma+1)\vec{B}_\perp + (1+G)\vec{B}_\parallel\} \times \vec{S}$$

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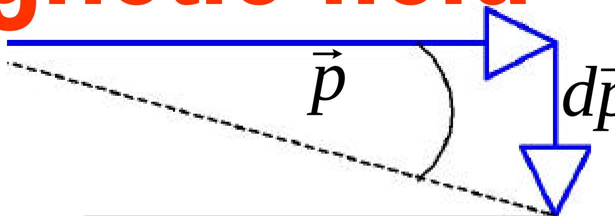
Spins in a transverse magnetic field



$$\frac{d\vec{p}}{dt} = \left(\frac{-q}{m\gamma}\right)\{ \qquad \vec{B}_\perp \qquad \} \times \vec{p}$$

$$\frac{d\vec{S}}{dt} = \left(\frac{-q}{m\gamma}\right)\{(G\gamma + 1)\vec{B}_\perp + (1+G)\vec{B}_{\parallel}\} \times \vec{S}$$

$$df_p = \frac{dp}{p} = -\frac{qB_\perp}{mg}\frac{dl}{v}$$

- Relative to the direction of the momentum, the spin rotation of relativistic particles does not depend on energy

Protons: 5.48 Tm rotate by

Deuterons: 137.2 Tm rotate by

Electrons: 4.62 Tm rotate by

$$df = -\frac{qG}{m}B_\perp\frac{dl}{v}$$

- Devices can be built which rotate spins independent of energy.

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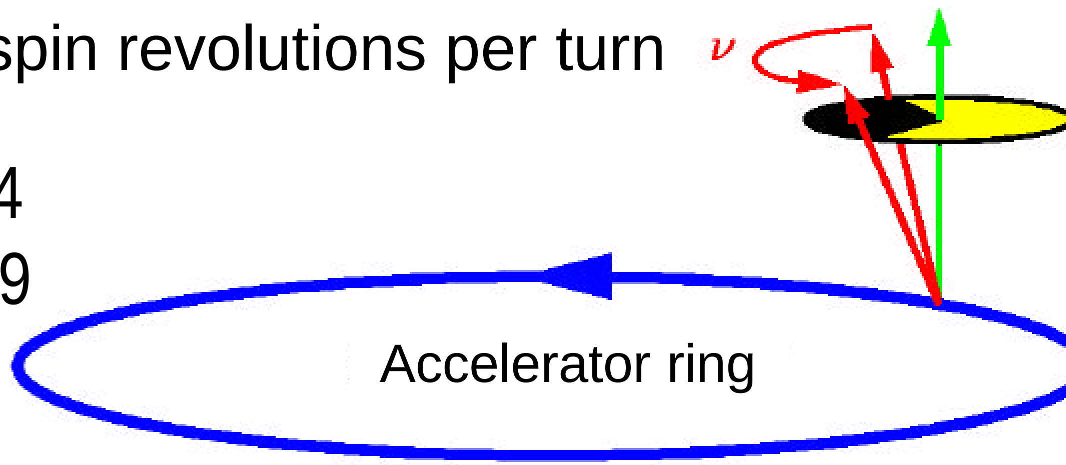
Spin-tune in a flat ring

$$\frac{d\vec{p}}{dt} = \left(\frac{-q}{m\gamma}\right)\{ \quad \vec{B}_\perp \quad \} \times \vec{p}$$

$$\frac{d\vec{S}}{dt} = \left(\frac{-q}{m\gamma}\right)\{(G\gamma+1)\vec{B}_\perp + (1+G)\vec{B}_\parallel\} \times \vec{S}$$

Spin-tune Gg: Number of spin revolutions per turn



COSY

3.3 GeV/c Protons: Gg = 6.54
3.3 GeV/c Deuterons: Gg =-0.29

HERA

920 GeV/c Protons: Gg =1756 and 1 more each 523 MeV
920 GeV/c Deuterons: Gg = -70 and 1 more each 13119 MeV
27.5GeV/c Electrons: Gg = 62.5 and 1 more each 442 MeV

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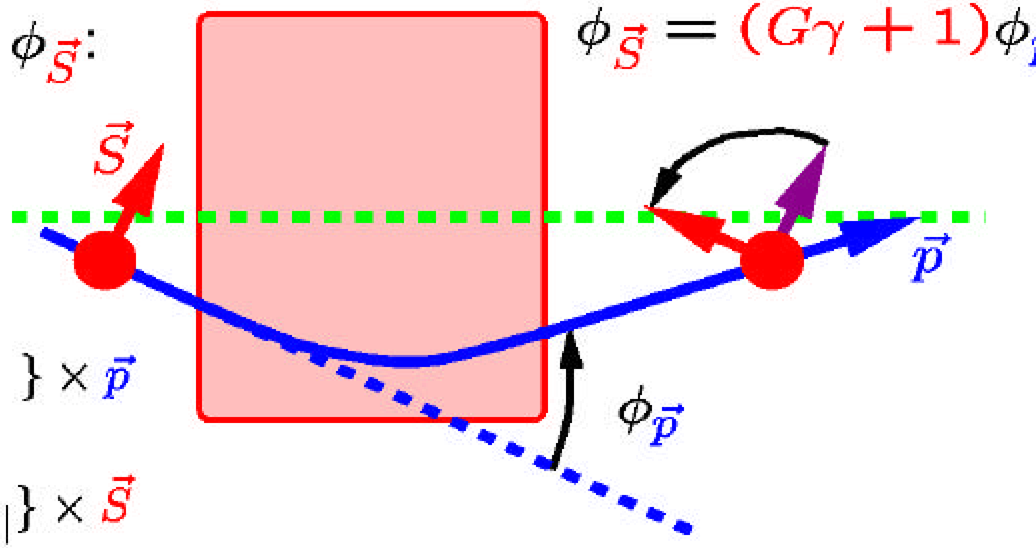
Spin-kick $\phi_{\vec{S}}$:



$$\frac{d\vec{p}}{dt} = \left(\frac{-q}{m\gamma}\right)\{\quad \vec{B}_\perp \quad\} \times \vec{p}$$

$$\frac{d\vec{S}}{dt} = \left(\frac{-q}{m\gamma}\right)\{(G\gamma + 1)\vec{B}_\perp + (1 + G)\vec{B}_\parallel\} \times \vec{S}$$

COSY	spin-kick	orbit deflection
3.3 GeV/c Protons:	/2	11.9 deg
3.3 GeV/c Deuterons:	/2	126.6 deg

HERA	spin-kick	orbit deflection
920 GeV/c Protons:	/2	0.89 mrad
920 GeV/c Deuterons:	/2	-22.1 mrad
27.5GeV/c Electrons:	/2	24.8 mrad

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Spin rotation in solenoids

$$\frac{d\vec{p}}{dt} = (\frac{-q}{m\gamma})\{\vec{B}_\perp\}\times\vec{p}$$

$$\frac{d\vec{S}}{dt} = (\frac{-q}{m\gamma})\{(G\gamma+1)\vec{B}_\perp + (1+G)\vec{B}_\parallel\}\times\vec{S}$$

$$df = -(1+G)\frac{q}{mg}B_\parallel\frac{dl}{v}$$

$$= -(1+G)\frac{qB_\parallel}{p}dl$$

COSY	**spin-rotation**	**solenoid field**
3.3 GeV/c Protons:		12.39 Tm
3.3 GeV/c Deuterons:		40.35 Tm

HERA	**spin-rotation**	**solenoid field**
920 GeV/c Protons:		3456 Tm
920 GeV/c Deuterons:		11250 Tm
27.5GeV/c Electrons:		288 Tm

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Orbit rotation in solenoids

$m g \ddot{\vec{r}} = q \dot{\vec{r}} \times \vec{B}$ **with** $\vec{B} = -B'_z \frac{r}{2} \vec{e}_r + B_z \vec{e}_z$ **so that** $\vec{\nabla} \cdot \vec{B} = 0$

$$\vec{r} = r\vec{e}_r + z\vec{e}_z , \qquad \ddot{\vec{r}} = (\ddot{r} - r\dot{j}^2)\vec{e}_r + (2\dot{r}\dot{j} + r\ddot{j})\vec{e}_f + \ddot{z}\vec{e}_z$$

$$\dot{\vec{r}} = \dot{r}\vec{e}_r + r\dot{j}\vec{e}_f + \dot{z}\vec{e}_z , \qquad = \frac{q}{mg}(\dot{r}\vec{e}_r + r\dot{j}\vec{e}_f + \dot{z}\vec{e}_z) \times (-B'_z \frac{r}{2}\vec{e}_r + B_z\vec{e}_z$$

component: $2\dot{r}\dot{j} + r\ddot{j} = -\frac{q}{mg}(\dot{r}B_z + \dot{z}\frac{r}{2}B'_z)$

$$\frac{d}{dt}(r^2\dot{j}) = -\frac{q}{mg}\frac{d}{dt}(\frac{r^2}{2}B_z)$$

Orbit

$$dj = -\frac{1}{2}\frac{qB_z}{p}dl$$

Spin

$$df = -(1+G)\frac{qB_\parallel}{p}dl$$

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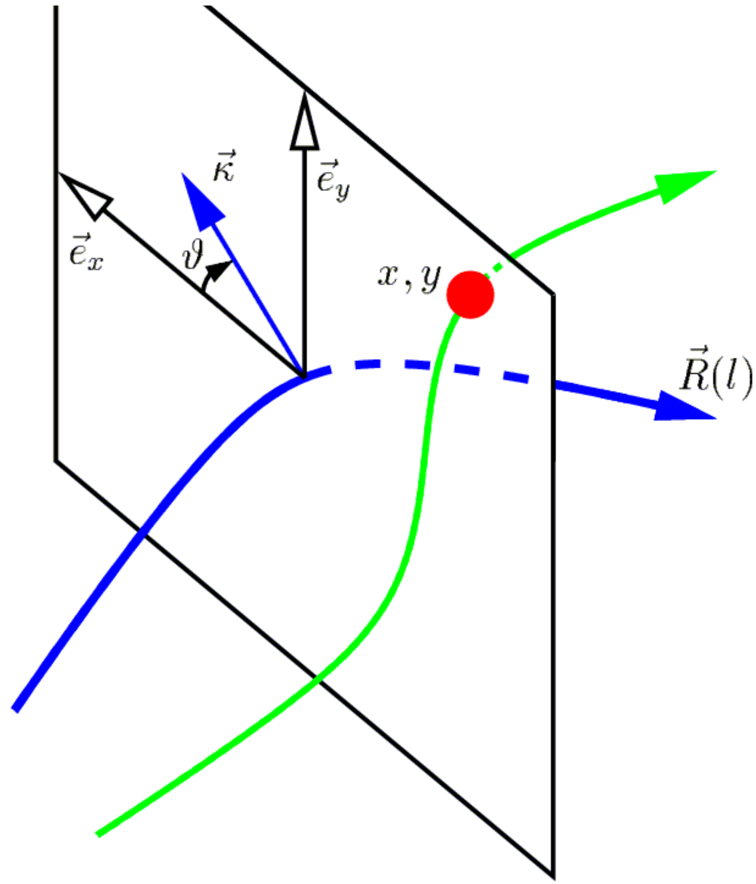
Spin motion in the Accelerator Frame

Independent variable: $t \Rightarrow l \Rightarrow q = 2p \frac{l}{L}$



$$\frac{d}{dl}\vec{e}_y = \frac{\sin J}{r}\vec{e}_l = k_y\vec{e}_l$$

$$\frac{d}{dl}\vec{e}_x = \frac{\cos J}{r}\vec{e}_l = k_x\vec{e}_l$$

$$\frac{d}{dl}\vec{e}_l = -\frac{1}{r}(\cos J\vec{e}_x + \sin J\vec{e}_y) = -k_x\vec{e}_x - k_y\vec{e}_y$$

$$\boxed{\vec{s} = S_x\vec{e}_x + S_y\vec{e}_y + S_l\vec{e}_l}$$

$$\frac{d}{dl}\vec{s} = (\frac{d}{dl}S_x - S_l k_x)\vec{e}_x + (\frac{d}{dl}S_y - S_l k_y)\vec{e}_y + (\frac{d}{dl}S_l + S_x k_x + S_y k_y)\vec{e}_l$$

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Spin motion in the Accelerator Frame

Independent variable: $t \Rightarrow l \Rightarrow q = 2p\,\frac{l}{L}$

$$\frac{dt}{dl} = \vec{e}_l \cdot \frac{d}{dl}\vec{r} \Big/ \vec{e}_l \cdot \frac{d}{dt}\vec{r}$$

$$(\frac{d}{dl}S_x - S_l k_x)\vec{e}_x + (\frac{d}{dl}S_y - S_l k_y)\vec{e}_y + (\frac{d}{dl}S_l + S_x k_x + S_y k_y)\vec{e}_l = \frac{dt}{dl}\vec{\Omega}_{BMT} \times \vec{s}$$

$$\frac{d}{dl}\vec{S} = [\frac{dt}{dl}\vec{\Omega}_{BMT}(\vec{r},\vec{p}) - \vec{k}\times\vec{e}_l] \times \vec{S} \quad \text{with} \quad \vec{S}\times(\vec{k}\times\vec{e}_l) = \vec{k}(\vec{S}\cdot\vec{e}_l) - \vec{e}_l(\vec{S}\cdot\vec{k}$$

$$\frac{d}{dq}\vec{S} = \vec{\Omega}(\vec{z},q)\times\vec{S}, \quad \vec{\Omega}(\vec{z},q) = \frac{L}{2p}[\frac{dt}{dl}\vec{\Omega}_{BMT}(\vec{r},\vec{p}) - \vec{k}\times\vec{e}_l]$$

$$\frac{d}{dq}\vec{z} = \vec{v}(\vec{z},q)$$

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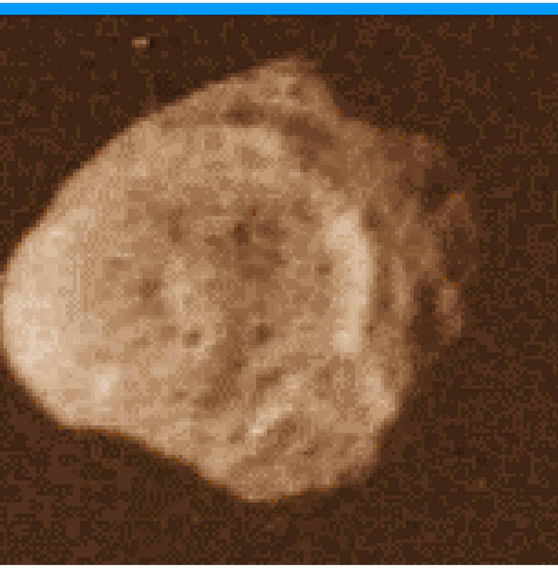


The tumbling of Hyperion

250km X 115km X 110km ($a=3(B-A)/2C=0.4$): largest non-round structure in the solar system

Unique due to large a and e=0.1

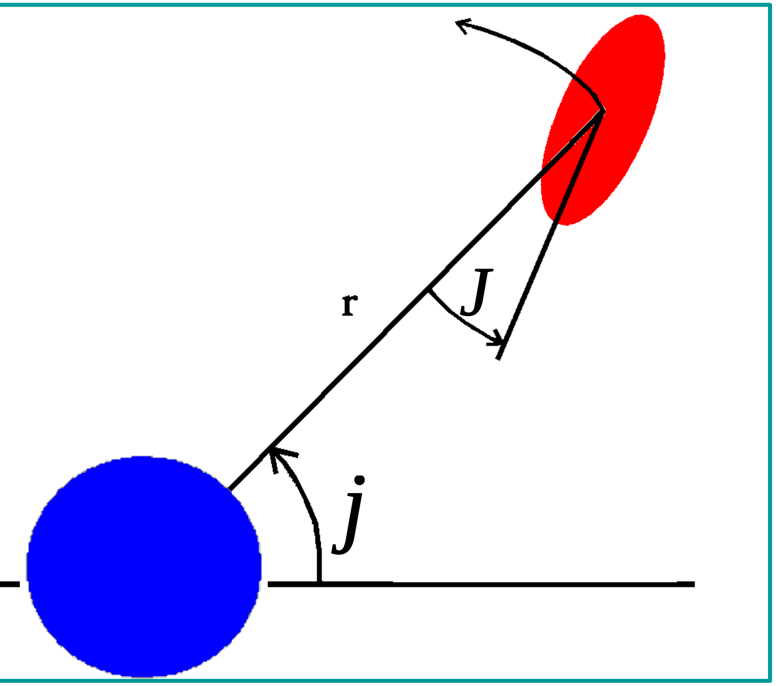
Voyager2 observed a rotation axis which was close to the orbit plane: no spin-orbit-coppling

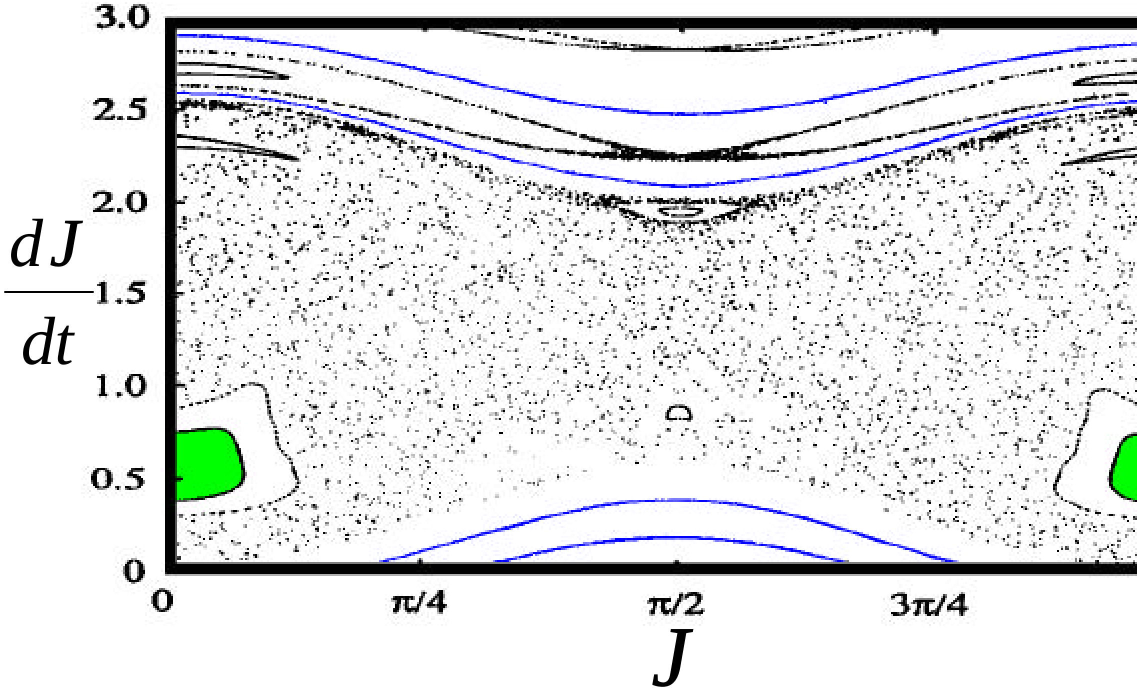
Change in luminousness shows a chaotic rotation

The only observed chaos in solar-system-dynamics

Model: rotation around the vertical

$$\frac{d^2(J + j(t))}{dt^2} = -a\left(\frac{a}{r(t)}\right)^3 \sin 2J$$





While changing from rotation to libration around spin-orbit-coupplun a large chaotic region has to be crossed.

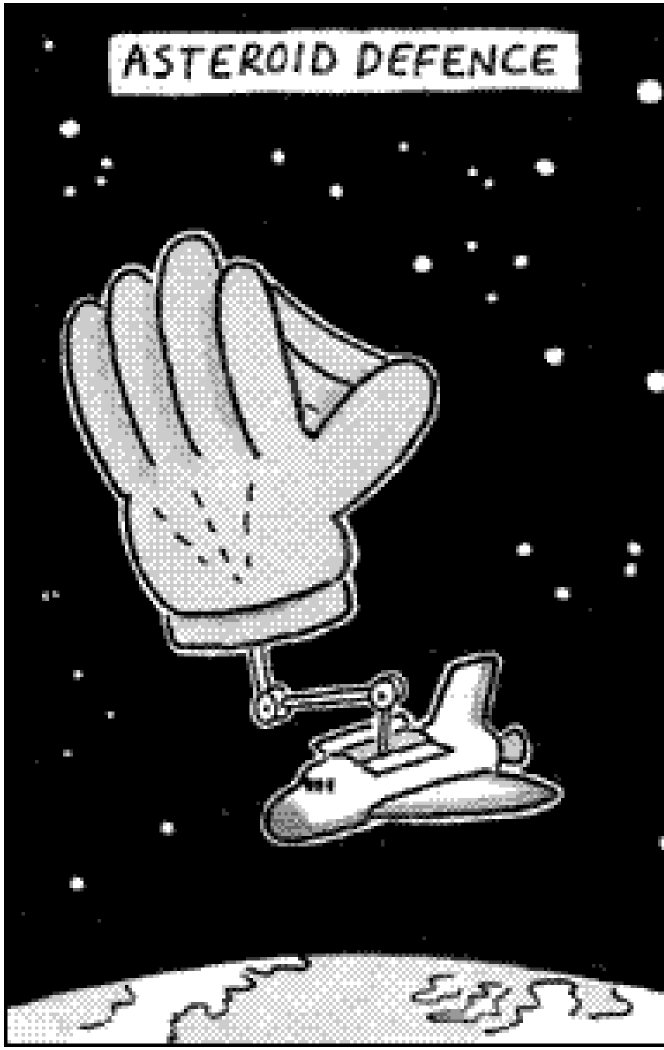
The Importance of Asteroids

The world ends on Feb 1 2019
(possibly)
(Filed: 25/07/2002, Telegraph)

Scientists have detected a giant asteroid heading towards Earth. It could wipe out humanity, but it could miss us altogether. Astronomers say a huge asteroid is scheduled to crash into the Earth at 11.47 am on Feb 1 2019.

Objects the size of NT7 only hit the Earth every one or two million years.

The dangers of NT7 have yet to be reviewed by the International Astronomical Union,
the main international body responsible for announcing such risks.



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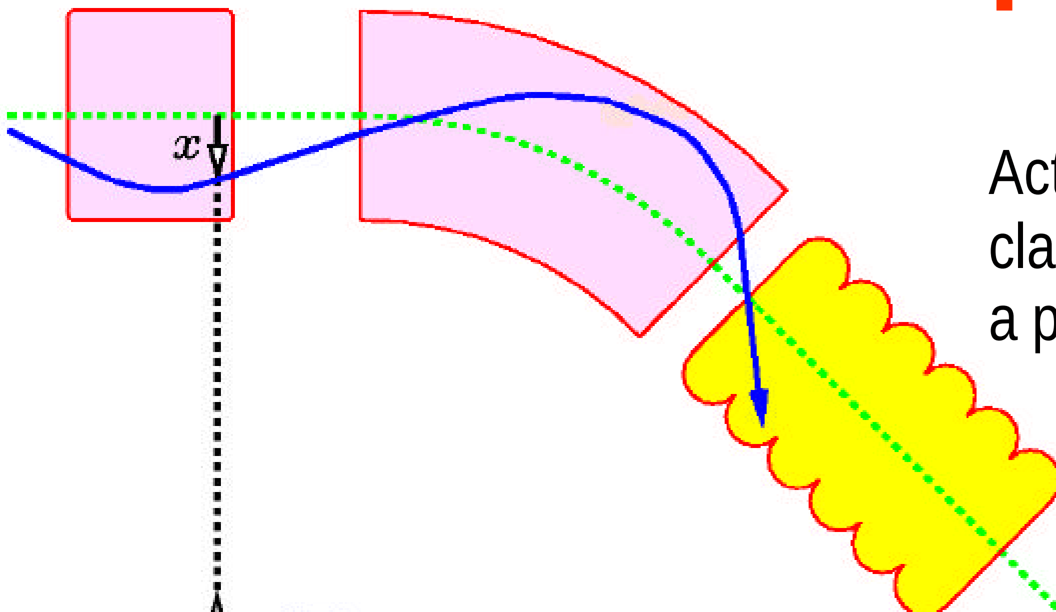
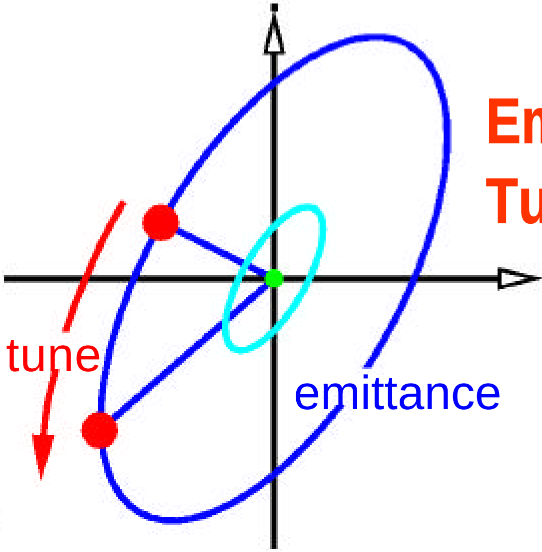
Phase space motion



Action-angle variables of a classical periodic system like a pendulum or planetary motion



Emittances: $\vec{e}: e_x, e_y, e_l$

Tunes: $\vec{Q}: Q_x, Q_y, Q_l$

Action variables: $\vec{\Phi} = \vec{Q}q$

Angle variables: $\vec{J} = \frac{1}{2}\vec{e}$

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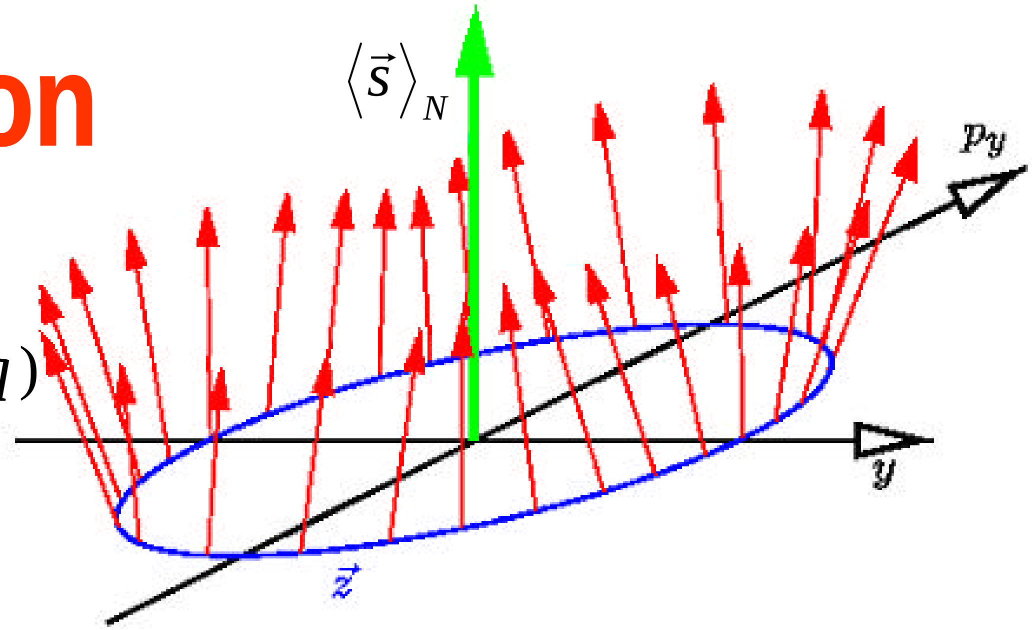
Equation of motion for spin fields



Spin field: Spin direction $\vec{f}(\vec{z},q)$ for each phase space point $\vec{z}$

$$\frac{d}{dq}\vec{f} = \partial_q \vec{f} + [\vec{v}(\vec{z},q)\cdot\partial_{\vec{z}}]\vec{f} = \vec{\Omega}(\vec{z},q)\times\vec{f}$$

The spin transport matrix EOM

$$\vec{S}(q) = \underline{R}(\vec{z}_i, q_0; q)\vec{S}_i$$

$$\vec{f}(\vec{z}, q) = \underline{R}(\vec{z}_i, q_0; q)\,\vec{f}(\vec{z}_i, q_0)$$

$$\frac{d\vec{S}}{dq} = \vec{\Omega} \times \vec{S}_i \Rightarrow \partial_q \underline{R}(\vec{z}_i, q_0; q) = \begin{pmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{pmatrix} \underline{R}(\vec{z}_i, q_0; q$$

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The spin transport matrix

Rotation vector: $\vec{e}$ **, Rotation angle** a **.**

$$\vec{S}(q) = \vec{e}(\vec{e}\cdot\vec{S}_i) + \cos a\,[\vec{S}_i - \vec{e}(\vec{e}\cdot\vec{S}_i)] + \sin a\cdot\vec{e}\times[\vec{S}_i - \vec{e}(\vec{e}\cdot\vec{S}_i)]$$

$$a_0 = \cos\tfrac{a}{2},\quad \vec{a} = \vec{e}\sin\tfrac{a}{2} \quad \text{with} \quad a_0^{\,2} + \vec{a}^{\,2} = 1$$

$$\vec{S}(q) = (a_0^{\,2} - \vec{a}^{\,2})\vec{S}_i + 2\vec{a}(\vec{a}\cdot\vec{S}_i)] + 2a_0\vec{a}\times\vec{S}_i$$

$$R_{ij} = (a_0^{\,2} - \vec{a}^{\,2})d_{ij} + 2a_i a_j - 2a_0 e_{ijk} a_k$$

$$Tr(\underline{R}) = 4a_0^{\,2} - 1,\quad e_{lmn} R_{mn} = -4a_0 a_l$$

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The spin transport quaternion

$$A = a_0\underline{1}_2 - i\vec{a}\cdot\underline{\vec{s}}, \qquad s_l s_m = ie_{lmn}s_n + d_{lm}$$

Propagation: $C = (b_0\underline{1}_2 - i\vec{b}\cdot\underline{\vec{s}})(a_0\underline{1}_2 - i\vec{a}\cdot\underline{\vec{s}})$

$$= (b_0a_0 - \vec{b}\cdot\vec{a})\underline{1}_2 - i(b_0\vec{a} + \vec{b}a_0 + \vec{b}\times\vec{a})\cdot\underline{\vec{s}}$$

Infinitesimal rotation: $B = \underline{1}_2 - i\frac{dq}{2}\vec{\Omega}\cdot\underline{\vec{s}}, \qquad C = A + dA$

$$dA = -\frac{dq}{2}[\vec{\Omega}\cdot\vec{a}\underline{1}_2 + i(\vec{\Omega}a_0 + \vec{\Omega}\times\vec{a})\cdot\underline{\vec{s}}]$$

$$= -\frac{dq}{2}[(\vec{\Omega}\cdot\underline{\vec{s}})(\vec{a}\cdot\underline{\vec{s}}) + i\vec{\Omega}\cdot\underline{\vec{s}}a_0] = -i\frac{dq}{2}\vec{\Omega}\cdot\underline{\vec{s}}A$$

$$\boxed{\frac{dA}{dq} = -i\frac{1}{2}\vec{\Omega}\cdot\underline{\vec{s}}A}$$

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Equation of motion for Spinors

$$\Psi(q) = (a_0 \underline{1}_2 - i\vec{a}\cdot\underline{\vec{s}})\Psi_i, \quad \Longrightarrow \quad \frac{d\Psi}{dq} = -i\frac{1}{2}(\vec{\Omega}\cdot\vec{s})\Psi$$

$$\vec{S}(q) = \Psi^+ \underline{\vec{s}}\Psi$$ **with** $$|y_1|^2 + |y_2|^2 = 1$$

since $$\frac{d\vec{S}}{dq} = i\frac{1}{2}\Psi^+[(\vec{\Omega}\cdot\underline{\vec{s}})\underline{\vec{s}} - \underline{\vec{s}}(\vec{\Omega}\cdot\underline{\vec{s}})]\Psi$$

$$= i\frac{1}{2}\Psi^+(\vec{\Omega}\times\underline{\vec{s}})\Psi = \vec{\Omega}\times\vec{S}$$

$$\vec{S} = \begin{pmatrix} \sin J \cos f \\ \sin J \sin f \\ \cos J \end{pmatrix} \Longleftrightarrow \Psi = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = e^{ix}\begin{pmatrix} \cos\frac{J}{2} \\ e^{if}\sin\frac{J}{2} \end{pmatrix}$$

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Spinor pase

Rotation around the spin direction: $\vec{\Omega} = \vec{S} a$

$$\frac{d\Psi}{dq} = -i\frac{a}{2}\vec{S}\cdot\underline{\vec{s}}\Psi = -i\frac{a}{2}\begin{pmatrix} \cos J & e^{-ij}\sin J \\ e^{ij}\sin J & -\cos J \end{pmatrix}\Psi$$

$$= -ia\begin{pmatrix} \cos^2\frac{J}{2} - \frac{1}{2} & e^{-ij}\sin\frac{J}{2}\cos\frac{J}{2} \\ e^{ij}\sin\frac{J}{2}\cos\frac{J}{2} & \sin^2\frac{J}{2} - \frac{1}{2} \end{pmatrix}\Psi$$

$$= -ia(\Psi\Psi^{+} - \tfrac{1}{2}\underline{1}_2)\Psi = -i\frac{a}{2}\Psi$$

$$\boxed{\Psi = e^{i\frac{a}{2}(q - q_0)}\Psi(q_0)}$$

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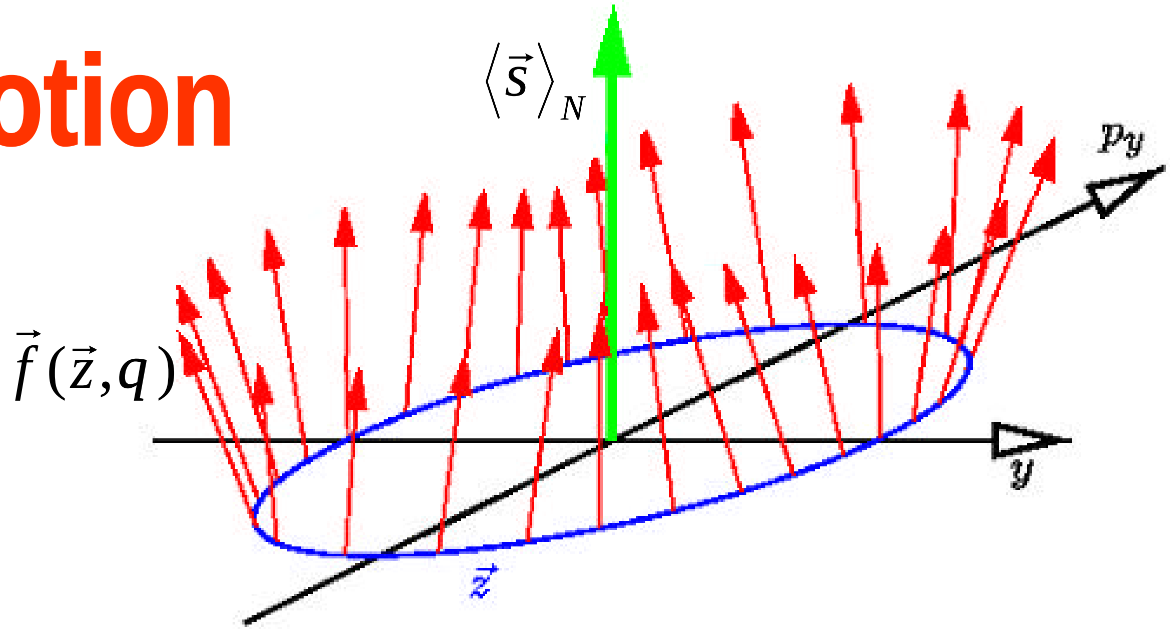
Equation of motion for spin fields



Spin field: Spin direction $\vec{f}(\vec{z},q)$ for each phase space point $\vec{z}$

$$\frac{d}{dq}\Psi = \partial_q \Psi + [\vec{v}(\vec{z},q)\cdot\partial_{\vec{z}}]\Psi = -\frac{i}{2}[\vec{\Omega}(\vec{z},q)\cdot\vec{s}]\Psi$$

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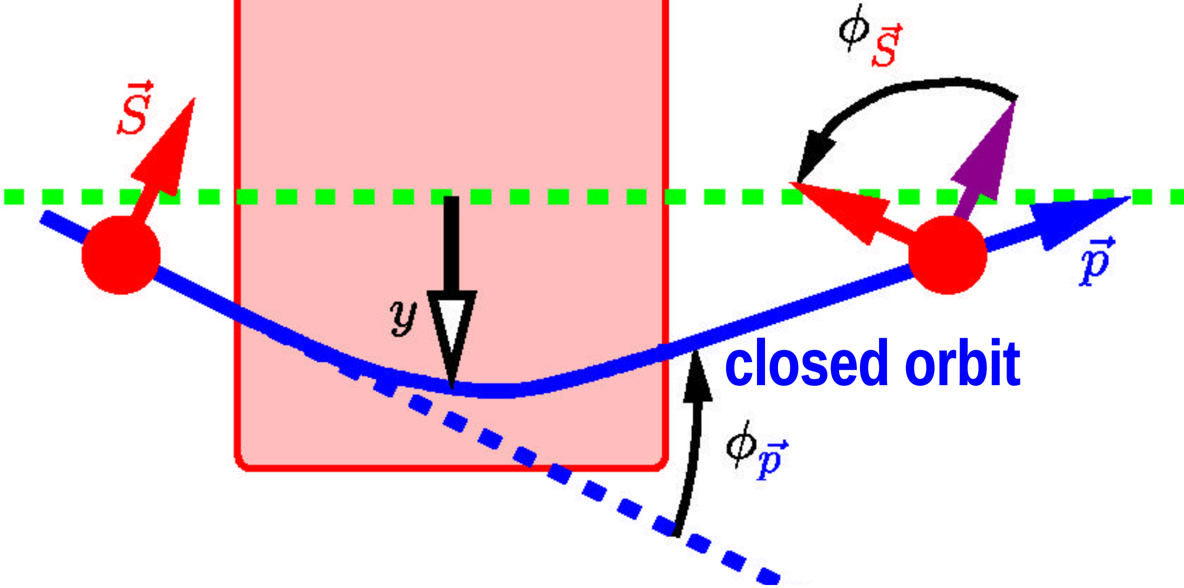
Spin perturbation on the closed orbit

Integer values of the closed orbit spin-tune n_0 lead to coherent disturbances of spin motion called imperfection resonances.



Remedy:
Partial snakes avoid resonances by avoiding an integer spin-tune n

$$f_{\vec{S}} \propto f_{\vec{p}} \propto y_0(q - q_0 \bmod 2p) = \sum_{k_0} a_{k_0} e^{ik_0 q}$$

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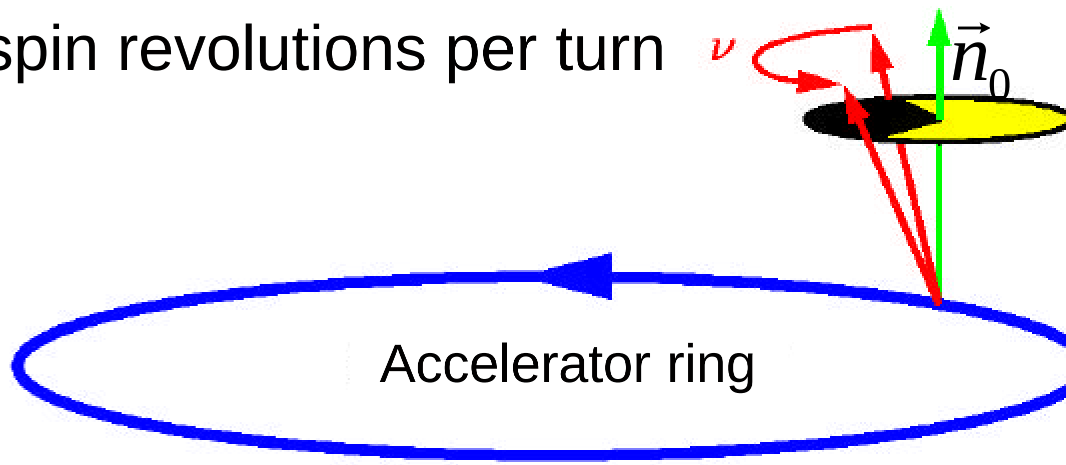
Periodic spin direction for the closed orbit

A particle traveling on the closed orbit has the closed orbit spin direction $\vec{n}_0$ if its spin is periodic after every turn.

$$\vec{n}_0 = \underline{R}(\vec{z}_0, q_0; q_0 + 2p)\vec{n}_0$$

Spin-tune **n_0**: Number of spin revolutions per turn



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Closed orbit spin motion

- When the design orbit spin tune $\mathbf{n}_0$ becomes integer, no net rotation has occurred after one turn, so that perturbations from the design orbit dominate the motion
- If the perturbations are very small, the resonance region can be crossed quickly and the spins hardly react
- If the perturbation is large, the closed orbit spin direction changes slowly and spins can follow adiabatic changes of the periodic spin direction on the closed orbit.
- When the perturbations have intermediate strength, the polarization will be reduced

Remedies:

- Correction of the closed orbit to reduce the perturbation
- Increase of spin perturbation (for example by a solenoid partial snake) to increase the perturbation

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The periodic coordinate system

Closed orbit precession vector: $\vec{\Omega}_0(q) = \vec{\Omega}_0(q + 2p)$

Closed orbit spin vector: $\frac{d\vec{n}_0}{dq} = \vec{\Omega}_0(q) \times \vec{n}_0 \,, \quad \vec{n}_0(q) = \vec{n}_0(q + 2p)$

Non-periodic Perpendicular vectors: $\frac{d\vec{m}_0}{dq} = \vec{\Omega}_0(q) \times \vec{m}_0 \,, \qquad \vec{l}_0 = \vec{n}_0 \times \vec{m}_0$

$$[\vec{m}_0 + i\vec{l}_0](q) = e^{i2pn_0}[\vec{m}_0 + i\vec{l}_0](q + 2p)$$

Non-periodic Perpendicular vectors: $[\vec{m} + i\vec{l}\,](q) = e^{iqn_0}[\vec{m}_0 + i\vec{l}_0](q)$

$$\frac{d(\vec{m} + i\vec{l}\,)}{dq} = [\vec{\Omega}_0(q) - n_0\vec{n}_0] \times (\vec{m} + i\vec{l}\,)$$

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EOM in the periodic system

$$\vec{S} = s_1\vec{m}(q) + s_2\vec{l}\,(q) + s_3\vec{n}_0(q)$$

$$\vec{\Omega}_0(q)\times\vec{S} = \frac{d}{dq}\vec{S} = \vec{m}\frac{ds_1}{dq} + \vec{l}\,\frac{ds_2}{dq} + \vec{n}_0\frac{ds_3}{dq} + [\vec{\Omega}_0 - n_0\vec{n}_0]\times\vec{S}$$

$$\hat{s} = s_1 + is_2\,, \quad \tfrac{d}{dq}\hat{s} = in_0\hat{s}\,, \quad \tfrac{d}{dq}s_3 = 0$$

Smooth rotation around the periodic spin direction !

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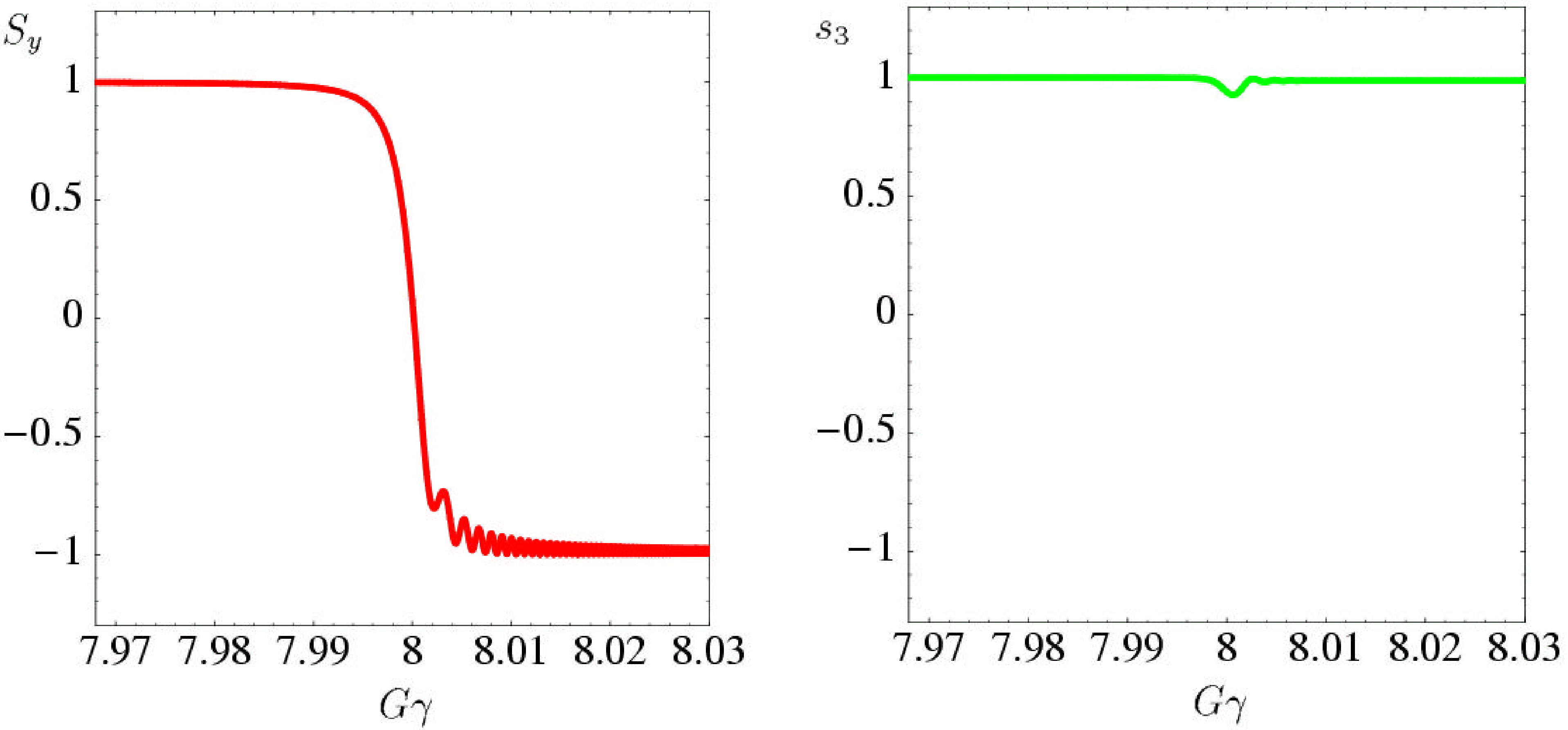
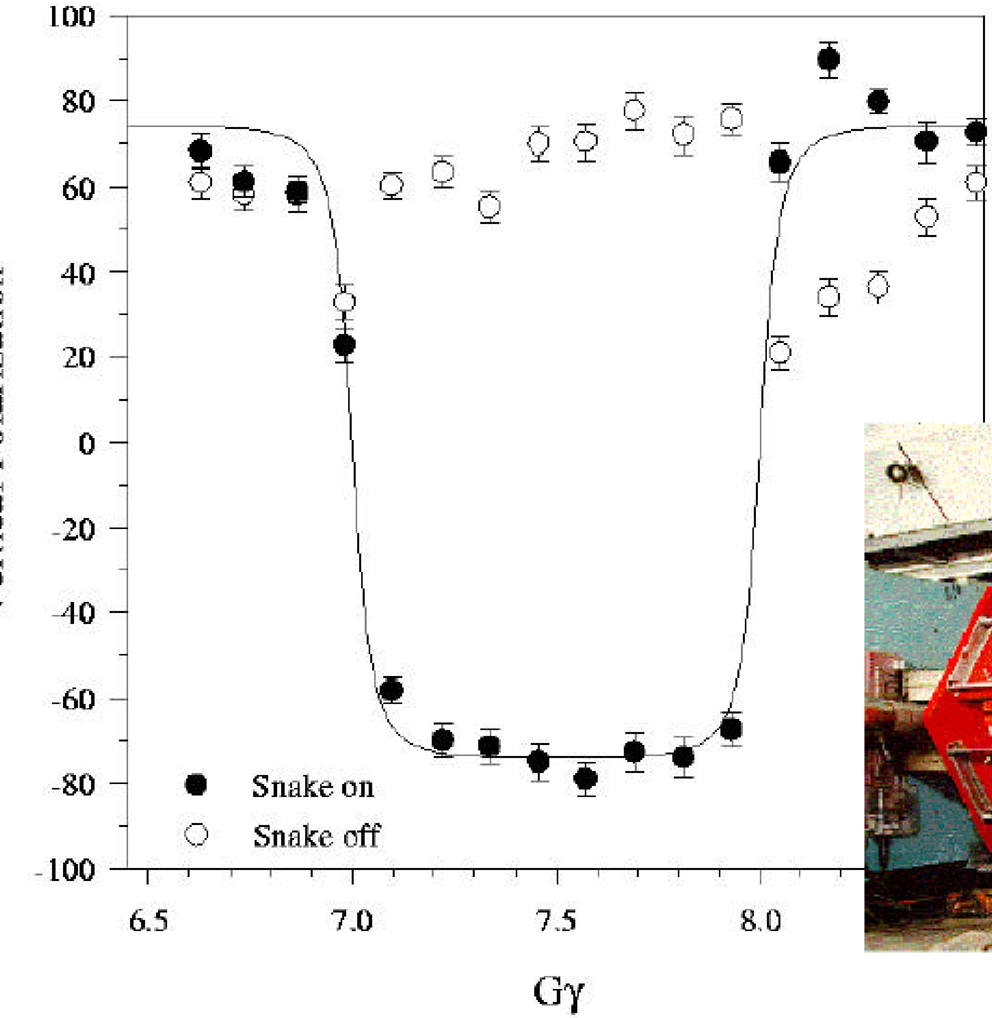
Adiabatic invariant on the closed orbit

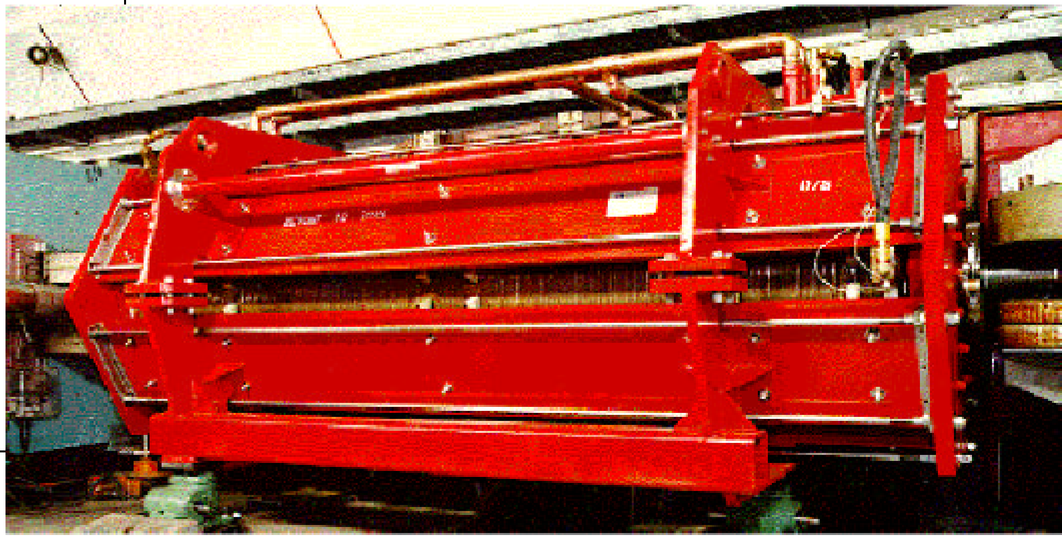


Spin flip during crossing of a resonance with a partial snake

Spin flip at imperfection resonances in the AGS



Courtesy T.Roser



Slowly varying periodic system

Slowly varying parameter $t = eq : \qquad \vec{\Omega}_0(q,t) = \vec{\Omega}_0(q+2p,t)$

Slowly rotating coordinate system: $\frac{d}{dt}(\vec{m},\vec{l},\vec{n}_0) = \vec{h}(q,t)\times(\vec{m},\vec{l},\vec{n}_0)$

$$\frac{d}{dq}\vec{S} = \vec{\Omega}_0(q)\times\vec{S}$$

$$= \vec{m}\frac{ds_1}{dq} + \vec{l}\frac{ds_2}{dq} + \vec{n}_0\frac{ds_3}{dq} + [\vec{\Omega}_0 - n_0\vec{n}_0]\times\vec{S} + (\frac{dt}{dq})\vec{h}\times\vec{S}$$

$$s_1 = \sqrt{1-s_3{}^2}\cos j \ , \quad s_2 = \sqrt{1-s_3{}^2}\sin j$$

$$\sqrt{1-s_3{}^2}\frac{d}{dq}j = -\sin j\frac{d}{dq}s_1 + \cos j\frac{d}{dq}s_2$$

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Slowly varying periodic system

$$\frac{d}{dq}\begin{pmatrix} s_3 \\ j \end{pmatrix} = n_0(t)\begin{pmatrix} e(h_2\sin j + h_1\cos j)\sqrt{1-s_3{}^2} \\ +e[(h_2\sin j + h_1\cos j)\dfrac{s_3}{\sqrt{1-s_3{}^2}} - h_3] \end{pmatrix}$$

$$\frac{d}{dq}\begin{pmatrix} t \\ \tilde{q} \end{pmatrix} = \begin{pmatrix} e \\ 1 \end{pmatrix}$$

Averaging of two frequency systems:

The system which is averaged over 2 **p** of the 2 fast variables describes the true motion of the slow variables up to $< c\sqrt{e}$ for $q < \frac{1}{e}$.

$$s_3(q) = s_3(q_0) + c\sqrt{e}$$ **is an adiabatic invariant**

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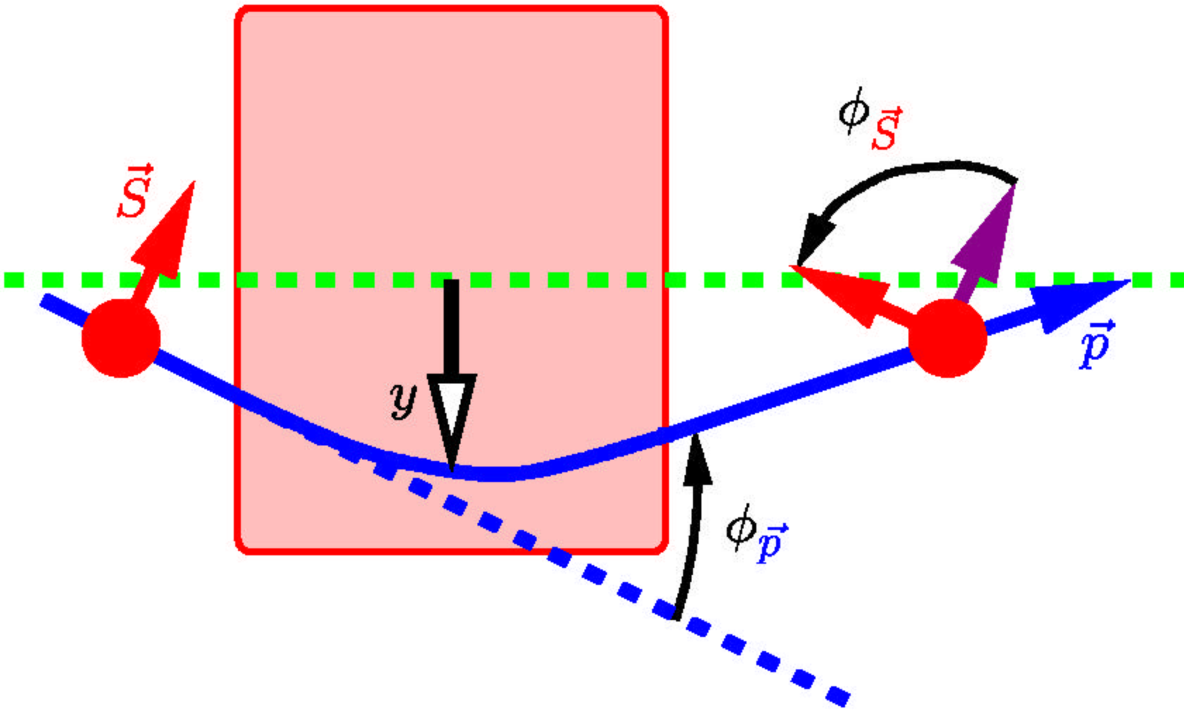
Driven spin perturbation on a trajectory

Integer values of spin-tune $n \pm$ tune n_y lead to coherent disturbances of spin motion



Remedy:

Siberian Snakes avoid resonances by making the spin-tune $n = ½$ independent of energy.

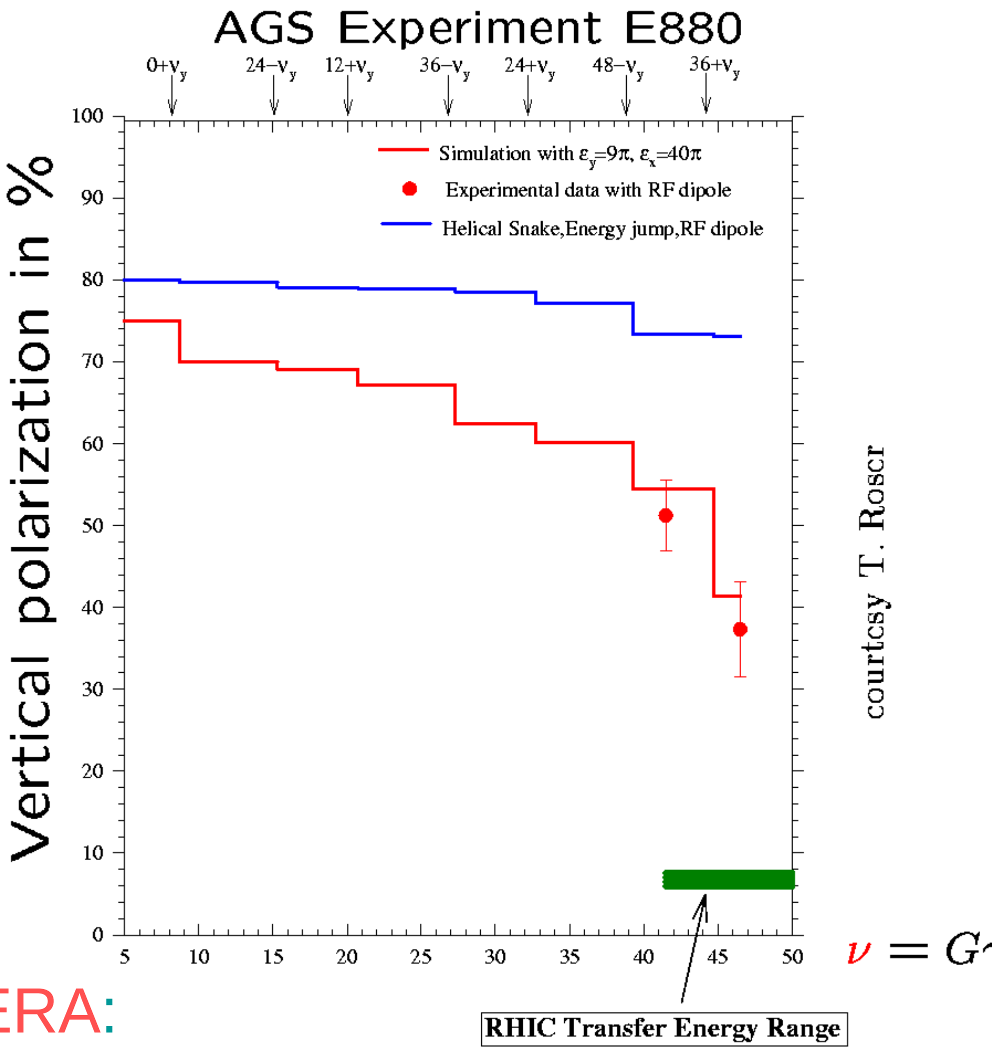
$$\phi_{\vec{S}} \propto \phi_{\vec{p}} \propto y = y_0 \sin(\psi_0 + nQ_y)$$

Crossing resonances

Remedy for DESY III:

Tune jump, energy jump,

and RF dipole excitation



Remedy for PETRA and HERA:

Fixing the spin tune to ½ by Siberian Snakes

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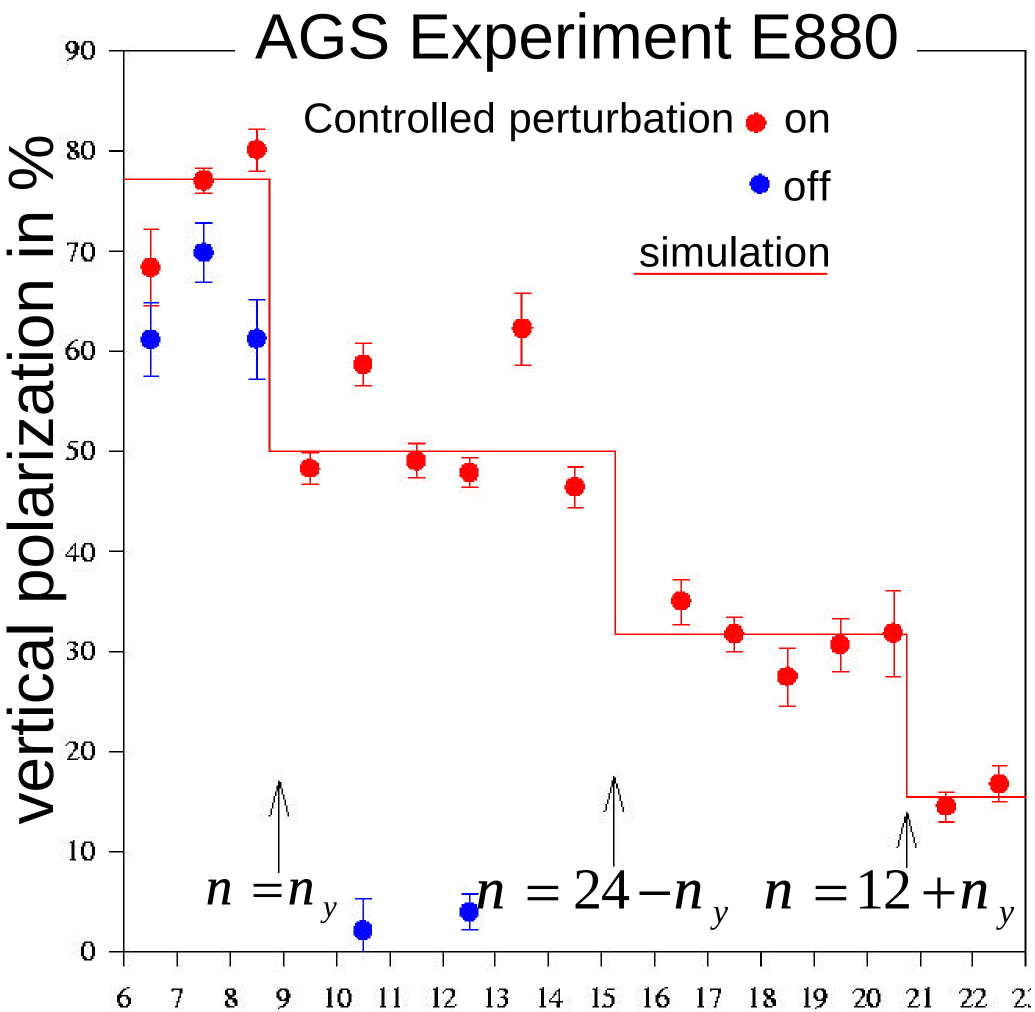
Problems with Resonances



At integer spin-tune n the spin returns without a change after one turn, and error fields add up.

Remedy: insert controlled perturbations of spin motion.

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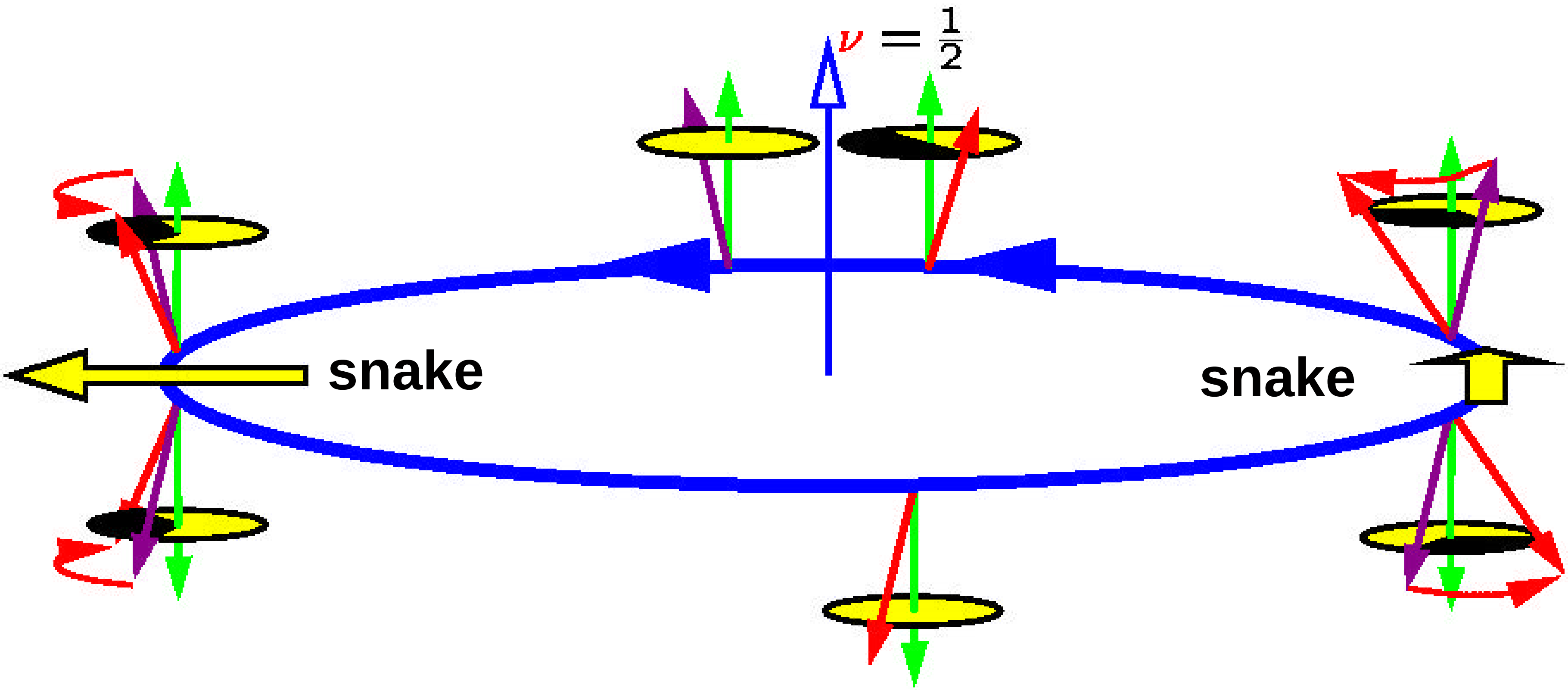
Siberian Snakes

Siberian Snakes rotate spins at each energy ½ times



Freedom: direction of the rotation axis in the horizontal

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CO spin motion with 1 Siberian Snake

$$A \quad = -i(s_1 \cos a + s_2 \sin a)(\cos Ggp - i \sin Ggp\, s_3)$$
$$= -i[s_1 \cos(a - Ggp) + s_2 \sin(a - Ggp)]$$

Spin direction after the snake:

$$\vec{n}_0 = \vec{e}_x \cos(a - \tfrac{Gg}{2}) + \vec{e}_l \sin(a - \tfrac{Gg}{2}) \Leftrightarrow \Psi(0) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i(a - \frac{Gg}{2})} \end{pmatrix}$$

$$\Psi(q) = (\cos \tfrac{Ggq}{2} - i \sin \tfrac{Ggq}{2}\, s_3) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i(a - Ggp)} \end{pmatrix} = \frac{e^{-i\frac{Ggq}{2}}}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i[a - Gg(p - q)]} \end{pmatrix}$$

Snake angle tunes spin direction anywhere in the ring, especially at $q = p$
it is independent of energy !

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CO spin motion with 2N Siberian Snake

$$A = \prod_{j=1}^{2N} i e^{-i\frac{y_j}{2}s_3}(s_1 \cos a_j + s_2 \sin a_j)$$

$$= i^N e^{-i\frac{y_{2N} \cdots - y_3 + y_2 - y_1}{2}s_3} \prod_{j=1}^{N} (s_1 \cos a_{2j} + s_2 \sin a_{2j})(s_1 \cos a_{2j-1} + s_2 \sin a_{2j-}$$

$$= i^N e^{-i\frac{\Delta y}{2}s_3} \prod_{j=1}^{N} [\cos(a_{2j} - a_{2j-1}) - i \sin(a_{2j} - a_{2j-1}) s_3]$$

$$A = i^N e^{-i\frac{\Delta y + 2\Delta a}{2}s_3} \Rightarrow$$

$$n_0 = \frac{\Delta y + 2\Delta a}{2p}$$

$$\vec{n}_0 = \vec{e}_y$$

$\Delta y = 0$, to make $_0$ independent of energy

$\Delta a = \frac{p}{2}$, to make $_0$=0.5

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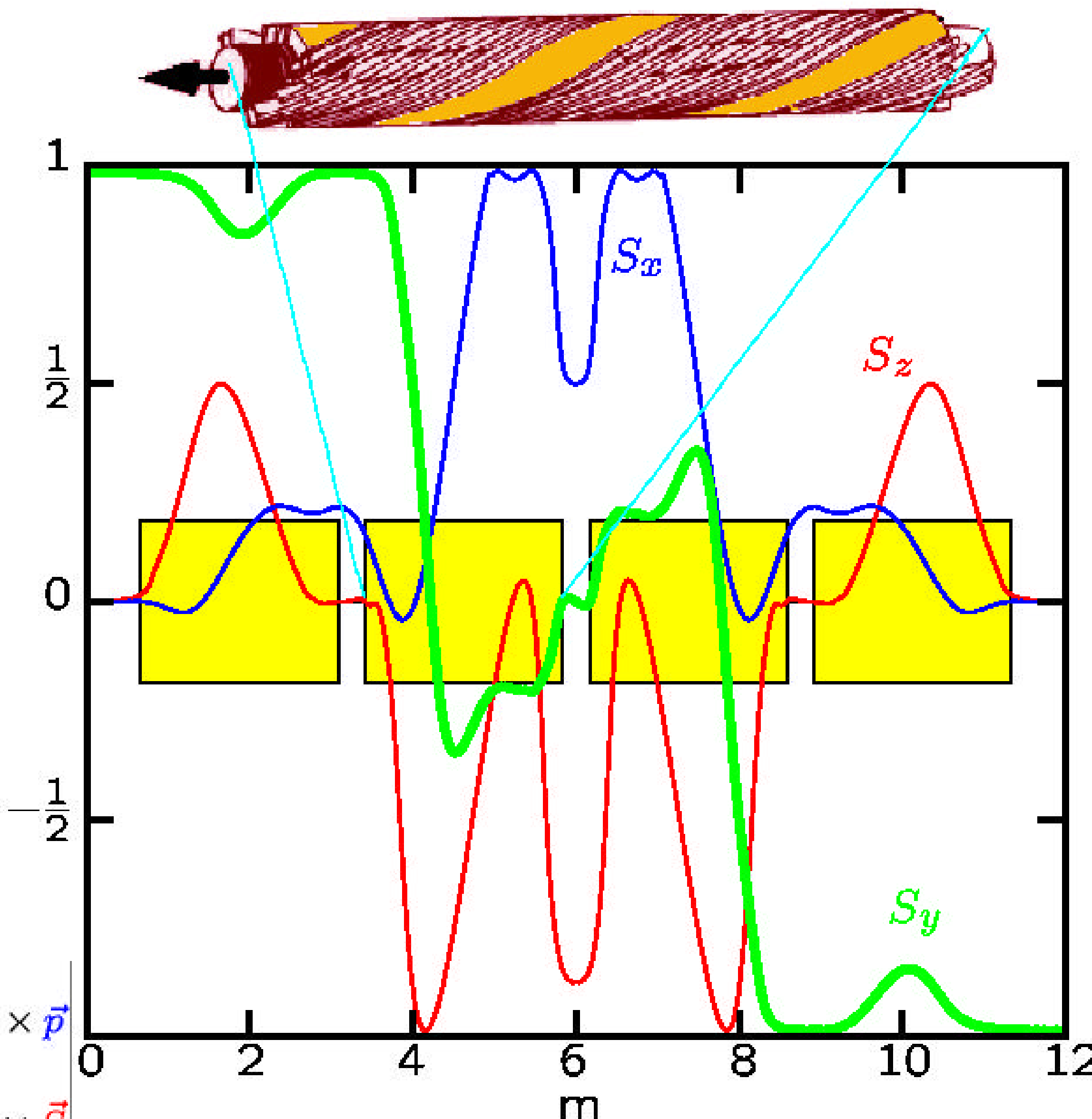
RHIC Siberian Snakes (A)

Spin Motion

- 2 helical dipoles
- 10 cm diameter
- Superconducting 4Tesla magnets



$$\frac{d\vec{p}}{dt} = \left(\frac{-q}{m\gamma}\right)\{ \quad \vec{B}_\perp \quad \} \times \vec{p}$$

$$\frac{d\vec{S}}{dt} = \left(\frac{-q}{m\gamma}\right)\{(G\gamma+1)\vec{B}_\perp + (1+G)\vec{B}_\parallel\} \times \vec{S}$$

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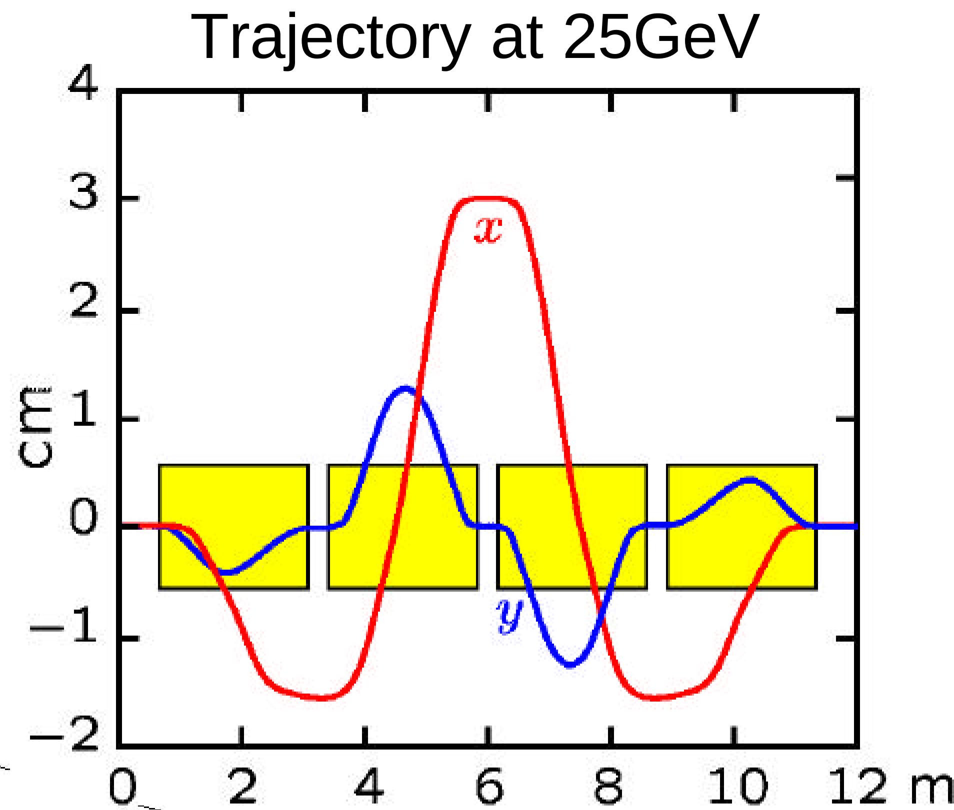
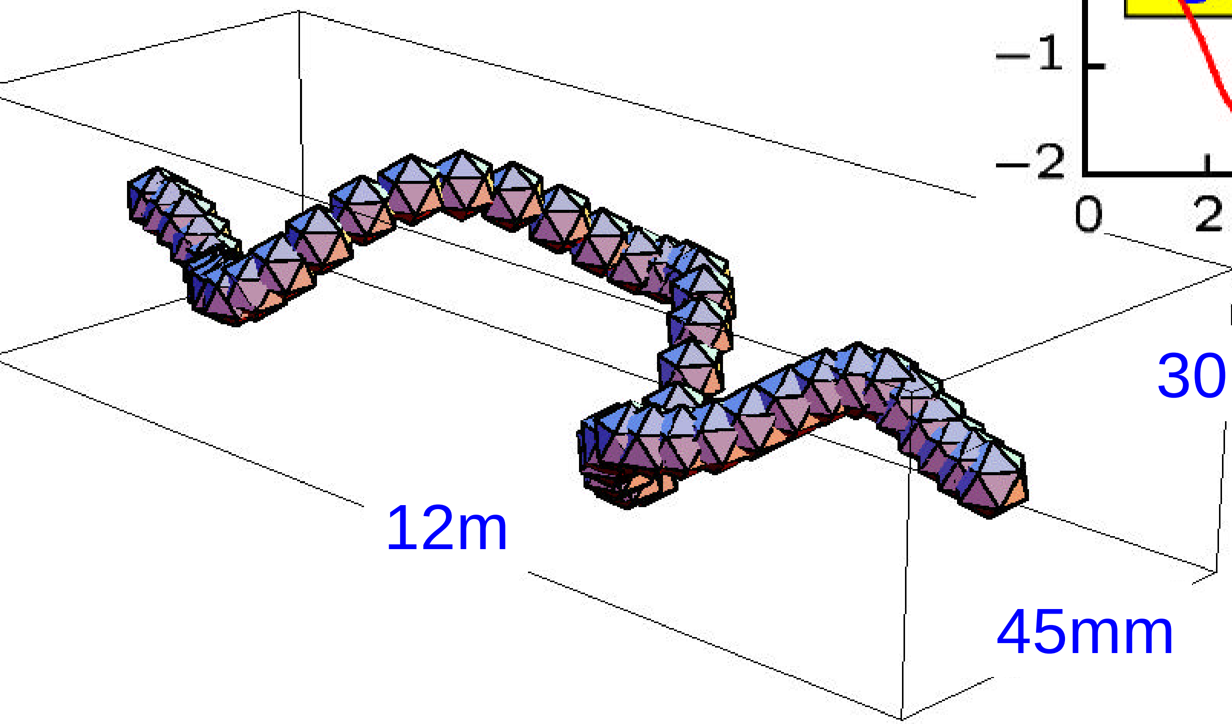
RHIC Siberian Snakes (B)

Particle Trajectories



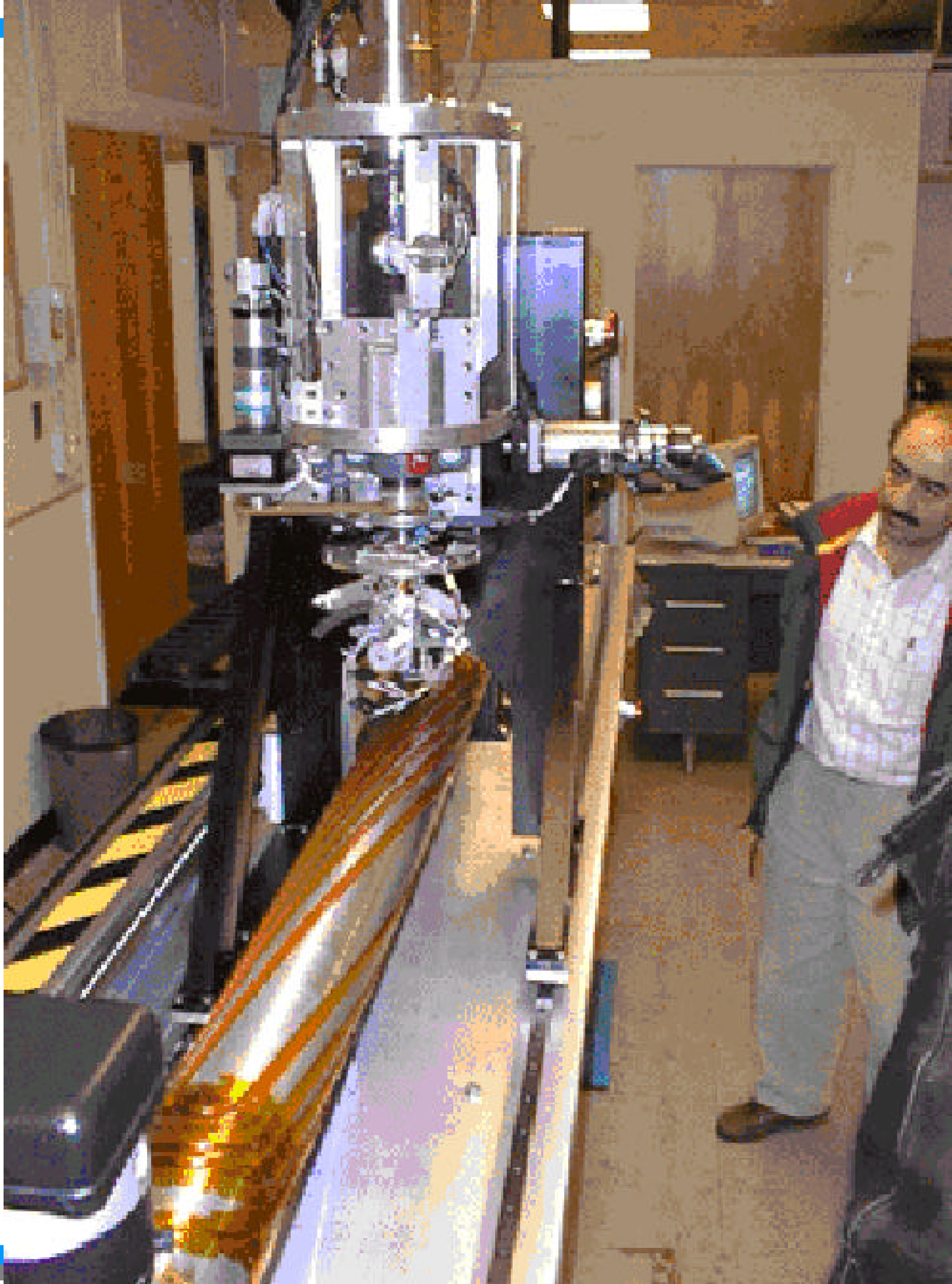


RHIC Siberian Snake (C)

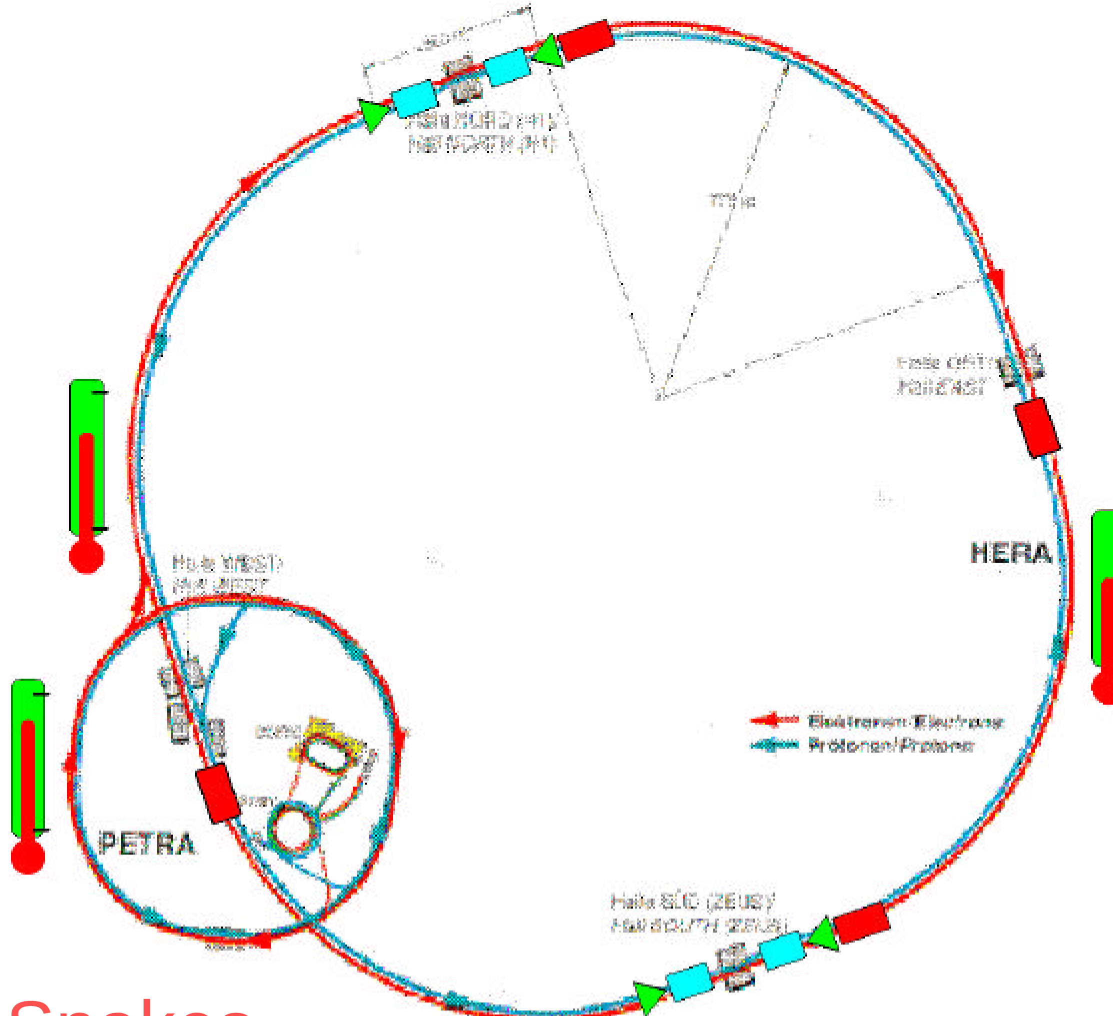
Production

$$\frac{d\vec{p}}{dt} = \left(\frac{-q}{m\gamma}\right)\{ \vec{B}_\perp \} \times \vec{p}$$

$$\frac{d\vec{S}}{dt} = \left(\frac{-q}{m\gamma}\right)\{(G\gamma + 1)\vec{B}_\perp + (1 + G)\vec{B}_\parallel\} \times \vec{S}$$



Required installations in HERA



Cost: about 30M Euro

- Polarimeters
- Flattening Snakes
- Spin rotators
- At least 4 Siberian Snakes

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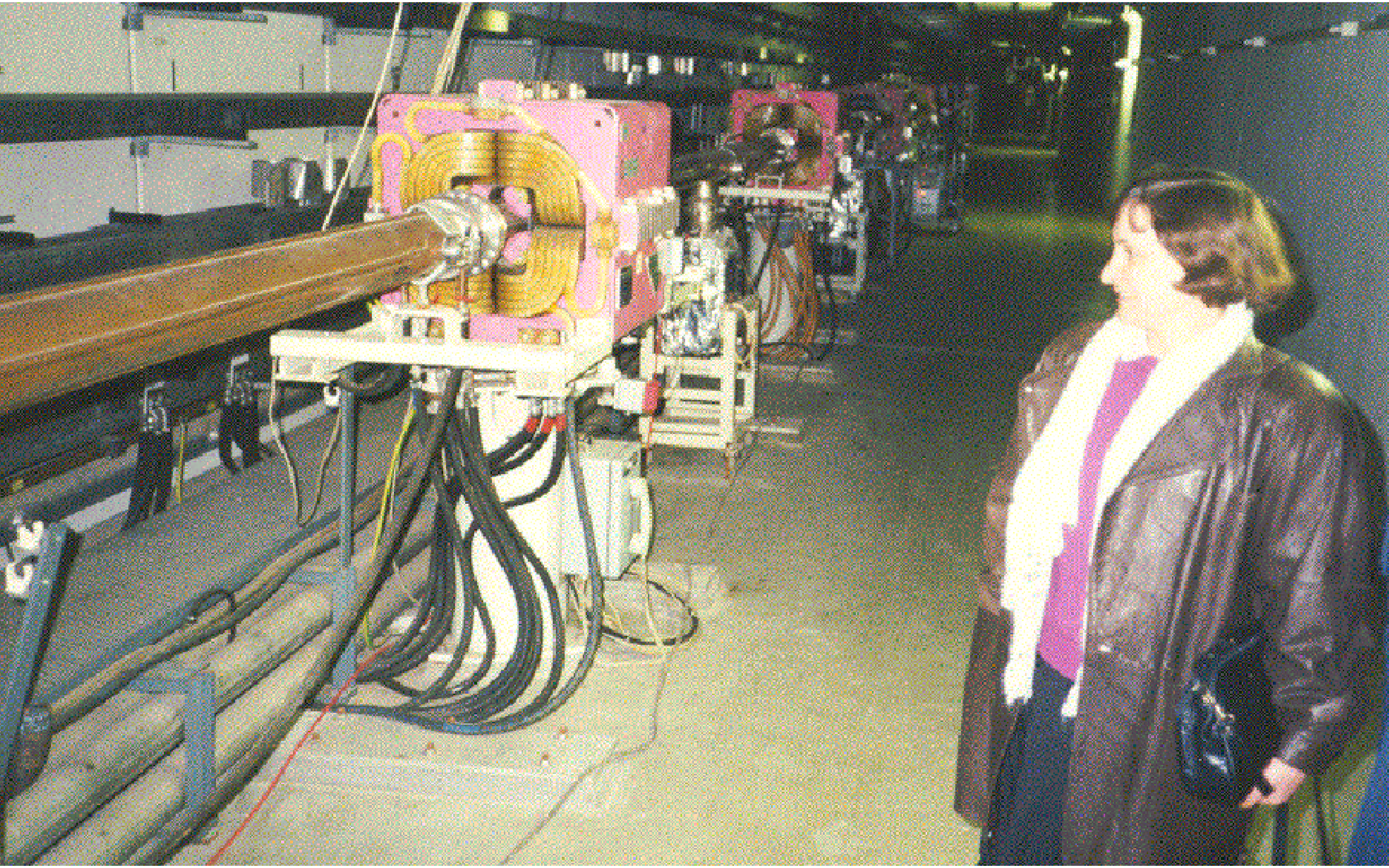
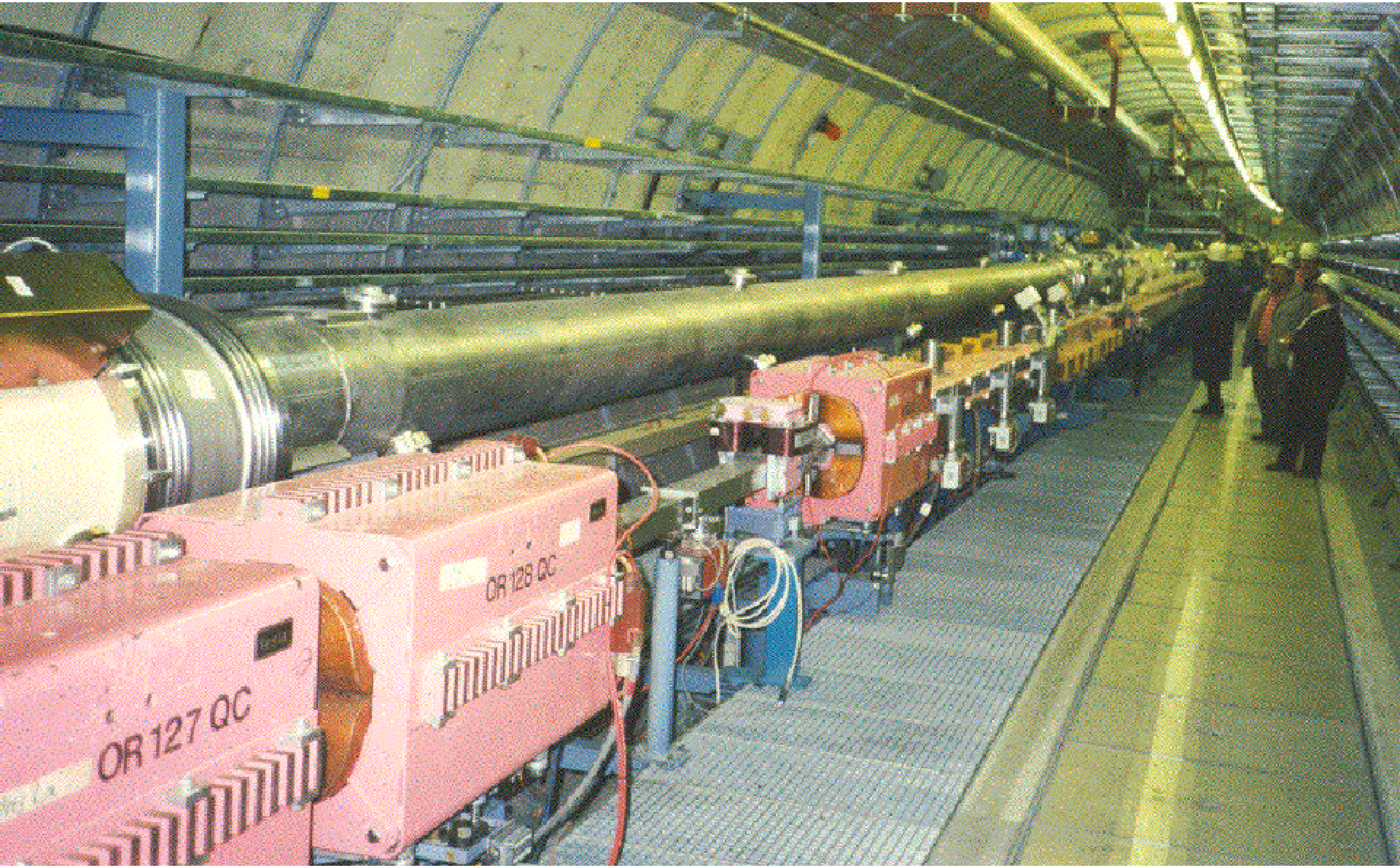
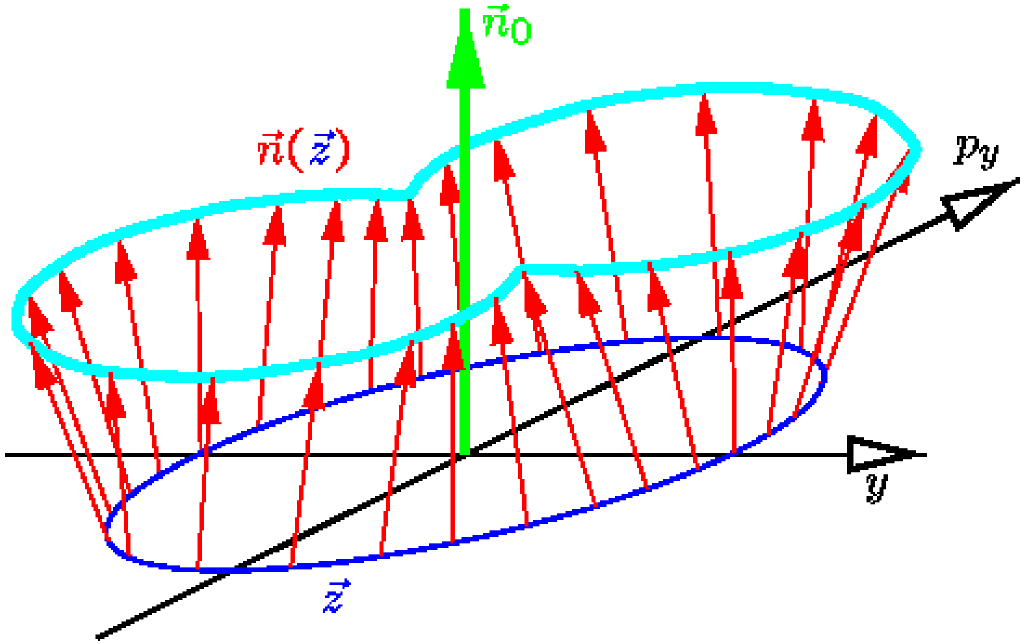
Space for PETRA's Siberian Snakes



Space for Siberian Snakes in HERA



The Invariant Spin Field



A) Maximum polarization: $\boldsymbol{P_{lim}} = < \vec{n}(\vec{z}) >_{\text{Phase space}}$

For a large divergence, the average polarization is small, even if the local polarization is 100%.

B) $\vec{n}(\vec{z}) \cdot \vec{S}$ is an adiabatic invariance !

Linearized $\vec{n}(\vec{z})$ can be analytically computed

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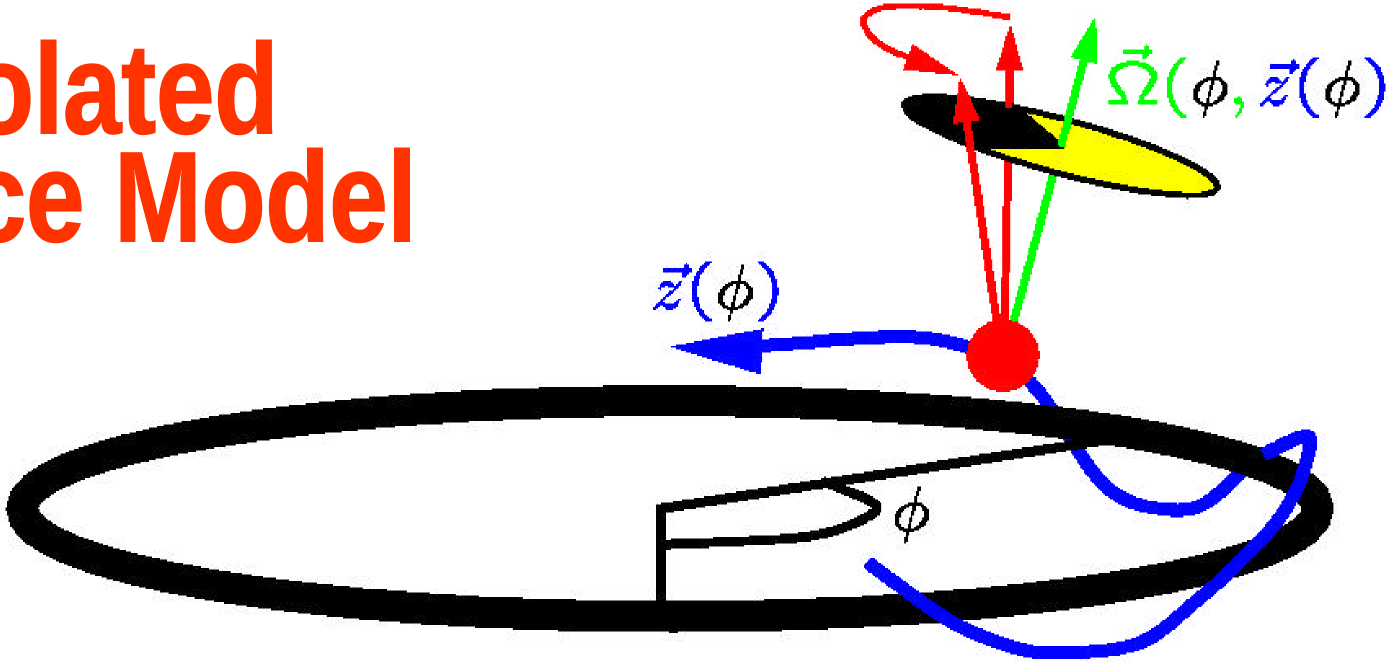
The Isolated Resonance Model



$$\vec{S}' = \vec{\Omega}(\phi, \vec{z}(\phi)) \times \vec{S}$$

All Fourier components of $\vec{\Omega}(\phi, \vec{z}(\phi))$, except the dominant one, are neglected.

Is this theory still applicable for HERA with 920 GeV ? ? ?

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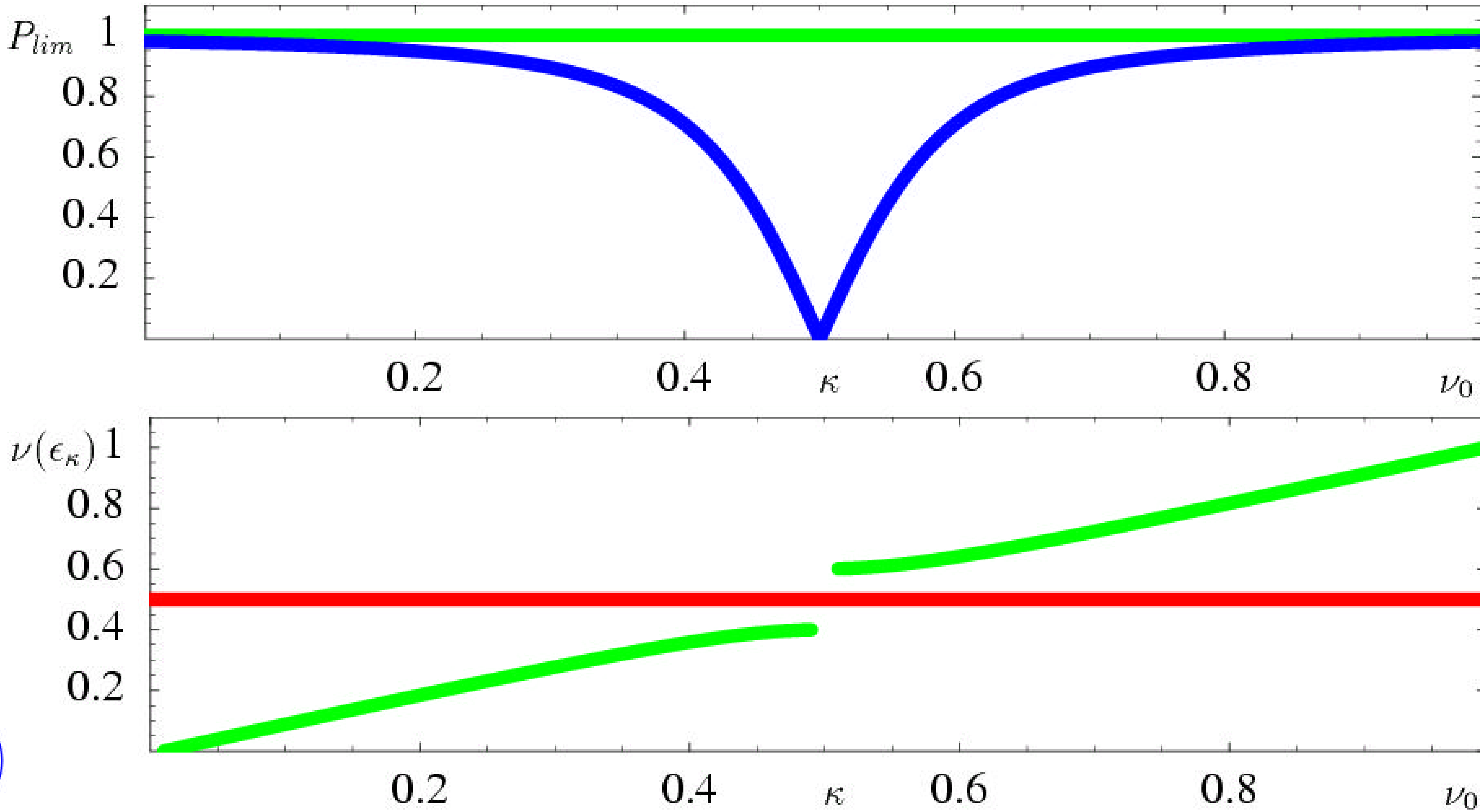
The Isolated Resonance Model

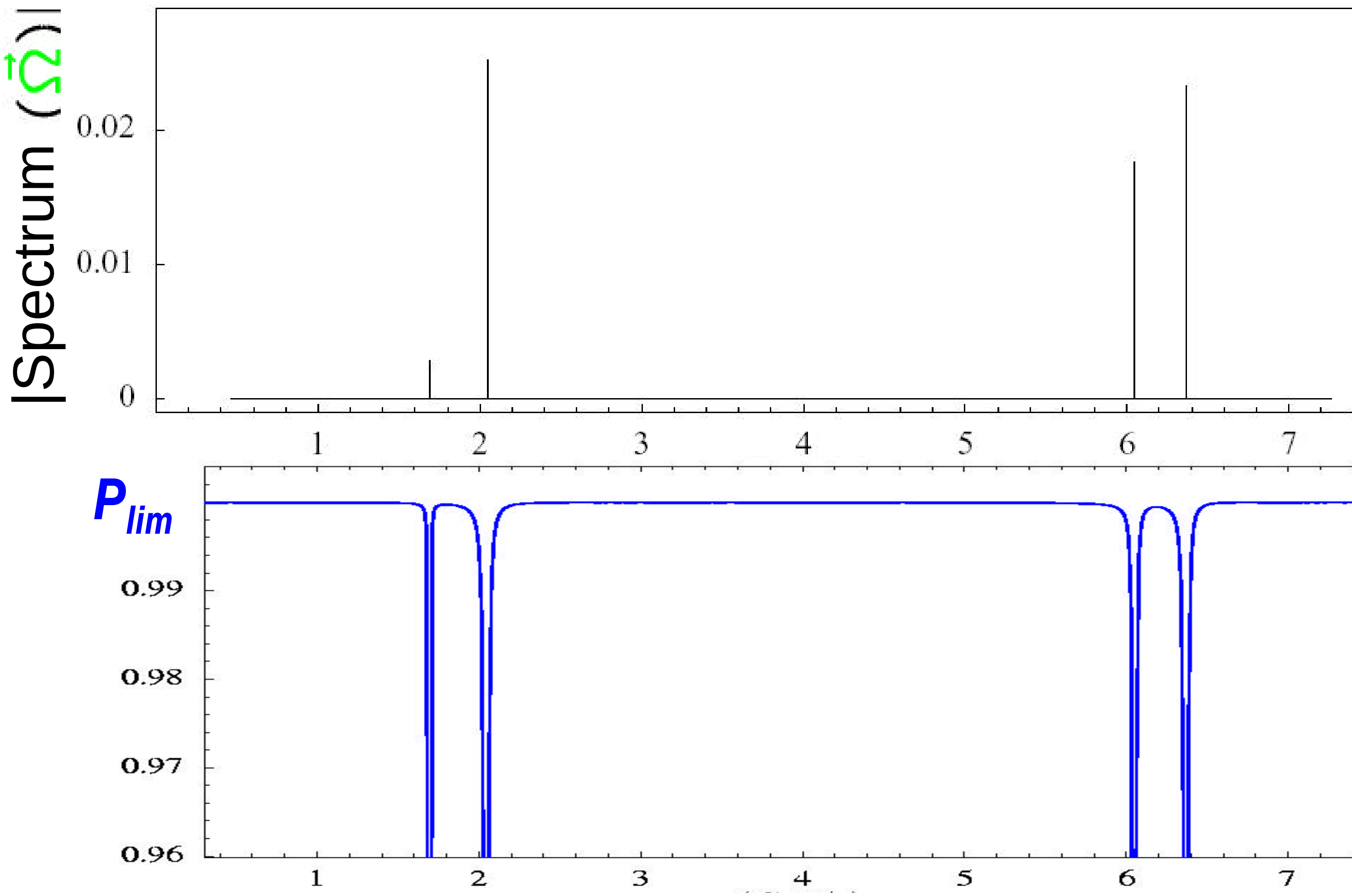
All Fourier components of $\vec{\Omega}(\phi, \vec{z}(\phi))$,
except the dominant one, are neglected.



First Order Theories A) DESY III

Isolated resonance model:

Linear spin-field theory:



Momentum (GeV/c)

Low energies: First order theories agree

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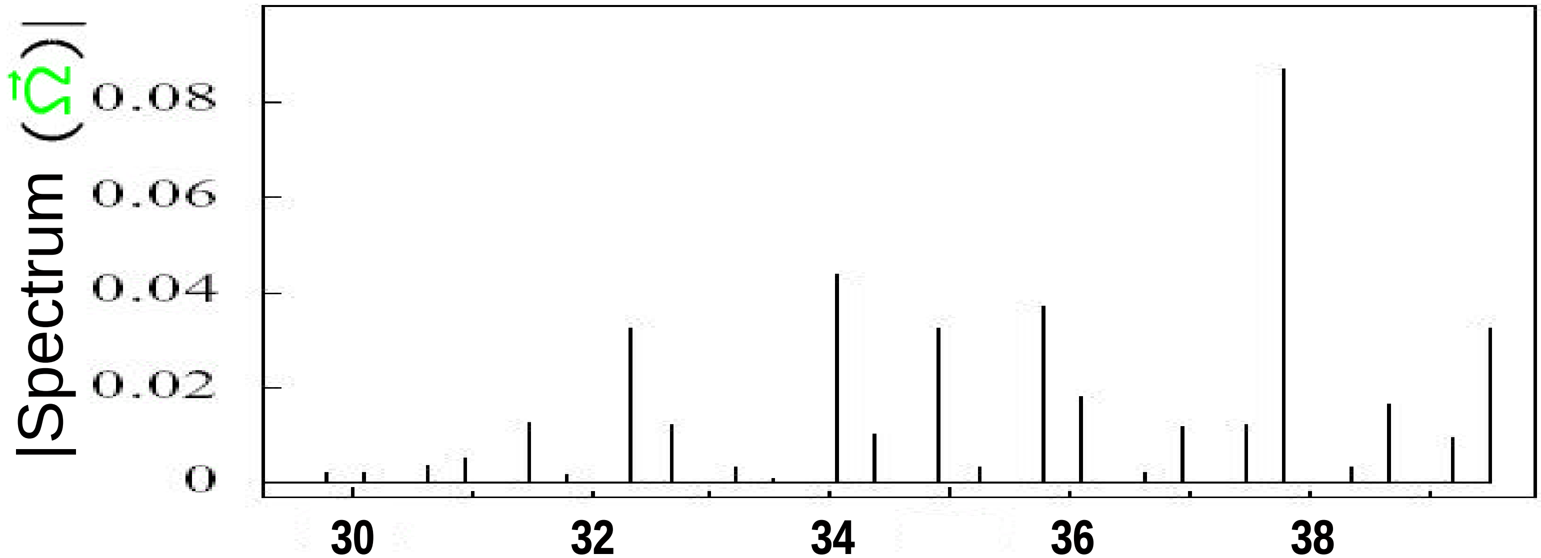
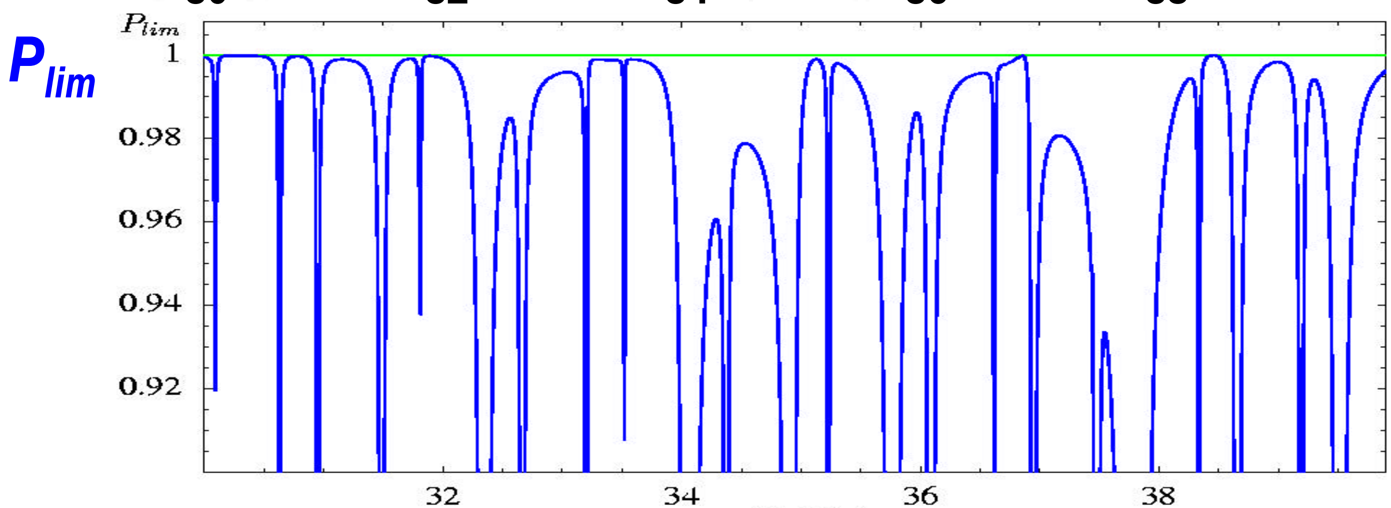
First Order Theories B) PETRA

Isolated resonance model:



Linear spin-field theory:



Momentum (GeV/c)

Medium energies: resonances still isolated

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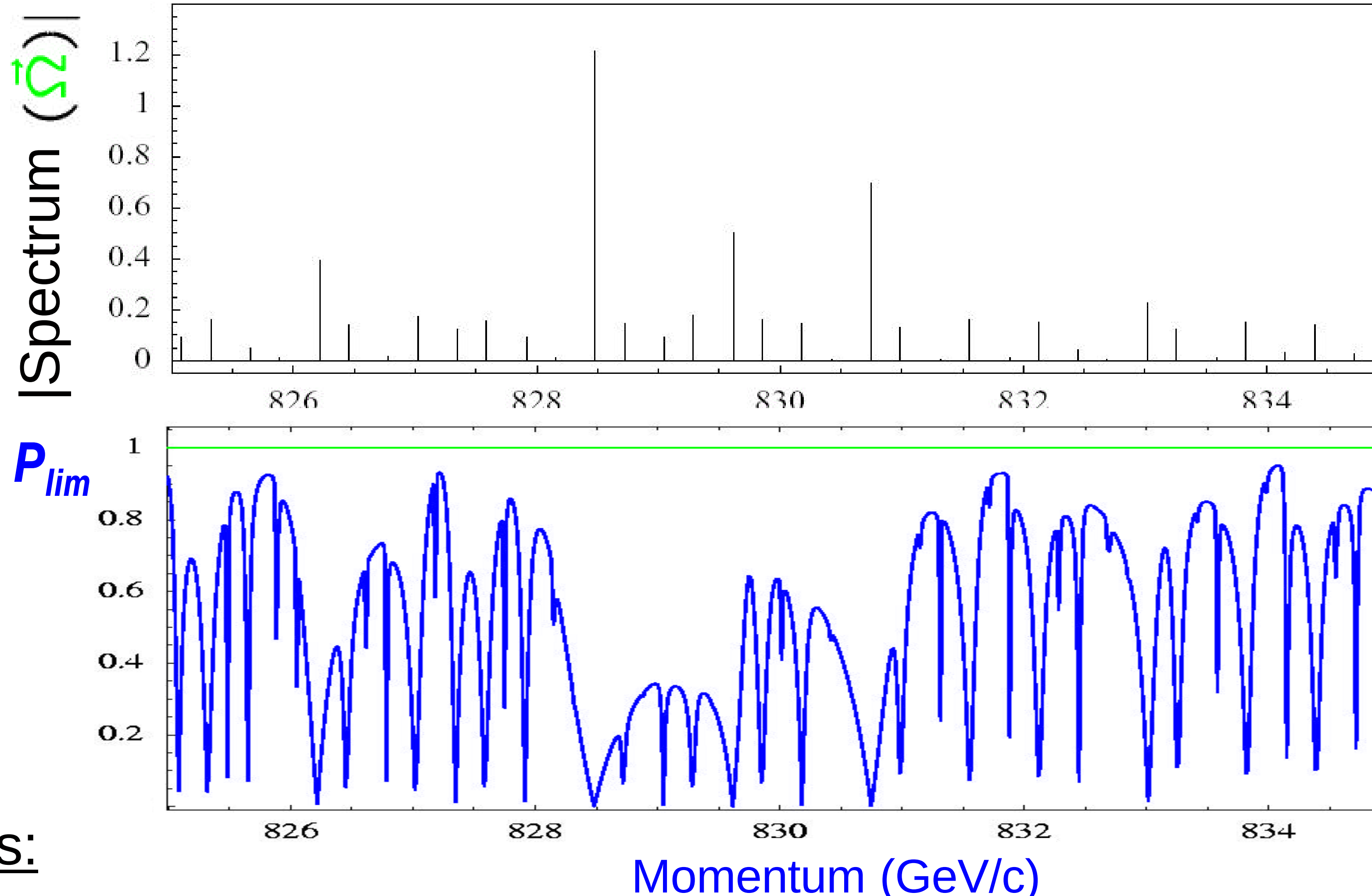
First Order Theories C) HERA

Isolated resonance model:

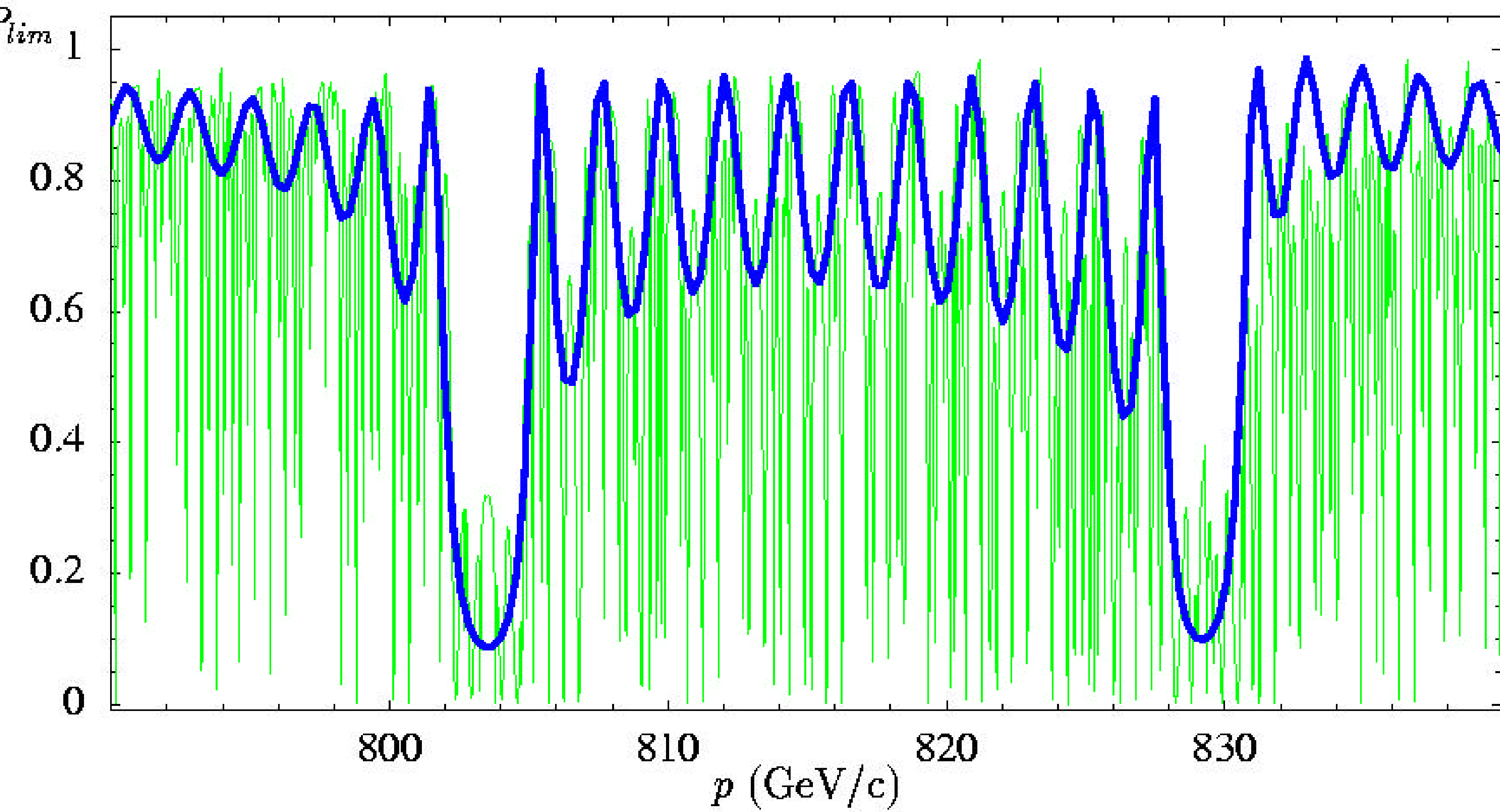


Linear spin-field theory:

High energies: Resonances are no longer isolated.

The isolated resonance model becomes invalid

Siberian Snakes and Resonances



Some structure of the 1st order resonances remains after Siberian Snakes have been installed.

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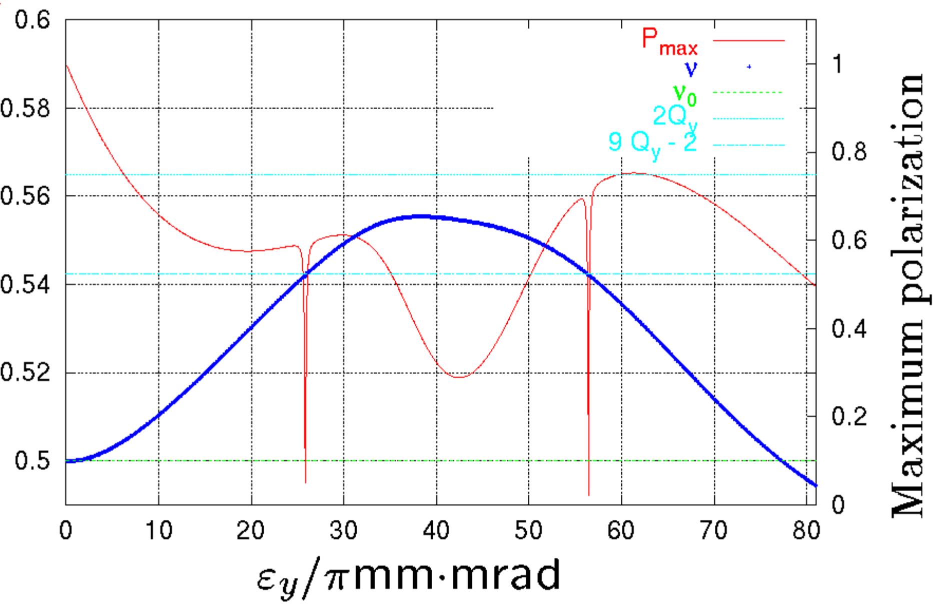
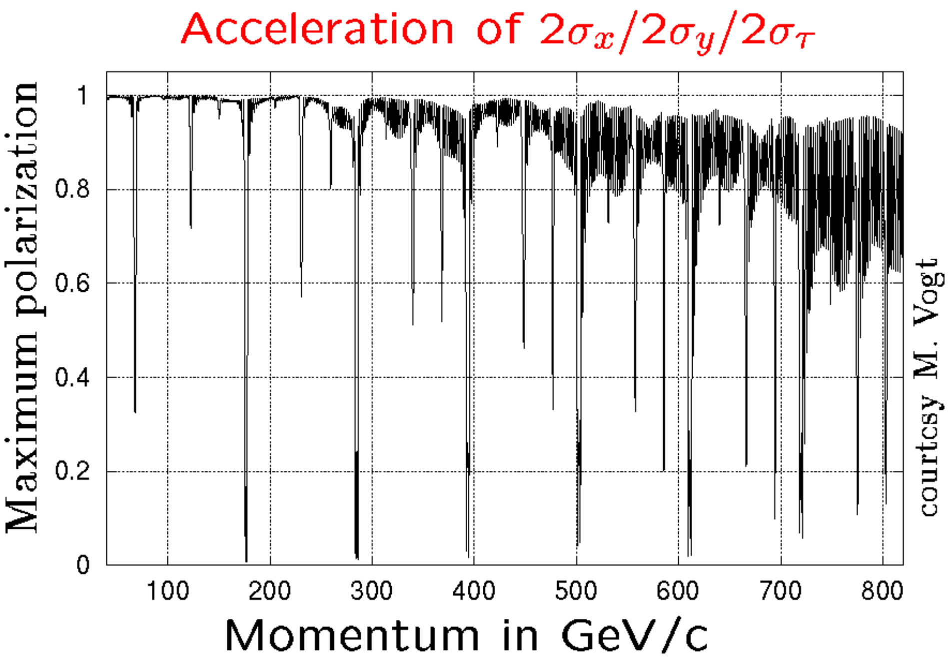
Amplitude dependent spin-orbit resonances





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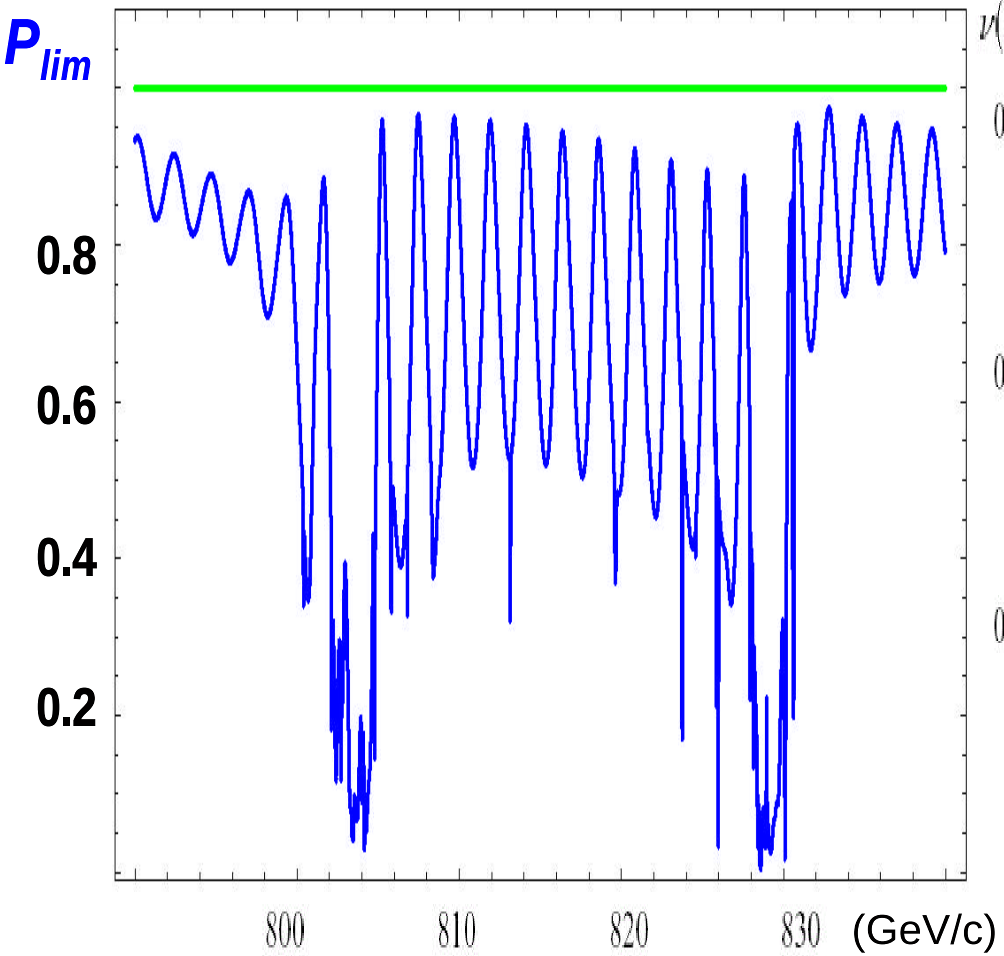
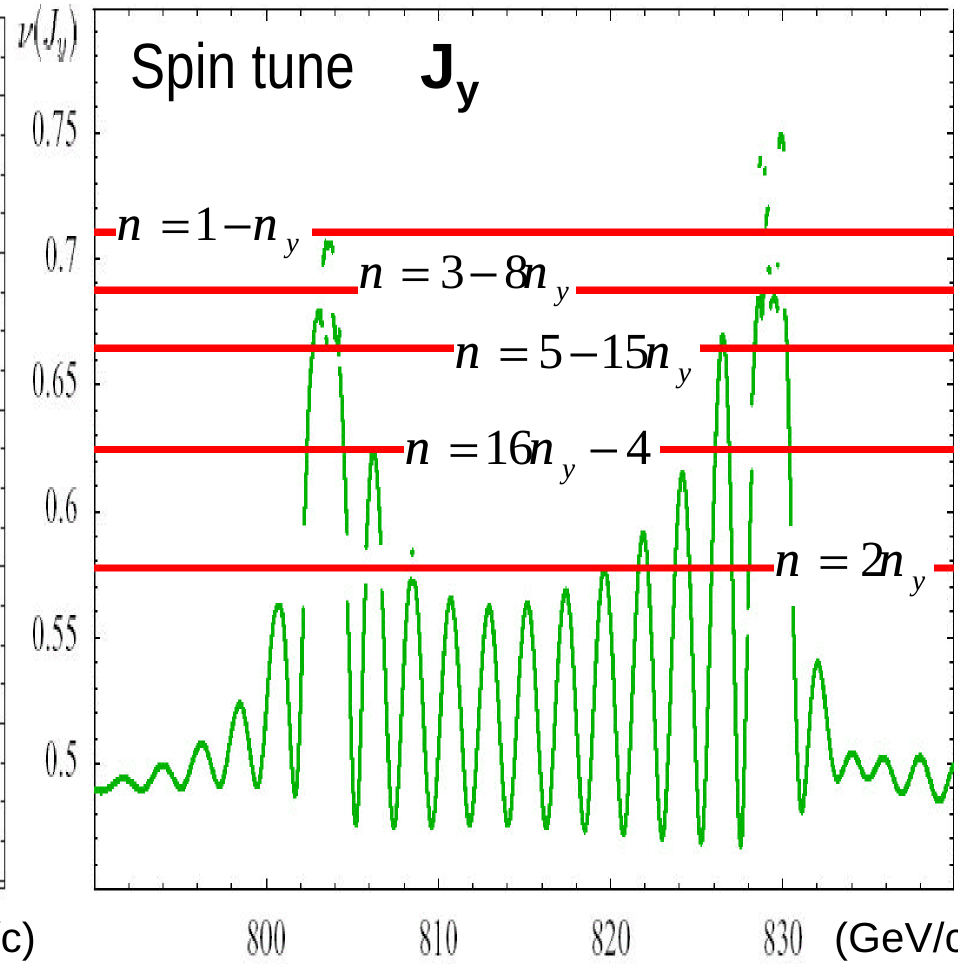
Spin Tune at Higher Order Resonance

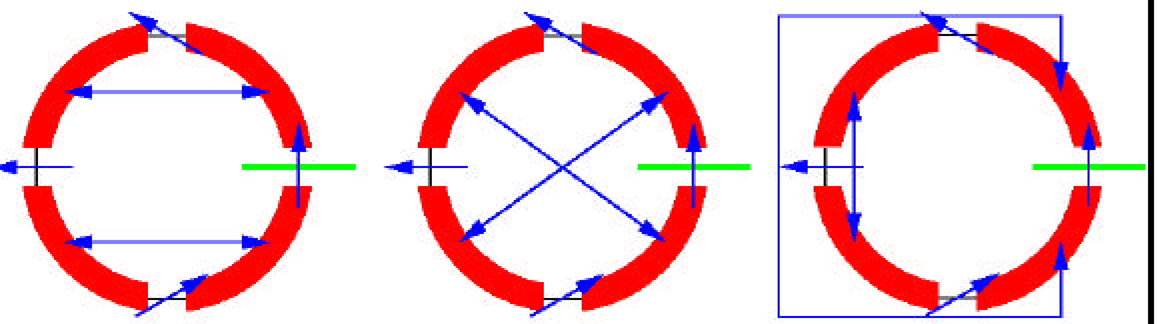




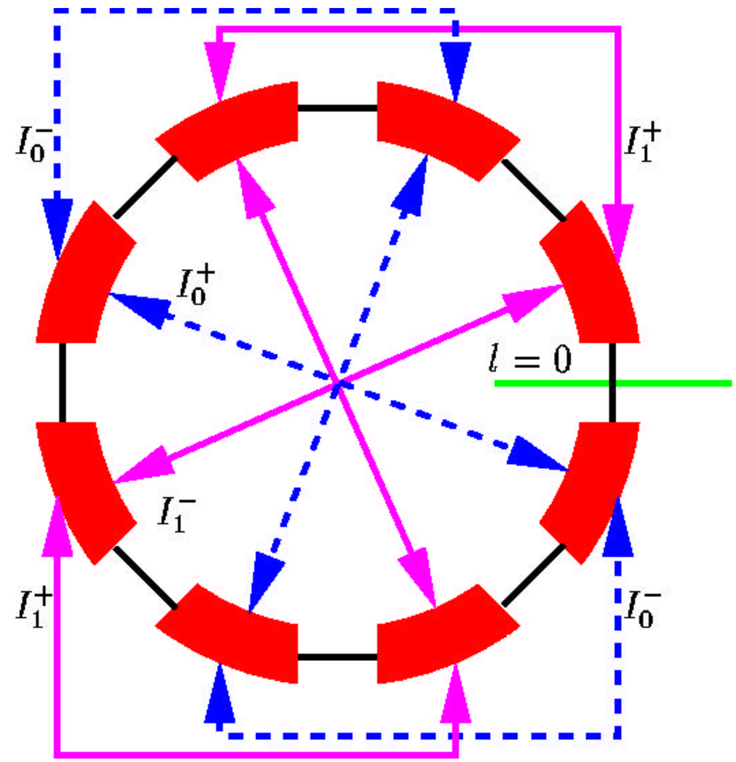
The spin tune deviates from ½ for particles which oscillate around the design trajectory with amplitude J_y.

Snake matching

4 Snakes:



8 Snakes:



1st Order: 4 harmonics of the spin perturbation in each section.

With 4 snakes only 2 can be compensated

With 8 snakes all 4 can be compensated

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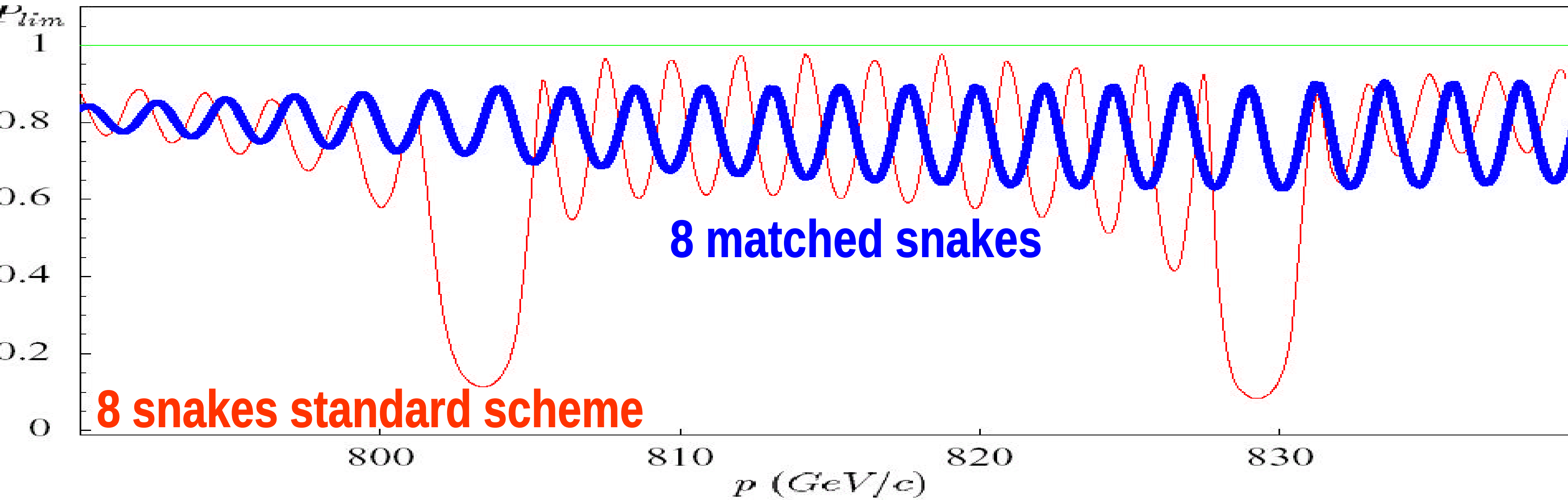
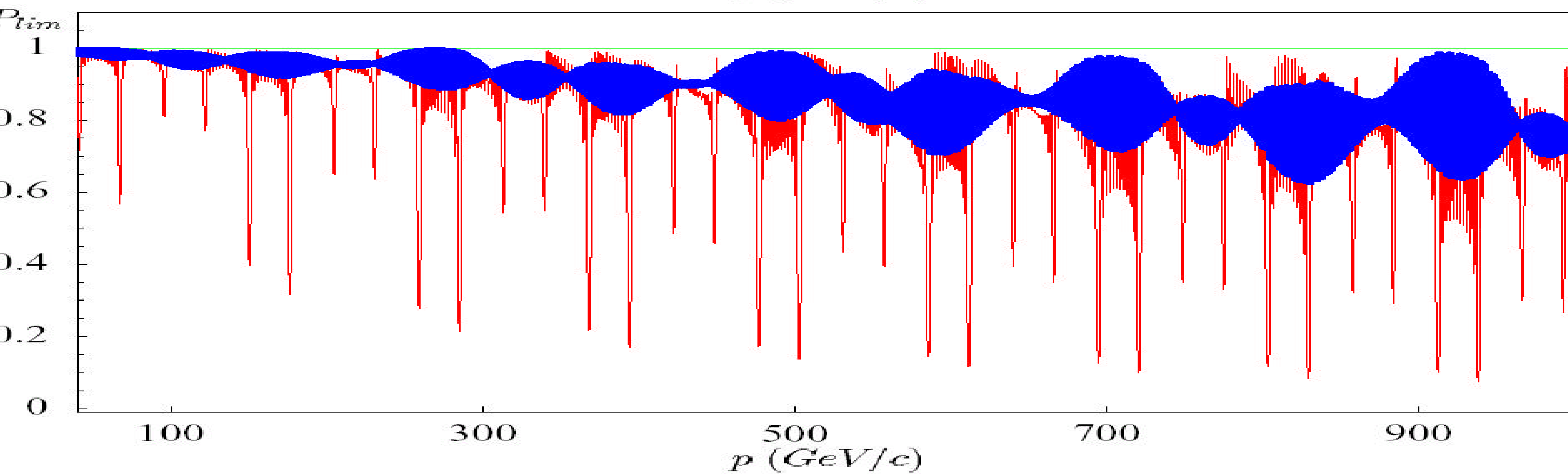
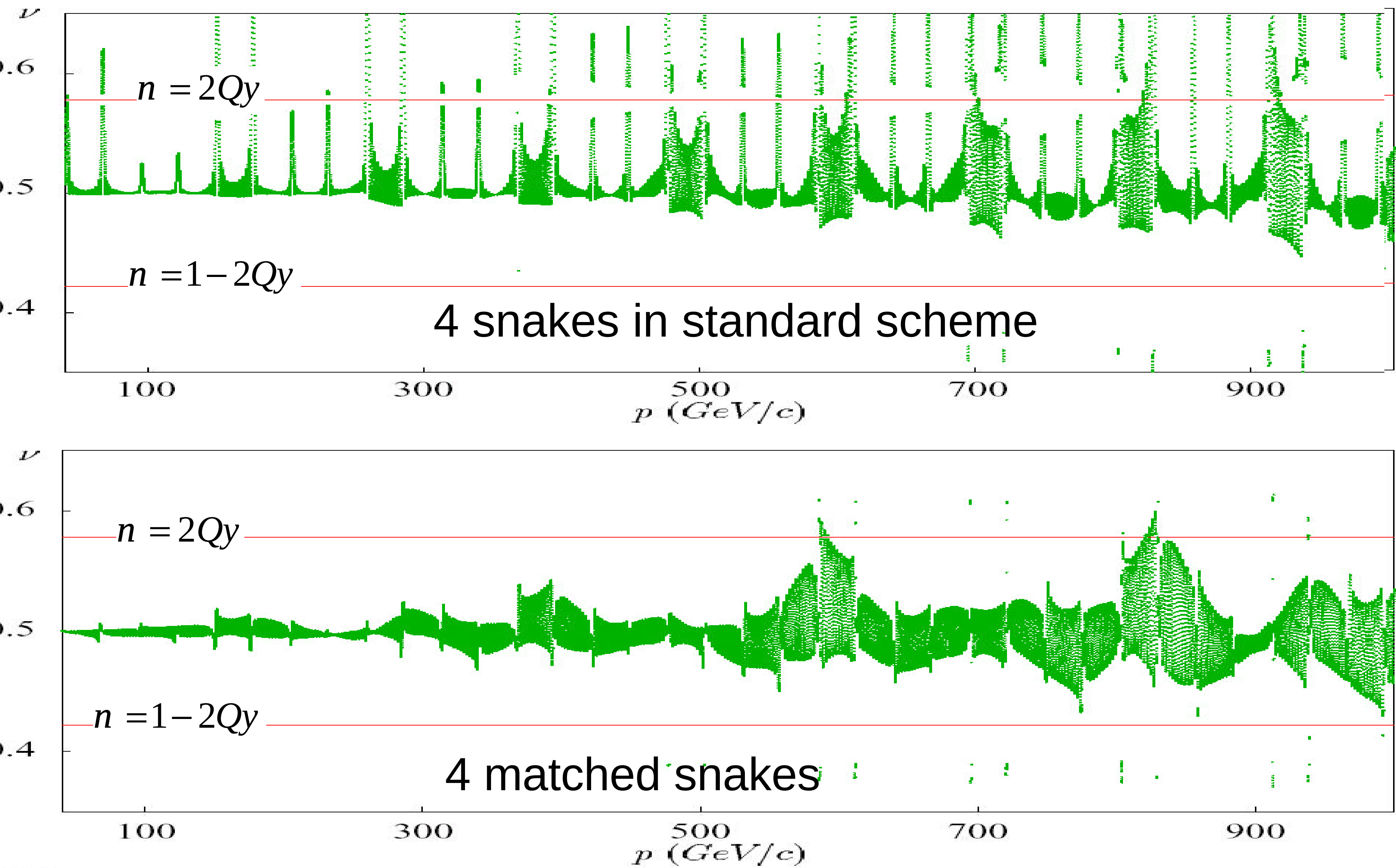
P_{lim} after Snake Matching

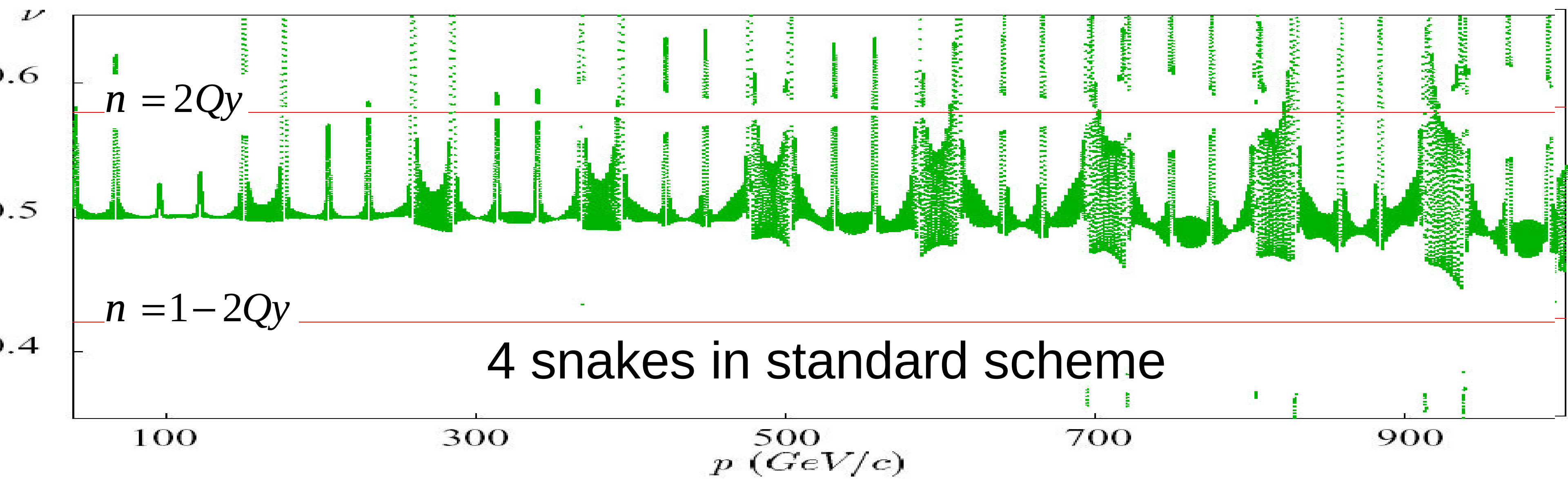


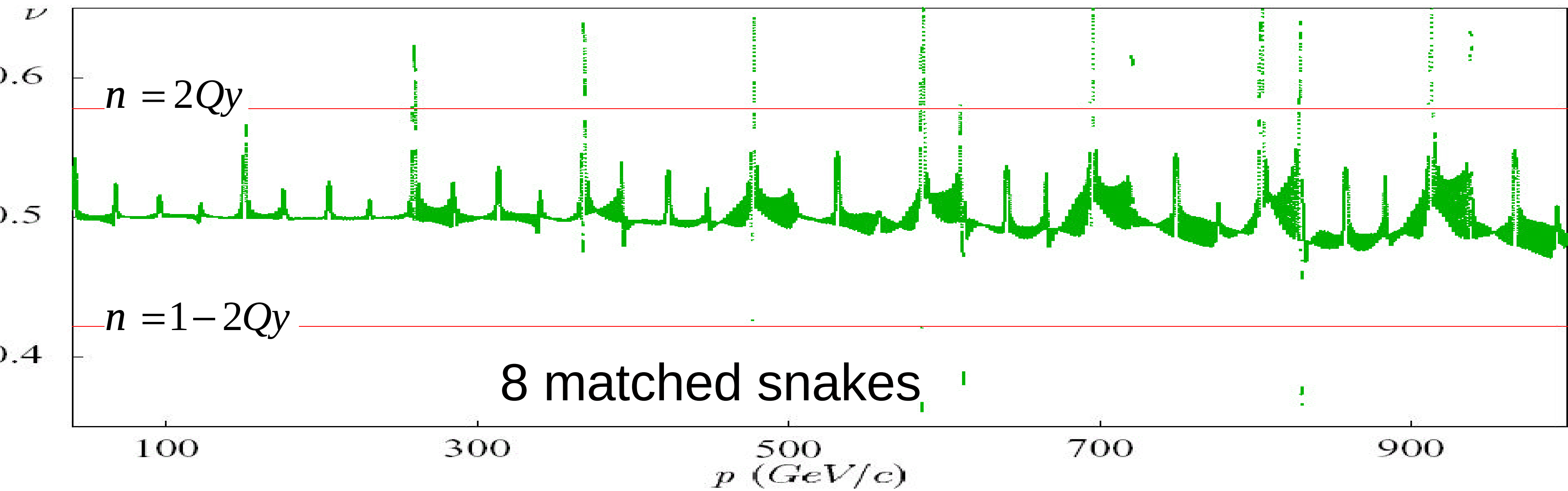


Spin Tune after Snake Matching



Spin Tune after Snake Matching



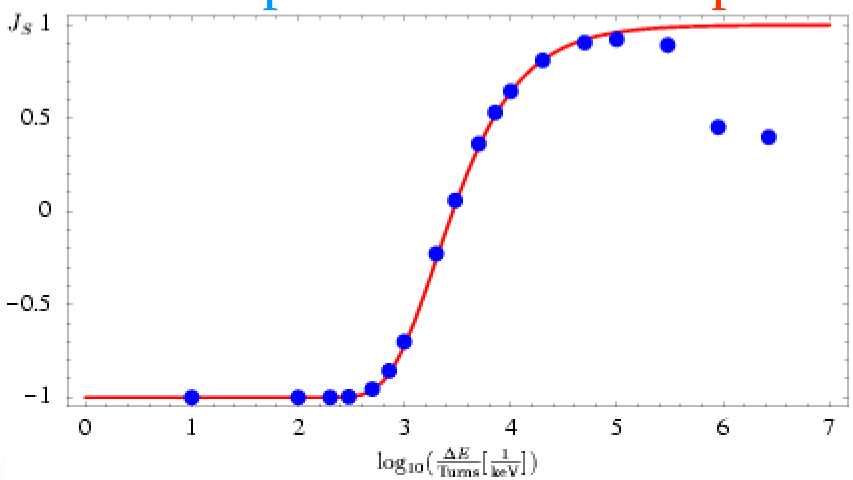


High Order Resonance Strength

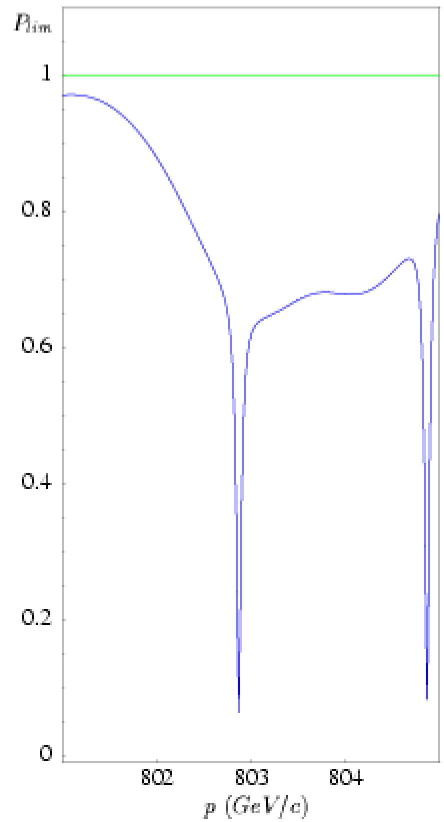
The higher order Froissart-Stora formula

- Resonances up to 19th order can be observed
- Resonance strength can be determined from tune jump.

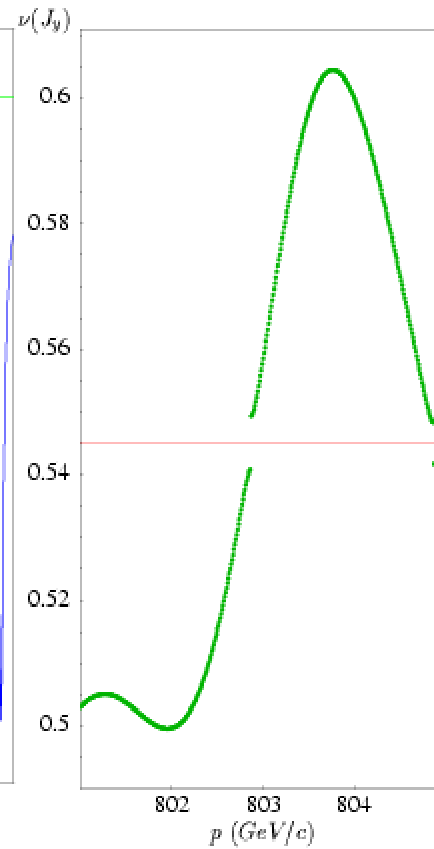
Tracked depolarization as expected



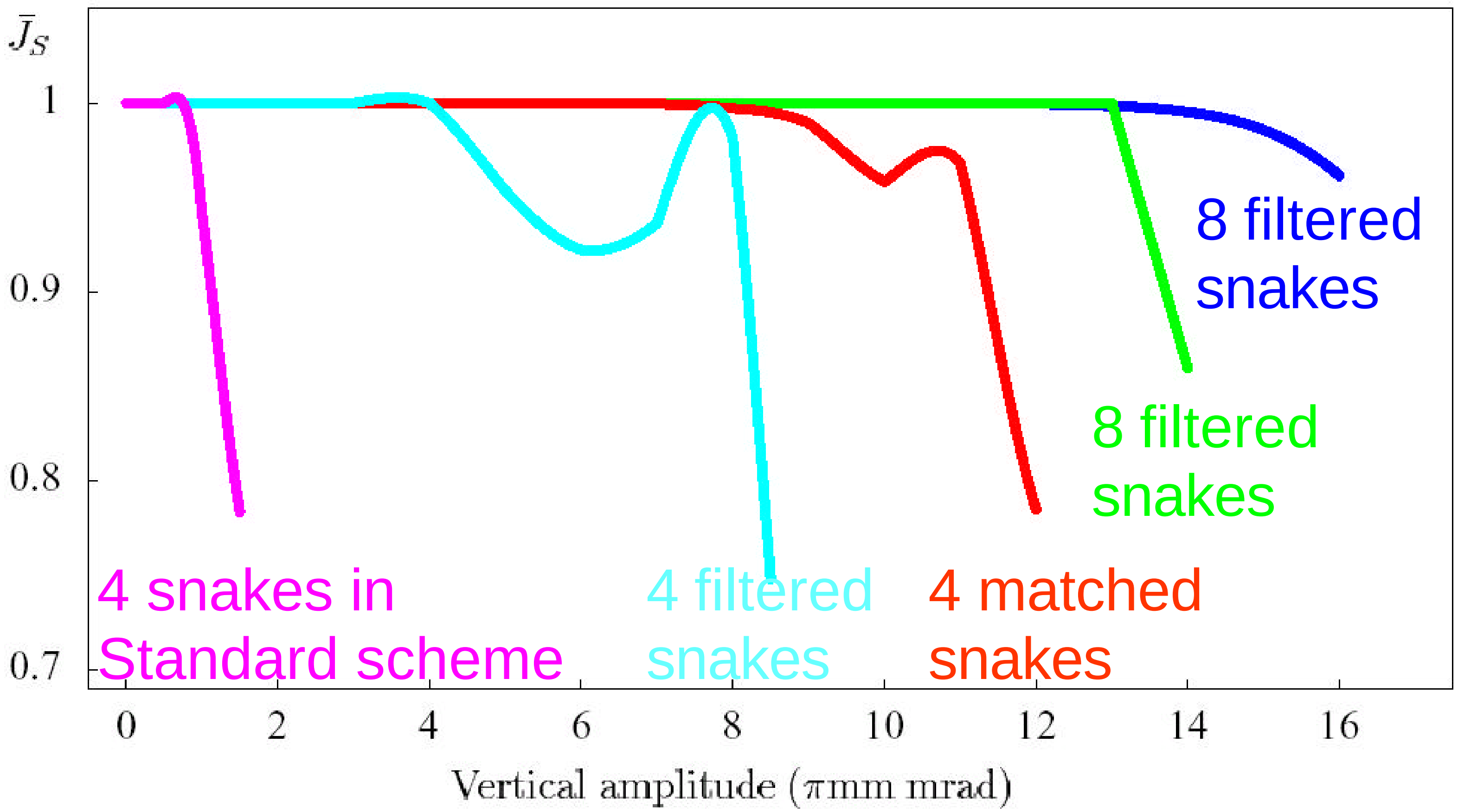
$$P_{\lim} = <\vec{n}(\vec{z})>$$



Spin tune



Allowed Beam Sizes



Snake matching allows to have significantly larger beams.

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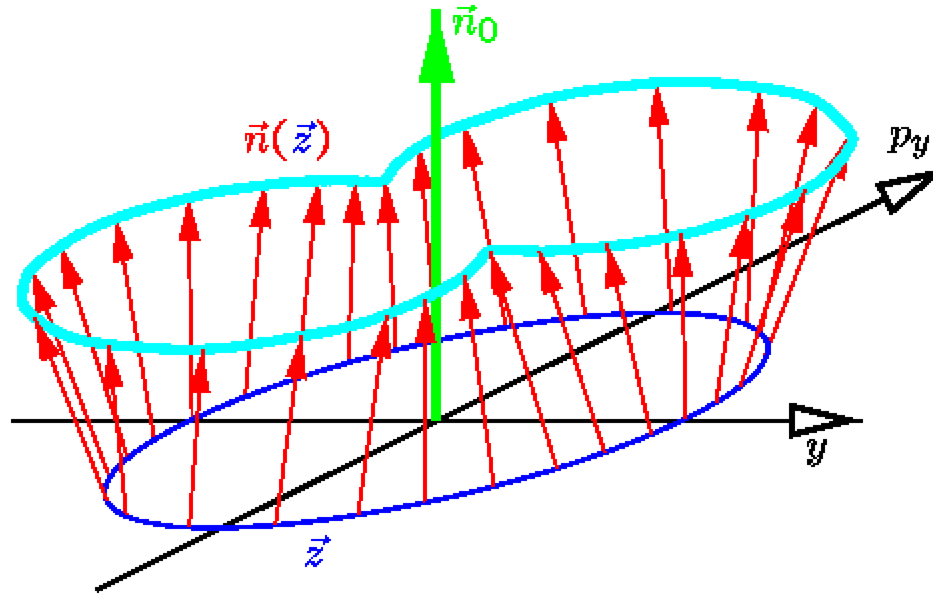
The Invariant Spin Field

Computation of the invariant spin field by analyzing tracking data:

- Fourier analysis
- Stroboscopic averaging
- Anti-damping
- Differential Algebra



$$\vec{S}_{n+1} = \underline{A}(\vec{z}_n)\vec{S}_n$$

defines the $\vec{n}$–axis

$$\vec{n}(\vec{z}_{n+1}) = \underline{A}(\vec{z}_n)\vec{n}(\vec{z}_n)$$

Computations performed in SPRINT, Hoffstaetter and Vogt, DESY/02

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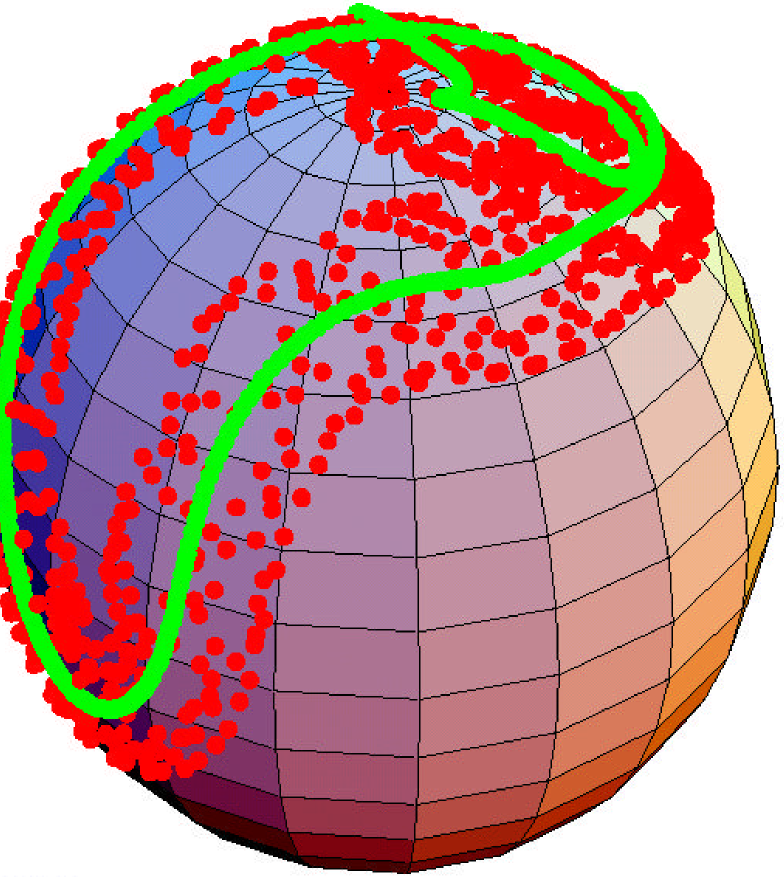
Higher Order Effects



- Overlapping resonances
- Deformation of the invariant spin field
- Resonances of very high order
- Nonlinear spin transfer matrix

Siberian Snakes avoid 1^{st} order resonances but higher orders become important for HERA.

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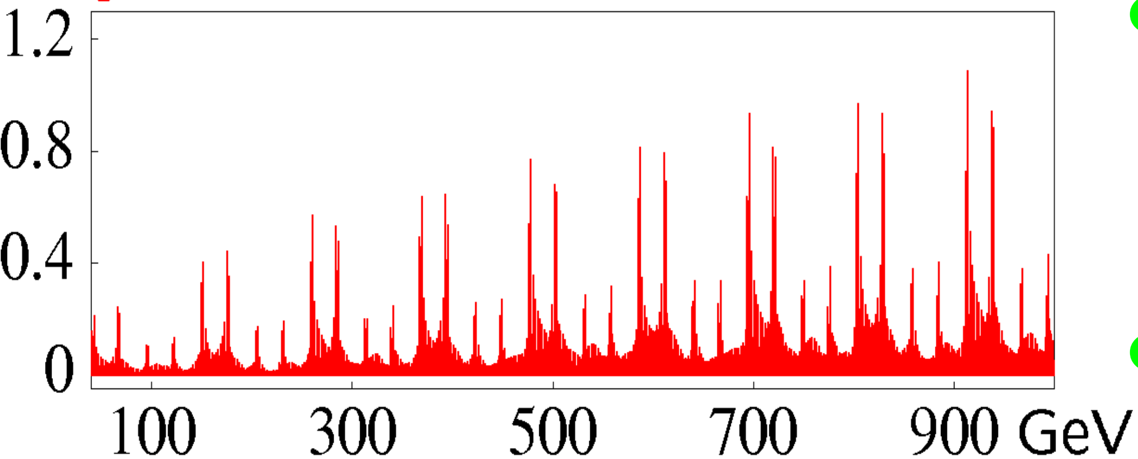
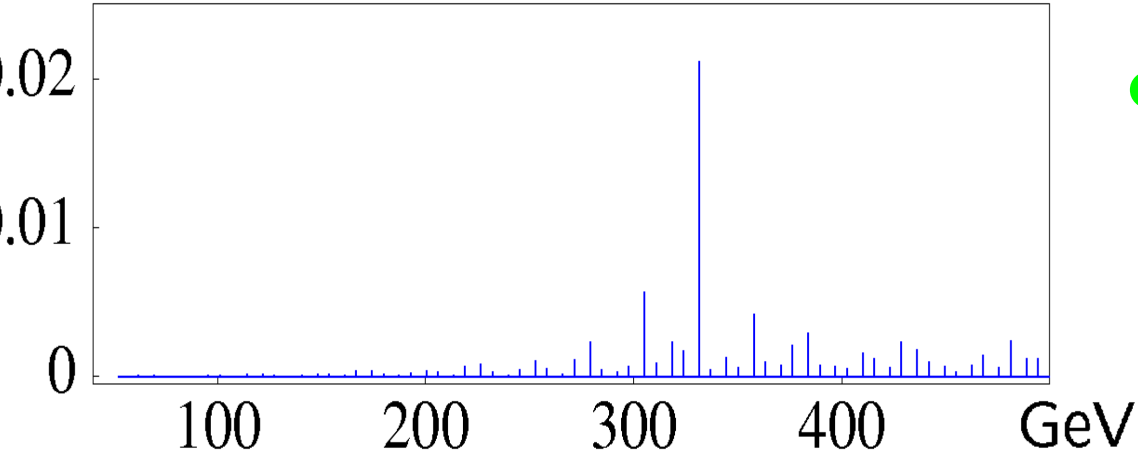
Polarized Deuterons

$\vec{p}$ depolarizing resonance strength



$\vec{D}$ depolarizing resonance strength



- Resonances are 25 times weaker and 25 times rarer for D than for p
- Transverse polarization could be achieved without Siberian Snakes
- Transverse RF dipoles could be used to rotate and stabilize longitudinal polarization

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