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Polarized Protons at High Energies

Polarized protons at HERA: the accelerator issues

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Abstract: The electrons in HERA become polarized at high energy by emitting spin flip synchrotron radiation. Proton beams at HERA energies do not emit enough synchrotron radiation to become polarized. Therefore polarized proton beams have to be produced by a polarized source and subsequently they have to be accelerated. The major problems occurring during this process for the DESY accelerator chain are outlined. Accelerating proton beams to high energy with little loss of polarization requires Siberian Snakes. A huge number of different snake schemes are possible for HERA, and the concept of filtering out optimal Siberian Snake angles is described. After polarized beams are accelerated to high energy, the polarization has to be stable for several hours in order to be useful for the experiments H1 and ZEUS. Furthermore the polarization in all parts of the beam has to be nearly parallel during all this time. This goal can again be achieved by filtering out optimal Siberian Snake angles. Finally, results about the polarization's sensitivity to misalignments are presented.

1 Introduction

Spin flip synchrotron radiation causes an electron beam to become polarized anti-parallel to the magnetic dipole field. If the beam is flat and disturbing effects of misalignments on the spin motion are compensated then electron polarization of over 60% can routinely be achieved in HERA. Around the HERMES experiment, spin rotators consisting of interleaved vertical and horizontal bends have been installed, which disturb the orbit only marginally but orient the polarization parallel to the beam direction in the HERMES experiment while it remains vertical in the arcs. Installing such spin rotators in a high energy storage ring while keeping a high degree of the self polarizing mechanism was only possible after spin matching of spin–orbit motion [1, 2]. It has not been achieved in any other laboratory and the attainment of longitudinally polarized electron or positron beams in HERA was and still is a unique achievement [3].

The HERMES experiment studies interactions of the polarized electron or positron beam with polarized nuclei in the HERMES gas jet target. The center of mass energy of these collisions is approximately 7GeV. A 920GeV/c polarized proton beam in HERA would allow the analysis of e-p collisions with center of mass energies of up to 318GeV in H1 and ZEUS. In a future experiment, collisions of the polarized protons with a polarized gas jet target at 42GeV could also be investigated. A very active group of many high energy physicists has already been studying which experiments could be performed with polarized protons and electrons at HERA energies [4, 5, 6, 7].

In contrast to high energy electrons, protons do not emit sufficient synchrotron radiation to become polarized at high energy. Therefore the energy of polarized proton beams has not increased along with the achievement of higher and higher energies for unpolarized proton beams. So far no polarized proton beam with energies above 25GeV/c has been produced [8]. But theoretical and numerical studies of very high energy polarized proton acceleration have been undertaken for high energy rings, namely for RHIC (250GeV/c) [9], TEVATRON (900GeV/c) [10], and HERA (920GeV/c) [11, 12, 13, 14].

2 The DESY accelerator chain for polarized protons

There are currently no alternatives to accelerating polarized protons which are produced in a polarized source H^- source. A proton beam at DESY is then accelerated by an RFQ to 750keV, then by the LINAC III to 50MeV, by the DESY III synchrotron to 7.5GeV/c, by the PETRA synchrotron to 40GeV/c and then by HERA to 920GeV/c. The four main challenges for the HERA polarized proton project are therefore

- Production of a 20mA pulsed H^- beam.
- Polarimetry at various stages in the acceleration chain.
- Acceleration through the complete accelerator chain with little loss of polarization.
- Storage of a polarized beam at the top energy over many hours with little depolarization.

Polarized protons are produced either by a polarized atomic beam source (ABS) or in an optically pumped polarized ion source (OPPIS). Pulsed beams with polarization of up to 87% for 1mA H^- beam current [15] and up to 60% for 5mA [16] respectively has been achieved with these sources. Compared with the 60mA of DESY's current source this sounds rather limited. However experts claim that currents of up to 20mA could be possible in OPPIS sources [17]. The maximal currents of 205mA in DESY III can already be achieved with such a source current. If DESY III were to become capable of handling more current, then the transfer efficiency of DESY's low energy beam transport (LEBT) and of the radio frequency quadrupole (RFQ) which is around 50% would have to be improved.

For polarization optimization, polarimeters will have to be installed at several crucial places in the accelerator chain. The source would contain a Lyman-α polarimeter. This does not disturb the beam [18]. Another polarimeter could be installed after the RFQ at 750keV [19]. This could not be operated continuously since it disturbs the beam. The transfer of polarized particles through LINAC III has to be optimized with yet another polarimeter which could be similar to that of the AGS LINAC [20].

Each of the following accelerator rings will need its own polarimeter. The polarimeter for DESY III could be similar to the AGS internal polarimeter [20]. Since polarization at the DESY III momentum of up to 7.5GeV/c has been achieved at several labs, the technology of all the polarimeters mentioned so far is well understood. It is different with the polarimeters required for PETRA and HERA energies; for these high energies there is no established polarimeter. Here one has to wait and see how the novel techniques envisioned and developed for RHIC [21, 22] will work.

Before polarization in DESY's high energy rings can be measured, care has to been taken that the polarization produced in the source is not lost during the acceleration and the beam transport process. To explain the problems involved, some concepts from polarization dynamics have to be introduced.

3 Spin motion in circular accelerators

3.1 The equation of spin motion

Spin motion through electromagnetic fields is described by the Thomas-BMT equation, which has some similarity to the Lorentz equation for particle motion through magnetic fields,

$$\frac{d\vec{p}}{dt} = -(\frac{q}{m\gamma})\{ \quad \vec{B}_\perp \quad \} \times \vec{p}\,, \tag{1}$$

$$\frac{d\vec{s}}{dt} = -(\frac{q}{m\gamma})\{(G\gamma+1)\vec{B}_\perp + (1+G)\vec{B}_{||}\} \times \vec{s}\,. \tag{2}$$

Here $G = 1.79$ is the gyromagnetic anomaly of the proton which moves through a magnetic field with components perpendicular and parallel to the particle's velocity. Several conclusions can immediately be drawn from these equations. (1) If the orbit is deflected by an angle ϕ in a transverse magnetic field, then the spin is rotated by an angle $(G\gamma+1)\phi$. A 1mrad orbit kick at the HERA momentum of 920GeV/c produces 100° of spin rotation. (2) In a flat 920GeV/c ring, the one turn orbit deflection angle of 2π leads to 1756 spin rotations around the vertical direction. (3) Whenever the energy is increased by 523MeV, the spin rotates once more per revolution around the ring. (4) The spin rotation in a dipole is proportional to the dipole field $B_\perp$ and a field integral of 2.74Tm always leads to a spin rotation of $\pi/2$ relative to the orbit motion. Therefore the spin rotation in a transverse field does not change during the acceleration process if the bending field is not changed. But of course the orbit deflection decreases.

3.2 Imperfection resonances for closed orbit distortions

The axis $\vec{n}_0$ around which spins rotate when a particle moves along the design orbit of a circular accelerator is vertical in a flat ring. The total spin transport for one turn around the ring can be described by a rotation matrix $\underline{R}(\vec{n}_0, \phi)$ which rotates spins around the vertical $\vec{n}_0$ axis by an angle ϕ . The spin tune $\nu_0 = \phi/2\pi$ describes the spin rotation angle and is equal to $G\gamma$ in a flat ring. When ν_0 is close to an integer, a case which is referred to as *imperfection resonance*, the rotation matrix is close to the identity and spin directions have hardly changed after one turn. Misalignments create horizontal field components on the closed orbit which produce precessions away from the vertical direction. For small misalignments, these rotations around the horizontal might be very small but they can still dominate spin motion when the main fields produce hardly any spin rotation during one turn close to integer values of ν_0. Thus the precession axis $\vec{n}_0$ for spins is vertical away from imperfection resonances and $\vec{n}_0$ is close to horizontal in their vicinity.

Since it is inadvisable to let misalignments dominate spin motion, imperfection resonances ought to be avoided. However, since $\nu_0 = G\gamma$ in a flat ring, the spin tune changes during acceleration and the crossing of imperfection resonances is unavoidable. There are three possible regimes for resonance crossing:

- The effects of misalignments are very small and the resonance is crossed so rapidly that the spins hardly react and depolarization is very weak.
- When the effect of misalignments is very strong, the precession axis $\vec{n}_0$ changes very slowly during acceleration since the precession around the horizontal fields of misaligned elements starts to dominate already far from an imperfection resonance. Then the spin can follow the slow change of $\vec{n}_0$ and depolarization is very limited.
- At intermediate strength the polarization will be reduced.

The following two strategies can therefore be used to limit depolarization when imperfection resonances are crossed:

- Careful correction of the closed orbit to limit horizontal field components.
- Increasing the horizontal field components, for example by introducing a solenoid. Devices which are deliberately used to increase the effect of imperfection resonances are referred to as partial snakes. A solenoid magnet has been installed in the AGS and very effectively avoids polarization loss at integer resonances of the spin tune as can be seen in figure 1 [8].

Since the closed orbit is not very well controlled in DESY III, a solenoid partial snake for overcoming the $\nu_0 = G\gamma = 8$ imperfection resonance at 4.08GeV/c can therefore probably not be avoided [13].

3.3 Intrinsic resonances for betatron amplitudes

Particles which oscillate around the closed orbit with a synchro–betatron amplitude experience fields which fluctuate with the orbital tunes ν_x, ν_y, ν_τ. If periodic field variations are in resonance with the spin tune ν_0, depolarization can occur,

$$\nu_0 = nN_{sym} + i\nu_x + j\nu_y + k\nu_\tau \quad , \quad n, N_{sym}, i, j, k \in \mathbb{N} \ . \tag{3}$$

The super period N_{sym} of a ring reduces the number of resonances. These resonances are called *intrinsic resonances* of order n for $n = |i| + |j| + |k|$. The depolarizing effect of these resonances has been experimentally verified in many low energy polarized proton accelerators. The first order intrinsic resonances are the dominant reason for depolarization after solenoids have been introduced to eliminate the effect of imperfection resonances.

The depolarization at imperfection resonances has been explained by the tilt of $\vec{n}_0$ from the vertical towards the horizontal which is produced by spurious fields at integer spin tunes. The depolarizing effect at intrinsic resonances can be understood in similar terms. Spins on the closed orbit do not change their direction from turn to turn if they are parallel to $\vec{n}_0$ since $\vec{n}_0$ is the rotation axis of the spin transport matrix for particles on the closed orbit. Similarly one can ask if the whole *field* of spin directions in phase space can be invariant from turn to turn. For particles which are not on the closed orbit, the one turn spin transport from some chosen

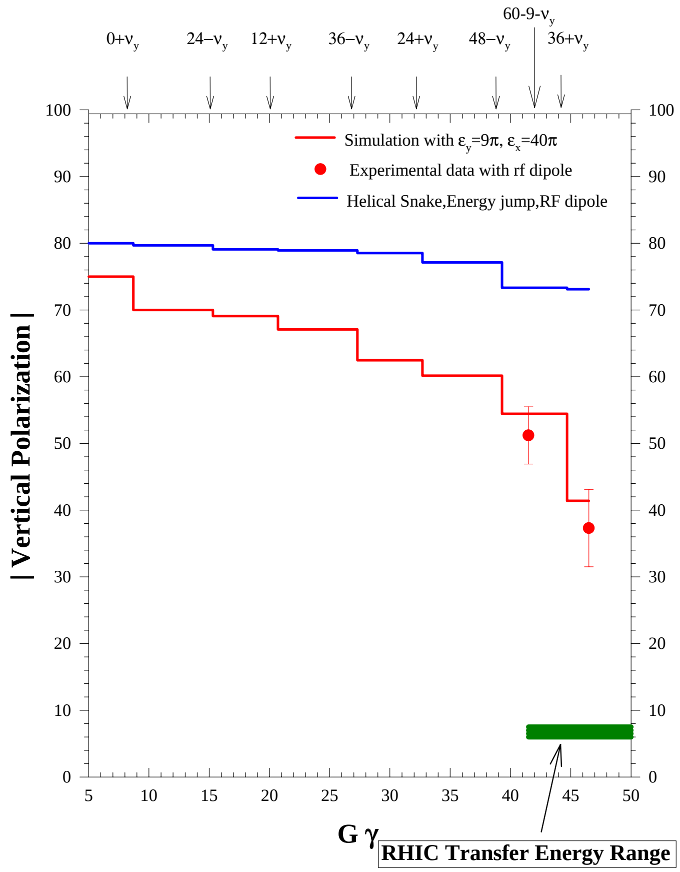

Figure 1: *Depolarization during acceleration in the AGS. A solenoid was used to avoid the imperfection resonances; depolarization at the strong intrinsic resonances* $0+\nu_y$, $12+\nu_y$, $36-\nu_y$, *and* $36+\nu_y$ *was avoided by RF dipole excitation. This figure is presented in [8].*

point of the ring is described by the rotation $\underline{R}(\vec{z}_i)$ which is a function of the particle's initial phase space point $\vec{z}_i$.

Invariance of the spin field $\vec{n}(\vec{z})$ does not require that spin vectors do not change from turn to turn, but rather that a spin in direction $\vec{n}(\vec{z}_i)$ at an initial phase space point $\vec{z}_i$ gets transported via $\underline{R}(\vec{z}_i)$ to a spin direction which agrees with $\vec{n}(\vec{z}_f)$ at the final phase space point after one turn around the ring,

$$\vec{n}(\vec{z}_f) = \underline{R}(\vec{z}_i)\vec{n}(\vec{z}_i) \ . \tag{4}$$

An invariant spin field $\vec{n}(\vec{z})$ is called a Derbenev–Kondratenko $\vec{n}$-axis [24]. Note that the polarization state of a particle beam is not invariant from turn to turn when all particles are completely polarized parallel to each other, but rather when each particle is polarized parallel to $\vec{n}(\vec{z})$ at its phase space point $\vec{z}$.

If a particle at $\vec{z}_i$ in phase space is polarized parallel to $\vec{n}(\vec{z}_i)$, then it will be parallel to $\vec{n}(\vec{z}_i)$ whenever it comes close to $\vec{z}_i$ during later turns around the ring as long as $\vec{n}(\vec{z})$ is sufficiently continuous. The spin rotation which takes place while two particles travel along the same trajectory does not change the angle between their two spins. When a particle initially is polarized with an angle φ to $\vec{n}(\vec{z}_i)$, then it will therefore have been rotated around $\vec{n}(\vec{z}_i)$ every time it comes close to $\vec{z}_i$ but it will still have the angle φ to the invariant spin direction. The time averaged polarization at $\vec{z}_i$ will therefore be parallel to $\vec{n}(\vec{z}_i)$ with an average degree of polarization which can only be 1 if the the spin was initially parallel to the invariant spin field. When the spins of all particles are initially in their optimal direction, then the beam is still not completely polarized but it has the polarization $< \vec{n} >$ where $< \ldots >$ denotes an average over the beam. The maximum polarization that can be stored in an accelerator at a given fixed energy is therefore determined by $< \vec{n} >$. Thus a large divergence in the directions of $\vec{n}(\vec{z})$ leads to low polarization.

The invariant direction $\vec{n}(\vec{z})$ will change during the acceleration process. Spins rotate ν_0 times around $\vec{n}_0$ during one turn with respect to the standard accelerator coordinate system if they travel along the closed orbit. This rotation is produced by the main guide fields of the accelerator. And far away from resonances spin motion is dominated by these main guide fields even for particles which oscillate around the closed orbit with the vertical betatron tune ν_y. The invariant spin field $\vec{n}(\vec{z})$ will then be close to $\vec{n}_0$, i.e. the vertical in a flat ring.

If the spin vectors are expressed in terms of a coordinate system which rotates by 2π around $\vec{n}_0$ during one betatron period of vertical motion, then in this coordinate system the main guide fields produce a rotation of spins by $2\pi(\nu_0-\nu_y)$ during one turn. At $\nu_0 = \nu_y$ the spin rotation due to the main guide fields vanishes and the remaining rotations are due to extra fields picked up by the oscillating trajectory some distance away from the closed orbit. At intrinsic resonances these spurious effects dominate over the effect of the accelerator's main guide fields and $\vec{n}(\vec{z})$ can be strongly distorted away from $\vec{n}_0$. If spins cannot follow this change of the invariant spin direction, the polarization is reduced.

During acceleration in DESY III only 4 strong intrinsic first order resonances have to be crossed. These are the $8-\nu_y$, $0+\nu_y$, $16-\nu_y$, $8+\nu_y$ resonances. When these resonances are crossed quickly enough, depolarization can be limited. Since the ramping speed cannot be increased by a sufficient amount, special quadrupoles can be inserted which temporarily and very quickly change the orbital tunes, leading to a crossing of the resonances in a few micro seconds [23]. At the AGS it is planned to cross resonances with the successfully tested RF dipole excitation of resonances [8]. The RF dipole has been used to slowly excite all the particle amplitudes coherently. Then the dominant resonances $0+\nu_y$, $12+\nu_y$, $36-\nu_y$, and $36+\nu_y$ shown in figure 1 were crossed with little loss of polarization. Finally the RF dipole was slowly switched off. No noticeable increase of emittance has been observed. With a solenoid partial snake and with the RF dipole excitation method, both successfully tested at the AGS, a polarized proton beam with high polarization could be extracted at 7.5GeV/c from the DESY III synchrotron.

4 Siberian Snakes in PETRA and HERA

During acceleration to 920GeV/c in a flat ring around 5000 resonances have to be crossed. Each resonance crossing, even if it is handled very well, adds to a grand total depolarization. In PETRA and HERA it therefore becomes necessary to avoid imperfection and intrinsic resonances completely by making the spin tune independent of energy.

After acceleration, the beam has to stay polarized over many hours. In HERA a 1σ energy spread of $\Delta_E/E = 1.6 \cdot 10^{-4}$ creates an associated spin tune spread of $\Delta\nu_0 = 0.28$. The spin tune can therefore easily cross resonances during every synchrotron period. The disruptive effect of this periodic crossing of resonances is shown in figure 2 (left). When the spin tune's dependence on the energy deviation is eliminated in figure 2 (right), no variations are observed. At high energy it is therefore essential to make the spin tune independent of energy deviations.

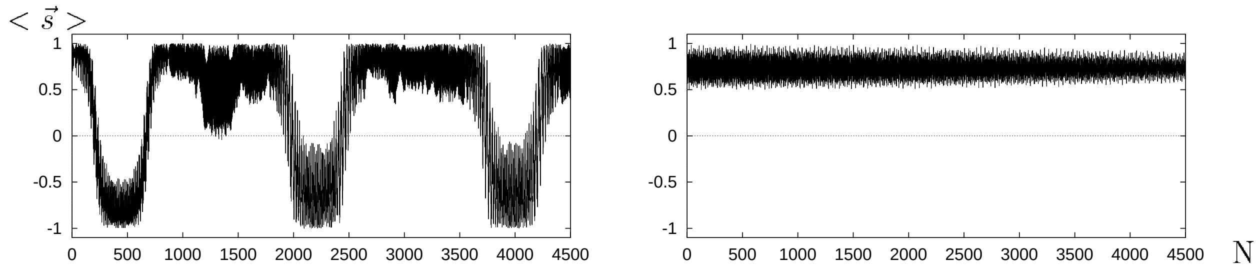

Figure 2: *Left: Disruptive effect of periodic crossing of the $\nu_0 + \nu_y$ resonance (after 400, 2000, and 3600 turns) and the $\nu_0 - \nu_y$ resonance (after 1200 and 2800 turns) in HERA at 820GeV/c. One synchrotron periods takes 1600 turns. Right: Without synchrotron motion no depolarization is observed. This has been shown in [28].*

This can be achieved by installing Siberian Snakes. These can make the spin tune independent of machine energy as well as independent of energy fluctuations during a synchrotron period. Siberian Snakes are combinations of magnets which rotate the spin by π around some usually horizontal fixed axis. The angle between the snake axis and the beam direction is referred to as the snake angle. The spin motion in a Siberian Snake is nearly independent of energy. This is possible since the spin rotation is proportional to the magnetic field in equation (2) but almost independent of energy for particle with high γ. A suitable insertion of Siberian Snakes causes the spin tune to become $\nu_0 = 1/2$ for all energies. The Siberian Snakes to be used for RHIC consist of four 2.4m long helical dipole magnets which create an orbit distortion of 3cm at 25GeV/c. The orbit and spin motion are shown in figure 3. Shorter Siberian Snakes of little more than 3m length can also be constructed [25].

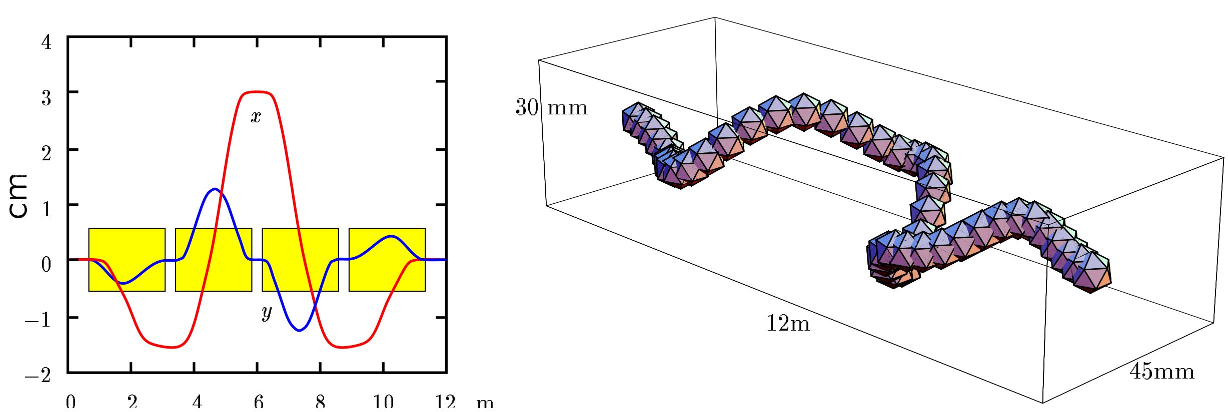

Figure 3: *Orbit motion in a helical snake designed for RHIC.*

Since the orbit distortions increase for fixed fields with decreasing energy, Siberian Snakes can only be used at rather high energies. For example a Siberian Snake which creates an orbit distortion of 15cm has been proposed [10] for an 8GeV/c ring. Similar devices would become necessary for the implementation of Siberian Snakes in PETRA at the low energy end. In HERA the orbit distortions would be much smaller due to the higher energy.

4.1 The required number of Siberian Snakes in HERA

So far no reliable formula has been established for determining the number of Siberian Snakes required for an accelerator. The following very simple argument is therefore often used: the Siberian Snakes should dominate the spin precession produced by closed orbit distortions and betatron oscillations. The resonance strength [26] ϵ is a measure for that precession and is shown in figure 4 for 1σ vertical betatron oscillations in HERA. This leads to the rule of thumb that the number of snakes, which has to be even, sufficiently larger than 5ϵ [27]. This would lead to more than four Siberian Snakes in HERA. From tracking analysis [28] it has also become clear that four snakes are not always enough. However, it has been shown that the efficiency of four snake arrangements depends very much on the rotation angles of the individual snakes [12]. Furthermore it has been shown that eight snakes are not necessarily better than four snakes for the non-flat HERA ring [29]. However, that analysis did not include misalignments.

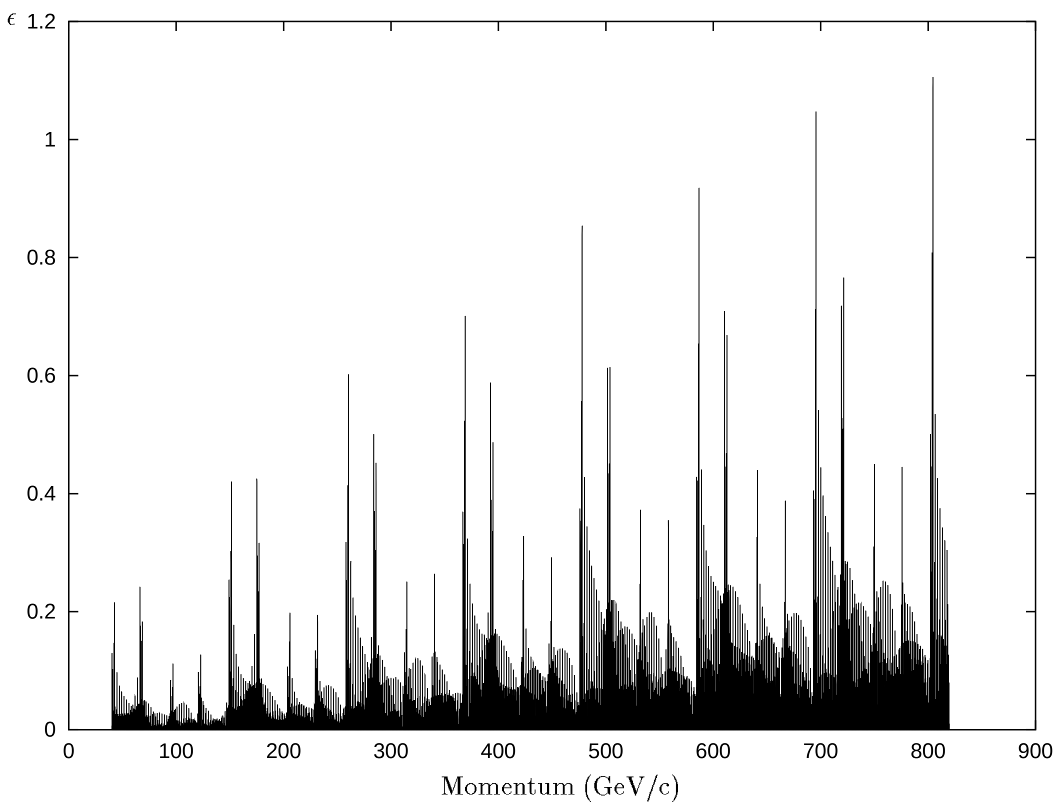

Figure 4: *Spin resonance strength for HERA computed for a 1σ vertical emittance of 4πmm mrad.*

To summarize, there are around 5000 spin tunes which satisfy equation 3 for imperfection and first order intrinsic resonances when particles are accelerated up to 920GeV/c in a flat ring. Siberian Snakes in PETRA and HERA avoid all first order intrinsic resonances. There is space for at least two Siberian Snakes in PETRA and there is space for four Siberian Snakes in HERA. Eight Siberian Snakes can only be inserted with severe and expensive changes in HERA. The number of remaining resonances in DESY III is reduced by mid plane symmetry and by the eight fold super period. Depolarization at the one remaining imperfection resonance in DESY III is avoided by a partial solenoid snake. Depolarization at the four remaining intrinsic resonances in DESY III can be avoided by RF dipole excitation of resonances.

5 Polarization during acceleration and storage

A polarized beam can be depolarized during acceleration even when Siberian Snakes avoid first order resonances. It was already explained in section 3.2 that spins of particles on the closed orbit precess around the rotation axis $\vec{n}_0$. A change of this precession direction can be either fast enough to be irrelevant or slow enough to allow the spins to follow; otherwise depolarization occurs. A similar effect occurs when the invariant spin field changes during acceleration. Figure 5 shows that the invariant spin field at different phase space points can be rather parallel (leading to high polarization) and suddenly diverges at a critical energy (leading to diminished polarization) only to become parallel again at higher energies. When this change in direction is fast enough, the spins do not react strongly. If it is slow enough, the spins follow the changing invariant spin field adiabatically. In both cases the beam will recover its polarization after the critical energy region is crossed. Only in intermediate cases is the beam polarization reduced. Such an effect not only occurs during ramping but also due to the energy change associated with synchrotron motion.

After acceleration in HERA, the polarized beam still has to be stored over several hours during which the polarization should be invariant and high. Therefore each particle should be polarized parallel to the invariant spin field $\vec{n}(\vec{z})$ at its phase space position $\vec{z}$. To have a high average beam polarization, the polarization direction $\vec{n}(\vec{z})$ additionally has to be almost parallel to the average polarization direction at every phase space point $\vec{z}$.

The analysis of proton polarization in HERA therefore started with the question: "How does the invariant spin field look at a fixed high energy [11]?" When spins at different phase space points in the beam point in significantly different directions, four problems occur. (1) If sudden changes of the invariant spin direction $\vec{n}(\vec{z})$ occur during acceleration, then the polarization of individual particles cannot follow and the average polarization is diminished. (2) The divergence of the polarization direction reduces the average polarization available to the particle physics experiments since $< \vec{n} >$ is the maximum polarization which can be stored in a ring. (3) It makes the polarization involved in each collision analyzed in a detector strongly dependent on the phase space position of the interacting particle. (4) Polarimeters which measure the polarization of particles in the tails do not yield accurate values for the average polarization of the beam.

5.1 Filtering out optimal snake schemes

The depolarization while accelerating will be small and all particles will be polarized in nearly the same direction at high energy if the accelerator has an invariant spin field which does not change much during the acceleration process and which is nearly parallel for all relevant phase space points. Achieving this is one of the non-trivial tasks of Siberian Snakes. However, as mentioned in section 4.1 there is so far no reliable formula for determining the number of Siberian Snakes required for an accelerator. To make things worse, for any given number of Siberian Snakes there are very many different possible combinations of the snake angles which lead to an energy independent spin tune of 1/2 and to a vertical $\vec{n}_0$ in the accelerator's arcs. And so far there is also no reliable formula for determining which of these snake schemes leads to the highest polarization. In the following we will describe the only established algorithm for finding snake angles which are optimal for the acceleration and storage of high energy polarized proton beams.

The simplest well established way of computing $\vec{n}(\vec{z})$ involves the linearization in spin and phase space variables used in the program SLIM [30]. At some energies this leads to a maximum of the polarization $< \vec{n} >$ which is nearly zero [12]. The maximum polarization is especially small near energies where first order resonances would occur if Siberian Snakes were not installed. Thus residual resonance structures are seen in the energy dependence of the phase space average $< \vec{n} >$ in figure 5 and 6.

In figure 5 $< \vec{n} >$ was computed by the linear approximation. For the top figure four Siberian Snakes have been installed in the ring. The resulting maximum polarization is shown for three different intuitive choices of the snake angles which were initially suggested because they seemed to be appropriate for HERA due to its approximate super period of 4. This figure shows that different snake schemes perform differently.

To find out whether other directions of the rotation axes of the four Siberian Snakes would be advantageous, the following automated filtering algorithm was introduced:

1. Determine a set of snake angles from which the best snake scheme should be chosen. We have investigated over 10^6 schemes.
2. Find all snake combinations for a flat ring which lead to a spin tune of $\nu_0 = 1/2$.
3. Find all snake combinations which additionally lead to vertical $\vec{n}_0$.
4. Compute the maximum polarization $< \vec{n} >$ over the whole energy range of HERA using linearized spin orbit motion and filter on high average values of the maximum polarization.
5. Compute the opening angle non-perturbatively by stroboscopic averaging for the snake combinations which are left after all these filters.

Figure 5 (bottom) shows that it was possible to increase the maximally storable polarization computed by linear theory.

During our investigations we allowed the snake angle of each of the four snakes to vary in steps of 22.5°. Snakes with vertical rotation direction, so called type III snakes, were also allowed. During the filtering analysis it was found for the first time that type III snakes can be advantageous in a storage ring [12].

To see whether the maximum storable polarization has really increased, we computed the $< \vec{n} >$ in a non-perturbative way as the 5th step of filtering and compare it to $< \vec{n} >$ of some non-filtered snake schemes. We usually use stroboscopic averaging [31] for non-perturbative computation of $\vec{n}$. Other methods do not converge close to resonances and for large phase space amplitudes [32], or are only practical for one degree of freedom [33]. Stroboscopic averaging

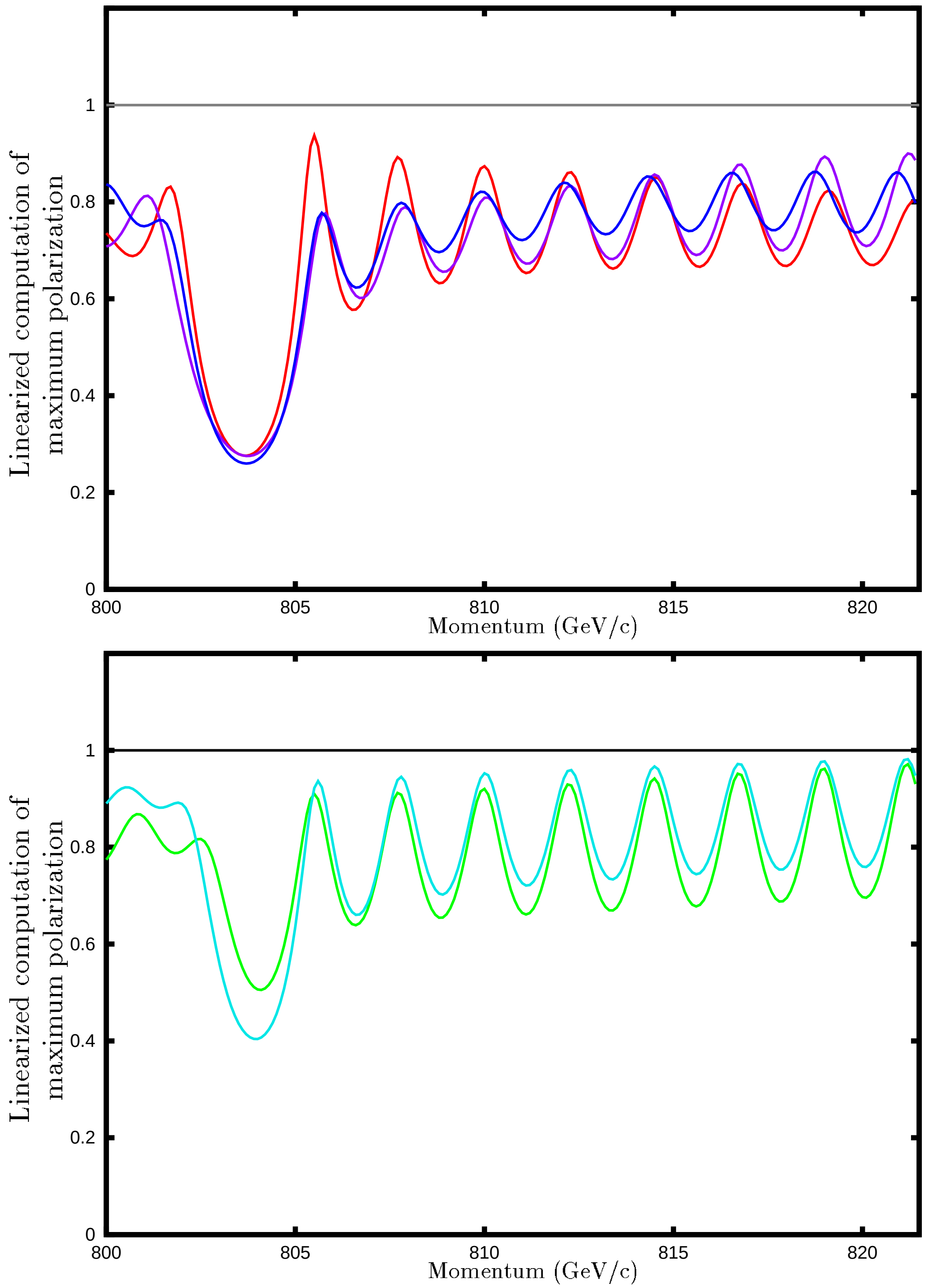

Figure 5: *Opening angle of the equilibrium spin field as computed with linearized spin motion for 1σ vertical amplitude with four Siberian Snakes in HERA. Top: Three different intuitively chosen snake schemes. Bottom: The two best filtered schemes of Siberian Snakes. The different snake angles are counted from the radial direction.*

red:	*longitudinal (East),*	*radial (South),*	*radial (West),*	*radial (North).*
purple:	*radial (East),*	$45°$ *(South),*	*radial (West),*	$45°$ *(North).*
blue:	$22.5°$ *(East),*	$-22.5°$ *(South),*	$22.5°$ *(West),*	$-22.5°$ *(North).*
green:	*radial (East),*	$67.5°$ *(South),*	*longitudinal (West),*	$-67.5°$ *(North).*
cyan:	*radial (East),*	*longitudinal (South),*	$-45°$ *(West),*	$-45°$ *(North).*

is non-perturbative and can therefore also be applied close to resonances, it only uses tracking data and can therefore easily be added to every spin tracking code. Furthermore it usually converges quickly. The result is shown in figure 6 and it is clearly very advantageous to use the filtered snake arrangement.

The following conclusions can be drawn from these filtering calculations: (1) Initial worries about big open angles of $\vec{n}(\vec{z})$ (and associated with it depolarization during acceleration and low storable polarization) are in general justified. (2) Special snake angles can be found where these opening angles are reduced. (3) In regions where these angles are small, the spread in polarization directions $\vec{n}(\vec{z})$ can be computed quite accurately by linearizing spin and orbit motion. This is an important point since now it is possible to apply the same linear spin matching techniques [1] for optimizing the invariant spin field which were so valuable to electron polarization in HERA.

6 The sensitivity of polarization to misalignments

The sensitivity of the average polarization of a proton beam to misalignments of beam line elements is shown in figure 7. The top figure shows the change of polarization during beam acceleration of an initially completely polarized beam with 1σ vertical betatron amplitude. In the middle figure the change of polarization of the same initial beam is shown after a closed orbit deviation has been produced by field errors and then has been corrected to rms deviations of 0.5mm in both planes. The depolarization is caused by the transverse amplitude of the particles, not by the closed orbit spin motion. This can be concluded from the bottom figure where the vertical polarization of a particle traveling on the closed orbit through the same misaligned ring is shown. The current closed orbit deviation in the HERA proton ring usually has an rms value of about 1mm in the horizontal and in the vertical plane. It therefore seems essential that the quality of the closed orbit quality in HERA be improved before polarized beams can be accelerated [34]. However, there are also schemes which reduce the sensitivity of the polarization on closed orbit distortions [35].

Acknowledgment

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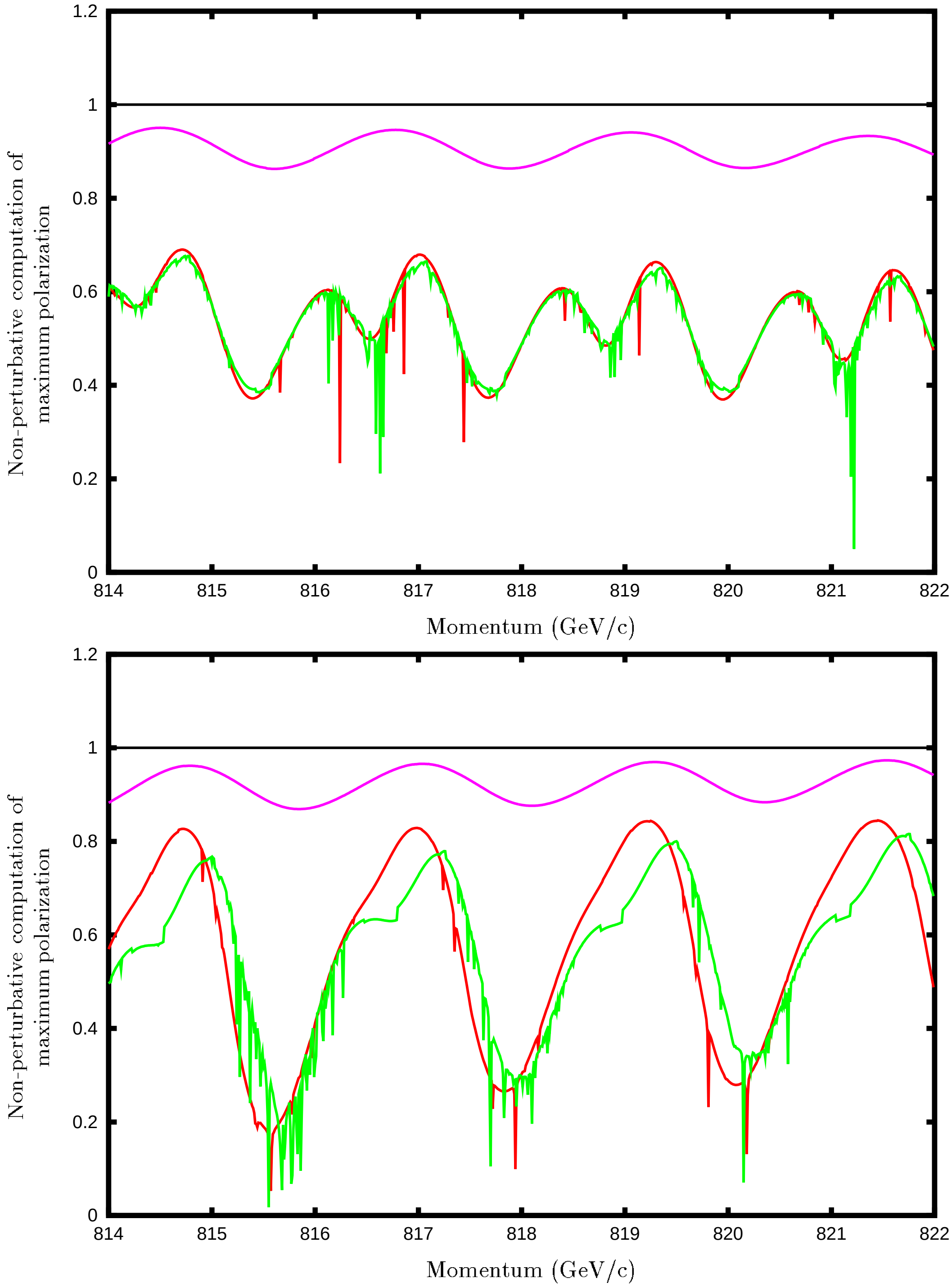

Figure 6: *Opening angle of the equilibrium spin field as computed with stroboscopic averaging for four Siberian Snakes in HERA. Top: The intuitively chosen scheme with snake angles longitudinal (East), radial (South), radial (West), radial (North). Bottom: The best filtered scheme with snake angles radial (East), 67.5° (South), longitudinal (West), −67.5° (North). The orbital amplitudes are*

purple: 1σ *in each degree of freedom.*
red: 2σ *vertical and horizontal motion.*
green: 2σ *in each degree of freedom.*

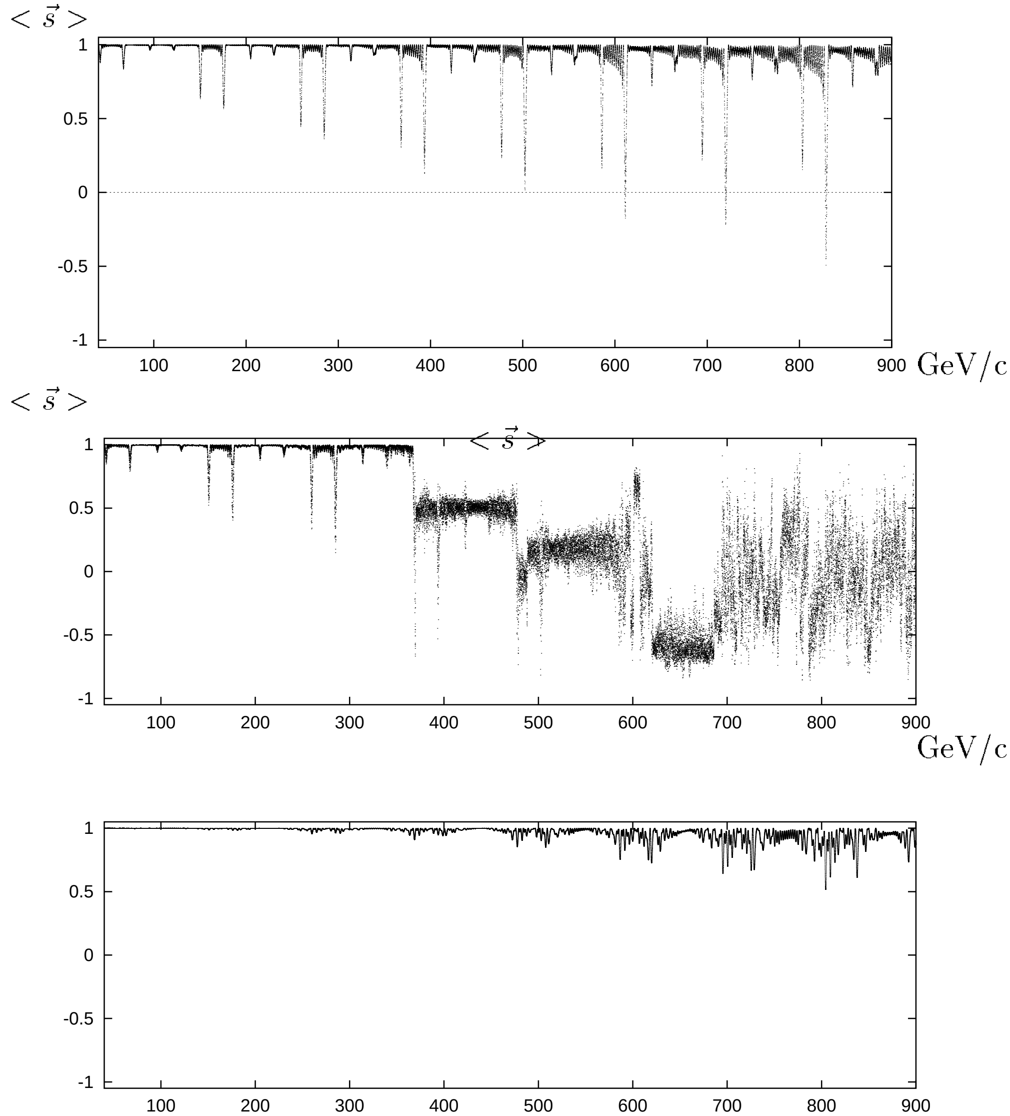

Figure 7: *Top: Change of polarization during beam acceleration of an initially completely polarized beam with 1σ vertical amplitude. Middle: Change of polarization of the same initial beam after a closed orbit deviation has been produced by field errors and then has been corrected to rms deviations of 0.5mm in both planes. Bottom: Vertical polarization of a particle on the closed orbit through the same misaligned ring. The depolarization shown in the middle figure is therefore caused by the transverse amplitude of the particles, not by the closed orbit spin motion. These figures were computed by N. Golubeva.*

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