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Stroboscopic averaging for computing the equilibrium spin direction in storage rings

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ABSTRACT

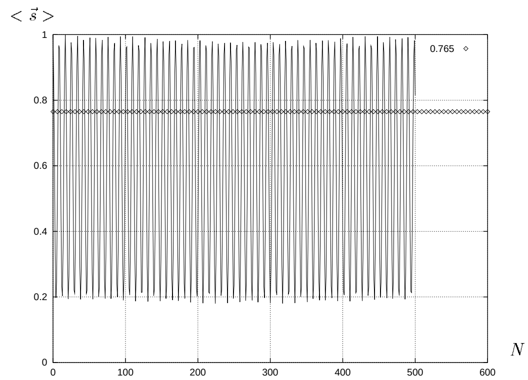
The equilibrium polarization direction in a high energy storage ring can vary substantially across the beam in the interaction region of a high energy experiment when no countermeasure is taken. Such a divergence of the polarization direction would not only diminish the average polarization available to the particle physics experiment, but it would also make the polarization involved in each collision analyzed in a detector strongly dependent on the phase space position of the interacting particle. In order to analyze and compensate for this effect, methods for computing the equilibrium polarization direction are needed. In this paper we explane the method of stroboscopic averaging [1], which computes this direction in a very efficient way, and we analyze the convergence speed. Since only tracking data is needed, stroboscopic averaging can be implemented easily in existing spin tracking programs.

The equilibrium polarization direction

HERA is the only high energy accelerator which provides longitudinally polarized electrons for a high energy experiment. Currently a collaboration of several international laboratories investigates the possibilities of complimenting this capability with a polarized proton beam [2,3,4]. Polarized protons, however, have never been accelerated to more than about 25 GeV and novel problems should be expected. Here we concentrate on a problem which arises at high energies, 820 GeV in the case of HERA. In horizontal dipoles, spins precess only around the vertical field direction. The quadrupoles have vertical and horizontal fields and additionally cause the spins to precess away from the vertical direction. If fields which are perpendicular to the particle's velocity cause the orbit to rotate by an angle $\delta\phi$, the particle's spin precesses by $(a\gamma+1)\cdot\delta\phi$. In the case of HERA, $(a\gamma+1)$ can reach 1557. For big γ the spin therefore rotates in a rather different fashion on different trajectories. From this it is clear that if an equilibrium spin distribution exists, it will vary across the orbital phase space. We describe this equilibrium distribution of spin directions in phase space by $\vec{n}(\vec{z})$, where $\vec{z}$ denotes the position in the six dimensional phase space of the beam. A particle with an initial spin $\vec{s}_i$ at an initial phase space point $\vec{z}_i$ will have a final spin $\vec{s}_f$ after it is transported to a final point $\vec{z}_f=\vec{M}(\vec{z}_i)$ during one turn around the storage ring. This change of the spin during one turn is described by a phase space dependent rotation matrix $\vec{s}_f=\underline{R}(\vec{z}_i)\vec{s}_i$. The periodicity condition for the equilibrium spin distribution $\vec{n}(\vec{z})$ can be written using this spin transport matrix

$$\underline{R}(\vec{z}_i)\vec{n}(\vec{z}_i) = \vec{n}(\vec{z}_f) = \vec{n}(\vec{M}(\vec{z}_i)) \ . \tag{1}$$

A common problem when tracking an ensemble of polarized particles is illustrated in the first figure which was computed for a typical HERA optic. When the spin of all particles is initially chosen to be parallel to the closed orbit spin direction $\vec{n}(\vec{z}=0)$, then the average polarization can fluctuate widely during the tracking simulation. When, however, the equilibrium distribution $\vec{n}(\vec{z})$ is known one should initialize tracking calculations with spins spread out over phase space according to this direction. This leads to constant average polarization as illustrated by the horizontal line at 0.765. The fact that these tracking data lead to very constant polarization already illustrates that our method of computing the $\vec{n}$-axis is very accurate.



An illustration of the usefulness of knowing the equilibrium spin distribution and an example of the accuracy of stroboscopic averaging.

Stroboscopic averaging

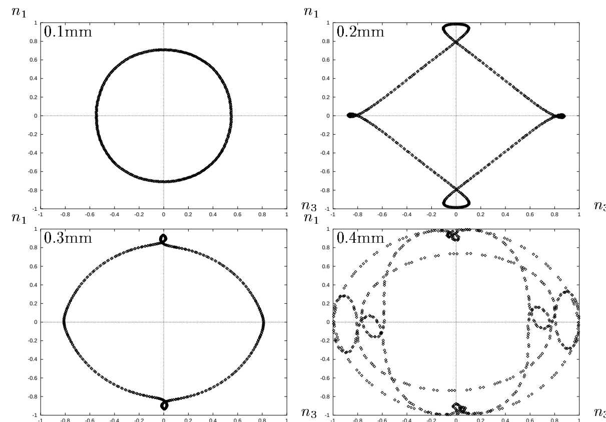
If a particle beam is approximated by a phase space density, disregarding its discrete structure, then we can associate a spin field $\vec{f}(\vec{z})$ with the particle beam. If one installs a point like 'gedanken' polarimeter at a phase space point $\vec{z}_0$, then this polarimeter initially measures $\vec{f}(\vec{z}_0)$. When the particle beam passes the polarimeter after one turn around the ring, the polarimeter measures $\underline{R}(\vec{M}^{-1}(\vec{z}_0))\vec{f}(\vec{M}^{-1}(\vec{z}_0))$. The inverse map $\vec{M}^{-1}$ indicates that a particle at the phase space point $\vec{M}^{-1}(\vec{z}_0)$ will be transported to $\vec{z}_0$ after one turn. After the beam has traveled around the storage ring N times and the polarization has been measured whenever the beam passed the 'gedanken' polarimeter, one averages over the different measurements to obtain the stroboscopic average

$$<\vec{f}>_N(\vec{z}_0) = \frac{1}{N+1}\{\vec{f}(\vec{z}_0) + \sum_{i=1}^{N}\underline{R}(\vec{M}^{-1}(\vec{z}_0))\cdot\ldots\cdot\underline{R}(\vec{M}^{-i}\vec{z}_0)\vec{f}(\vec{M}^{-i}(\vec{z}_0))\} \ . \tag{2}$$

A polarized proton beam can only be useful for high energy experiments when the time average of the polarization $<\vec{f}>_N$ in every part of the beam does not converge to zero. It turns out that the stroboscopic average satisfies the periodicity condition (1) rather well

$$\begin{aligned} &\underline{R}(\vec{z}_0)<\vec{f}>_N(\vec{z}_0) - <\vec{f}>_N(\vec{M}(\vec{z}_0)) \qquad (3)\\ = \ &\frac{1}{N+1}\{\underline{R}(\vec{z}_0)\cdot\ldots\cdot\underline{R}(\vec{M}^{-N}(\vec{z}_0))\vec{f}(\vec{M}^{-N}(\vec{z}_0)) - \vec{f}(\vec{M}(\vec{z}_0))\} \ . \end{aligned}$$

For big N the stroboscopic average violates the periodicity condition only by a small amount which converges to 0 like $1/N$. We can normalize to $\vec{n}_N(\vec{z}) = <\vec{f}>_N (\vec{z}_0)/|<\vec{f}>_N (\vec{z}_0)|$ for calculating the $\vec{n}$-axis. The convergence property of this quantity is discussed in detail in [1] where stroboscopic averaging has first been introduced. There we also describe a more efficient way of performing stroboscopic averaging from single particle tracking data. But the procedure mentioned here suffices to illustrate the underlying principles. Typically about 2000 tracking points are needed to compute the $\vec{n}$-axis to an accuracy of about a tenth of a percent. For linear orbital motion with one degree of freedom the tracked phase space points all lie on an ellipse. The tracked spins, however, can be spread all over the unit sphere. When on the other hand the initial spin is $\vec{n}(\vec{z}_0)$, then the tracked spins all lie on a closed curve on the unit sphere. The fact that they lie on a curve can be use as a check for the accuracy of stroboscopic averaging. The second figure shows projections of such curves on the s_x–s_y plane for four different emittances in HERA. Stroboscopic averaging has become a major tool at DESY for analyzing changes to the HERA lattice for polarized proton storage [5].



For one degree of freedom, tracked spins do not spread out over the unit sphere but lie on one curve if the initial spin is $\vec{n}(\vec{z}_0)$. The four different loci of $\vec{n}$-axese corresponds to four different initial vertical excursions of the beam.

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