Proceedings of SPIN96
Amsterdam, 1996

Calculations of the Equilibrium Spin Distribution for Protons at High Energy in HERA

D.P.Barber[1], K.Heineman[1], G.H.Hoffstätter[2], M. Vogt[1]

(1)DESY–MPY, Notkestr. 85, 22603 Hamburg, FRG
(2)TH–Darmstadt / Inst. f. Angew. Phys, Hochschulstr. 6, 64289 Darmstadt, FRG

ABSTRACT

We present calculations of the distribution across phase space of equilibrium spin directions at high energy in the HERA proton ring obtained using a completely new and non–perturbative algorithm [1].

1 Introduction

The phase space density $\rho(\vec{z},\theta)$ of a stable bunch in a storage ring satisfies the periodicity condition $\rho_{eq}(\vec{z},\theta) \equiv \rho_{eq}(\vec{z},\theta+2\pi)$. For high luminosity we require ρ to be large (large number of particles) and narrow (small beam cross section).

In analogy to the orbital case, at equilibrium the distribution of spin polarization $\vec{P}(\vec{z},\theta)$ satisfies $\vec{P}_{\text{eq.}}(\vec{z},\theta) \equiv \vec{P}_{\text{eq.}}(\vec{z},\theta+2\pi)$. We can factor $\vec{P}_{\text{eq}}$ into its modulus P_{eq} and its unit direction $\vec{n}$ which also satisfies the condition $\vec{n}(\vec{z},\theta) = \vec{n}(\vec{z},\theta+2\pi)$. If we require large polarization we need

1. large value of *local* polarization $P_{\text{eq}}(\vec{z},\theta)$ at each interaction point $\theta_{\text{i.p.}}$ and for all $\vec{z}$ where $\rho(\vec{z},\theta)$ doesn't vanish.

2. at the IPs the equilibrium polarization *directions* at each point in phase space must be almost parallel to the *average* polarization direction $<\vec{n}(\theta_{\text{i.p.}})>_{\text{p.s.}}$. (Regardless whether it is longitudinal or vertical.)

The polarization received by the experiments is proportional to :

$$\vec{P}_{\text{beam}}(\theta_{\text{i.p.}}) \equiv \frac{1}{N_{\text{part.}}}\int_{\text{p.s.}} \vec{P}_{\text{eq.}}(\vec{z},\theta_{\text{i.p.}})\rho_{\text{eq.}}(\vec{z},\theta_{\text{i.p.}})\, d^6\vec{z} \ . \tag{1}$$

So even if the *local* polarization is large the effective measured polarization will be limited by the spread of the equilibrium spin directions.

Let $\vec{z}(\theta)$ be a solution of the Lorentz force equation for a particle moving through the accelerator in *machine coordinates*. Then the T–BMT equation (for spin precession) reads :

$$\frac{d\vec{S}(\vec{z}(\theta),\theta)}{d\theta} = \vec{\Omega}(\vec{z}(\theta),\theta)\times\vec{S}(\vec{z}(\theta),\theta) \quad \Rightarrow \quad \vec{S}_f = \mathbf{R}(\vec{z}_i;\theta_f,\theta_i)\,\vec{S}_i \ ; \ \mathbf{R}\in\mathbf{SO}(3) \ . \tag{2}$$

For transverse magnetic fields $\vec{\Omega}$ is given by $\frac{eL}{2\pi mc\gamma}(1+a\gamma)\vec{B}_\perp(\vec{z}(\theta),\theta)$ with $a=\frac{g-2}{2}$. We can then conclude that if an external transverse magnetic field kicks the orbit by an amount $\delta\vartheta_{\rm orbit}$ the resulting spin precession is $\delta\vartheta_{\rm spin} = a\gamma\ \delta\vartheta_{\rm orbit}$. In a flat ring with no misalignment, in the absence of snakes and for a particle on the design orbit the number of precessions per turn (spin tune) is given by $\nu_{\rm sp}^{(0)} \equiv a\gamma$.

The $\vec{n}$–axis is a single valued vector field on $\mathbb{R}^{6+1}$ (phase space $\times$ azimuth domain). It obeys (2) in that if we evaluate $\vec{n}$ along a particle trajectory we obtain a 'spin trajectory' which is a solution of the initial value problem given by (2) with $\vec{S}(\vec{z}_0(\theta_0),\theta_0) = \vec{n}(\vec{z}_0,\theta_0)$. For the orbital transport map $\mathbf{T} \in \mathbf{Sp}(6)$ and $\vec{z}_f = \mathbf{T}(\theta_f,\theta_i)\vec{z}_i$ single–valuedness on phase space reads as :

$$\vec{n}(\vec{z}_f,\theta_f) = \mathbf{R}(\vec{z}_i;\theta_f,\theta_i)\ \vec{n}(\vec{z}_i,\theta_i) \quad . \tag{3}$$

On the closed orbit the vector $\vec{n}_0(\theta) \equiv \vec{n}(\vec{0},\theta)$ is the fixed point of $\mathbf{R}(\vec{0};\theta+2\pi,\theta)$.

The stroboscopic averaging method takes advantage of the fact that the time domain (turn–by–turn) average of a hypothetical initially unidirectional spin ensemble taken at a certain phase space point $\vec{z}_0$ will asymptotically give the average local polarization direction at this point. Hence the normalized average denoted by $\vec{n}_N(\vec{z}_0,\theta)$ approaches $\vec{n}(\vec{z},\theta)$ in the limit $N \to \infty$. If we suppress all θ–dependencies the 'first' particle will contribute $\vec{S}(\vec{z}_0)$, the 'second' : $\mathbf{R}(\mathbf{T}^{-1}\,\vec{z}_0)\vec{S}(\mathbf{T}^{-1}\,\vec{z}_0)$. Summing up all contributions we obtain :

$$\vec{n}_N(\vec{z}_0) \sim \vec{S}(\vec{z}_0) + \sum_{i=1}^{N}\prod_{j=1}^{i}\mathbf{R}(\mathbf{T}^{-j}\,\vec{z}_0)\vec{S}(\mathbf{T}^{-i}\,\vec{z}_0) \quad . \tag{4}$$

This method, described in [1] in more detail, only requires single particle multi-turn tracking (for one phase space point) and is implemented in the computer code `SPRINT`.

2 Results for the HERA–p Lattice

We now investigate the equilibrium spin distribution at 820 GeV in HERA–p under luminosity conditions. For this energy the 'naive' spin tune, $a\gamma$, is about 1557 which means that 1 mrad of orbit deflection will result in 90° spin precession angle. Furthermore the HERA lattice is not flat; downstream and upstream of the East-, North- and South–IPs the proton beam is first bent vertically by 5.74 mrad then twice horizontally by 30.2 mrad and then -5.7 mrad vertically. In a flat and perfectly aligned machine the $\vec{n}_0$–axis is vertical but with vertical bending magnets it's direction would be strongly azimuth and energy dependent. Therefore it is necessary to insert special 'flattening snakes' [4] into the vertical bend sections to compensate them. Figures 1 and 2 show the dependence on energy of the average polar angle deviation of $\vec{n}$ from the mean, $< \angle(\vec{n}(\vec{J},\vec{\Psi}), < \vec{n}(\vec{J},\vec{\Psi}) >_{\vec{\Psi}}) >_{\vec{\Psi}}$, (called opening angle) on a fixed invariant phase space torus. Figure 1 shows the dependence on the reference energy of the opening angle for the current *unmodified* luminosity optic and for a modified design by Golubeva/Balandin [3], Anferov/Barber/Hoffstätter

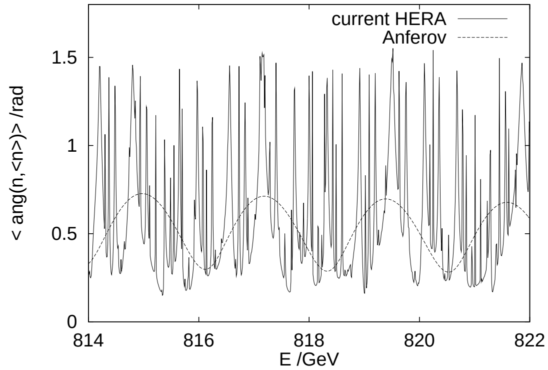

Figure 1: Energy scans for the current HERA–p luminosity optic and for the Anferov design [3],[4]

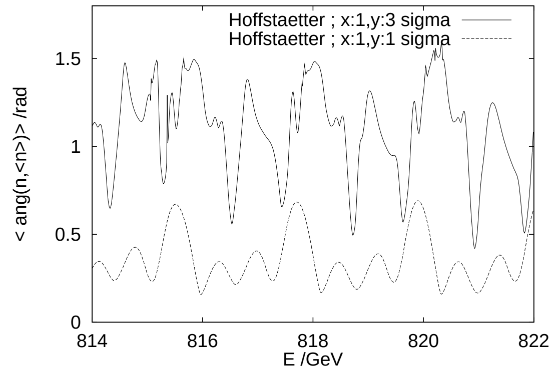

Figure 2: Two energy scans for the Hoffstätter design [5] on two different phase space tori

[4]. It has flattening snakes in the vertical bend sections, a longitudinal snake near the East- and 3 *radial* snakes near the South-, West-, and North–IP. Both curves are for 1 sigma horizontal and vertical ellipses. Longitudinal phase space motion is neglected. Figure 2 shows two energy scans of the opening angle for a snake design by G.Hoffstätter [5]. In which the radial snakes near the North-, South-, and West–IP are replaced by snakes with *vertical* rotation axis. The lower curve is for 1 sigma horizontal and vertical ellipses whereas the upper refers to 1 sigma (hor.) and 3 sigma (vert.). Figure 1 shows that even with moderate betatron excitation and without modification, the directional spread of the spin distribution is much too large and wildly fluctuating. Both snake layouts widen the energy region over which the opening angle is small but as we see in figure 2, at large amplitudes the opening angle is large even with snakes. The results are discussed in greater detail in [2]. A decrease of transverse beam emittance would certainly improve polarization at this level. In [2] we also give a list of essential further simulations.

[1] K. Heinemann, G.H. Hoffstätter; DESY 96-078, to appear Oct. 96 in *Phys Rev. E.*

[2] D.P. Barber, K. Heinemann, G.H. Hoffstätter, M. Vogt; DESY HERA 96-07.

[3] V. Balandin, N. Golubeva, D.P. Barber; DESY M 96-04.

[4] V.Anferov; to appear in Acceleration of Polarized Protons to 820 GeV at HERA, UM HE 96-20.

[5] G.H. Hoffstätter; in Proceedings of DESY M Group Seminar '96, DESY Report 96-05.